PRICING OF SWING OPTIONS IN A MEAN REVERTING MODEL WITH JUMPS

MATS KJAER
Göteborg University

Abstract. We investigate the pricing of swing options in a model where the logarithm of the spot price is the sum of a deterministic seasonal trend and an Ornstein-Uhlenbeck process driven by a jump diffusion.

First we calibrate the model to Nord Pool electricity market data. Second, the existence of an optimal exercise strategy is proved, and we present a numerical algorithm for computation of the swing option prices. It involves dynamic programming and the solution of multiple parabolic partial integro-differential equations by finite differences.

Numerical results show that adding jumps to a diffusion may result in 2-35% higher swing option prices, depending on the moneyness and timing flexibility of the option.

1. Introduction

One type of derivative that is common on the electric power and natural gas markets, is the swing option. It allows flexibility in delivery with respect to both the timing and amount of energy delivered. For many years, it was available in the over the counter (OTC) markets, before its complexity was fully understood. This paper is about the pricing of swing options, with examples taken from the Nord Pool electricity market. However, the proposed pricing framework is applicable on other commodity markets as well.

To the best of our knowledge, the first paper on this topic is the 1995 paper by Thompson [19]. He proposes a lattice based algorithm to price take-or-pay contracts, which is a simple type of swing option. Generalising this approach, Jaillet, Ronn and Tompaidis [12] propose a multi-level lattice method to price swing options on natural gas. They use a trinomial tree discretisation of a continuous time model, where the logarithm of the spot price is a one-factor mean reverting process driven by a Wiener process. This is one of the models used by Lucia and Schwartz [14] to price electricity forwards. Recently, comparably efficient Monte Carlo methods for American options have been developed. These are applied by Ibáñ [11] in the context of swing options, also assuming the model by Lucia and Schwartz [14]. A different approach is taken by Dahlgren [5]. He works in a general one-factor diffusion setting, and shows that under some technical conditions on the drift and

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volatility, the pricing problem can be transformed into solving a set of variational inequalities.

This paper aims at extending the papers cited above in three directions. First, we will allow discontinuous spot price trajectories. Second, the amount of electricity to be delivered is chosen from a closed interval, rather than from a discrete set. Third, at each exercise date, the swing option holder has to fix a vector of amounts for multiple deliveries rather than a scalar amount for a single delivery.

More specifically, we use a model by Deng [7], where the logarithm of the spot price is the sum of a seasonality term and an Ornstein-Uhlenbeck process driven by a jump diffusion. To start with, we note that in the chosen spot price model, the price of a simple European derivative is given by the unique solution to a parabolic partial integro-differential equation (PIDE). Next, we prove the existence of an optimal exercise strategy, and as in the papers cited above, the proof is based on dynamic programming. By combining these two results, we obtain a numerical method for the pricing of swing options. To solve the resulting PIDEs numerically, we modify an operator splitting finite difference method proposed in Cont and Tankov [4].

For model estimation, we use historical Nord Pool spot price data and employ a combination of the least squares method presented in Lucia and Schwartz [14], and the Fourier transform based maximum likelihood approach by Singleton [18]. Traded forwards are used to estimate the market price of risk.

This paper is organised as follows: First we introduce swing options in Section 2. This is followed by a brief discussion of the mechanisms of Nord Pool in Section 3, followed by some general mathematical assumptions and notation in Section 4. The spot price model is introduced in Section 5, where derivatives pricing is also discussed. Section 6 deals with model estimation and section 7 is about the pricing of swing options. Here we give a proof of the existence of an optimal exercise strategy. A numerical algorithm for the computation of the price is proposed in Section 8. Swing option pricing results together with estimated model parameters are presented in Section 9. Finally, Section 10 concludes the paper and gives suggestions for future research.

2. PAYOFF OF A SWING OPTIONS

Before defining the payoff of a swing option formally, we give an example of a swing option payoff.

**Example 1.** The contract runs for one year. Every Friday, the holder decides on which days the following week he wants to buy 1 MWh of electricity at 30 EUR/MWh. At least 50 MWh and at most 100 MWh have to be bought in total during the year. The contract is financially settled with net payments every day.

Example 1 is generalised in Definition 2.1 below.

**Definition 2.1.** Swing option. A swing option of class $\mathcal{P}$ is a financial contract with the following payoff characteristics.

1. **Maturity date:** The contract runs over the period $[0, T]$.
2. **Strike:** The fixed price $K$ EUR/MWh.
3. **Swing action times:** The times when the holder is allowed to make decisions are denoted by $\{T_n\}_{n=1}^N$, where $0 \leq T_1 < T_2 < \ldots < T_N < T$. 
(4) **Swing action:** At each swing action date $T_n$, the holder decides on the amount of energy $B_n^d$ MWh to be bought at the fixed price $K$ EUR/MWh over each of the $D$ periods $[T_n^d, T_{n+1}^d]$, $1 \leq d \leq D$. Here $T_n = T_1^d < T_2^d < \ldots < T_n^D = T_{n+1}$, and $T_{D+1}^N = T$.

(5) **Allowed amounts per period:** We assume that $B_n^d \in \mathcal{O} \subseteq [0, \infty)$, where $\mathcal{O}$ is either a closed interval $\mathcal{O} = [B, \overline{B}]$ or a discrete set. This means that the holder of the swing contract is not allowed to short energy.

(6) **Allowed amount in total:** The holder may buy at least $\underline{M}$ MWh and at most $\overline{M}$ MWh in total. To be of interest, $\overline{NDB} < \underline{M} \leq \overline{M} < ND\overline{B}$.

(7) **Settlement:** All swing contracts are financially settled. To reduce the consequences of a default of the counterpart, net payments occur at times $T_n^d$, $1 \leq d \leq D$.

In Example 1, $D = 7$, $T_n^d$ is the beginning of day $d$, and $\mathcal{O} = \{0, 1\}$. Dahlgren [5], Jaillet, Ronn and Tompaidis [12] and Ibanz [11] all focus on the case when $D = 1$. By setting $\mathcal{O} = \{0, 1\}$, $\underline{M} = 0$, $\overline{M} = 1$ and $D = 1$, we see that Bermudan call options belong to class $\mathcal{P}$.

There are endless ways of generalising the swing option class $\mathcal{P}$. For example, one could allow under- or overdrafts at a penalty fee. Making the strike price $K$ depend on calendar time would also be natural and feasible. Finally, one could have contracts where the holder is allowed either to sell or to buy energy, thereby creating a virtual electric energy storage facility.

