Valuation of Electricity Futures: Reduced-Form vs. Dynamic Equilibrium Models

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Abstract

In the recent literature, reduced-form models and equilibrium models have been proposed for pricing electricity futures. We present the first empirical comparison between a one-factor reduced-form model by Lucia and Schwartz (2002) and an equilibrium model which is a dynamic extension of Bessembinder and Lemmon (2002). The latter one models three groups of agents – producers, retailers, and end-users – and explicitly considers the production and cost structure. This allows to better meet the distribution of spot prices, generate price spikes without modelling exogenous jumps, and endogenously derive a term structure of the risk premium. Our analysis is based on 50 months of futures prices from the Scandinavian electricity exchange Nord Pool. We estimate both models out-of-sample for risk-neutral and risk-averse participants. We find that considering risk premia in futures prices improves the forecasting quality for both models. The equilibrium model better explains futures prices in volatile markets, i.e. when price jumps occur. We also find empirical evidence for the derived structure of the risk premia. The much larger effort for estimating the equilibrium model pays off by a better handling of price spikes when evaluating electricity futures.

JEL classification: G13

Keywords: Electricity futures, risk premium, reduced-form models, equilibrium models.

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1 Introduction

Even though the liberalization of electricity markets started about 5 to 15 years ago in most industrial countries, valuation of electricity futures is still a challenging topic. From the non-storability of electricity two major consequences arise: First, Spot prices exhibit seasonal patterns, a large volatility, and occasional price peaks. Second, futures cannot be evaluated by the cost-of-carry relationship that is a well-known technique for other traded commodities.

Instead of the cost of carry, electricity forward prices contain a risk premium on top of the expected spot price, cf. Geman (2005, p. 270). Determining the risk premium is the major theoretical challenge. However, in order to evaluate models empirically, also the problem of estimating expected spot prices has to be solved.

The literature on evaluating electricity forwards can be classified into econometric models, reduced-form models and equilibrium models.

Econometric models determine futures prices from historical prices and fundamental data. They focus on the second problems and identify the most important determinants of spot and futures prices and quantify their impact. Partly, these models use also econometric techniques to estimate futures prices including implicit risk premia (cf. Fleten and Lemming (2001) or Elliott et al. (2003)).

Reduced-form models build a bridge between econometric and equilibrium models. They are more parsimonious with regard to the number and the properties of the determinants of futures prices. Typically, up to three risk factors are considered which are modelled as diffusion or jump processes. They take into account equilibrium aspects by incorporating market prices of risk for these risk factors. Their major advantage are closed-form solutions for futures prices that are convenient for implementation and estimation. However, their analytical solutions rely in general on factor distributions that do not reflect the observed stochastic properties of spot prices. For typical reduced-form models we refer to Pilipović (1997), Kellerhals (2001), Geman and Vasicek (2001), Koekebakker and Ollmar (2001), Audet et al. (2002), Lucia and Schwartz (2002), Villaplana (2003), Geman and Roncoroni (2003), Hinz and Weber (2005), and Benth et al. (2006).

Equilibrium models focus on modelling the production and demand structure. Spot prices and their distribution are achieved endogenously. Based on assumptions on the risk prefer-
ences of agents in the market, they show whether risk premia exist and how they behave. In general, they are more flexible than reduced-form models. Their major advantage lies in the explanation of prices and their stochastic properties. On the other hand side, they are more difficult to apply to empirical data than reduced-form models. A fundamental dynamic approach by Routledge et al. (2001) considers the storage of different types of fuel and its sequential conversion into electricity. The static model by Bessembinder and Lemmon (2002) focusses on the hedging activities of risk-averse electricity producers and retailers. A dynamic extension of this model is proposed by Bühler and Müller-Merbach (2004).

The dynamic extension provides several advantages: First, we endogenously derive the term structure of futures prices and its risk premia. Comparative statics show that the dynamic approach can explain increasing, decreasing, and hump-shaped term structures of risk premia even when there is no seasonal structure in the prices. Second, it explains empirical stochastic characteristics as the increasing volatility and right-skewness of futures prices for decreasing time to maturity. Third, the dynamic setting is necessary to evaluate cascade futures as they are common at Nord Pool.

Most of the proposed reduced-form models have been applied to empirical data to test their pricing quality, but not the equilibrium models. We contribute to the empirical literature by the first comparative estimation and evaluation of a reduced-form model and an equilibrium model on the same set of empirical data. The main question we want to answer is whether the detailed modelling of the production structure pays off for the valuation of electricity futures. We choose a one-factor model out of each group, the reduced-form model by Lucia and Schwartz and our dynamic extension of the Bessembinder/Lemmon-model. In the equilibrium model the demand volume is the exogenous factor. We estimate the cost structure in order to translate demand volumes into spot prices. In the reduced-form model, the spot price is exogenous. For both models we consider the strong seasonal patterns of the exogenous variables.

We conduct our empirical analysis with data from the Scandinavian exchange Nord Pool which is one of the oldest electricity exchanges in Europe. Nord Pool not only provides price data, but also time series of end-user demand and of the water reservoir level. These have a major price impact in the hydro plant dominated Scandinavian electricity industry. We test and compare the two models with weekly observations of futures prices during a period of 50 months.

We find that under a risk-neutral setting the equilibrium model provides better out-of-sample
futures price explanations than the reduced-form model. After implicit estimation of the risk parameters for each model this difference diminishes for the in-sample evaluation. Applying those estimated risk parameters for out-of-sample predictions again gives better results for the equilibrium model. The equilibrium model is especially more reliable when price peaks occur.

Our paper is organized as follows. Section 2 shortly reviews the two models. Section 3 describes the spot and futures market at Nord Pool and Section 4 provides descriptive statistics for the used time series. In Section 5 we explain the estimation design and procedure. We present the empirical results in Section 6. Section 7 concludes.

2 Review of the Models

In this section we shortly review the proposed models. For a more detailed description we refer to the original papers.

2.1 The Reduced-Form Model by Lucia and Schwartz

In their paper, Lucia and Schwartz (2002) propose two different one-factor models as well as two different two-factor models. Apart from the number of factors the approaches differ in modelling the spot price or the log-spot price as a diffusion process. In their empirical study with Nord Pool data, Lucia and Schwartz conclude that the one-factor models “for the log-price do a worse job in explaining futures and forward prices.” We therefore restrict our analysis to their spot price model which is shortly reviewed below.

The spot price $\tilde{P}_t$ equals the sum of a deterministic (seasonal) component $f(t)$ and a stochastic component $\tilde{X}_t$ that follows an Ornstein-Uhlenbeck process

$$d\tilde{X}_t = -\kappa \tilde{X}_t dt + \sigma^X d\tilde{Z}_t$$

where $\tilde{Z}_t$ is a Wiener process. $\sigma^X$ denotes the constant volatility and $\kappa$ the speed of adjustment.

The risk-neutral version of (1) with a market price of risk, $\lambda$, that is assumed constant, is
given by

\[ d\tilde{X}_t = \kappa \left( -\lambda \frac{\sigma X}{\kappa} - \tilde{X}_t \right) dt + \sigma X \, d\tilde{Z}_t^* \]  

(2)

Under the risk-neutral measure (denoted by *), the futures price must equal the expected spot price at the futures’ maturity date \( T \). Since \( \tilde{P}_t = f(t) + \tilde{X}_t \), the futures price \( F_{t,T} \) at date \( t \) reads:

\[ F_{t,T} = E_t^* \{ \tilde{P}_T \} = f(T) + (P_t - f(t))e^{-\kappa(T-t)} - \lambda \frac{\sigma X}{\kappa}(1 - e^{-\kappa(T-t)}) \]  

(3)

The futures price in (3) consists of three additive components. The first one, \( f(T) \), describes the unconditionally expected future spot price. The second component, \( (P_t - f(t))e^{-\kappa(T-t)} \), is the difference between the current spot price and its deterministic component. This difference is discounted with the speed of adjustment \( \kappa \) and reflects the mean-reverting behavior: The best forecast is to expect that the actual deviation of the spot price from \( f(t) \) continuously fades out. Finally, the third component is a correction for the spot price risk that is monotonous in the time-to-maturity and proportional to the market price of risk, \( \lambda \).

The second and the third term in (3) can be positive or negative.

### 2.2 The Dynamic Equilibrium Model

In Bühler and Müller-Merbach (2004) we develop a multi-period extension of the static model by Bessembinder and Lemmon (2002). The general framework of this equilibrium model is outlined in Section 2.2.1. It has several degrees of freedom that we specify in Section 2.2.2.

#### 2.2.1 Model Framework

The market structure is characterized by competitive, risk-averse producers \( G \) and retailers \( R \) of electricity. Both groups are aggregated to representative agents. We model their risk aversion by a mean-variance approach on terminal wealth \( \tilde{W}_\Theta \) at some future date \( \Theta \). At date \( t \), they maximize the objective function

\[ E_t \{ \tilde{W}_\Theta^A \} - \frac{1}{2} \lambda^A \text{Var}_t \{ \tilde{W}_\Theta^A \} , \quad A \in \{ G, R \} \]  

(4)

The terminal wealth is determined by the cash flows that the representative agents receive from producing or retailing electricity, respectively, as well as from their trading activities.
Both agents must trade in the spot market to satisfy the exogenous end-user demand $\tilde{D}_t$ at each date $t$. As electricity is not storable we assume that the demand is equal to the production volume. Both agents may trade in the futures market to increase their risk-adjusted terminal wealth (4).

When not hedged, at date $t$ the producer receives the cash flow

$$C'_t(\tilde{D}_t)\tilde{D}_t - C_t(\tilde{D}_t).$$

(5)

$C_t(D)$ denotes the variable production cost as a (possibly time-varying) function of the demanded and produced quantity of electricity, $D$, and $C'_t(D)$ denotes the marginal cost function. In a competitive equilibrium marginal costs equal the spot price. We assume that $C'_t$ is strictly increasing in the production volume.

