Long-term Forward Prices and Seasonal Effects of Wheat in the EU and US

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Herve OTT

European Commission

Joint Research Centre

IPTS - Institute for Prospective Technological Studies

AGRILIFE unit

Seville - Spain

EUROPEAN COMMISSION

Background Information I

- **E.U. post-2013 CAP: more liberalized**
- **Need to hedge against risk**
- **European agricultural commodity market will expand**
  - Trend will keep expanding:
  e.g. Paris milling wheat volume 98: 41 000; 09: 2 mill.
- **Importance to estimate a long-term forward curves**
  - To price long-terms swaps accurately
  - To price long-term agricultural insurance

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Background Information I

- **Agricultural commodities: the challenge**
  - Black and Black-Scholes option pricing models assume price volatility increases proportionally to the square root of time. But Bessembinder *et al*. find mean reversion to production costs for ag commodities
  - Simple arbitrage formulae not possible
    - Crude oil and Eurodollars trade futures with maturities as far as ten years through exchanges
    - Forward curve can be obtained by simple arbitrage formulae for other products (e.g., cost of carry for gold, interest rate minus dividend for stocks, and the interest rate differential for currencies)

Background Information II

- **Large literature focusing on extensions of Black-Scholes to model interest rate dynamics**
  - Dai and Singleton, Duffee, Piazzesi, Duffie, Pan, and Singleton
- **Affine models quite popular to model interest rates**
  - No arbitrage opportunities
- **Affine models can be applied to study spot and futures commodity prices**
  - Allows for multiple source of uncertainty, time varying heteroscedasticity and price jumps
  - Relatively few studies: Schwartz, Casassus and Collin-Dufresne

Objective I

- **Estimate the forward curve of the wheat futures prices in the European Union (E.U.) and the United States (U.S.) by fitting affine model**
  - E.U. milling wheat prices at Euronext: illiquid market
  - U.S. prices at Chicago Mercantile Exchange (CME): liquid market
- **Analyze and compare the E.U. and US forward curve of the wheat futures prices**
  - Common pattern
  - Idiosyncracies

Affine Model I

- **Advocated affine model:**
  - Closed-form solution for both futures and options
  - the resulting price structure does not allow for arbitrage opportunities
  - Deals with agricultural commodity idiosyncracies: seasonality and price mean reversion
  - Fitted using Bayesian Markov Chain Monte Carlo (MCMC) methods
    - Confidence intervals including both estimation (“parameter”) risk and model risk are easily computed

Affine Model II

- **Two-factor affine Gaussian theoretical model of interest rates recently developed by Collin-Dufresne, Goldstein, and Jones**

- **Adapted to represent commodity futures by Lence, Hart, and Hayes**
  - Seasonality
  - Spot mean reversion

- **Entire future curve determined by two factors:**

  $Y_1(t)$ = Natural log of spot price at time t
  $Y_2(t)$ = Expected risk-neutral change in Y_1 at time t

- **Historical Process (without seasonality):**

$$\begin{bmatrix} dY_1(t) \\ dY_2(t) \end{bmatrix} = \left(\begin{bmatrix} \kappa_{10} \\ \kappa_{20} \end{bmatrix} - \begin{bmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{bmatrix} \begin{bmatrix} Y_1(t) \\ Y_2(t) \end{bmatrix} \right) dt$$

$$+ \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}^{1/2} \begin{bmatrix} dW_1(t) \\ dW_2(t) \end{bmatrix}$$

Affine Model V

- **Market Prices of Risk (without seasonality):**

$$\begin{bmatrix} \Lambda_1(t) \\ \Lambda_2(t) \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}^{-1/2}$$

$$\left(\begin{bmatrix} \lambda_{01} & 0 \\ 0 & \lambda_{02} \end{bmatrix} - \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \begin{bmatrix} Y_1(t) \\ Y_2(t) \end{bmatrix} \right)$$

Affine Model VI

- **Change of measure:**

$$\begin{bmatrix} dW_1(t) \\ dW_2(t) \end{bmatrix} = -\begin{bmatrix} \Lambda_1(t) \\ \Lambda_2(t) \end{bmatrix} dt + \begin{bmatrix} d\tilde{W}_1(t) \\ d\tilde{W}_2(t) \end{bmatrix}$$

Affine Model VII

- **Risk-Neutral Process (without seasonality):**

$$\begin{bmatrix} dY_1(t) \\ dY_2(t) \end{bmatrix} = \left(\begin{bmatrix} \tilde{\kappa}_{10} \\ \tilde{\kappa}_{20} \end{bmatrix} - \begin{bmatrix} \tilde{\kappa}_{11} & \tilde{\kappa}_{12} \\ \tilde{\kappa}_{21} & \tilde{\kappa}_{22} \end{bmatrix} \begin{bmatrix} Y_1(t) \\ Y_2(t) \end{bmatrix} \right) dt$$

$$+ \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}^{1/2} \begin{bmatrix} d\tilde{W}_1(t) \\ d\tilde{W}_2(t) \end{bmatrix}$$