3. **Nord Pool**

The spot market on Nord Pool is an auction based day ahead market, where suppliers and consumers from the entire Nordic region place bids for each individual hour during the next day. Calls on eight hour blocks are also allowed. By looking at where supply meets demand, a so-called system price is calculated for the entire region. This is not a trivial process since the introduction of block bidding.

The average system price over 24 hours is called the base load price. Most derivatives are written on the base load price, from now on referred to as the spot price. In particular, forwards for delivery of 1 MWh at a constant load during one day, one week, one month and longer periods are available. These start trading six days, seven weeks and five months prior to the first delivery date, and are financially settled with daily net payments during delivery, making them swaps rather than forwards. Trading is mainly concentrated to those contracts with longer delivery periods. In addition, put and call options on these forwards are offered, but their liquidity is in general very low. Consequently, potential model calibration procedures do not have access to reliable forward curves or implied volatility surfaces. More information about Nord Pool is available at [http://www.nordpool.com](http://www.nordpool.com).

4. **General assumptions and notation**

Before introducing the spot price model, we make some general assumptions of a mathematical nature and fix some notation.

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space equipped with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions as defined in Protter [17]. We refer to $P$ as the historical or real world probability measure, and expectations with respect to this measure
are denoted $\mathbb{E}_P$. To begin with, the spot price of electricity $S_t$, $t \geq 0$, is assumed to be a positive semi-martingale belonging to this filtered probability space, but its dynamics will be specified further in Section 5. In addition, it is assumed that the no-arbitrage conditions for the fundamental theorem of asset pricing (see Delbaen and Schachermayer [6] for details) are fulfilled. Hence there exists at least one risk neutral probability measure $Q$. Risk neutral expectations are simply denoted $\mathbb{E}$.

We also introduce the riskless bank account $B_t$ with dynamics

$$B_t = B_0 e^{rt},$$

for some fixed $r > 0$.

Having fixed a risk neutral measure $Q$, general derivatives pricing theory (see for example Bingham and Kiesel [3]) shows that the arbitrage free price $V(t)$ of a derivative paying $Y \in L^1(\Omega, \mathcal{F}_T, Q)$, $Y$ bounded from below by a constant, at time $T > t$, is given by

$$V(t) = e^{-r(T-t)} \mathbb{E}[Y|\mathcal{F}_t]. \quad (4.1)$$

Forward prices $F(t, T)$ at time $t$ for delivery at time $T \geq t$ are defined by

$$F(t, T) = \mathbb{E}[S_T|\mathcal{F}_t],$$

and forward contract prices $G(t, T)$ by

$$G(t, T) = e^{-r(T-t)}(F(t, T) - K),$$

where $K$ is the strike price. In the case when $S_t$ is a Markov process under $Q$, we will sometimes write $G(t, T, s)$ and $F(t, T, s)$ to emphasise the dependence on the current spot price $s = S_t$.

5. Spot price model and derivatives pricing

Electric power differs from most other commodities in that at this moment of writing, it is not possible to store large amounts of electric energy in a feasible manner. This means that financial derivatives written on spot electricity cannot be hedged by non-producers. Inelastic demand and absence of stocks to smooth supply shocks result in a price dynamics characterised by seasonality, mean reversion and sudden spikes as seen in Figure 1 below. The model introduced in this section tries to catch some of these features, while still being possible to calibrate in the absence of a liquid vanilla options market.
As a model for $S_t$, we will use a slightly modified version of the one presented by Deng [7], from now on called the Deng-model. Here the spot price follows the $P$-dynamics

$$\begin{align*}
S_t &= \exp(f(t) + X_t) \\
\frac{dX_t}{X_t} &= -\alpha dt + dL_t,
\end{align*}$$

(5.1)

where $\alpha > 0$ is fixed, $f(t)$ is a deterministic seasonal trend, and $L_t$ is a compensated jump diffusion belonging to the filtered probability space defined in Section 4. More specifically, let $\{W_t\}_{t \geq 0}$ be a $P$-Wiener process, $\sigma > 0$ be fixed, and $\{U^J_t\}_{t \geq 0}$ a compound Poisson process independent of $\{W_t\}_{t \geq 0}$, with jump size density $f_J$ under $P$. Then

$$L_t = \sigma W_t + U^J_t - \lambda_J \mathbb{E}_P[J]t.$$

In addition, we assume that $f_J$ satisfies

$$\int_{\mathbb{R}} e^{2y} f_J(y) dy < \infty,$$

(5.2)

which according to Cont and Tankov [4] is sufficient for $S_t$ to have finite second moments under $P$ for all $t \geq 0$.

Setting $\lambda_J = 0$ in (5.1) retrieves the model by Lucia and Schwartz [14], from now on referred to as the LS-model. It will be used as a reference model to study the impact on swing option prices from the introduction of jumps.
Here all parameters are positive and \( \lambda \) is generally not known explicitly, but the transform analysis by Duffie, Filipovic and Schachermayer [8] shows that the conditional characteristic function under \( P \) of \( X_{t+\tau}, \tau > 0 \), given \( X_t = x \) is

\[
\varphi_{X_{t+\tau}}(u) = \exp \left( i u x e^{-\alpha \tau} - \frac{\sigma^2 u^2}{4 \alpha} (1 - e^{-2 \alpha \tau}) \right) \times \exp \left( \lambda J \int_0^\tau \mathbb{E}_P \left[ \exp(iuJe^{-\alpha s}) - (1 + iuJe^{-\alpha s}) \right] ds \right), \quad u \in \mathbb{R}.
\]

In most cases the integral inside (5.4) cannot be evaluated in closed form, but has to be computed numerically. One exception is the two-sided exponential jump size distribution, with density

\[
f_J(x|\lambda_1, \mu_1, \lambda_2, \mu_2) = \begin{cases} \frac{\lambda_1}{\lambda_1 + \lambda_2} e^{-x/\mu_1}, & x \geq 0 \\ \frac{\lambda_2}{\lambda_1 + \lambda_2} e^{x/\mu_2}, & x < 0. \end{cases}
\]

Here all parameters are positive and \( \lambda_J = \lambda_1 + \lambda_2 \). Moreover, (5.4) simplifies to

\[
\varphi_{X_{t+\tau}}(u) = \exp \left( i u \left\{ x e^{-\alpha \tau} - \frac{\lambda_1 \mu_1 - \lambda_2 \mu_2}{\alpha} (1 - e^{-\alpha \tau}) - \frac{\sigma^2 u^2}{4 \alpha} (1 - e^{-2 \alpha \tau}) \right\} \right) \times \left( \frac{1 - i u \mu_1 e^{-\alpha \tau}}{1 - i u \mu_2 e^{-\alpha \tau}} \right)^{\lambda_1/\lambda_2}, \quad u \in \mathbb{R}.
\]

Equation (5.6) could be interpreted as \( U_t^J \) being the difference between two compound Poisson processes with intensities \( \lambda_1 \) and \( \lambda_2 \), and exponentially distributed jump sizes with parameters \( \mu_1 \) and \( \mu_2 \) modelling up and down jumps. In this case, the moment condition (5.2) translates into \( \mu_1, \mu_2 < 1/2 \). This is the jump distribution that we are going to use in this paper.