The retailer receives the cash flow

$$p\tilde{D}_t - C'_t(\tilde{D}_t)\tilde{D}_t.$$\

(6)

$p$ represents the fixed end-user price per unit of electricity which is assumed to be independent of the consumed quantity of electricity.

By trading futures, both representative agents can reduce their risks by adding compounded cash flows to their final wealth from the marking-to-market. In equilibrium the following futures prices clear the market:

$$F_{t,T} = E_t\{\tilde{F}_{t+1,T}\} - \xi \sum_{\theta=t+1}^{\Theta} e^{r(\Theta-\theta)}\text{Cov}_t\{p\tilde{D}_\theta - C_\theta(\tilde{D}_\theta); \tilde{F}_{t+1,T}\}, \quad t < T \leq \Theta$$

(7)

where $\xi = \frac{\lambda R}{\lambda R + \lambda G}$ is the combined risk aversion factor and $r$ the constant interest rate. In the last period before maturity, (7) can be written as

$$F_{T-1,T} = E_{T-1}\{\tilde{F}_{T,T}\} - \xi \sum_{\theta=T-1}^{\Theta} e^{r(\Theta-\theta)}\text{Cov}_{T-1}\{p\tilde{D}_\theta - C_\theta(\tilde{D}_\theta); C'_T(\tilde{D}_T)\}, \quad t < T \leq \Theta$$

(8)

where $F_{T,T} = C'_T(D_T)$ is the final settlement price and equal to the spot price $P_T$.

The equilibrium futures price in (7) is equal to the expected next period’s futures price or spot price, respectively, minus a risk premium. This risk premium is the sum of the covariances between future cash flows and the next period’s futures or spot price. The term
The stochastic component $\tilde{S}_t$ that we apply in the empirical test is described in detail in Section A in the Appendix.

We assume that the stochastic process of $\tilde{S}_t$ is modelled as a first-order autoregressive process:

$$\tilde{S}_t = \rho \tilde{S}_{t-1} + \sigma^S \tilde{\epsilon}_t^S, \quad \tilde{\epsilon}_t^S \sim \mathcal{N}(0, 1) \text{ i. i. d.} \quad (9)$$

$p\tilde{D}_t - C_t(\tilde{D}_t)$ denotes the aggregate net cash flow that the whole electricity sector receives in the economy. The representative retailer receives $p\tilde{D}_t$ and the producer has to pay $C_t(\tilde{D}_t)$ for producing during the time interval $[t, t + 1)$. The spot and the futures market allocate the aggregate cash flow and the cash flow risk between the producer and the retailer.

### 2.2.2 Model Specification

Two factors in the equilibrium model are yet to be specified: The process of the state variable, i.e. the end-user demand $D_t$, and the cost structure.

We assume that the stochastic process of $\tilde{D}_t$ is the sum of a deterministic component $\bar{D}_t$ and a stochastic component $\tilde{S}_t$. $\bar{D}_t$ reflects the characteristic seasonal patterns. Its functional form that we apply in the empirical test is described in detail in Section A in the Appendix. The stochastic component $\tilde{S}_t$ is modelled as a first-order autoregressive process:

$$\tilde{S}_t = \rho \tilde{S}_{t-1} + \sigma^S \tilde{\epsilon}_t^S, \quad \tilde{\epsilon}_t^S \sim \mathcal{N}(0, 1) \text{ i. i. d.} \quad (9)$$
This choice is motivated by two reasons: First, we found that a sophisticated estimation of 
the deterministic component $\bar{D}_t$ strongly reduces the higher order autocorrelations of the 
residuals $\tilde{S}_t = \tilde{D}_t - \bar{D}_t$ on a weekly basis so that an autoregressive process of first-order is 
sufficient according to the Akaike criterion. Second, the stochastic component of the spot 
price, $X_t$, in the reduced-form model by Lucia and Schwartz has to be estimated as AR(1) 
by construction. In order to conduct a fair comparison we adapt this structure.

The cost function and the marginal cost function were defined on the end-user demand that 
equals the production volume. We assume an exponential function for the marginal cost.\(^1\)

The form of the cost function changes over time if the production capacity of power plants 
within the Nord Pool area does. Especially the production plan of hydro plants must regularly be adjusted to the actual level of the water reservoirs in order not to run out of water 
and not to fail in future delivery obligations of hydro plants. The total water reservoir level 
exhibits a deterministic pattern due to the average seasonal precipitation throughout a year 
and the periods of melting and freezing. The expected level of this seasonal reservoir level 
determines the constant production of electricity by hydro plants. Deviations from this base 
level are caused by not anticipated components of water reservoir levels like unusual rainy or 
dry periods, e.g. during summer 2002. Unexpected deviations from the expected seasonal 
level result in an immediate reduction or extension of the hydro production with the goal to 
smooth the total future production.

As hydro plants represent the cheapest technology for generating electricity an adjustment 
of their production reduces or extends the first and constant section of the marginal cost 
function. In other words, the marginal cost function shifts by an amount that depends on 
the deviation from the median reservoir level, cf. Figure 1. This is achieved by defining a 
reservoir-corrected end-user demand $D_t^*$ as follows:

$$D_t^* = D_t + \gamma^*WRD_t$$

\(^{10}\)

$WRD_t$ denotes the deviation from the median water reservoir level and the parameter $\gamma^*$ 
translates a surplus (lack) of stored potential energy measured in percentage-points into 
additional (missing) production capacity. $WRD_t$ is not modelled as a second stochastic state 
variable, but as an observable quantity at date $t$.

\(^{1}\)In an earlier version of this paper, we assumed a continuous, monotonously increasing, piecewise linear 
marginal cost function with empirically determined kinks. This function provided better estimations of the 
futures prices, but the estimation of the kinks lead to unstable results.
Figure 2: This figure shows the term structure of the risk premium in the equilibrium model with an exponential function of marginal cost for three states of demand. For $D = 60$ GW, the spot price is equal to 270.43 NOK, for the median case of $D = 40$ it is 148.41, and for $D = 20$ the spot price is equal to 81.45.

This translation of the demand and production quantity allows us to use the marginal cost function

$$C'(D_t) = \exp(c_0 + c_1 D_t + \gamma W R D_t), \quad \gamma = \gamma^* c_1$$

(11)

with time-invariant coefficients.

2.3 Comparative Statics

As the recursive futures price formula (7) cannot directly be interpreted, we provide brief comparative statics on the term structure of the risk premium. Figure 2 shows the term structure for three different states of demand along the time to maturity.

The term structure typically exhibits an inner extremum a few periods before maturity which is a maximum for large levels of demand and a minimum for low levels. It approaches to a small value close to zero for large maturities. This value can be positive or negative,
depending on the parameter settings.

For the reduced-form model, the risk premium $-\lambda \frac{\sigma_X}{\kappa}(1 - e^{-\kappa T})$ is analytically tractable. It is monotone and asymptotically approaches $-\lambda \frac{\sigma_X}{\kappa}$. As $\sigma_X$ and $\kappa$ are strictly positive, the sign of $\lambda$ determines whether the risk premium is positive or negative.

3 Contracts at Nord Pool

For our empirical study we choose the Scandinavian electricity market because it is one of the most early liberalized electricity markets in Europe. Trading at the Scandinavian electricity exchange Nord Pool began in 1992. Furthermore, Nord Pool not only provides price data, but also time series on electricity consumption and production volumes and on the water reservoir level of the Scandinavian hydro plants. These plants account for about 55% of the production.

For our purposes, two market segments at Nord Pool are interesting: The spot market and the so-called financial market. At the latter one, futures, forwards, and options are traded. All prices used in our study are given in NOK/MW.

In the spot market electricity is traded for physical delivery during each single hour of the subsequent day (or days, if weekends or exchange holidays follow). The spot market is organized as a single auction market. All traders have to provide their buy or sell offers for each hour of the subsequent day. Nord Pool then calculates 24 hourly market clearing prices. The equally weighted average of those prices is called the system price which we refer to as the daily spot price hereafter.

In the financial market futures and forwards are traded continuously. The underlying of all contracts is the 24-hour-delivery of electricity per day at a constant rate during a specified delivery period. All contracts are cash settled. The delivery periods of the listed contracts range from one day to one year. Futures are used for shorter delivery periods, forward contracts are listed for delivery periods of one month and longer. The time-to-maturity of the listed contracts, i.e. the time until the delivery period begins, ranges from two days up to three years.

In detail, the following contracts are listed or were listed during our observation period:

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2 Between 2003 and 2006 Nord Pool subsequently switched from NOK to Euro.
• *Day futures* with a delivery period of one day are listed with a time-to-maturity between two and a maximum of ten days. These contracts show a rather thin liquidity.

• *Week futures* have a delivery period from Monday through Sunday. They may have a time-to-maturity up to eight weeks. They are the most actively traded contracts at Nord Pool, however, liquidity decreases with increasing time-to-maturity.

• *Block futures* cover a delivery period of four weeks. The listed maturities comprise the time interval from eight up to 48 weeks. Block futures were actively traded but were successively replaced by moth forwards in 2003.

• *Month forwards* were introduced in 2003. Their delivery periods equal the calendar months, i.e. 28 to 31 days. The listed maturities cover the subsequent six months.

• For longer delivery periods, *quarter*, *season*, and *year forwards* are or were traded at Nord Pool. With sometimes only one trade per week, these contracts are the least liquid ones.

Some of these contracts have a special cascade feature that is not found in other commodity markets. A certain time before maturity they are split up into into a volume-equivalent bundle of contracts with shorter maturities. In detail, block futures are split into the corresponding four week futures contracts eight weeks before its delivery period begins. For the longer maturities, year forwards were split into season forwards and are nowadays split into quarter forwards that in turn are split into month forwards. This cascade structure allows the market participants to hedge their exposures more precisely for closer maturities. Note that forward contracts are not split into futures.

