Late-Mover Advantage and Product Diffusion*

Sandeep Kapur

Department of Economics, Birkbeck College, Gresse St, London W1P 1PA, UK

Abstract
The diffusion of a new product, whose price is expected to fall progressively, is modelled as a sequence of Wars of Attrition. The likely pattern of diffusion is found to be reasonably consistent with the empirically observed S-shape.

JEL Classification: O33

1. Introduction
One possible reason for the slow diffusion of new products may lie in the expectation, on part of the potential buyers, that the price of these products will fall in the future. A new product may be intrinsically desirable and yet not acquired immediately if the advantage of future price-reduction outweighs the subjective cost of delaying purchase. This note considers the case where the price of a new product is expected to fall, not as a function of time, but as a function of the degree of its market penetration. Usually, those who buy early bear the 'cost' of early imperfections; subsequent buyers tend to benefit from remedial product improvements and, effectively, face a lower price [see Rosenberg (1976)]. Alternatively, the buyers might expect prices to fall with the extent of market penetration if the product is supplied by a price-discriminating monopolist who believes his market to be heterogeneous. In either case, we have an externality: individuals who are later in the acquisition sequence gain from price reductions induced by early purchasers. I show how such a structure could lead to a pattern of sequential acquisitions even when all buyers are, in fact, identical. Further, the pattern is reasonably consistent with the empirically observed S-shaped diffusion curves.

2. The Model
Suppose there are N identical potential buyers for a new product. Let the product be such that no individual would ever buy more than one unit (that is, there are no repeat purchases), and that it has no resale value. The private benefit from acquiring the product (the money value of the discounted sum of utility over the infinite horizon) is uniform for all individuals and is denoted by the constant $G > 0$. The price of the product depends on a buyer's rank in the acquisition sequence. In particular, it is assumed that those who buy after others pay less. Consider the stage where $i$ individuals have acquired the product, $i < N$. The price at this stage is given as

* I thank Frank Hahn for his comments. Any errors are mine.
\[ C(i) = \lambda c \quad \text{where } \lambda \in (0, 1) \text{ and } 0 < c < G \quad (1) \]

The restriction \( \lambda < 1 \) ensures that the price declines along the acquisition sequence. The net benefit from acquiring the product at stage \( i \) is given by \( \pi(i) = G - \lambda ic \). The restriction that \( c < G \) ensures that the product is desirable even for the initial buyer.

The diffusion of the product is modelled as a sequence of 'stage-games'. Consider the \( i \)-th stage, where \( i \) individuals have already acquired the product and \( n = N-i \) potential buyers remain in the market. The acquisition behaviour of these \( n \) individuals is treated as an \( n \)-player War of Attrition. The game ends as soon as one or more of these \( n \) players acquires the product. If, say, one individual acquires at this stage, the process moves on to the \( (i+1) \)th stage, and the \( n-1 \) remaining players engage in another War of Attrition, and so on. A priori, simultaneous acquisitions are not ruled out: if \( k \) individuals acquire the product simultaneously, the process moves directly to stage \( (i+k) \). I refer to each War of Attrition as a stage-game.

The \( i \)-th stage-game begins at time \( t_i \) and ends as soon as at least one of the \( n \) players in that stage-game acquires the product. At time \( t_i \), each of the \( n \) players (simultaneously) chooses a time \( t \) to make the purchase, where \( t \in (t_i, t_i+\Delta, t_i+2\Delta, \ldots) \). All players discount the future - the (common) instantaneous rate of discount is given by \( \rho > 0 \) - so, clearly, buying early has the obvious advantage. However, delaying the purchase creates the possibility that some other individual(s) might buy first, enabling cheaper purchase at a later stage. It is assumed that the induced reduction in purchase price is instantaneous, and that the interval \( \Delta \) is small enough for there to be some conceivable gain from waiting. In particular, we impose the following condition.