Even in the LS-model, which is driven by one Wiener process only, the inability to store spot electricity means that the market is incomplete and \( Q \) is not unique. By the Girsanov Theorem (see Karatzas and Shreve [13] or Protter [17]), one family of risk neutral measures is characterised by a market price of spot price risk function of the form \( \lambda(t, S_t) \). For the sake of analytical tractability, Lucia and Schwartz [14] choose \( \lambda \in \mathbb{R} \) constant.

The introduction of jumps makes this class of measures much larger. According to Theorem 9.6 in Cont and Tankov [4], we may change both the jump intensity and jump distribution to any distribution absolutely continuous with \( f_J \). We follow the route taken in Merton [15] for equity derivatives, and do not price jump risk. This may be a dubious assumption for electricity, but in the absence of liquid vanilla options, we feel that pricing the jump risk separately would be difficult. Pricing diffusive risk as in the LS-model gives the \( Q \)-dynamics for \( S_t \) as

\[
\begin{align*}
S_t &= \exp(f(t) + X_t) \\
\frac{dX_t}{\sigma \lambda - \alpha X_t} &= dt + d\tilde{L}_t,
\end{align*}
\]

\[
(5.7)
\]
with \( \hat{L}_t = \sigma \hat{W}_t + U_t - \lambda J \mathbb{E}_P[J] t \), and \( \hat{W}_t = W_t - \lambda t \) being a \( Q \)-Wiener process. Another consequence of this choice of \( Q \) is that the jump intensity and distribution are invariant under this change of measure, and hence the condition for \( S_t \) having second moments under \( Q \) is still given by (5.2).

Forward prices are also given by the transform analysis in Duffie, Filipovic, and Schachermayer [8] as

\[
F(t, T) = \exp \left( f(T) + (\log S_t - f(t))e^{-\alpha(T-t)} \right) \times \exp \left( -\frac{\sigma \lambda}{\alpha} (1 - e^{-\alpha(T-t)}) + \frac{\sigma^2}{4\alpha} (1 - e^{-2\alpha(T-t)}) \right) \times \exp \left( \lambda J \int_0^{T-t} \mathbb{E}[\exp(\lambda t) - (1 + \alpha t)] ds \right)
\]

\[
= F_{\text{season}} \times F_{\text{diffusion}} \times F_{\text{jump}}.
\]

showing that forward prices are products of three factors originating from the seasonality trend, diffusion and jumps respectively.

In the double exponential model (5.5), (5.8) simplifies to

\[
F(t, T) = \exp \left( f(T) + (\log S_t - f(t))e^{-\alpha(T-t)} \right) \times \exp \left( -\frac{\sigma \lambda}{\alpha} (1 - e^{-\alpha(T-t)}) + \frac{\sigma^2}{4\alpha} (1 - e^{-2\alpha(T-t)}) \right) \times \left( \frac{1 - \mu_1 e^{-\alpha(T-t)}}{1 - \mu_1} \right) \times \left( \frac{1 + \mu_2 e^{-\alpha(T-t)}}{1 + \mu_2} \right) \times \exp \left( -\frac{\lambda_1 \mu_1 - \lambda_2 \mu_2}{\alpha} (1 - e^{-\alpha(T-t)}) \right).
\]

By (4.1) and the Markov property of \( X_t \), the price at time \( t \leq T \) of a simple European derivative with payoff \( Y = H(S_T) \) satisfies \( V(t) = V(t, x) \), with

\[
V(t, x) = e^{-r(T-t)} \mathbb{E}[H(\exp(f(T) + X_t^x))].
\]

We will conclude this section by giving a Feynman-Kac formula, which states that \( V(t, x) \) in (5.10) is the unique solution to a parabolic partial integro-differential equation (PIDE). The exact conditions on the payoff \( H \) under which this result holds as well as a proof are given in Appendix A.

Let \( u \) be a \( C^{1,2} \) function\(^1\) with a bounded first \( x \)-derivative. Then \( u \) is in the domain of the operators \( D_x \) and \( I_x \) defined as

\[
D_x u = \frac{\sigma^2}{2} \frac{\partial^2 u}{\partial x^2} + (-\sigma \lambda - \alpha x) \frac{\partial u}{\partial x},
\]

\[
I_x u = \int_{\mathbb{R}} \left\{ u(t, x + y) - u(t, x) - y \frac{\partial u}{\partial x} \right\} f_j(y) dy.
\]

Moreover, \( u \) is in the domain of the infinitesimal generator of \( X_t \), which is given by \( \mathcal{L}_x = D_x + \lambda J I_x \).

\(^1\)This means that \( u \) has one continuous \( t \)-derivative and two continuous \( x \)-derivatives.
Under some technical conditions on the payoff function $H$, the option price (5.10) is the unique solution to the PIDE
\[
\begin{align*}
\frac{\partial V}{\partial t} + \mathcal{L}_x V - rV &= 0 \\
V(T, x) &= H(e^{f(T)+x}).
\end{align*}
\] (5.13)

In the absence of jumps, $\mathcal{L}_x = D_x$, and the PIDE (5.13) reduces to a PDE.

6. Model estimation

In this section we discuss estimation of the model parameters for the Deng and LS models. We use the paradigm of Lucia and Schwartz [14], where the model is first estimated under $P$ using historical spot price data, followed by an estimation of the market price of risk $\lambda$ from traded forward contracts.

We postulate a periodic seasonal trend of the form
\[
f(t|\Theta) = A_0 + \sum_{n=1}^{N} A_n \cos(2\pi f_n t + B_n),
\] (6.1)
with the frequencies $f_n$ corresponding to cycles with lengths of one year, three months, one month, one week and three days, capturing the four seasons and the work week. The parameter vector $\Theta = (A_0, A_n, B_n)_{n=1}^{N}$ is unknown and is to be estimated from data.

The stationary covariance function of the logarithm of the spot price $Y_t = f(t|\Theta) + X_t$ is given by
\[
\text{Cov}(Y_{t+\tau}, Y_t) = e^{-\alpha \tau} \text{Var}(Y_t),
\]
so $\alpha$ is estimated from sample covariances and variances.