Nord Pool does not provide intra-day prices for the financial market but only so called closing prices that are applied for the daily settlement of futures. The closing price is the last registered trading price at a randomly chosen point in time within the last ten minutes before trading ends. If there were no trades at a certain exchange day, Nord Pool uses several procedures to estimate a closing price. Nord Pool also provides the daily trading volumes for each contract so that we can identify closing prices that are based on trading.

\[ \textsuperscript{3}\text{The types and the cascade structure of the listed derivatives at Nord Pool are subject to current changes.}\]

\[ \textsuperscript{4}\text{See Nord Pool ASA (ed.) (2004) for details.}\]
4 Descriptive Statistics

At the Scandinavian electricity exchange Nord Pool daily spot prices are available since 1992. We do not use spot prices before 11/01/1996 because Finish electricity firms gained access to Nord Pool in October 1996.\textsuperscript{5}

The available times series of daily production, consumption, import, and export volumes start at 03/29/1999.\textsuperscript{6} The water reservoir level and its deviation from the historical median are measured once a week and are available from 1996. Daily futures prices and trading volumes are available from 1995. We only consider transaction prices but not prices that were set by Nord Pool for settlement purposes only.

For the fixed tariff rate $p$ in the equilibrium model we use the average price of “1-year/new fixed-price contracts” for households and industrial customers that is provided as a quarterly time series by Statistics Norway. For the interest rate $r$ we use the 3-month-NIBOR (Norwegian Interbank Offered Rate) provided by the central bank of Norway.

All time series end on 08/04/2004. This provides us with a period of 64 months for the shortest time series, the production and consumption volumes. As we need a period of over one year to estimate the parameters of the models, we test them during a period of 50 months from from 05/24/2000 to 08/04/2004 (cf. Section 5 for details).

In the next subsections we present descriptive statistics of daily spot prices, daily demand, and the weekly measured water reservoir level for the 64-months period from 03/29/1999 until 08/04/2004. Quantities will be given in GW, prices in NOK/MW if not denoted otherwise.

4.1 Daily Spot Prices

Figure 5 in the Appendix plots the level of the daily spot price and its first difference during the five-year period and Table 1 shows descriptive statistics. As mentioned before, the spot price exhibits some seasonality in that the prices during the winter time are usually higher than in the preceding summer. However, this seasonality is superimposed by strong changes in the mean level of the spot price. Table 1 shows that in the last two 12-month periods of our observation period the yearly mean as well as the median were about twice as high as

\textsuperscript{6}Prior to that date, the referred time series were available for only some of the countries.
Table 1: Descriptive statistics for the daily spot price $P_t$ and its first differences at Nord Pool in period between 11/01/1996 and 08/10/2004 and for subperiods of 12 months each.

<table>
<thead>
<tr>
<th></th>
<th>Daily Spot Price, $P_t$ [NOK/MW]</th>
<th>Daily Spot Price Differences, $P_t - P_{t-1}$ [NOK/MW]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Obs.</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>365</td>
<td>365</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>130.27</td>
</tr>
<tr>
<td></td>
<td>Std. dev.</td>
<td>29.14</td>
</tr>
<tr>
<td></td>
<td>Skewness</td>
<td>-0.018</td>
</tr>
<tr>
<td></td>
<td>Kurtosis</td>
<td>-0.29</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>58.21</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>234.25</td>
</tr>
</tbody>
</table>

in earlier periods. We also observe a fluctuating standard deviation.

The rather high values for the kurtosis in combination with the mostly positive skewness reflect the occasional spikes in the spot price process. The price spikes usually occur in the winter time. In Figure 5 we marked two examples of single spikes, A and B, and a period of high prices and frequent spikes, C. Spikes A (387.78 NOK/MW) and B (633.36 NOK/MW) happened on 01/24/2000 and 02/05/2001, respectively, and disappeared one day later. They are related with large electricity demands on those two days. This can be seen in Figure 6 that plots the daily demand volumes, and in Figure 7.

The third example, C, covers a period of about four months of record high prices that cannot be explained by demand alone. We argue that these large spot prices are caused by unusually low water reservoir levels. The period of low precipitation began in summer 2002. Figure 8 shows that during this period the water reservoir deviation from the long term median is negative and decreasing. As a consequence the production capacity of hydro plants was reduced and producers were forced to run plants with higher variable costs.⁷ This period will be a challenge for both models.

The high absolute values of the minimum and maximum in Table 1 are an obvious consequence of the price spikes. The high kurtosis and the mostly positive skewness do not support the assumption of normal distribution. However, note that the values in Table 1 reflect also the seasonal patterns in the spot prices. We will discuss the distribution of spot prices without the deterministic (seasonal) component in Section 6.

We tested the spot price series for unit roots with the Augmented Dickey-Fuller (ADF) test, including a constant. With a test value of $-3.007$ the hypothesis of a unit root is rejected on the 5%-level if we include up to 35 lags.

However, in four cases we will empirically find a unit root in the spot price series after having extracted the deterministic components on a weekly basis (cf. Section 6.2).

### 4.2 Futures Prices

Futures prices also exhibit seasonal patterns as well as an increasing trend during the observation period. We therefore abstain from presenting descriptive statistics on futures prices, but focus on the differences between futures and spot prices.

Following Longstaff and Wang (2004), we calculate ex-post differences between futures prices $F_{t,T}$ observed at time $t$ and realized spot prices $P_{T}$ during the delivery periods of the futures contracts. These differences provide a first empirical insight into the term structure of the risk premia.

As we only want to consider traded futures prices, but do not want the analysis biased by liquidity which is larger for short-termed contracts, we take weekly averages of traded prices for each contract instead of daily prices. By doing so, contracts with only one transaction during a certain week are considered to the same extent as actively traded contracts.

Table 2 shows the results. $Wx$ denotes week futures with $x$ weeks until maturity, $Bx$ denotes block futures with $x$ 4-week periods until decomposition in four equivalent week futures. We find positive ex-post differences in average over all contracts. This result is remarkable as the spot prices in average showed an increasing trend during the observation period. If that trend was not expected by the market participants, one might expect negative ex-post differences.

Furthermore, we find positive ex-post differences for week futures, but differences almost equal to zero for block futures. The differences increase with maturity for the week futures.
Table 2: This table shows the ex-post calculated differences between observed futures prices before maturity and realized spot prices during the delivery period. We used weekly average futures prices during the period 11/01/1996 until 08/10/2004.

We observe positive, but decreasing ex-post differences for the first four block futures and slightly negative ones for the block futures with longer times to maturity. Note that there are only few observations for the W8 and the B11 contract.

The standard deviation of the ex-post differences increases with maturity. This reflects the increasing forecasting uncertainty, but leads to insignificance of the ex-post differences of the block futures. In detail, when applying a t-test with Newey/West correction, only the ex-post differences for the W1, W2, W3, W6, and W7 contracts are significant at the 5%-level.

Nevertheless, the term structure of the ex-post differences as estimates for the risk premia supports the theoretical derived term structure of the equilibrium model.

### 4.3 Daily Electricity Production and Demand

For the equilibrium model we need the total electricity consumption or production in the Nord Pool area. If this was a closed market, the end-user consumption of electricity had to equal the production (after transmission losses) at each point in time. However, up to 10% of the average daily used electricity can be exported from or imported into the Nord Pool area. Since the spot price results from market clearing of total demand and supply, we define the electricity supply as the production in the Nord Pool area plus the electricity import. Analogously, we refer to the demand as the consumption in the Nord Pool area plus
Table 3: Descriptive statistics for the daily electricity demand in the Nord Pool area, measured in GW. Statistics are given for the whole sample that comprises the 64-month period between 29/03/1999 and 08/10/2004 (1961 observations) and for subperiods of 12 months.

<table>
<thead>
<tr>
<th>Daily Electricity Demand, $D_t$ [GW]</th>
<th>01.04.99–</th>
<th>01.04.00–</th>
<th>01.04.01–</th>
<th>01.04.02–</th>
<th>01.04.03–</th>
<th>whole</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs.</td>
<td>366</td>
<td>365</td>
<td>365</td>
<td>365</td>
<td>366</td>
<td>1961</td>
</tr>
<tr>
<td>Mean</td>
<td>38.58</td>
<td>40.28</td>
<td>40.14</td>
<td>40.67</td>
<td>39.41</td>
<td>39.44</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>6.69</td>
<td>6.98</td>
<td>7.15</td>
<td>6.95</td>
<td>7.05</td>
<td>6.96</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.104</td>
<td>0.351</td>
<td>0.002</td>
<td>-0.017</td>
<td>0.053</td>
<td>0.181</td>
</tr>
<tr>
<td>Kurtosis</td>
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<th>01.04.00–</th>
<th>01.04.01–</th>
<th>01.04.02–</th>
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<td>6.92</td>
<td>6.44</td>
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the export into countries outside of this area. According to the non-storability of electricity, these two quantities must be equal at each date. On a daily basis, we find that those two quantities differ by less than 1% in 99% of all days. The correlation between them is 99.96 % and 99.77 % for the first differences in the observation period. We attribute these minor differences to measurement errors.

Figure 6 shows the daily demand as defined above. This time series exhibits a very strong seasonal component. The differences between demand volumes in the same period of different years are much smaller in absolute and percentage terms than for the time series of spot prices. Table 3 underlines this observation. Compared to Table 1 for the spot prices, the characteristics of the sample distribution are much more stable for the yearly subperiods. Like the spot prices the demand shows downward spikes for the week-ends. The strongest downward spike can be observed at the Scandinavian holiday midsummer (end of June).

As for the spot price we conducted an ADF-test for the demand level. The hypothesis of a unit root is rejected at the 1%-level (test value of −3.716) if 35 lags are included.

By visual comparison of the two time series we find that the demand volume not only explains
the two price spikes A and B, but also part of the general behavior of the spot price as both rise in the autumn and winter time and decrease in the spring time until summer. The correlation between daily demand and the spot price is 43.7% and 48.8% for their first differences.