$$\kappa_{ij} = \tilde{\kappa}_{ij} + \lambda_{ij}$$

Affine Model VIII

- **Affine Structure:**

$$P(t) = exp[\phi_0 + \phi_1 \, Y_1(t) + \phi_2 \, Y_2(t)]$$

$\Rightarrow$

$$ln[P(t)] = \phi_0 + \phi_1 \, Y_1(t) + \phi_2 \, Y_2(t)$$

$\Rightarrow$

$$F(t, \, \tau) = \tilde{E}_t[P(t+\tau)]$$

$$ln[F(t, \, \tau)] = A(\tau) + B_1(\tau) \, Y_1(t) + B_2(\tau) \, Y_2(t)$$

- **Affine Structure:**

$$\frac{dB(\tau)}{d\tau} = -\tilde{K}^{\mathrm{T}} B(\tau)$$

$$\frac{dA(\tau)}{d\tau} = \tilde{K}_0^{\mathrm{T}} B(\tau) + \frac{1}{2} \sum_{i=1}^{2} ([(\Sigma^{1/2})^{\mathrm{T}} B(\tau)]_i)^2$$

$$B(0) = [\phi_1 \ \phi_2]^{\mathrm{T}}$$

$$A(0) = \phi_0$$

$$\tilde{K} \equiv [\tilde{\kappa}_{11} \ \tilde{\kappa}_{12} ; \tilde{\kappa}_{21} \ \tilde{\kappa}_{22}]$$

$$\tilde{K}_0 \equiv [\tilde{\kappa}_{01} \ \tilde{\kappa}_{02}]^{\mathrm{T}}$$

Affine Model X

- **Bivariate Gaussian process has a maximum of 12 identifiable parameters (Dai and Singleton)**
- **Collin-Dufresne, Goldstein, and Jones proposed normalization:**

$$\tilde{K}_0 = \begin{bmatrix} 0 \\ \tilde{\kappa}_{20} \end{bmatrix}$$

$$\tilde{K} = \begin{bmatrix} 0 & -1 \\ \tilde{\kappa}_{21} & \tilde{\kappa}_{22} \end{bmatrix}$$

Affine Model XI

- **Identifiable parameters under Collin-Dufresne, Goldstein, and Jones' model:**

$$\{\tilde{\kappa}_{20},\ \tilde{\kappa}_{21},\ \tilde{\kappa}_{22},\ \kappa_{10},\ \kappa_{20},\ \kappa_{11},\ \kappa_{12},\ \kappa_{21},\ \kappa_{22},\ \sigma_1,\ \sigma_2,\ \rho_{12}\}$$

$$\{\tilde{\kappa}_{20},\ \tilde{\kappa}_{21},\ \tilde{\kappa}_{22},\ \lambda_{10},\ \lambda_{20},\ \lambda_{11},\ \lambda_{12},\ \lambda_{21},\ \lambda_{22},\ \sigma_1,\ \sigma_2,\ \rho_{12}\}$$

$$\kappa_{ij} = \tilde{\kappa}_{ij} + \lambda_{ij}$$

Seasonality

- **Seasonality is incorporated through trigonometric functions in the intercepts:**

$$\tilde{\kappa}_{20}(t) = \tilde{\kappa}_{200} + \sum_{k=1}^{2}\left[\tilde{\kappa}_{20k,sin}\ sin\left(\frac{2\pi k}{\delta}t\right) + \tilde{\kappa}_{20k,cos}\ cos\left(\frac{2\pi k}{\delta}t\right)\right]$$

$$\lambda_{i0}(t) = \lambda_{i00} + \sum_{k=1}^{2}\left[\lambda_{i0k,sin}\ sin\left(\frac{2\pi k}{\delta}t\right) + \lambda_{i0k,cos}\ cos\left(\frac{2\pi k}{\delta}t\right)\right]$$

δ is the length of the periodic time interval

Estimation data I

- **E.U.:**
  Milling wheat futures at Euronext (NYSE Liffe Paris)
  - January, March, May, August, and November maturities
  - Euros per metric ton
  - Delivery in Rouen, France
- **U.S.:**
  #2 Soft Red Winter wheat futures at Chicago Mercantile Exchange
  - March, May, July, September, and December maturities
  - Cents of dollar per bushel
  - Delivery at par in Chicago, U.S.A.

Estimation data II

- **Monthly data, Jan. 1999 through Dec. 2010**
- **Prices for traded contracts on day 15th (16th, 14th, 17th, or 13th if closer day to 15th not available)**
- **n_T = 144 observation dates**
- **Longest time to maturity:**

  n_τ = 20 months for E.U.

  n_τ = 36 months for U.S.

Estimation data III

- **Potential observations:**

  $n_{T\tau} = 2880\ (= 144 \times 20)$ for E.U.

  $n_{T\tau} = 5184\ (= 144 \times 36)$ for U.S.

- **Actual observations**

  725 for E.U.

  1185 for U.S.

Estimation Method

- **Parameters estimated using Bayesian MCMC approach**
- **Define**

$$\tilde{\kappa} \equiv \{ \tilde{\kappa}_{200} , \tilde{\kappa}_{201,sin} , \tilde{\kappa}_{201,cos} , \tilde{\kappa}_{202,sin} , \tilde{\kappa}_{202,cos} , \tilde{\kappa}_{21} , \tilde{\kappa}_{22} \}$$

$$\lambda \equiv \{\lambda_{100}, \lambda_{101,sin}, \lambda_{101,cos}, \lambda_{102,sin}, \lambda_{102,cos}, \lambda_{200}, \lambda_{201,sin}, \lambda_{201,cos}, \lambda_{202,sin}, \lambda_{202,cos}, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}\}$$

$$\sigma \equiv \{\sigma_1, \sigma_2, \rho_{12}\}$$

$$s \equiv \{s_e, r_e\}$$

Results I

Table 1. Estimates of Means and Standard Deviations for Parameters of Affine Two-Factor Model of Wheat Futures