Having found $\alpha$, $\Theta$ is estimated by the non-linear least square method proposed by Lucia and Schwartz [14]. An explicit Euler discretisation of the SDE (5.1) with $y_t = \log S_t$ denoting the sampled process, and time step $\Delta t = \text{1 day}$, yields
\[
y_t = (1 - \alpha)y_{t-1} + f(t|\Theta) - (1 - \alpha)f(t-1|\Theta) + \epsilon_t, \quad t = 2, \ldots, T.
\]
Here $T$ is the sample size and $\{\epsilon_t\}_{t=2}^{T}$ a sequence of i.i.d. random variables with mean zero and finite variance. Finally, a least squares estimator can be obtained by minimising
\[
F(\Theta) = \frac{1}{T-1} \sum_{t=2}^{T} \left| y_t - \left[ f(t|\Theta) - (1 - \alpha)f(t-1|\Theta) + (1 - \alpha)y_{t-1} \right] \right|^2
\]
with respect to $\Theta$.

Next we subtract the estimated seasonal function $f(t|\Theta)$ from $y_t = \log S_t$, which leaves us with discrete observations $x_t$ of the process $X_t$. These are used to estimate the remaining parameters with the maximum likelihood method. Here we note that the stochastic variables $z_t \equiv x_{t+1} - x_t e^{-\alpha}$ are i.i.d. in both the LS and Deng models. From (5.8), we see that in the LS model, these variables are normally distributed with mean 0 and variance $\frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha})$. A standard maximum likelihood estimation is then used to find $\sigma$. 
In the Deng model, inverse Fourier transformation of the conditional characteristic function (5.4) yields
\[ f(x_{t+1}|x_t)(x) = \int_{\mathbb{R}} e^{iux_t}e^{-\alpha-\alpha_1x_1-\lambda_2\alpha_2x_2(1-e^{-\alpha})-\frac{\sigma^2}{4\alpha}(1-e^{-2\alpha})+\int_0^1 \varphi_J(we^{-\alpha}s)du} e^{-iu(x-x_t)e^{-\alpha}} \frac{du}{2\pi} \]
\[ = \int_{\mathbb{R}} e^{-iu\lambda_1}\varphi_Z(u) e^{-\alpha_2\lambda_2\alpha_2\alpha_2(1-e^{-\alpha})-\frac{\sigma^2}{4\alpha}(1-e^{-2\alpha})+\int_0^1 \varphi_J(we^{-\alpha}s)du} e^{-iu(x-x_t)e^{-\alpha}} \frac{du}{2\pi} \]
\[ \equiv \int_{\mathbb{R}} \varphi_Z(u) e^{-\alpha_2\lambda_2\alpha_2\alpha_2(1-e^{-\alpha})-\frac{\sigma^2}{4\alpha}(1-e^{-2\alpha})+\int_0^1 \varphi_J(we^{-\alpha}s)du} e^{-iu(x-x_t)e^{-\alpha}} \frac{du}{2\pi}, \]
where \( Z \) is a random variable with the same law as \( z_t \), and characteristic function \( \varphi_Z(u) \) under \( P \).

We may now form the log likelihood function as
\[ L = -\frac{1}{T} \sum_{t=1}^{T-1} \log \left( \int_{\mathbb{R}} \varphi_Z(u) e^{-\alpha_2\lambda_2\alpha_2\alpha_2(1-e^{-\alpha})-\frac{\sigma^2}{4\alpha}(1-e^{-2\alpha})+\int_0^1 \varphi_J(we^{-\alpha}s)du} e^{-iu(x-x_t)e^{-\alpha}} \frac{du}{2\pi} \right), \] (6.2)
with \( \alpha \) fixed. During the search for the optimal parameters, the Fourier integral (6.2) is approximated by a discrete Fourier transform, which is then computed by the FFT-algorithm. The resulting discretely sampled density is then linearly interpolated when evaluating the likelihood function. More details about approximation of continuous Fourier transforms by discrete ones, and the FFT algorithm can be found in Folland [9]. A similar method is suggested in Singleton [18], but there a Gauss-Hermite quadrature is used instead of FFT to compute the integrals in (6.2).

It remains to estimate the market price of risk \( \lambda \). Let \( \hat{F}(t,T_k) \), \( 1 \leq k \leq K \) be the market prices of the traded forward prices for delivery of 1 MWh during one day starting at time \( T_k \). Contracts with longer delivery times may be regarded as a portfolio of these one day delivery forwards. The market price of risk, estimated at time \( t \), is then chosen such that
\[ \sum_{k=1}^{K} |\hat{F}(t,T_k) - F(t,T_k)|^2 \] (6.3)
is minimised. Here \( F(t,T_k) \) are the model implied forward prices given by (5.8).

7. Pricing of swing options

In this section, we show how to price swing options of class \( \mathcal{P} \) introduced earlier in Section 2, and derive upper and lower bounds for these prices. The pricing methodology could easily be modified to cope with the other types of swing options discussed at the end of Section 2.

Without loss of generality, we assume that \( B = 0 \) for the remainder of this paper. Any swing option with \( B > 0 \) may be decomposed into a portfolio of \( B \) of each of the forward contracts \( G(T_n,T^d_n) \), \( 1 \leq n \leq N \), \( 1 \leq d \leq D \), plus a swing option equal to the original one, but with \( \mathcal{O} \) replaced by \([0,B] \), \( \overline{M} \) by \( \overline{M} - NDB \), and \( \underline{M} \) by \( \underline{M} - NDB \).

Initially, we determine the actual payoff induced by each swing action. A time \( T_n \) decision to buy \( B^d_n \) MWh during the period \( (T^d_n,T^{d+1}_n] \) at \( K \) EUR/MWh is equivalent to receiving \( B^d_n \) forward contracts \( G(T_n,T^d_n) \) which deliver 1 MWh each over this period at the fixed price \( K \) EUR/MWh. Consequently, a swing action is described
by a $D$-vector $\{B_1^n, \ldots, B_D^n\} \in \mathcal{O}^D$. Below we show how to derive the payoff in the case when $\mathcal{O}$ is a closed interval, but discrete state spaces are dealt with similarly.