The relation between the spot price and the demand volume is not linear. Figure 7 shows the scatter plot of the daily spot price and the demand volume. We draw two conclusions from this plot. First, for the same demand volume the spot price varies considerably. E.g. an electricity demand of 40 GW is related with spot prices between 80 and 350 NOK/MW. Therefore, there must exist at least one more variable that has an important impact on the spot prices. In the next subsection we will identify the deviation of the water reservoir level from its median as an additional determinant. Second, under the assumption of a competitive market among producers the spot price equals the marginal cost of the last produced unit of electricity. Given this assumption, Figure 7 indicates qualitatively that the marginal costs are slightly, progressively increasing in the daily production volume.

There are at least three possible reasons why the spot prices could deviate from the unobservable marginal cost function:

1. The system prices at Nord Pool are usually determined one day, sometimes several days before delivery. Therefore, the two quantities are determined asynchronously.

2. By using daily data we average across 24 hourly data of demand volumes and spot prices, i.e. points of the marginal cost function. As the marginal cost function is presumably convex, the average spot price is an upward biased estimator of the spot price for the average demand volume. The absolute amount of this bias depends on the production level.

3. The marginal cost function may vary over time. Apart from temporary plant outages and from building new or breaking down old power plants, the level of the water reservoirs and thus the production capacity of hydro plants has a major impact.

In our analysis we will not model the first two of the above mentioned effects, but take into account the third one as we consider it to be the most crucial one.
The water reservoir level for Norway, Sweden, and Finland is published once a week in percentage points of the total reservoir capacity in the Nord Pool area. Figure 8 in the Appendix shows the time series of percentage levels and the deviation from its median in percentage points of the total reservoir capacity. Table 4 presents the key sample statistics of the deviation. Note that the seasonal pattern in the level series is caused by the melting period starting in April and the freezing period starting around November. The first one increases the inflow into the water reservoirs while in the freezing period the inflows are reduced. The seasonal component of the production volumes, i.e. the water outflow, seems to have only a minor impact on the reservoir level.

Market participants know and anticipate the seasonal fluctuation of the reservoir level. Therefore, as the scatter plot in Figure 9 shows, the absolute reservoir level contributes only little to the explanation of spot prices. The correlation between the first differences of the spot price and the reservoir level is only $-3.7\%$.

Next we analyze the weekly deviation in percentage-points between the observed reservoir level and its long-term median. We choose the median instead of the mean because this value is published by Nord Pool and serves as a reference for all market participants.

This difference between the observed and the median level is also plotted in Figure 8. The sample characteristics in Table 4 show that the reservoir deviation largely varies between years.

The scatter plot of the weekly spot prices and the reservoir deviations from its median in
Figure 10 shows a clear impact of the reservoir deviation. Compared to Figure 9 it presents a much stronger relationship between the water reservoir characteristics and the spot price. For low reservoir deviations of about $-20\%$-points together with a high consumption the electricity demand has to be served by expensive plants like gas-fired turbines, i.e. the marginal costs are large. Vice versa, if there is an unexpected reservoir surplus, a larger portion of demand can be served by hydro plants. As hydro power is the cheapest technology for generating electricity, Figure 10 reflects also the increasing behavior of the marginal cost function.

5 Estimation Procedure

The basic structure of our estimation procedure is described in Figure 3.

Figure 3: This figure shows the moving time windows for estimating the deterministic and the stochastic part of the spot prices and for valuation of futures prices. We evaluate futures prices on 220 Wednesdays, i.e. $n = 1, \ldots, 220$, whereof we skip three that were holidays. The first valuation day is Wednesday, 05/24/2000, the last one is Wednesday, 08/04/2004.

The total test period runs from 05/24/2000 to 08/04/2004, i.e. 220 Wednesdays. We observe futures prices on 216 of these Wednesdays, four of them are holidays. We take futures prices from the following Thursday if that is not a holiday, which is only once the case, and skipped
a week otherwise. This leaves us with 217 observations. For each of these valuation days we determine theoretical futures prices using data up to the preceding Tuesday. These theoretical futures prices are compared with observed prices.

We choose Wednesdays as valuation dates for two reasons: First, Wednesdays provide us with the most observed traded futures prices because very few holidays happen to be on Wednesdays. Second, the weekly reservoir data are published on Wednesdays at 1 p.m. These data refer to the reservoir level of the preceding Monday. Therefore, futures prices of Wednesdays are the earliest ones that incorporate the new information on the water reservoir level.

For each of the 217 valuation days we perform the following estimation steps. First, we estimate the deterministic component of the spot price in the reduced-form model, $f(t)$, and of the end-user demand in our model, $\bar{D}(t)$, using daily data. This data frequency is used to capture as much information as possible. We apply dummy variables for day-of-week and holiday effects and a continuous, piecewise linear function to represent the seasonal patterns.\footnote{Lucia and Schwartz (2002) use two alternative methods: a cosine-function and a piecewise constant function.} These estimated daily demands and spot prices are averaged to weekly deterministic time series.

Second, we average the observed daily spot prices and demand volumes on a weekly basis. Those weekly data are used to determine the parameters of the stochastic components of the spot price and the demand volume. By two reasons we consider only weekly data in our estimation procedure: First, the computation time for the equilibrium model increases with the order $O(N^2)$ in the number of time steps. Using daily data would lead to unsatisfactory long computation times. Second, we only consider week and block futures as those are the most liquid ones. Thus daily calculations are not necessary.

As explained in Section 2.2.2, for the equilibrium model we also consider the deviation from the median water reservoir as an additional determinant for the marginal cost function. However, we do not model this deviation as a second stochastic factor, but as an explanatory variable. We therefore use the value of the water reservoir deviation that we observe on a valuation day as predictor for all future water reservoir deviations.

The weekly differences between the observed and the deterministic values of the spot price and the demand volume, respectively, are the stochastic deviations $X_t$ and $S_t$ that the further procedure is based on.
All regressions are performed with standard least squares techniques or, in the case of autoregressive error terms, with non-linear least squares.

5.1 Estimating the Reduced-form Model

Our procedure to estimate the parameters of the reduced-form model consists of the following seven steps:

1. We estimate the deterministic components of the spot price process with daily data. The yearly seasonality is modelled by 12 overlapping triangular functions. To account for the week-end effects we introduce dummy variables for Fridays, Saturdays, and Sundays. We furthermore add dummy variables for a specified set of 24 holidays days. For each of the 217 Wednesdays (valuation days), we use an estimation window from 11/01/1996 until the preceding Tuesday. A more detailed description of this step is given in Section A of the Appendix.

2. Based on the 217 regression results of Step 1 we calculate time series of deterministic spot prices, \( f(T) \), for the future maturity dates \( T \) of futures contracts up to 08/04/2005.

3. We aggregate the forecasted deterministic spot prices as well as the observed spot prices to obtain weekly mean prices. The difference of these prices yields the weekly time series of the spot price deviation, \( X_t \).

4. We model the time series \( X_t \) as a first order autoregressive process

\[
X_t = \phi X_{t-1} + \epsilon_t^X
\]  

and use the estimate \( \hat{\phi} \) as a proxy for the speed of adjustment, \( \hat{\kappa} = 1 - \hat{\phi} \), and the standard deviation of the regression as a proxy for \( \sigma^X \).

5. Finally we calculate prices for week futures according to (3), setting \( \lambda \) to zero. The prices of block futures are computed as the average of the prices of the underlying week futures.

The next step of implicitly estimating the market price of risk, \( \lambda \), is described in Section 5.3.
In comparison to the estimation design by Lucia and Schwartz (2002), ours differs in two details: First, we use a different approach for extracting the deterministic components. Second, we aggregate daily data into weekly data.

5.2 Estimating the Equilibrium Model

We conduct the estimation of the equilibrium model in the following steps:

1. The seasonal component of the daily end-user demand in the Nord Pool area is estimated analogously to the seasonal component of the spot price in the reduced-form model. The estimation window begins on 03/29/1999 because Nord Pool does not provide a longer times series for demand or production data. The estimation window ends at the Tuesday of each particular valuation week.

2. We sum up the estimated daily seasonal volumes to obtain weekly seasonal demand data. Analogously, we obtain the observed demand volumes per week. The differences out of both time series provides us with the stochastic component $\tilde{S}_t$ on a weekly basis.

3. We estimate the parameters $\hat{\rho}$ and $\hat{\sigma}^S$ of the AR(1)-process $\tilde{S}_t$ as defined in (9).

4. For the estimation of the marginal cost function $C_t'(D_t^*)$ we simultaneously estimate its slope coefficients $c_i$ and the shift variable $\gamma$ of the water reservoir deviation. Taking the logarithm on (11) leads to the linear estimation equation

$$\ln(P_t) = c_0 + c_1 D_t + \gamma \text{WRD}_t + \epsilon_t^c$$

where autocorrelation is considered in $\epsilon_t^c$.

5. We receive the cost function by integrating over the marginal cost function. We cannot observe the fixed costs of production, however, as those are independent from the demand, they are not needed for evaluating the risk premia.

6. The last necessary parameters are the tariff rate $p$ that retailers charge their customers and the interest rate. We linearly interpolate between the quarterly published tariff data in order to achieve a weekly time series. The calculated weekly tariff prices vary between 130 and 227 NOK/MW. For the interest rate we use daily observations.

Finally, we determine the theoretical price from (7) with $\xi = 0$. 
5.3 Evaluation of Futures Prices

For the implicit estimation of the market price of risk $\lambda$ and the risk parameter $\xi$, and to evaluate the models’ pricing ability we only use trade based futures prices. It is well known that the implicit estimation of $\lambda$ and $\xi$ using daily prices results in highly unstable parameter values. We therefore use pooled prices of eight subsequent Wednesdays to estimate these parameters.

Our valuation analysis consists of three steps:

First, in study A, we evaluate both models out-of-sample as if market participants were risk-neutral, i.e., we assume $\lambda = 0$ for the reduced-form model and $\xi = 0$ for the equilibrium model.