**Condition W.** \[ e^{-\rho \Delta} \pi(N-1) > \pi(i) \]

Here, \( \pi(N-1) \) is the maximal net benefit, that is, the net benefit from buying after everyone else. The condition requires that its value, discounted for the lag \( \Delta \) exceed the net benefit from immediate acquisition at stage \( i \). By assuming that \( \Delta \) is small enough for condition W to hold, or more accurately, by confining attention to those stages \( i \) where it holds, we ensure that there are some conceivable gains from waiting.

For an arbitrary player, a behaviour strategy is given by \( B = [\beta(t_1), \beta(t_1+\Delta), \ldots] \), where \( \beta(t) \) denotes the probability of acquisition by that player at time \( t \), conditional on the game reaching period \( t \). In principle, each stage-game is an infinite-horizon game - the game lasts forever if no individual ever buys the product. The payoffs are symmetric and all players are assumed to be risk-neutral. I consider the symmetric mixed-strategy Nash equilibrium of this acquisition timing game. At this equilibrium, each player is indifferent between various acquisition times in the support of her randomisation. Clearly, acquisition at time \( t \) yields a benefit of \( \pi(i) \) at that instant. On the other hand, if acquisition is delayed by (exactly) one period \( \Delta \), the expected benefit depends on what the other potential buyers do in the interim. If \( r \) rivals buy at time \( t \), the net benefit of buying in the next period is \( \pi(i+r) \); therefore, the expected payoff to waiting for one period is given by

\[ e^{-\rho \Delta} \sum_{r=0}^{n-1} \text{probability (} r \text{ rivals acquire at } t \text{ ) } [\pi(i+r)] \]

Given symmetry, let the probability of acquisition at time \( t \) be given by \( \beta(t) \) for each of the \( n-1 \) rival players. Since in equilibrium each player must be indifferent between buying at \( t \) or at \( t+\Delta \), \( \beta(t) \) must be such that
\[ \pi(i) = e^{\rho \Delta} \sum_{r=0}^{n-1} \binom{n-1}{r} \beta^r (1-\beta)^{n-r-1} \left[ G - \lambda i + rc \right] \]  

(2)

Solving this for \( \beta \), we get \[ \beta^*(t) = \frac{1}{\Delta(1 - \lambda)} \left( \frac{1 - V(i)}{\lambda^c} \right), \] where \[ V(i) \equiv G(1 - e^{\rho \Delta}) + e^{\rho \Delta} \lambda^c. \]

Condition W ensures that \( \lambda < V(i) < 1 \), so that \( \beta^*(t) \in (0, 1) \); the equilibrium acquisition probability is well defined. Further, since \( \beta^*(t) = \beta^* \) for all \( t \), the equilibrium strategies are stationary. Lastly, it can be argued that the equilibrium is subgame perfect.\(^1\)

Now, consider what happens to the equilibrium acquisition probability as the length of the decision interval, \( \Delta \), shrinks. Define \( \alpha(i) \), the equilibrium intensity of acquisition as

\[ \alpha(i) \equiv \lim_{\Delta \to 0} \frac{\beta^*}{\Delta} = \lim_{\Delta \to 0} \frac{1 - V(i)}{\Delta(1 - \lambda)} \]

\[ = \frac{1}{n-1} \frac{\rho G - \lambda^c}{(1 - \lambda) \lambda^c} = \frac{1}{n-1} \frac{\rho \pi(i)}{\pi(i+1) - \pi(i)}. \]

This suggests that when the interval \( \Delta \) is small, simultaneous acquisition is non-generic. Clearly, the probability of acquisition in the small interval \( \Delta \) is approximately \( \alpha \Delta \); the probability of simultaneous acquisition by \( k \) individuals in the interval \( \Delta \) is of order \( \Delta^k \), which can be ignored for \( k > 1 \) when \( \Delta \) is small. In general, therefore, acquisitions tend to be staggered. Note that \( \alpha \) is increasing in \( \rho \) [the rate of discount], and in \( \pi(i) \), the net benefit from immediate acquisition. It is decreasing in \( \pi(i+1) - \pi(i) \), the incremental benefit from delaying acquisition by one stage. Since \( \alpha \) is a Poisson-type parameter, the waiting time till acquisition (for any particular individual in the stage game) is distributed exponentially with parameter \( \alpha \). Stating this formally, we have

**Proposition 1.** In the symmetric mixed-strategy Nash Equilibrium of the \( i \)-th stage-game, each player chooses a strategy such that the distribution of the waiting time up to her acquisition tends to an exponential distribution with parameter \( \alpha(i) \).