By maximising the value of the forward contracts $G(T_n, T_n^d)$ received, it is possible to condense this vector into one number $\Delta_n = \sum_{d=1}^D B_d^n$, representing the amount of energy bought due to this swing action. The state space, to which $\Delta_n$ belongs, is denoted $\mathcal{S}$, and when $\mathcal{O} = [0, D]$, we have that $\mathcal{S} = [0, D]$. Since the forward contracts received are normally not available on the market, they must be priced theoretically by (5.9). More specifically, let $\tilde{G}_d(T_n, s)$, $1 \leq d \leq D$, denote the time $T_n$ prices of the $D$ forward contracts with delivery during $(T_n, T_{n+1}]$, sorted by their theoretical value. Here we employ the convention that $d = 1$ corresponds to the most expensive contract. Fixing $\Delta_n \in [(k-1)B, kB)$, where $1 \leq k \leq D$, and buying as much of the most expensive contracts as possible result in a maximised payoff $g$ of

$$g(T_n, s, \Delta_n) = \sum_{d=1}^{k-1} B \tilde{G}_d(T_n, s) + [\Delta_n - (k-1)B] \tilde{G}_k(T_n, s). \tag{7.1}$$

By this construction, $g(\cdot, \cdot, \Delta)$ is continuous, piecewise linear and concave, implying that it attains its maximum and minimum on $\mathcal{S}$. This claim holds trivially for finite state spaces. Finally, note that the ordering of the forward contracts according to their theoretical values may change with $s$, and that if $D = 1$, $g(T_n, s, \Delta_n) = \Delta_n(s - K)$.

Under these assumptions, a swing action means choosing $\Delta_n$ without violating the contract constraints, and thereby receiving the amount $g(T_n, s, \Delta_n)$. This is stated formally in Definition 7.1 below.

**Definition 7.1.** The set of admissible swing action strategies $\mathcal{A}$ consists of all sequences $\{\Delta_n\}_{n=1}^N$ such that

1. $\Delta_n \in \mathcal{S}$,
2. $\Delta_n$ is $\mathcal{F}_{T_n}$-measurable,
3. $\sum_{n=1}^N \Delta_n \in [\underline{M}, \overline{M}]$.

This definition reflects the fact that the decision must depend on known information only. To keep track of whether the constraint (3) above has been broken or not, we define $Z_t = \sum_{n=1}^t \Delta_n$ to be the amount of energy bought up to time $t \in (T_j, T_{j+1}]$. Since $S_t$ and $Z_t$ are both Markov processes, the swing option value $V(t)$ satisfies $V(t) = V(t, s, z)$, where $s = S_t$ and $z = Z_t$.

Before proceeding to the main pricing theorem, we give upper and lower bounds on the swing option price $V(t, s, z)$ in Proposition 7.2 below. Here $c(t, s, T_n, T_n^d)$ denotes the price at time $t \leq T_n$ of an option that pays

$$Y = e^{-r(T_n^d - T_n)} \max(F(T_n, T_n^d) - K, 0)$$

at time $T_n$. Due to the affine structure of the model, these option prices may be computed by evaluating Fourier transforms (see Duffie, Filipovic and Schachermayer [8] for details).

In analogy with the definition of $\tilde{G}_d(T_n, s)$, we introduce $\tilde{G}_k(T_n, s)$, where $1 \leq k \leq D(N - n)$, as all the forward contracts maturing in $[T_n, T_N]$, sorted by their theoretical value. Finally, $[x]$ is the integer part of $x \in \mathbb{R}$.
Proposition 7.2. Let $T_{j-1} \leq t < T_j$, $2 \leq j \leq N$. Then the swing option price $V(t, s, z)$ satisfies

\[
V(t, s, z) \leq \mathcal{B} \sum_{n=j}^{N} \sum_{d=1}^{D} c(t, s, T_n, T_n^d) .
\]

\[
V(t, s, z) \geq \mathcal{B} \sup_{M \in [M-z, M-z]} \left\{ \sum_{k=1}^{[M/B]} G_k(T_n, s) + (M/B - [M/B])G_{[M/B]+1}(T_n, s) \right\}
\]

Proof. Upper bound: Setting $\mathcal{M} = 0$ and $\mathcal{M} = N\mathcal{B} \mathcal{D}$ clearly increases the flexibility and hence the value of the option. In this case, the optimal strategy is trivial: At each swing action date $T_n$, pick $\mathcal{B}$ of the forward contracts maturing in $[T_n, T_{n+1})$ with positive value and nothing of the others. This strategy is clearly equivalent to a portfolio of call options.

Lower bound: This is the value of a strategy which involves fixing the entire remaining swing action strategy $\{\Delta_n\}_{n=j}^{N}$ at time $t$, which is clearly suboptimal. First we fix $M \in [M-z, M-z]$, which is the total number of MW to be bought in the remaining life of the swing option, $[T_j, T_N]$. Next, allocate these $M$ MW in a feasible manner among the remaining futures, such that the most expensive ones are chosen. Finally we take the supremum over $M$.

We now state and prove the main theorem of this section.

Theorem 7.3. Consider a swing option of class $\mathcal{P}$, and let $1 \leq j \leq N$. Then the value $V(t, s, z)$ of this swing option is given by

\[
V(t, s, z) = \begin{cases} 
\sup_{\{\Delta_n\}_{n=j}^{N} \in \mathcal{A}} \sum_{n=j}^{N} e^{-r(T_n-T_j)} \mathbb{E} \left[ g(T_n, S_{T_n}, \Delta_n) | \mathcal{F}_{T_j} \right], & t = T_j, \\
 e^{-r(T_j-t)} \mathbb{E} \left[ V(T_j, S_{T_j}, z) | \mathcal{F}_t \right], & T_{j-1} < t < T_j, \ j > 1, \\
 & t < T_1, \ j = 1.
\end{cases}
\]

Moreover, there exists at least one optimal swing action plan $\{\Delta_n^*\}_{n=j}^{N} \in \mathcal{A}$ such that the supremum is attained.

Proof. The claim is trivial in the case of a finite state space $\mathcal{O}$, since the number of swing action strategies is finite due to the Bermudan structure of the option. Below we assume that $\mathcal{O} = [0, \mathcal{B}]$, and prove the claim by backwards induction.

Step 1: The claim holds for $j = N$. At the last swing action date $t = T_N$, we choose the allowed swing action $\Delta_N$ that maximizes the payoff, or

\[
V(T_N, s, z) = \sup_{\Delta_N \in \mathcal{A}} g(T_N, s, \Delta_N).
\]  

(7.2) By the continuity of $g(\cdot, \cdot, \Delta)$, the supremum (7.2) is attained by some $\Delta_N^*$ on the compact set $\mathcal{S}$. Note that in general, $\Delta_N^* = \Delta_N^*(s, z)$. We also have that $V(T_N, s, z)$ is continuous in $z$. To prove this claim, observe that the definitions of $g$ and $G_k$ imply that

\[
|V(T_N, s, z_2) - V(T_N, s, z_1)| \leq |z_2 - z_1|G_1(T_N, s),
\]

so the continuity follows by letting $|z_2 - z_1| \to 0$. 