In the second step, study B, we introduce risk aversion. For both models we estimate the implicit risk aversion parameters by minimizing the root mean squared error (RMSE) between the observed and the model prices by using observed futures prices of four subsequent Wednesdays. We use only traded week- and block-futures, but no forward contracts. For calculating the RMSE and other deviations, we weight each observation by the volume of the underlying. A block futures price thus has four times the weight of a week futures price.

For the equilibrium model we implemented the state space of the end-user demand as a grid with 21 points for each time step. The reservoir deviation is set to its most recent value. We identify the planning horizon $\Theta$ with the longest futures’ maturity plus four more time steps. An earlier analysis revealed that this amount suffices in order to suppress finite horizon effects.

In the third step, study C, we conduct another out-of-sample test by incorporating the previously estimated parameters $\lambda$ and $\xi$. The comparison of the out-of-sample result assuming risk-neutral or risk-averse agents allow us to draw conclusions about the importance of the risk premia in electricity futures prices.

6 Results

This section presents the estimation results. According to the design lined out in Section 5 we first present results of our estimation of the deterministic components in both models, cf. Section 6.1. Second, in Section 6.2 we present the parameters of the stochastic processes.
6 Results

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<td>Estimation window</td>
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<td>Observations</td>
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Table 5: Key results of the deterministic components for daily data in average and for the last estimation, i.e. the whole sample.

$\hat{S}_t$ and $\hat{X}_t$, i.e. the autocorrelation and the standard deviation, for the weekly processes. In Section 6.3 we present the estimation results for the marginal cost function. In Section 6.4 we evaluate futures under the assumption of risk neutral market participants. With these results, we then implicitly estimate the risk parameters by minimizing the RMSE as presented in Section 6.5. Finally, in Section 6.6 the estimated risk aversion parameters are used for calculating futures prices out-of-sample.

6.1 Deterministic Components

In Table 5 we present some results for the estimation of the deterministic components for the whole time series. Detailed results for the whole observation period of spot prices and demand volumes are given in Table 12 in the Appendix.

We find that the holiday effects are more pronounced in the demand volume than in the spot price, leading to 22 out of 24 days that are significant for all demand series on the 1%-level, but only 10 for the spot price series. On the 5%-level, all 24 days were significant for the demand series and 14 for the spot price series.

According to the Akaike criterion, we set the order of autocorrelation lags to 4 for the spot prices and to 3 for the demand.

The $R^2$ is higher for the demand volume than for the spot price. The standard error of the regression, standardized on the mean of spot prices or daily demand, respectively, is larger...
for the spot price than for the demand. These results reflect the intuition from Figures 5 and 6 that the seasonal patterns are much stronger in the demand series than in the spot price series.

In particular, the coefficients for the overlapping triangular functions are all significant for the demand, but only four are significant for the spot price on the 5%-level. The dummy variables for Fridays, Saturdays, and Sundays are all highly significant for the spot price as well as for the demand volume.

In Figures 11 and 12 in the Appendix the deterministic components for both models are plotted exemplary for the estimation period. Visual inspection affirms that the deterministic component of the demand explains most of its general behavior. For the spot price the deterministic component provides less contribution to forecasting future prices. The residuals of the spot price may even exceed the deterministic component. The regular downward spikes in the deterministic components as well as in the observed data are due to the end-of-week effects. We like to point out that the use of monthly dummy variables instead of triangular functions would result in a rather irregular behavior of the stochastic component.

### 6.2 Stochastic Processes

As described in Section 5 we estimate the AR(1)-processes of the residuals $X_t$ and $S_t$, respectively, from weekly data by non-linear least squares. The upper part of Table 6 shows descriptive statistics of $X_t$ and $S_t$, the lower part the main estimation results used at the last valuation day, i.e., for the whole estimation window without the last observation, and descriptive statistics of the residuals.

The higher moments and the Jarque-Bera-Statistic in Table 6 show that the assumption of normal distribution is acceptable for the stochastic demand $S_t$, but not for the stochastic spot price $X_t$. Especially the large kurtosis of $X_t$ reflects the fat tails in the distribution spot prices. The Durbin-Watson-Statistic supports the modelling of autoregressive processes for both series.

For the 217 time series of the stochastic component of the spot price, $X_t$, we find a mean autocorrelation coefficient $\hat{\phi}$ of 0.906. For the whole estimation window $\hat{\phi}$ is equal to 0.951. For four out of the 217 estimation windows, $\hat{\phi}$ is larger than 1. A Dickey-Fuller test (four lags, without a constant) does not reject the hypothesis of a unit root in five cases (significance level of 5%-level). The estimation windows for those seemingly non-stationary processes all
Table 6: Descriptive statistics and estimation results of the weekly AR(1)-processes of the stochastic component of the spot price, $X_t$, and of the demand, $S_t$, respectively, for all estimations in average and for the last estimation (whole sample).

end between 12/04/2002 and 01/15/2003. This is the period when daily spot prices at Nord Pool reached new record levels, cf. the period C in Figure 5.

The speed of adjustment, $\hat{\kappa}$, becomes negative if $\hat{\phi}$ is greater than 1. This result is not compatible with the theoretical model. We did not exclude these cases.

For the stochastic component of the demand, $S_t$, the mean autocorrelation coefficient, $\hat{\rho}$, equals 0.526; it is 0.580 for the whole estimation window. With a maximum across all estimation windows equal to 0.632, the process $S_t$ is always stationary. We find rather low values of $\hat{\rho}$ around 0.20 for the first few estimation intervals which are artefacts due to the short observation periods. We did not exclude those values neither.

The autocorrelation coefficient $\hat{\phi}$ for the stochastic spot price $X_t$ is much larger than $\hat{\rho}$ for the stochastic demand $S_t$. Also the $R^2$ is much larger for the first one than for the latter one. This result corresponds to the findings in Section 6.1 that a great amount of the demand variation can be explained by seasonal patterns, while the spot price fluctuates more randomly. Figures 11 and 12 show the residuals $X_t$ and $S_t$, respectively, on a daily basis. The first one exhibits only a weak mean-reversion characteristic, but the second one quickly oscillates around zero. Thus, for the spot price the autocorrelation coefficient has to capture deviations that are larger and more persistent than those of the demand.
Table 7: This table shows key statistics for the estimated coefficients of the marginal cost function in average over all estimation periods and for the whole sample, i.e. the last period. The estimates for the whole sample are based on 281 observations between 03/31/1999 and 08/10/2004.

For evaluation purposes, we use the estimated $\hat{\rho}$ as given in (9). We use $1 - \hat{\varphi}$ as an estimate for the speed of adjustment, $\kappa$, in the reduced-form model (cf. (1)). The standard errors of the regressions serve as estimates for $\sigma^X$ and $\sigma^S$, respectively.

After the estimation of the autoregressive processes we find a negative skewness combined with a larger kurtosis in the residuals of both time series. The Jarque-Bera-Statistic of the residuals increases for both time series when compared to the results before estimation. The hypothesis of normal distribution is rejected at usual significance levels, but the specification failure is much less for the stochastic component of the demand.

The Durbin-Watson-Statistic for the residuals indicates that the AR(1)-approach is more suitable for the stochastic component of the demand than for the one of the spot price.

### 6.3 Marginal Cost Function

Table 7 shows the parameter results for the marginal cost function in average as well as for the last estimation, i.e. across the whole sample. The parameter values of $c_0$ and $c_1$ are positive and thus affirm the assumption of an increasing marginal cost function.

The estimates of $\hat{\gamma}$ are negative as expected. The mean of $-0.0135$ means a reduction of the aggregate hydro plant production by $0.0135/0.0322 = 0.42$ GW if the water reservoir level is one percentage point below its median for a particular day. Since we determine $\hat{\gamma}$ on a weekly basis, a production of 0.42 GW during one week is equivalent to 70.6 GWh. The total reservoir capacity of the hydro plants in the Nord Pool area amounts to approximately 123500 GWh, i.e. one percentage-point is equal to 1235 GWh. Thus, in case of a negative water reservoir deviation our estimate of $\hat{\gamma}$ implies that hydro plant operators reduce their production so that approximately 6% of the deviation is recovered after one week.
6.4 Risk-neutral Evaluation (Study A)

In the first step – study A – of testing the models, we determine theoretical futures prices under the assumption of risk neutrality. This case is considered as a benchmark of how well the seasonal components and stochastic parameters of the processes are identified.

In Table 8 we report statistics valuation errors $f_{t,T}^{A,RF/EQ} = \hat{F}_{t,T}^{A,RF/EQ} - F_{t,T}$ and absolute valuation errors $|f_{t,T}^{A,RF/EQ}|$ between estimated futures prices $\hat{F}_{t,T}^{A,RF/EQ}$ and observed futures prices $F_{t,T}$. The indexes EQ und RF denote the equilibrium model and the reduced-dorm model, respectively.

We find that the equilibrium model explains futures prices slightly better than the reduced-form model in terms of the mean absolute error, but not in terms of the median. Both models clearly underestimate the futures, as the mean and median error show.

Even though the mean and median results do not clearly favor on specific model, the variation does. The error range for the reduced-form model is roughly three times as large as for the equilibrium model. Also the standard deviations show a more robust performance of the equilibrium model.

In order to gain an optical impression of the results, we calculated the mean deviation (MD) and the mean absolute deviation (MAD) across all observations during rolling intervals of eight weeks, see Figure 17 in the Appendix. We find that the reduced-form model provides

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<td>$f_{t,T}^{A,RF}$</td>
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Table 8: This table shows descriptive statistics of the valuation errors $f_{t,T}^{A,RF/EQ} = \hat{F}_{t,T}^{A,RF/EQ} - F_{t,T}$ and absolute valuation errors $|f_{t,T}^{A,RF/EQ}|$, given in NOK, during the valuation period 05/03/2000 – 08/10/2004 in study A. The study is based on 2046 futures prices.