	EURONEXT		CME	
	Mean	(Std. Deviation)	Mean	(Std. Deviation)
Risk-Neutral Parameters				
$\tilde{\kappa}_{20}$	-0.0025	(0.0018)	0.00073	(0.00045)
$\tilde{\kappa}_{201,sin}$	-0.00730	(0.00044)	-0.00289	(0.00030)
$\tilde{\kappa}_{201,cos}$	0.00211	(0.00046)	-0.00086	(0.00026)
$\tilde{\kappa}_{202,sin}$	0.0068	(0.0015)	0.0040	(0.0016)
$\tilde{\kappa}_{202,cos}$	-0.0117	(0.0014)	0.00082	(0.00089)
$\tilde{\kappa}_{21}$	-0.00056	(0.00037)	0.000138	(0.000070)
$\tilde{\kappa}_{22}$	0.021	(0.012)	0.0837	(0.0038)

Results II

Table 1. Estimates of Means and Standard Deviations for Parameters of Affine Two-Factor Model of Wheat Futures

	EURONEXT		CME	
	Mean	(Std. Deviation)	Mean	(Std. Deviation)
Risk-Premiums				
λ_{10}	0.02	(0.13)	0.06	(0.14)
$\lambda_{101,sin}$	-0.0059	(0.0092)	-0.006	(0.013)
$\lambda_{101,cos}$	-0.0170	(0.0092)	-0.011	(0.013)
$\lambda_{102,sin}$	0.0023	(0.0092)	0.006	(0.013)
$\lambda_{102,cos}$	0.0072	(0.0092)	0.006	(0.013)
λ_{11}	0.002	(0.026)	0.011	(0.024)
λ_{12}	0.006	(0.018)	0.01	(0.10)
λ_{20}	0.016	(0.011)	-0.002	(0.012)
$\lambda_{201,sin}$	0.00081	(0.00065)	0.0011	(0.0010)
$\lambda_{201,cos}$	0.00132	(0.00086)	0.0014	(0.0010)
$\lambda_{202,sin}$	0.00220	(0.00099)	-0.0017	(0.0011)
$\lambda_{202,cos}$	0.00383	(0.00084)	0.0017	(0.0012)
λ_{21}	0.0035	(0.0023)	-0.0006	(0.0020)
λ_{22}	0.176	(0.046)	0.098	(0.033)

Results III

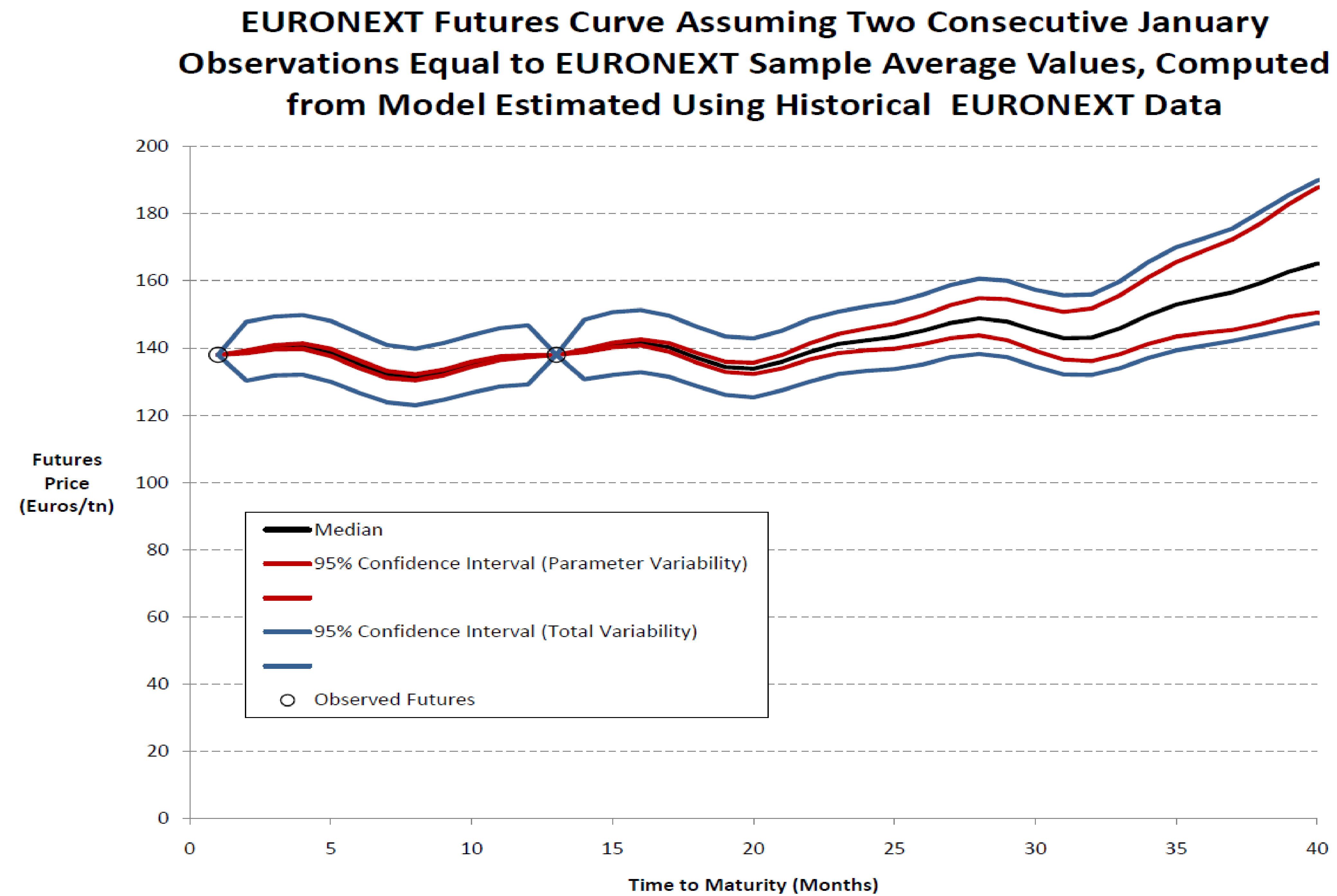
Table 1. Estimates of Means and Standard Deviations for Parameters of Affine Two-Factor Model of Wheat Futures