For other times \( t \in (T_{N-1}, T_N) \) in this period, we have a European derivative with payoff \( V(T_N, s, z) \) at time \( T_N \). Consequently,

\[
V(t, s, z) = e^{-r(T_N-t)} \mathbb{E}[V(T_N, S_{T_N}, z) | \mathcal{F}_t],
\]

by (4.1), proving the claim for \( n = N \).

**Step 2:** Induction assumption. Let \( j \) be arbitrary such that \( 1 < j < N \), and assume the following:

1. \( V(T_j, s, z) \) is given by
   \[
   V(T_j, s, z) = \sup_{\{\Delta_n\}_{n=j}^N} \sum_{n=j}^N e^{-r(T_n-T_j)} \mathbb{E}\left[g(T_n, S_{T_n}, \Delta_n) | \mathcal{F}_{T_j}\right].
   \]
2. \( V(T_j, s, z) \) is continuous in \( z \).
3. Given \( s \) and \( z \), there exists an optimal swing option strategy \( \{\Delta^*_n\}_{n=j}^N \) such that the supremum (7.4) is attained.

**Step 3:** The claim holds for \( j - 1 \), given the induction assumption in Step 2. When choosing \( \Delta_{j-1} \), we receive a payoff of \( g(T_{j-1}, s, \Delta_{j-1}) \) plus a new swing option with \( \Delta_{j-1} \) fewer exercise rights, and a first swing action date at time \( T_j \). Combining this with the first assumption of Step 2 yields

\[
V(T_{j-1}, s, z) \equiv \sup_{\Delta_{j-1} \in \mathcal{A}} \{g(T_{j-1}, s, \Delta_{j-1})
+ e^{-r(T_{j-1}-T_j)} \mathbb{E}[V(T_j, S_{T_j}, z + \Delta_{j-1}) | \mathcal{F}_{T_{j-1}}]\}.
\]

It remains to prove the continuity of \( V(T_{j-1}, s, z) \) and the existence of a \( \Delta^*_{j-1} \), such that the supremum (7.5) is attained. The continuity of \( g(T_{j-1}, s, z) \) implies that these two claims follow if

\[
\psi(z) \equiv \mathbb{E}[V(T_j, S_{T_j}, z) | \mathcal{F}_{T_{j-1}}]
\]

is continuous. However

\[
|\psi(z_2) - \psi(z_1)| \leq \mathbb{E}[|V(T_j, S_{T_j}, z_2) - V(T_j, S_{T_j}, z_1)| | \mathcal{F}_{T_{j-1}}],
\]

where the expression inside the expectation (7.7) goes to zero by the induction assumption in Step 2. The integrand may be bound by twice the upper bound given in Proposition 7.2 with \( z = 0 \), which is integrable. Hence the continuity of \( \psi \) follows by the Dominated Convergence Theorem.

For other times \( t \) in this period, we have a European derivative with payoff \( V(T_{j-1}, s, z) \) at time \( T_{j-1} \). The theorem now follows by induction.

Note that the optimal swing action strategy \( \{\Delta^*_n\}_{n=1}^N \) may not be unique, since the function \( g(\cdot, \cdot, \Delta) \) does not have to be monotone.

In order to use the recursive algorithm implicit in the proof of Theorem 7.3, we need to compute risk neutral conditional expectations of the type (7.3). A simple but time consuming approach by Ibánêz [11], is to use Monte Carlo simulation. However,
by Section 5, it can also be computed by solving the PIDE (5.13) with end condition 
\[ H(s) = V(T_n, s, z) \] for \((t, s) \in [T_{n-1}, T_n) \times [0, \infty)\) with \(z\) fixed.

8. Numerical pricing algorithm

In this section we combine the dynamic programming algorithm induced by the proof of Theorem 7.3 and a finite difference method into an algorithm used to compute swing option prices numerically.

In practice, the pricing problem can only be solved for discrete values of \((t, s, z)\). If the state space \(\mathcal{O}\) is finite, then the \(z\)-variable is already discrete. Otherwise, since we know that \(z \in [M, \overline{M}]\), this interval is sampled uniformly at \(N_z\) points \(z_m = m\Delta z\), where \(\Delta z = (\overline{M} - M)/(N_z - 1)\) and \(m = 0, \ldots, N_z - 1\).

As remarked in Section 5, we use \(x = X_t\) rather than \(s = S_t\) as state variable for the reason of numerical efficiency. We start by truncating \(x = X_t\) such that \(x \leq x \leq \overline{x}\), and this interval is sampled uniformly at \(N_x\) points \(x_l\) similarly to the \(z\)-variable. In the same fashion, each time interval \([T_{n-1}, T_n]\) between two swing action dates is sampled at \(N_t\) points \(t_k\). The spot price \(s_t\) at time \(t_k\) is then given by \(s_t = \exp(f(t_k) + x_l)\), so the \(x\)-grid is uniform, but the \(s\)-grid is not. Finally, the dynamic programming algorithm implicit in the proof of Theorem 7.3 is implemented as three nested for-loops over the indices \(n, l, m\), where checks are performed in each iteration to see whether a swing action is admissible or not.

To solve the PIDE (5.13) numerically, we use finite differences. When solving the PDE arising in the LS-model, we use a standard Crank-Nicholson scheme, which practical implementation is described in Wilmott [20]. Omitting the index \(m\), and writing \(v_k^l = \tilde{V}(t_k, x_l, z_m)\) and \(v_k = (v_k^1, \ldots, v_k^{N_x})^T\), the resulting linear system of equations may be written as

\[ A_1 v_k = A_2 v_{k+1}, \quad (8.1) \]

where the elements of the matrices \(A_1\) and \(A_2\) do not depend on time, thus enabling pre inversion of \(A_1\). Here we have assumed that the solution is linear in \(s\) at both boundaries, implying \(\frac{\partial^2 V}{\partial s^2} = 0\), or \(\frac{\partial^2 V}{\partial x^2} - \frac{\partial V}{\partial x} = 0\). Important properties of this method are unconditional stability and an error proportional to \((\Delta x)^2\) and \((\Delta t)^2\).