### 3. Diffusion Curves

What does this imply for the aggregate pattern of diffusion for the product? We begin by defining \( \Lambda(i) \), the aggregate acquisition intensity for stage \( i \), as the sum of the individual acquisition intensities for all individuals in that stage-game. That is,

\[ \Lambda(i) = \frac{n}{n-1} \frac{\rho \pi(i)}{\pi(i+1) - \pi(i)}. \]

(3)

Now, the duration of any stage can be thought of as the waiting time up to the earliest acquisition at that stage. From the standard properties of the exponential distribution, it follows that the stage duration is distributed exponentially with parameter \( \Lambda(i) \). We have

**Proposition 2.** The duration of the \( i \)-th stage-game, beginning at time \( t_b \), is distributed exponentially with parameter \( \Lambda(i) \). Further, the mean duration of the stage equals \( [\Lambda(i)]^{-1} \).

\(^1\) Note that \( \beta^*(t) \) is strictly less than 1 for all \( t \). Hence, there is a non-zero probability that the game does not end in any finite period, which means that the equilibrium path reaches every information set with a positive probability. That establishes subgame-perfectness.
Lastly, observe how $\Lambda(i)$ depends on the stage index. In expression (3) the first part $n/(n-1)$, which is identical to $(N-i)/(N-i-1)$, clearly increases with $i$. Given that prices have been assumed to decline geometrically, the second part too can be shown to increase in $i$. This is formalised as Proposition 3.

**Proposition 3.** The equilibrium aggregate acquisition intensity, $\Lambda(i)$, increases with $i$.

Propositions 2 and 3 have very marked implications for the likely pattern of product diffusion. As the product diffuses, say, the system moves from stage $i$ to stage $i+1$, the individuals that remain switch their acquisition intensity from $\alpha(i)$ to $\alpha(i+1)$, and the aggregate intensity changes from $\Lambda(i)$ to $\Lambda(i+1)$. It follows from Proposition 3 that the aggregate intensity must rise. But then, by Proposition 2, the expected duration of the stage tends to decrease. We have the following recursive structure,

$$E(t_{i+1}) = t_i + [\Lambda(i)]^{-1}, \quad (4)$$

If we plot the number of acquisitions against elapsed time, the smooth-line curve joining these points would describe an *expected diffusion curve*. As the product diffuses, successive acquisitions follow each other more closely - the curve rises gently at first and then more sharply. Ultimately, when all individuals have acquired the product, the process ends. Alternatively, if the process reaches a stage where there are no further gains from waiting (that is, Condition W does not hold), the mechanism described here has little to say; we will need to supplement the analysis with other mechanisms. Figure 1 provides a sketch of an expected diffusion curve of the type that is generated here, which is not inconsistent with the empirically observed S-shape, especially in its early stages.

**4. Discussion**

This note describes a simple model of product diffusion in which the anticipation of falling prices induces potential buyers to postpone their acquisition. Equivalently, one could model the situation as one where the price is fixed but the quality of the product is expected to improve gradually; either of these is especially applicable to the early stages of the product cycle. If the benefit of lower price (or quality improvements) accrues to those who buy after others, there exists a late-mover advantage and each individual might be expected to behave strategically. Modelling the strategic interaction as a sequence of Wars of Attrition, the expected duration of each stage is determined endogenously. If the price is expected to fall geometrically, as in the model presented here, we have one possible explanation for the rising part of the S-shaped diffusion curve. This shape may also be supported by other forms of late-mover advantage, some of which are developed in Kapur (1992).

REFERENCES