	EURONEXT		CME	
	Mean	(Std. Deviation)	Mean	(Std. Deviation)
Covariance Matrix				
σ_1	0.0775	(0.0048)	0.1113	(0.0071)
σ_2	0.00506	(0.00046)	0.00863	(0.00060)
ρ_{12}	-0.777	(0.037)	-0.810	(0.031)

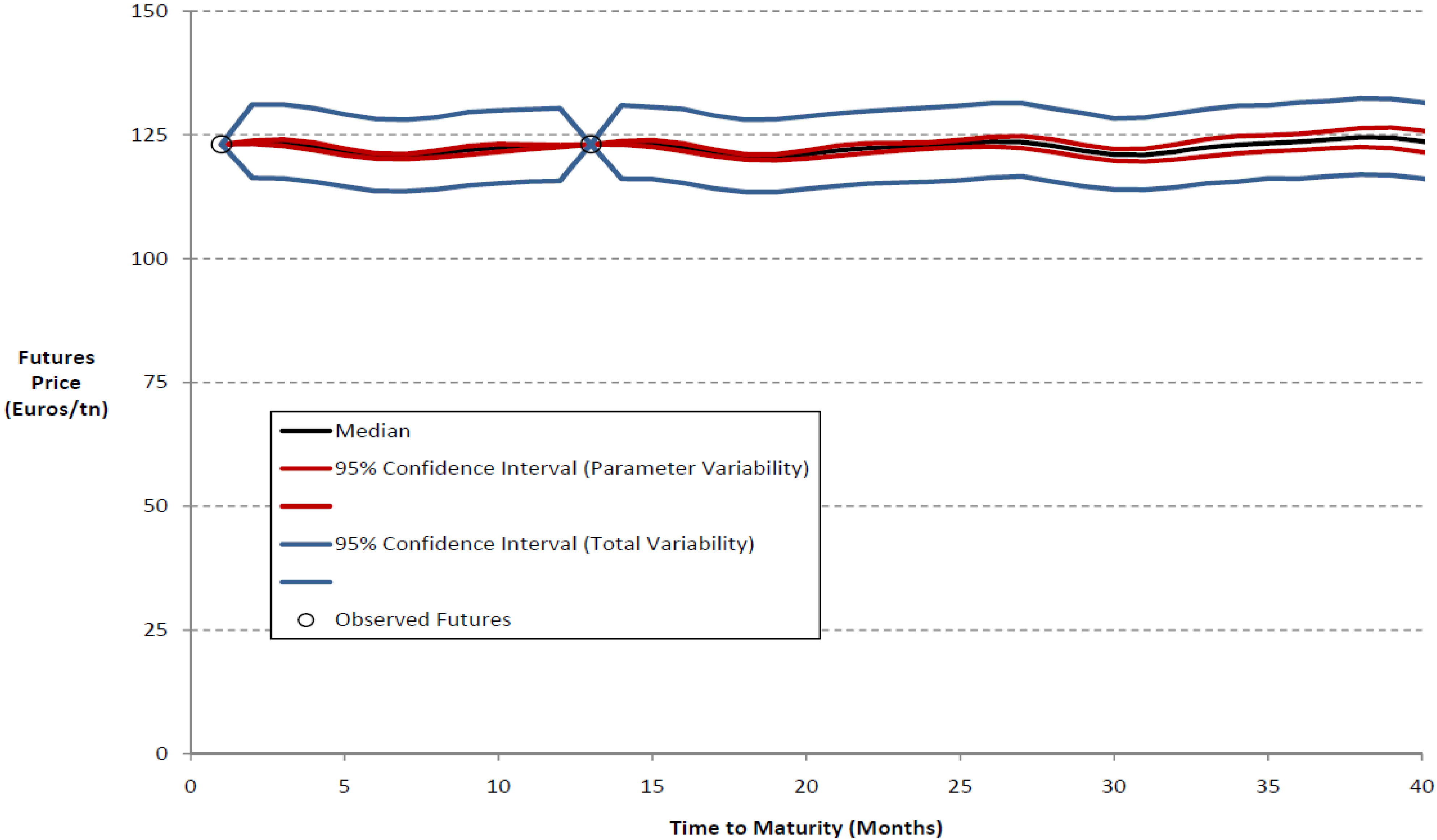
Results IV

Table 1. Estimates of Means and Standard Deviations for Parameters of Affine Two-Factor Model of Wheat Futures