Figure 13 in the Appendix plots the relation between $D_t$ and the weekly spot price, and 14 exemplary shows $D_t^*$ versus the weekly spot price and the estimated marginal cost functions for the last valuation day.
good estimates in periods when the electricity market is in a generally calm condition, e. g. during summer 2000. The equilibrium model provides better results when the spot price exhibits its characteristic spikes. During the winter 2002/03 when extreme weather conditions in combination with a very low reservoir level led to record-high prices, the equilibrium model explains futures prices better than the reduced-form model.

We argue that there are two main reasons for the dominance of the production based equilibrium approach during volatile markets: First, the reduced-form model cannot reflect the distribution of the spot price deviations $X_t$. As shown in Table 6 the Jarque-Bera statistic indicates that residuals from the AR(1)-estimation of the stochastic spot price do not behave normally. They exhibit the well-known fat tails in the distribution and are positively skewed. The production based approach in the equilibrium model does introduce the skewness of spot prices by assuming normally distributed demand deviations, $S_t$, that are transformed into right-skewed spot prices by the increasing function of marginal cost.

The second reason is that the seasonal component of the spot price is harder to estimate than the one of the demand. Consequently, the stochastic deviation of the spot price, $X_t$, has to explain a large amount of spot and futures prices in the reduced-form model, cf. Figure 11. However, it is unclear if a large value of $X_t$ is due to a singular event (e. g., a week of cold weather) that does not affect futures prices or if it is caused by lasting changes in the determinants of electricity prices (e. g., a change in the water reservoir deviation) that does affect futures prices. The first case would require a small value of $\kappa$, the second a large value of $\kappa$. The estimated $\hat{\kappa}$ is supposed to lie somewhere in between.

Even though the demand is similarly modelled as an AR(1)-process the described problem is of less importance: First, changes in the stochastic demand residual, $S_t$, are mostly caused by weather conditions and thus will in general fade out quickly. This is reflected by the low value of $\hat{\rho}$, cf. Table 6. Second, the deterministic component of the demand provides good estimates of future demand in comparison to the deterministic component of the spot price, cf. Figure 12. $S_t$ therefore has a smaller influence on forecasted demands and marginal costs than $X_t$ has. We conclude that a large absolute value of $X_t$ may lead to large deviations between the theoretical and the observed futures prices.

Figure 17 shows that the value of the Mean Deviation MD is mostly negative for both models, restating that both models tend to underestimate the observed futures prices. However, for the time between Dec. 2002 and Jan. 2003 that comprise those valuation days with estimates $\hat{\kappa} < 0$ (non-stationary processes of $X_t$) futures prices are strongly overestimated
in the reduced-form model.

A further analysis of the prices generated by the reduced-form model reveals that typically the futures prices of shorter maturities are quite accurately evaluated and sometimes overestimated, but futures of longer maturities are clearly underestimated. However, we find a significant relationship between time to maturity and the valuation error only if we exclude all observations during the winter 2002/03 (Dec. 2002 until Feb. 2003). We do not find a corresponding relationship for the equilibrium model.

We finally compare the results by the correlation of the valuation errors. The correlation between the errors of both models amounts to 27.1%, between the absolute errors it is 37.1%. The positive values correspond to the partial parallel evolution of the errors as shown in Figure 17.

We conclude that the assumption of risk neutrality is not valid. As both models underestimate futures prices, there exist implicit premia in futures prices. These are positive in average, i.e., the seller of a futures contract can earn a surplus. This conclusion confirms with the empirical findings of Longstaff and Wang (2004) who empirically analyze forward prices at the Pennsylvania-New Jersey-Maryland market.

6.5 In-sample Estimation (Study B)

We implicitly estimate the market price of risk, $\lambda$, and the risk parameter of the equilibrium model, $\xi$, by minimizing the root mean squared error between the theoretical futures prices and the traded prices of all block- and week-futures observed during a particular valuation interval of eight weeks.

In the case of the reduced-form model, we expect the market price of risk, $\lambda$, to be negative in average according to (3) and our considerations in Section 6.4. For the equilibrium model, the risk parameter has to be positive by construction if market participants are risk-averse. In order to compensate the underestimation of the futures prices in this model by a positive $\xi$, the sum of the covariances in (7) has to be negative on average.

Table 9 shows the aggregate results for the estimated risk parameters.

The market price of risk, $\lambda$, is negative as expected. Note that the values for $\lambda$ are much larger in absolute terms than those that Lucia and Schwartz (2002) report. Furthermore, Lucia and Schwartz (2002) report mostly positive $\lambda$. 
Table 9: This table shows the results of the implicitly estimated parameters of risk aversion.

<table>
<thead>
<tr>
<th></th>
<th>Reduced-form Model</th>
<th>Equilibrium Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>−0.2389</td>
<td>0.1569</td>
</tr>
<tr>
<td>Median</td>
<td>−0.2632</td>
<td>0.0270</td>
</tr>
<tr>
<td>Minimum</td>
<td>−0.7998</td>
<td>−0.6566</td>
</tr>
<tr>
<td>Maximum</td>
<td>1.0204</td>
<td>1.9497</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>0.3175</td>
<td>0.4092</td>
</tr>
</tbody>
</table>

The larger absolute values of $\lambda$ in our study are due to our approach of estimating the autocorrelation on a weekly basis. Recall that the risk premium in the reduced-form model is equal to $-\lambda \frac{\sigma_X}{\kappa} (1 - e^{-\kappa(T-t)})$. In order to achieve a certain risk premium, $\lambda$ must increase if the fraction $\frac{\sigma_X}{\kappa}$ decreases, c. p. From our weekly approach we receive larger estimates for the speed of adjustment, $\hat{\kappa}$, and for the spot price volatility, $\hat{\sigma}_X$, than Lucia and Schwartz (2002) do. Our estimates for $\hat{\kappa}$ are in average around 0.09 per week, and for $\hat{\sigma}_X$ around 20 NOK per week (cf. Table 6), i. e. $\hat{\sigma}_X \hat{\kappa} = 222$. Lucia and Schwartz (2002) report a $\hat{\kappa}$ of 0.01 per day, and a value of 9 for $\hat{\sigma}_X$, i. e. $\hat{\sigma}_X \hat{\kappa} = 900$. This larger value results from not considering autocorrelation of higher lags in the daily residuals, leading to an overestimation of $\hat{\sigma}_X$. If one assumes that the futures price forecasts without risk premia were equal for the daily and the weekly estimation approach, $\lambda$ then had to be larger for the latter one due to the estimates of $\hat{\kappa}$ and $\hat{\sigma}_X$.

The different test periods also account for part of the difference in absolute values and especially for negative instead of positive values of $\lambda$. Lucia and Schwartz (2002) test their model during a twelve-month period in 1998/1999 when spot prices were lower in average than in the years before. Therefore, their estimated deterministic component $f(t)$ of the spot price overestimates the spot prices in the test period, leading to mostly positive values of $\lambda$. We encountered the opposite situation. Table 1 shows that the twelve-month average spot price increased until 2003 and stayed on a high level until the end of our available time series. From an ex-post perspective, the deterministic component of the spot price then inevitably underestimates the observed and future spot prices. If market participants can partly predict such increasing spot prices – e. g. by considering water reservoir levels – and evaluate futures accordingly, the reduced-form model must underestimate futures prices when risk-neutrality is assumed. A negative $\lambda$ that is appropriately large in absolute terms can compensate such estimation errors.

As an exception in our sample, $\lambda$ adopts large positive values in the intervals covering...
December 2002 and January 2003 when spot prices were extremely high. This effect is caused by negative estimates of $\hat{\kappa}$ in those particular intervals.

Depending on the sign of $\lambda$, the term $-\lambda \sigma^2(1 - e^{-\kappa(T-t)})$ is either strictly positive or strictly negative for all maturities, it increases and asymptotically converges to a constant with increasing time to maturity ($T - t$). Thus, it matches the curve of valuation errors that basically increases with time-to-maturity as described in the previous section. Figure 18 in the Appendix shows a scatter plot of the in-sample estimated risk premia in the reduced-form model. In the majority of observations, those are clearly positive and adopt values up to 180 NOK. Especially during the winter 2002/03, the risk premia reach enormous negative values that do not seem economically meaningful.

The time series of $\lambda$ features a level-autocorrelation of 0.98. We therefore conclude that $\lambda$ contains valuable information and contributes to forecasting futures prices even when applied out-of-sample as we test in the next section even though we observe eight changes of sign in the time series. The means between $\lambda$ in warm and cold periods (April until September vs. October until March) show little difference. We conclude that $\lambda$ does not incorporate seasonal effects that discriminate between summer and winter.

The estimated values of $\xi$ in the equilibrium model are mostly and in average positive as expected. However, in 36 out of 217 cases $\xi$ is negative. The negative values mostly occur in subsequent valuation intervals. The level-autocorrelation of $\xi$ amounts to 0.95, i.e. $\xi$ is supposed to improve futures pricing out-of-sample, too. However, the time series of $\xi$ exhibits six changes of sign. For those valuation dates we expect the results from the out-of-sample test in Section 6.6 to be worse than those from the risk-neutral evaluation.

The technical reason for negative $\xi$ lies in the covariance term of (7). In the majority of cases, the expected future spot price is smaller than the observed futures price as study A showed. In order to generate a positive risk premium with a positive value of $\xi$, the sum of covariances in (7) must be negative. To meet this requirement, the covariances between the future demands $D_t$ times the end-user price and the next period’s futures price have to be smaller than the covariances between the future production costs $C(D_t)$ and the futures price.

For low demand volumes the slope of the cost function (i.e. the marginal cost function) is smaller than the average end-user price. Those low demand volumes would therefore contribute positively to the covariance term. If the probability of higher demand volumes is
too small, the whole covariance term might be positive.

According to the reasoning above, negative values of $\xi$ should be possible when only futures maturing in the summer time are evaluated, but not necessarily when the valuation day lies in the summer time as it is the case in our results.