For the PIDE (5.13) an operator splitting method similar to the one proposed in Chapter 12:4 of Cont and Tankov [4] is used. The diffusion part

\[ \frac{\partial V}{\partial t} + \mathcal{D}_x V - rV \]

is discretised by the Crank-Nicholson method as described above for the LS-model, whereas the integral operator \(\mathcal{I}_x\) given in (5.12) is discretised using an explicit scheme. More specifically, if we define

\[ v_n = \int_{\Delta x(n-\frac{1}{2})}^{\Delta x(n+\frac{1}{2})} f_J(y)dy, \]
then $\mathcal{I}_x$ may be approximated as

$$\mathcal{I}_x V(t_{k+1}, x_l, z) = \lambda J \int_{\mathbb{R}} \left[ V(t_{k+1}, x_l + y, z) - V(t_{k+1}, x_l, z) - y \frac{\partial V}{\partial x} \right] f_j(y) \, dy$$

$$\approx \lambda J \sum_{n=-N}^{N} \left[ v_{k+1}^{l+n} - v_{k+1}^l - \frac{n}{2}(v_{k+1}^{l+1} - v_{k+1}^{l-1}) \right] \nu_n.$$  

The density $f_j$ is thus truncated at $\pm N \Delta x$, where we set $N$ such that $\sum_{n=-N}^{N} \nu_n$ is close to one. Due to the non-local nature of the integral operator, it may happen that $x_{k+1}^{l+n} \notin [x_l, x]$ for some $n$, implying that extrapolation is necessary outside this interval. To be consistent with the assumed boundary condition $\frac{\partial^2 V}{\partial x^2} - \frac{\partial V}{\partial x} = 0$, we set

$$v_{k+1}^{l+n} = \begin{cases} a e^{\xi_{l+n}} + b, & x_{l+n} > x, \\ c e^{\xi_{l+n}} + d, & x_{l+n} < x, \end{cases}$$  

where the unknown coefficients $a$, $b$, $c$ and $d$, are selected to get continuous first derivatives at $x$ and $x$ respectively. Below we let $v_{k+1}^e$ denote $v_{k+1}$ extrapolated by this procedure.

To summarise, the introduction of jumps transforms the linear system (8.1) into

$$A_1 v_k = A_2 v_{k+1} + B v_{k+1}^e,$$  

with $A_1$ and $A_2$ being the same matrices as in (8.1). The matrix $B$ is a dense matrix such that

$$B(l,:) v_{k+1}^e \approx \mathcal{I}_x V(t_{k+1}, x_l, z),$$

where $B(l,:)$ denotes the $l$'th row of $B$.

9. Results

In this section we present the estimated model parameters and prices of some sample swing options.

First the seasonal function and jump diffusion parameters are estimated using the procedure described in Section 6. The data is daily Nord Pool spot prices over the period 1 January 2002 to 30 September 2004. To estimate the market price of risk, the 15 March 2005 closing prices for forwards with delivery during weeks 10 to 15 are used. The estimated parameters are presented in Tables 1 to 2. Model implied forward curves for both models are displayed in Figure ?? together with the seasonal part of the forward curve, $F_{season}$, defined in (5.8).

<table>
<thead>
<tr>
<th>$f_n$</th>
<th>$f_0 = \infty$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_n$</td>
<td>3.3873</td>
<td>0.2930</td>
<td>0.0411</td>
<td>-0.0251</td>
<td>-0.0310</td>
<td>-0.0133</td>
</tr>
<tr>
<td>$B_n$</td>
<td>n.a.</td>
<td>0.4063</td>
<td>1.2807</td>
<td>0.9712</td>
<td>0.6386</td>
<td>1.7629</td>
</tr>
</tbody>
</table>

Table 1: The parameters $A_0$ and $(A_n, B_n)$, $1 \leq n \leq 5$, of the seasonal function $f(t|\Theta)$ defined in (6.1) estimated by the procedure of Section 6. The frequencies are $f_0 = \infty$, $f_1 = 1/365$, $f_2 = 4/365$, $f_3 = 12/365$, $f_4 = 52/365$ and $f_5 = 104/365$ cycles per day. The frequency $f_0$ corresponds to the constant $A_0$. 
Table 2: The parameters \( \alpha, \sigma, \lambda_1, \mu_1, \lambda_2, \mu_2 \) of the Deng and LS models together with the market price of spot price risk \( \lambda \) (MPR) estimated by the procedure of Section 6. The time is measured in days.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \alpha )</th>
<th>( \sigma )</th>
<th>( \lambda_1 )</th>
<th>( \mu_1 )</th>
<th>( \lambda_2 )</th>
<th>( \mu_2 )</th>
<th>( \lambda ) (MPR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lucia-Schwartz</td>
<td>0.0211</td>
<td>0.0711</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
<td>0.0095</td>
</tr>
<tr>
<td>Deng</td>
<td>0.0211</td>
<td>0.0370</td>
<td>0.1432</td>
<td>0.0897</td>
<td>0.2355</td>
<td>0.0556</td>
<td>0.0199</td>
</tr>
</tbody>
</table>

In Lucia and Schwartz [14], the model calibration is also made with daily Nord Pool data. Although the time period and seasonal function are different, they report \( \alpha = 0.0140 \) and \( \sigma = 0.086 \), which is similar to the values reported in Table 2.

Not very surprising, the introduction of jumps results in a smaller diffusive volatility \( \sigma \), implying that variance has been transferred from the diffusion to the jumps. On average, there are one up (Type 1) jump and two down (Type 2) jumps every week since \( 1/\lambda_1 \approx 7 \) and \( 1/\lambda_2 \approx 3.5 \). Moreover, the up jumps have a mean of \( \mu_1 = 0.0897 \), which is almost three standard deviations of the diffusive shock over one day, 0.0370. Consequently, jumps are large and frequent and \( \mu_1 \) and \( \mu_2 \) satisfy the moment condition (5.2).

Another effect of the introduction of jumps is that the market price of risk rises. This effect is due to the transfer of variance from the priced volatility, to the unpriced jumps. To maintain the risk premium, the market price of spot price risk per unit of volatility has to increase.

We now turn our attention to the pricing of swing options. Here we use a Matlab-implementation of the numerical method presented in Section 8. The aim is to investigate the impact on swing option prices in both models, for different strike prices \( K \), number of decision dates \( N \), and total allowed amount \( \overline{M} \). All swing options run for one year (\( T = 1 \)) and are priced at \( t = 0 \). The model parameter values all come from Tables 1 and 2, and the continuously compounded interest rate is \( r = 3\% \) per annum.

Table 3: Dependence on the number of swing action dates \( N \). Current spot price: \( s = 30 \) EUR/MWh. Option parameters: \( \overline{M} = 0 \), \( \overline{M} = 100 \), \( O = \{0,1\} \) and \( K = 30 \) EUR/MWh. Model parameters from Tables 1 and 2.