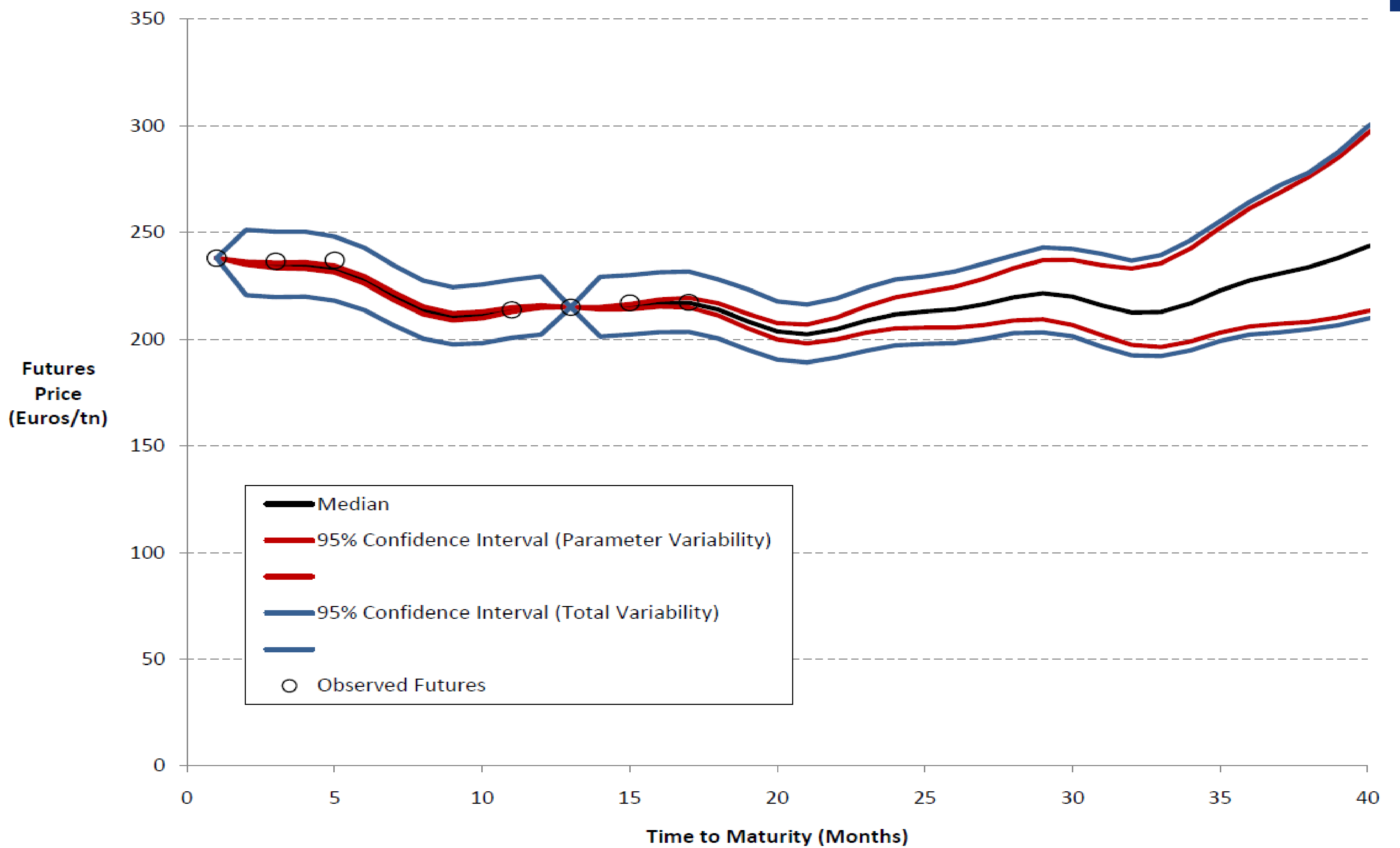
	EURONEXT		CME	
	Mean	(Std. Deviation)	Mean	(Std. Deviation)
Residual Errors				
σ_e	0.0326	0.0014	0.03054	0.00079
r	0.001	0.078	0.002	0.041


EURONEXT Futures Curve Assuming Two Consecutive January Observations Equal to EURONEXT Sample Average Values, Computed from Model Estimated Using Historical EURONEXT Data
Futures Price (Euros/tn)
200
180
160
140
120
100
80
60
40
20
0
0
5
10
15
20
25
30
35
40
Time to Maturity (Months)
Median
95% Confidence Interval (Parameter Variability)
95% Confidence Interval (Total Variability)
Observed Futures