However, the estimation of the marginal cost function used at those particular valuation dates is dominated by observations at summer dates. This effect arises from the beginning of each estimation period at the end of March 1999. Therefore, each estimation of the marginal cost function is based on more summer than winter observations. This effect especially holds for the earlier valuation dates. Furthermore, the estimation of the marginal cost functions before winter 2002/03 in general lacks observations of extremely large spot prices. Both facts lead to estimates of the marginal cost function with a slope that is too low.

Our analysis of the risk premia at those particular days has shown that the underestimated marginal cost function forces the parameter $\xi$ to be negative in such cases. Still, for higher production levels or low reservoir levels, the cost function is steep enough to support positive values of $\xi$.

Generally, in the equilibrium model a non-zero value of $\xi$ does not shift the whole futures curve into one direction as the reduced-form model does, but adds large premia for short maturities and lower premia for long maturities. Also a combination of positive premia at the short end and negative premia for longer maturities may occur. Figure 19 in the Appendix shows estimated risk premia along time to maturity. The risk premium in the equilibrium model adopts its extreme values for some short maturity. It approaches zero for long maturities and therefore cannot compensate theoretical miss-pricing of long-term futures. It indirectly depends on seasonal effects as it considers covariances between futures prices on the one hand side and production costs, marginal production costs, and demand volumes on the other hand side.

Thus, if the calculation with $\xi = 0$ shows a large difference between calculated and observed prices along the whole futures curve, even a large absolute value of $\xi$ contributes only little to the reduction of the estimation errors for long maturities. On the other hand side, if the estimation of $\xi$ is based on few futures contracts with only short maturities, i.e. after August 2003 when block futures were delisted and only week futures are available, the compensation of estimation errors for only those shorter futures is feasible. From a technical point of view, the compensation then works in a similar way as in the reduced-form model. As Figure 16
Table 10: This table shows descriptive statistics of the valuation errors \( f_{C,RF/EQ}^{t,T} = \hat{F}_{C,RF/EQ}^{t,T} - F_{t,T} \) and absolute valuation errors \( |f_{C,RF/EQ}^{t,T}| \), given in NOK, during the valuation period 06/28/2000 – 08/10/2004 in study C. The study is based on 1971 futures prices.

exemplary shows, the risk premium in both models increases with time-to-maturity for the first few weeks.

However, the estimates of \( \xi \) become less reliable if fewer futures prices enter the estimation procedure. Figure 21 shows that the implicitly estimated \( \xi \) varies stronger after August 2003. Also the variation of \( \lambda \) increases, but to a much less extent.

### 6.6 Risk-adjusted Evaluation (Study C)

We finally conduct a second out-of-sample test of futures prices by incorporating the risk parameters \( \lambda \) and \( \xi \). We apply the values of \( \lambda \) and \( \xi \) that were implicitly estimated in the previous valuation interval. The results are presented in Table 10.

We find that incorporating risk premia improves in average the forecasting quality of the reduced-form model only in the mean and the median of the valuation errors. The standard deviation and the error range, defined as the difference between maximum and minimum error, decreases for the equilibrium model, but increases for the reduced-form model, i.e. it even looses robustness.

In comparison to each other, the models show almost equal values of the absolute valuation error, but we find a lower median for the reduced-form model. As can be seen from Figure 20, the reduced-form model gains forecasting ability for calm market situations than the equilibrium model, but fails during volatile periods. This leads to a more than twice as high standard deviation of the errors of the reduced-form model than of the equilibrium model. Specifically, this result is driven by some extreme forecasting errors that derive from large prices in the winter 2002/03.
Table 11: This table shows the results of regressing the estimated risk premia of study C on the valuation errors of Study A.

Both models continue to underestimate the futures prices in average, even though to a much lesser extent. We conjectured in the previous sections that the different functional forms of the risk premium terms structures will allow the reduced-form model to better compensate for mispricing of long-term futures. As in Study A we therefore regress the valuation error on the time to maturity. We do not find a significant relationship for the equilibrium model. However, we find a significant positive relationship for the reduced-form model, i.e. the underpricing decreases in average with increasing time to maturity.

We again examine the correlations between the error measures of both models. For the valuation error we find a correlation of 26.6%, for the absolute valuation error it is 31.9%. Both dropped slightly compared to study A. This confirms that the adjustment by incorporating risk premia works differently in both models.

As discussed the risk premia necessarily contain a fraction due to misestimation of expected futures prices and another one for actual, but unobservable risk premia. These two cannot be separated. However, if we assume that the errors of the estimation of the expected spot prices are symmetrically distributed with a constant – presumably negative – mean, than the estimated risk premia should represent fraction of the actual risk premia plus a constant.

In order to evaluate the estimated risk premia, $\hat{\Lambda}_{t,T}^{C,RF/EQ} = \hat{F}_{t,T}^{C,RF/EQ} - \hat{F}_{t,T}^{A,RF/EQ}$, we regress them on the valuation errors $f_{t,T}^{A,RF/EQ}$ from Study A:

$$\hat{\Lambda}_{t,T}^{C, RF/EQ} = \hat{\zeta}_0^{RF/EQ} + \hat{\zeta}_A^{RF/EQ} f_{t,T}^{A,RF/EQ} + u_{t,T}^{RF/EQ}$$

with $u_{t,T}^{RF/EQ}$ i.i.d. distributed. The results are given in table 11.

As expected, the estimated constants are positive, indicating that the risk premia cover up part of the undervaluation. Surprisingly, the slope $\hat{\zeta}_A^{RF}$ for the reduced-form model does not significantly differ from zero. Thus, in average the risk premium in the reduced-form model does not capture any structure of the errors in study A. However, we did the same regression for subsamples and found a significant negative slope if we leave out all observations of the
winter 2002/03.

For the equilibrium model, we find a significant negative slope for the whole sample as well as for subsamples. We conclude that the risk premium given by the equilibrium model is always able to explain some of the structure of the valuation errors.

7 Conclusion

We empirically compared the pricing quality of a one-factor reduced-form model by Lucia and Schwartz (2002) and a one-factor equilibrium model by Bühler and Müller-Merbach (2004). Our analysis is based on 217 daily observations of all traded week and block futures prices at the Scandinavian electricity exchange Nord Pool.

The major difference between the two models lies in the exogenous variable – the spot price in the reduced-form model versus the end-user demand in the equilibrium model. Especially the demand, and to a lower extent also the spot price, exhibit seasonal patterns which might dominate possible risk premia in futures prices. We therefore carefully estimated the seasonal components.

In case of the equilibrium model, we furthermore estimated the marginal cost function that translates the symmetrically distributed demand (that equals the production volume) into right-skewed spot prices. The reduced-form model implies a symmetric distribution of spot prices.

When estimating futures prices out-of-sample under the assumption of risk neutrality, we find that the production-based equilibrium approach is more powerful in explaining futures prices.

In order to evaluate risk premia, we estimated the implicit risk parameter of each model by minimizing the RMSE between observed and theoretical futures prices, and applied those estimates out-of-sample. Both models improved their futures price estimations with a slight advantage in the mean for the reduced-form model. However, it fails when price spikes occur, leading to a much larger standard deviation of the valuation errors.

Our empirical study is based on weekly average spot prices. In doing so, the time series of spot prices becomes less volatile and the probability and frequency of price spikes decreases. Even so we found four instances for which the estimated process of the spot price in the reduced-form model became non-stationary due to extreme prices. Such a result contradicts
the model assumptions and leads to miss-pricing of futures in the reduced-form model.

We also analyzed the estimated risk premia which was endogenously derived in the equilibrium model, but arbitrarily set in the reduced-form model by the ad-hoc assumption of an invariant market price of risk. We find evidence that the theoretical term structure of the risk premia in the equilibrium model is supported by observed data, but no evidence for the one of the reduced-form model.

According the estimation effort, the equilibrium model is more costly than the reduced-form model. Both models require time-series of spot prices. Time series of futures prices are required if risk parameters are to be estimated. The equilibrium model further requires time series of demand volumes and needs additional input variables concerning the production and the end-user prices.

We conclude that the equilibrium model better explains risk premia and has a stronger ability to capture price spikes which are a key characteristic in electricity markets. If applied to another market than the hydro-dominated Scandinavian market, the marginal cost function has to be appropriately specified. Consideration of more than just one factor is possible.

Considering the cost for estimation and implementation, a reduced-form approach might be more suitable. A second factor that incorporates the risk of price spikes seems inevitable then. Among others, Villaplana (2003) and Geman and Roncoroni (2003) provide examples of jump-diffusion models. However, when introducing jumps in the spot price process, the appeal of reduced-form models concerning straightforward implementation diminishes.

References


To model the piecewise linear function of the seasonal pattern, we define 12 overlapping triangular functions \( M_{i,t} \) around certain equidistant calendar days \( \bar{t}_i \).

The selection of triangular functions \( M_{i,t} \) as regressors yields a piecewise linear and continuous function for the seasonal component, cf. Figure 4. Note that this procedure does not require more coefficients than conventional dummy variables.

The effects of specific days on the spot price or the daily demand are captured by dummy variables \( \text{FRI}_t \), \( \text{SAT}_t \), and \( \text{SUN}_t \) for week-end days, and \( H_{h,t} \) for holidays and for additional days with a significant impact on the endogenous variable. The residual in the regression is assumed to follow an autoregressive process whose lag is determined by the Akaike criterion.