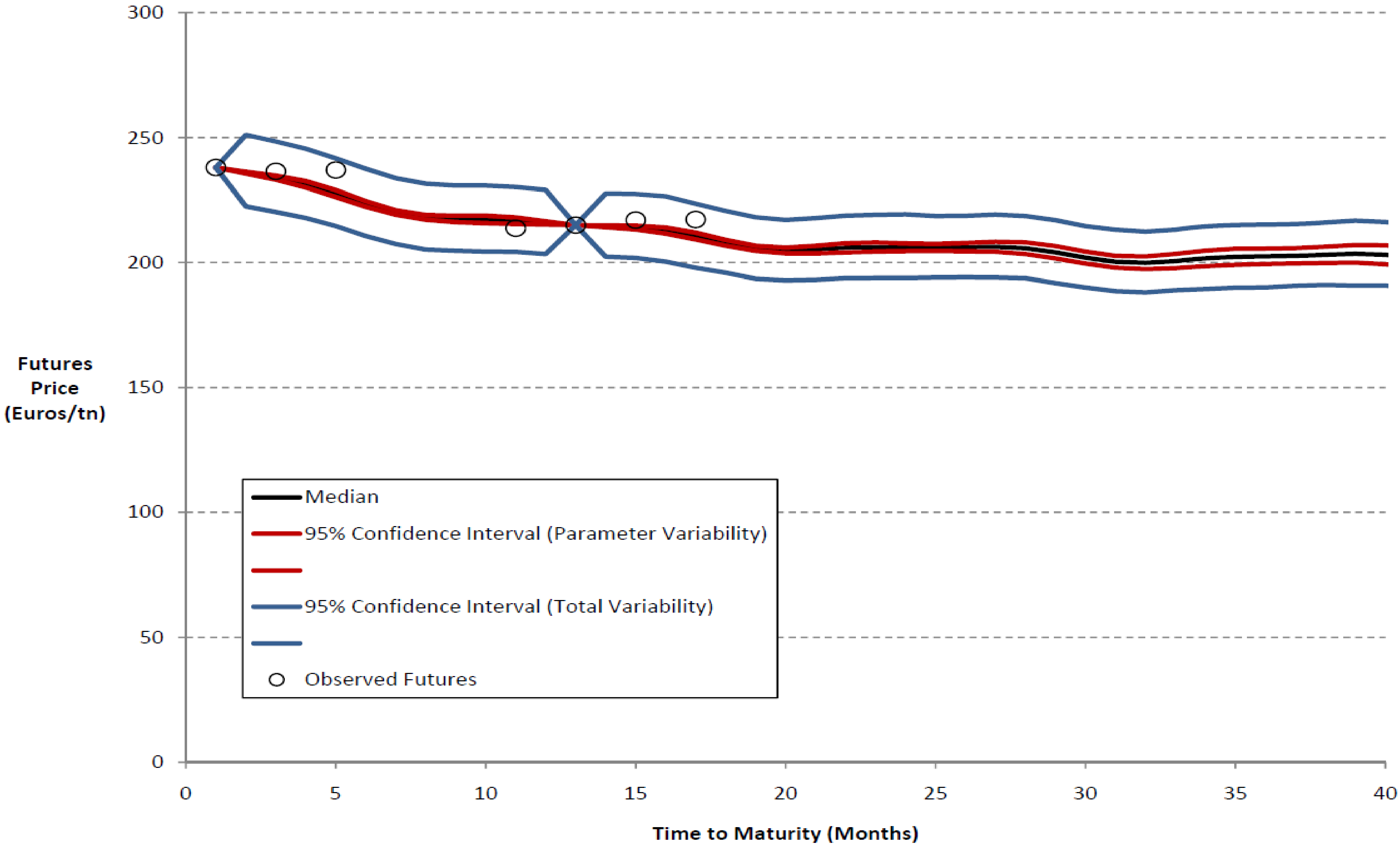
CME Futures Curve Assuming Two Consecutive January Observations Equal to CME Sample Average Values, Computed from Model Estimated Using Historical CME Data



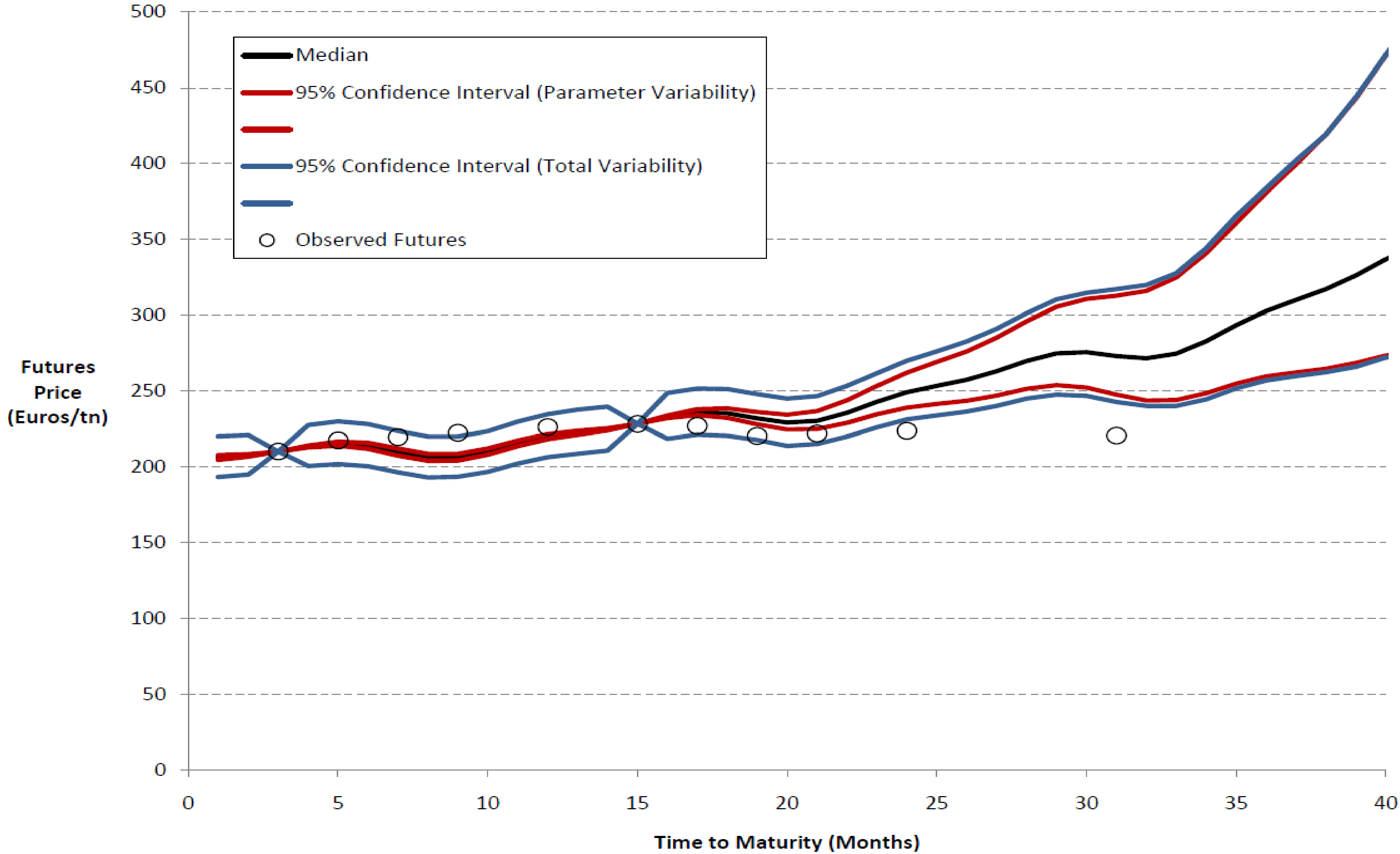
EURONEXT Futures curve on 12/15/2010, Computed from Model Estimated Using Historical EURONEXT Data



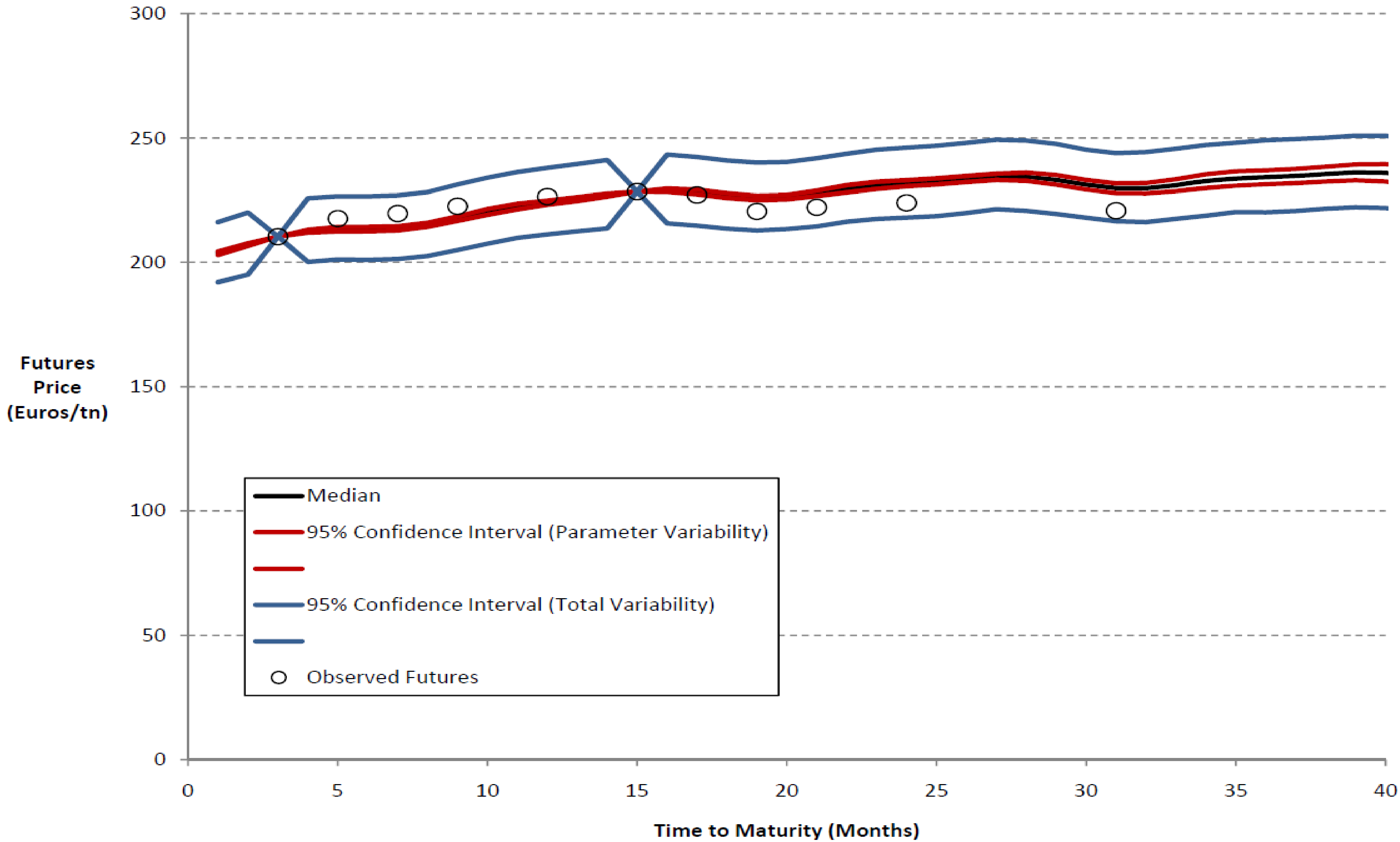
EURONEXT Futures curve on 12/15/2010, Computed from Model Estimated Using Historical CME Data