The triangular functions are defined as follows:

\[
M_{i,t} = \frac{1}{28} \begin{cases} 
\Delta t - (\bar{t}_i - t), & 0 \leq \bar{t}_i - t < \Delta t \\
\Delta t - (t - \bar{t}_i), & 0 < t - \bar{t}_i < \Delta t \\
0, & \text{elsewhere}
\end{cases}
\]  

(15)

with \( \Delta t = 28 \) days and the following calendar days \( \bar{t}_i \):

<table>
<thead>
<tr>
<th>( \bar{t}_1 )</th>
<th>01/01</th>
<th>( \bar{t}_2 )</th>
<th>01/29</th>
<th>( \bar{t}_3 )</th>
<th>02/26</th>
<th>( \bar{t}_4 )</th>
<th>03/26</th>
<th>( \bar{t}_5 )</th>
<th>04/23</th>
<th>( \bar{t}_6 )</th>
<th>05/21</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{t}_7 )</td>
<td>06/18</td>
<td>( \bar{t}_8 )</td>
<td>08/14</td>
<td>( \bar{t}_9 )</td>
<td>09/11</td>
<td>( \bar{t}_{10} )</td>
<td>10/09</td>
<td>( \bar{t}_{11} )</td>
<td>11/06</td>
<td>( \bar{t}_{12} )</td>
<td>12/04</td>
</tr>
</tbody>
</table>

We use this procedure for both models. We choose 13 calendar days (instead of 12 corresponding to the calendar months) for the simple reason that 365 mod 13 = 1, i.e. we can cover one year more regularly when applying 13 instead of 12 intervals. July 16 and 17 are used as benchmark days, i.e. not covered by any of the triangular functions. We choose these days as they are the ones with the lowest production in the long-term average.

The regression equation for the reduced-form model is given by:

\[
P_t = \beta_0 + \sum_{i=1}^{12} \beta_i M_{i,t} + \beta_{\text{FRI}} \text{FRI}_t + \beta_{\text{SAT}} \text{SAT}_t + \beta_{\text{SUN}} \text{SUN}_t + \sum_h \beta_h H_{h,t} + \epsilon_t^P \tag{16}
\]

with \( \epsilon_t^P = \phi_1 \epsilon_{t-1}^P + \phi_2 \epsilon_{t-2}^P + \phi_3 \epsilon_{t-3}^P + \ldots + \epsilon_t^P \), \( \epsilon_t^P \sim \mathcal{N}(0, \sigma_P^2) \) \tag{17}

The time index \( t \) in (16) and (17) runs from 11/01/1996 until the Tuesday preceding the valuation day (Wednesday). For the equilibrium model the endogenous variable \( P_t \) in (16) is
Figure 4: This figure illustrates the principle of describing the deterministic components in the spot price and in the demand series.
replaced by the daily demand $D_t$ and the autoregressive residuals have three lags only. The estimation results for the total observation period of spot prices and daily demand volumes are given in Table 12.

<table>
<thead>
<tr>
<th>Estimation Window No. of obs.</th>
<th>Spot Prices (Reduced-form Model) 11/01/1996–08/03/2004 2833</th>
<th>Electricity Demand (Equilibrium Model) 03/29/1999–08/03/2004 1955</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$ (Const.)</td>
<td>125.00</td>
<td>30.83</td>
</tr>
<tr>
<td>$\beta_1$ (01/01)</td>
<td>137.08</td>
<td>19.93</td>
</tr>
<tr>
<td>$\beta_2$ (01/29)</td>
<td>74.33</td>
<td>18.91</td>
</tr>
<tr>
<td>$\beta_3$ (02/26)</td>
<td>69.97</td>
<td>18.26</td>
</tr>
<tr>
<td>$\beta_4$ (03/26)</td>
<td>42.88</td>
<td>13.40</td>
</tr>
<tr>
<td>$\beta_5$ (04/23)</td>
<td>34.72</td>
<td>8.22</td>
</tr>
<tr>
<td>$\beta_6$ (05/21)</td>
<td>25.40</td>
<td>9.16</td>
</tr>
<tr>
<td>$\beta_7$ (06/18)</td>
<td>34.40</td>
<td>4.96</td>
</tr>
<tr>
<td>$\beta_8$ (08/14)</td>
<td>74.33</td>
<td>5.57</td>
</tr>
<tr>
<td>$\beta_9$ (09/11)</td>
<td>69.97</td>
<td>18.91</td>
</tr>
<tr>
<td>$\beta_{10}$ (10/09)</td>
<td>42.88</td>
<td>13.40</td>
</tr>
<tr>
<td>$\beta_{11}$ (11/06)</td>
<td>64.78</td>
<td>8.22</td>
</tr>
<tr>
<td>$\beta_{12}$ (12/04)</td>
<td>102.59</td>
<td>9.16</td>
</tr>
<tr>
<td>$FRI$</td>
<td>$-3.40$</td>
<td>$-0.38$</td>
</tr>
<tr>
<td>$SAT$</td>
<td>$-14.40$</td>
<td>$-3.56$</td>
</tr>
<tr>
<td>$SUN$</td>
<td>$-19.01$</td>
<td>$-4.32$</td>
</tr>
<tr>
<td>No. of significant holidays ($\alpha = 5%$)</td>
<td>14</td>
<td>24</td>
</tr>
<tr>
<td>$\phi_1$ (AR(1))</td>
<td>0.77</td>
<td>1.33</td>
</tr>
<tr>
<td>$\phi_2$ (AR(2))</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>$\phi_3$ (AR(3))</td>
<td>0.27</td>
<td>0.11</td>
</tr>
<tr>
<td>$\phi_4$ (AR(4))</td>
<td>$-0.08$</td>
<td>$-0.08$</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.965</td>
<td>0.986</td>
</tr>
<tr>
<td>S. E.</td>
<td>17.04</td>
<td>0.81</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>$-12034.7$</td>
<td>$-2337.6$</td>
</tr>
<tr>
<td>Residuals $u_t^p$, $u_t^D$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min.</td>
<td>$-162.61$</td>
<td>$-3.27$</td>
</tr>
<tr>
<td>Max.</td>
<td>367.36</td>
<td>3.45</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>16.91</td>
<td>0.80</td>
</tr>
<tr>
<td>Skewness</td>
<td>4.55</td>
<td>0.05</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>124.46</td>
<td>4.56</td>
</tr>
<tr>
<td>Jarque-Bera-Stat.</td>
<td>$1.75 \cdot 10^7$</td>
<td>199.52</td>
</tr>
</tbody>
</table>

*Table 12:* This table shows the results of the daily estimation of both models, cf. (16) and (17). The estimation windows comprises the whole available time series.
Figure 5: This figure shows the daily spot price ("system price") and its first differences at Nord Pool for the period from 03/31/1999 until 08/04/2004. The price spikes A and B and the period of high prices, C, correspond to those marked in Figures 6, 7, and 8.
This figure shows the daily electricity demand and its first differences in the Nord Pool area (Norway, Sweden, Finland) for the period from 03/31/1999 until 08/04/2004. The price spikes A and B and the period of high prices, C, correspond to those marked in Figures 5, 7, and 8.
Figure 7: This figure shows the daily spot price and the daily demand in a scatter plot for the period from 03/31/1999 until 08/10/2004. The correlation between the spot price and the production is 39.2%, for their first differences it is 49.7%. The points A and B correspond to those marked in Figure 5 and 6. The data points in the upper left area mostly correspond to the period of high prices, C, which is not marked in this figure.
Figure 8: This figure shows the water reservoir level and its deviation from the historical median (280 data points from 03/31/1999 until 08/10/2004). It is measured in percentage points of the total reservoir capacity. The correlation between the spot price and the deviation from the median reservoir level amounts to $-75.9\%$. The time period C corresponds to those in the previous Figures.
Figure 9: The weekly spot price and the water reservoir level in a scatter plot (280 data points from 03/31/1999 until 08/10/2004). The correlation between the spot price and the reservoir level is $-37.1\%$, but only $-3.7\%$ for the first differences.
Figure 10: This figure shows a scatter plot of the weekly spot price and the water reservoir deviation from its median in %-points (280 data points from 03/31/1999 until 08/10/2004). The correlation between the spot price and the deviation from the reservoir median is $-75.9\%$, for the first differences it is $-13.8\%$. 
Figure 11: This figure shows the daily spot price, $P_t$, its estimated deterministic component, $f(t)$, for the reduced-form model, and the daily residual (estimation window: 11/01/1996–08/04/2004).
Figure 12: This figure shows the daily electricity demand $D_t$, its estimated deterministic component $\tilde{D}_t$ used in the equilibrium model, and the residual $S_t$ (estimation window: 03/29/1999–08/04/2004).
Figure 13: This figure shows the scatter plot of weekly average demand $D_t$ and the weekly average spot prices $P_t$ during the observation period.
Figure 14: This figure shows the estimated marginal cost function $C'(D^*) = \exp(4.64 + 0.0209D^*)$ where $D^* = D - 0.4498 \text{WRD}$ and the scatter plot of the adjusted demand $D^*$ and the weekly average spot prices. The parameters of the marginal cost function are estimated on the whole observation period, i. e. the figure shows an in-sample comparison.
Figure 15: This figure shows the weekly average spot price, the price of the week futures contract with the shortest available maturity (1-Week Futures Contract), of the block futures contract with the shortest available maturity (1-Block Futures Contract), and of the block futures contract that matures in $6 \cdot 4 = 24$ weeks (6-Block Futures Contract). Note that block futures were delisted in August 2003.
Figure 16: This figure shows the observed futures curve, the theoretical futures curves and the theoretical futures premia at 05/29/2002. The grey sections of the observed futures curve denote settlement prices that were not traded and thus not considered in the implicit estimation of risk parameters.
Figure 17: This figure shows the MAD and the MD for both models in the case of risk neutrality ($\lambda = 0$, $\xi = 0$) and the 8-week average spot price.
Figure 18: This Figure shows a scatter plot of the in-sample estimated risk premia of the reduced-form model along the time to maturity.
Figure 19: This Figure shows a scatter plot of the in-sample estimated risk premia of the equilibrium model along the time to maturity.
Figure 20: This figure shows the MAD and the MD for both models with the estimated $\lambda$ and $\xi$ applied out-of-sample, and the 8-week average spot price.
Figure 21: This figure shows the implicitly estimated $\lambda$ and $\xi$ of the reduced-form model and the equilibrium model, respectively.