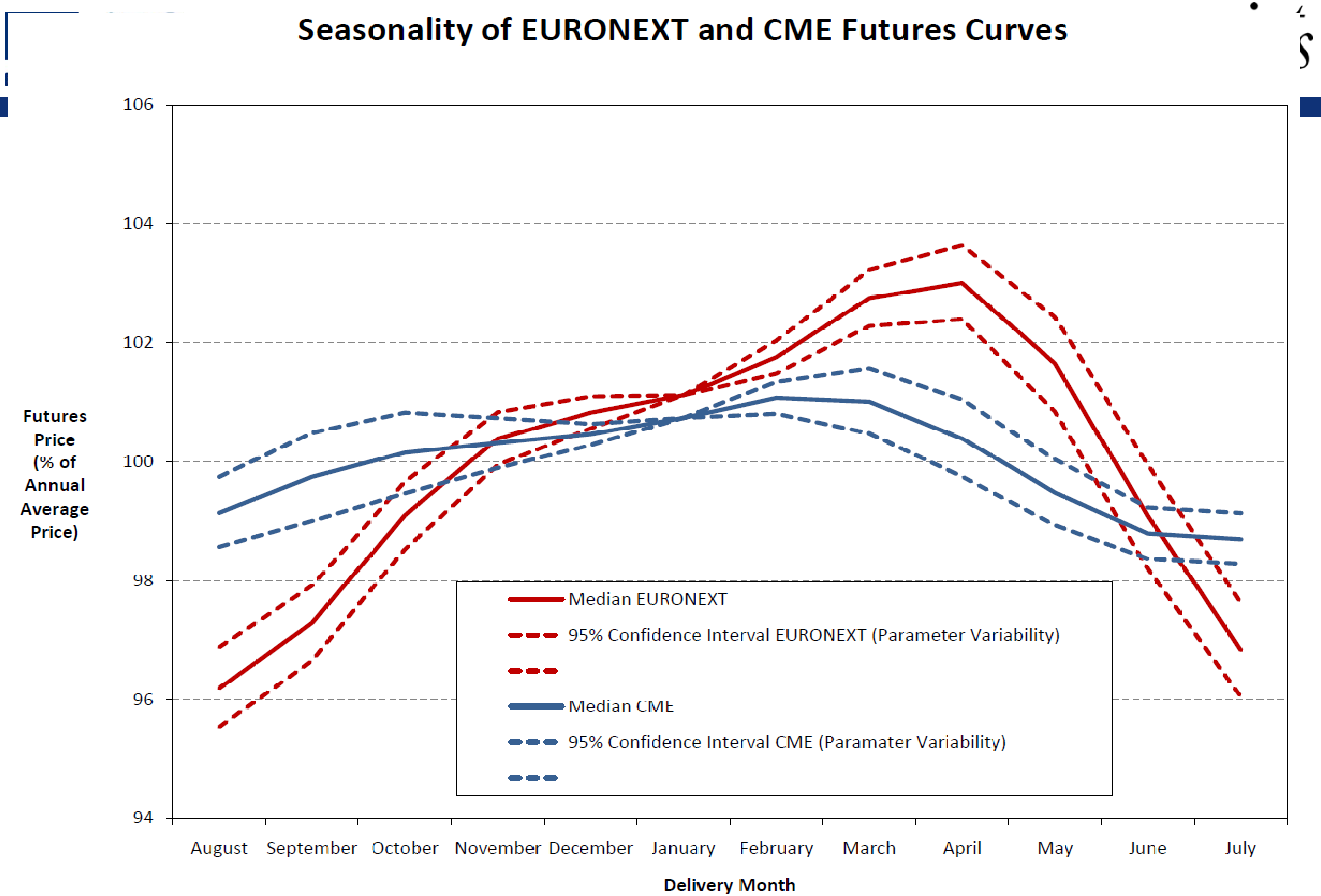


CME Futures curve on 12/15/2010, Computed from Model Estimated Using Historical EURONEXT Data



CME Futures curve on 12/15/2010, Computed from Model Estimates Using Historical CME Data





Seasonality of EURONEXT and CME Futures Curves
106
104
102
100
98
96
94
Futures Price (% of Annual Average Price)
Median EURONEXT
95% Confidence Interval EURONEXT (Parameter Variability)
Median CME
95% Confidence Interval CME (Paramater Variability)
August
September
October
November
December
January
February
March
April
May
June
July
Delivery Month

Conclusions I

- **Euronext future curve is non-stationary but stationary at CME**
  - CME futures curve tends to revert back to a long-run mean value, with futures curve displaying a negative (positive) trend when nearby futures are above (below) such value
  - Euronext futures curve does not show tendency for futures to revert to a long-term equilibrium value

Conclusions II

- **the speed of the expected risk-neutral drift tends to go back to its long-term mean if it is away from it is about four times faster in the CME than in Euronext**
  - CME futures curve is much more likely to be characterized by a long-term equilibrium than its counterpart at Euronext
  - Reason: CME highly liquid market whereas Euronext still illiquid

Conclusions III

- **Best option to estimate the US wheat forward curve**
  - Unrestrictive mean speed in all cases
- **Best option to estimate the EU wheat forward curve**
  - Cannot be derived from the CME parameter estimates
  - Unrestrictive mean of speed reversion for near-term maturities
  - Imposing a random walk for longer maturities

Conclusions IV

- **Futures curve exhibits stronger seasonality in the the Euronext**
  - Shape of seasonality is relatively similar
  - Seasonal minimum and maximum points in the futures curve occur one month later in Euronext compared to the CME (different marketing year)
  - futures curve at Euronext has a much more marked seasonality than the CME futures curve because the CME is an international market whereas Euronext is more isolated