<table>
<thead>
<tr>
<th>( N )</th>
<th>Deng</th>
<th>LS</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>842</td>
<td>841</td>
<td>0.12%</td>
</tr>
<tr>
<td>4</td>
<td>887</td>
<td>884</td>
<td>0.34%</td>
</tr>
<tr>
<td>12</td>
<td>1024</td>
<td>1010</td>
<td>1.39%</td>
</tr>
<tr>
<td>52</td>
<td>1197</td>
<td>1167</td>
<td>2.57%</td>
</tr>
<tr>
<td>364</td>
<td>1264</td>
<td>1228</td>
<td>2.93%</td>
</tr>
</tbody>
</table>

Figure 2 and Tables 3 to 5 suggest that the introduction of jumps increases the price of the swing option by 2-6% when \( K = 30 \), and by up to 35% when \( K = 60 \). An explanation for this could be the that the fatter tails of the jump diffusion have a big effect on the price only when the swing option is out of the money. This effect seems to be larger when the number of exercise rights \( \overline{M} \) is small and the number of swing action dates \( N \) is large for a given \( B \). This probably depends on the resulting
Figure 2: Dependence on the current spot price for $K = 30$ EUR/MWh (left) and $K = 60$ EUR/MWh (right). Option parameters: $M = 0$, $\bar{M} = 100$, $N = 52$ and $\mathcal{O} = \{0, 1\}$. Model parameters from Tables 1 and 2.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$M$ & 10 & 50 & 100 & 200 & 364 \\
\hline
Deng & 197 & 774 & 1197 & 1518 & 1559 \\
LS & 185 & 746 & 1167 & 1494 & 1536 \\
Difference & 6.49\% & 3.75\% & 2.57\% & 1.61\% & 1.50\% \\
\hline
\end{tabular}
\caption{Dependence on $M$ for $M = 0$. Current spot price: $s = 30$ EUR/MWh. Option parameters: $N = 52$, $\mathcal{O} = \{0, 1\}$ and $K = 30$ EUR/MWh. Model parameters from Tables 1 and 2.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$M = \bar{M}$ & 10 & 50 & 100 & 200 & 364 \\
\hline
Deng & 193 & 748 & 1131 & 1281 & 176 \\
LS & 181 & 721 & 1104 & 1262 & 171 \\
Difference & 6.63\% & 3.74\% & 2.45\% & 1.51\% & 2.92\% \\
\hline
\end{tabular}
\caption{Dependence on $\bar{M}$ for $M = \bar{M}$. Current spot price: $s = 30$ EUR/MWh. Option parameters: $N = 52$, $\mathcal{O} = \{0, 1\}$ and $K = 30$ EUR/MWh. Model parameters from Tables 1 and 2.}
\end{table}

higher timing flexibility, which in combination with the mean reversion allows the holder “to pick the peaks”.

From Table 3, it is evident that swing options with more flexibility in terms of more swing action dates $N$, and a non-binding lower limit $\underline{M}$, are more expensive. Tables 4 and 5 also show that the option value increases with $\bar{M}$ if $M = 0$, but starts to decrease from $\bar{M}$ around 170 if $M = \bar{M}$. Having to buy at an expensive price clearly decreases the value of the option.
10. Conclusion and final remarks

First we introduced the Lucia-Schwartz and Deng models, and suggested a time-series approach to their calibration. Estimation results show that jumps are frequent and of significant magnitude. Moreover, model implied forward prices are dominated by the seasonality part, which suggests that the seasonal function must be modelled with great care.

Second, we proved the existence of an optimal swing action strategy and proposed an algorithm for its computation. The results show that the introduction of jumps could make swing options 2-7% more expensive if in the money. For out of the money options, the difference could be up to 35%. Apart from moneyness, the difference in price between the LS and Deng models seems to depend on the timing flexibility of the swing actions.

The proof of Theorem 7.3 is generic in the sense that it is not restricted by the market model used in this paper. Changing the market model basically means changing the way risk neutral conditional expectations are computed. With this observation in mind, an interesting extension of this paper would be the pricing of electricity swing options in more complex spot price models. This could mean adding spikes, like in Andreasen and Dahlgren [1], making volatility stochastic, as has been done for equities by Heston [10], or having a stochastic seasonality trend as suggested by Lucia and Schwartz [14]. The main obstacle for the use of these models on Nord Pool, is the current lack of liquidity on its forward and options markets, which makes calibration difficult. However, for swing options on electricity markets with more liquid derivatives, such an extension would be feasible.

When the dimensionality increases, PDE/PIDE methods become more time consuming. Instead, Monte Carlo simulation becomes the method of choice, even for American contracts, and this is the route taken by Ibáñez [11]. Hence, further development of Monte Carlo methods for American options would be another natural extension of this paper.

References


**Appendix A. Validity of the Feynman-Kac Formula**

The reasons why we cannot use the Feynman-Kac results from Bensoussan and Lions [2] or Cont and Tankov [4] directly are the unbounded coefficient in front of the first order derivative in $L_x$ in combination with the insufficient regularity of many financially important payoff functions $H$.

In order to solve this problem, we change state variable from $X_t$ to $Z_t = \log F(t,T)$, with $F(t,T)$ given in (5.8). An application of Itô’s Lemma for jump diffusions (see Protter [17]) gives the $Q$-dynamics for $Z_t$ as

$$dZ_t = \left( -\frac{\sigma^2}{2} e^{-2\alpha(T-t)} + \lambda J \mathbb{E} \left[ 1 - \exp \left( Je^{-\alpha(T-t)} \right) \right] \right) dt + \sigma e^{-\alpha(T-t)} dW_t + e^{-\alpha(T-t)} dU^J_t. \tag{A.1}$$

If we define

$$\sigma(t) = \sigma e^{-\alpha(T-t)},$$

$$\gamma(t) = -\frac{\sigma^2(t)}{2} - \lambda \int_\mathbb{R} \left[ \exp \left( ye^{-\alpha(T-t)} \right) - 1 - ye^{-\alpha(T-t)} \right] f_J(y)dy,$$

and let $v$ be a $C^{1,2}$ function with bounded first $z-$derivative, the infinitesimal generator $L_z$ of $Z_t$ is given by

$$L_z v = \frac{\sigma^2(t)}{2} \frac{\partial^2 v}{\partial z^2} + \gamma(t) \frac{\partial v}{\partial z} + \lambda J \int_\mathbb{R} \left[ v(t, z + ye^{-\alpha(T-t)}) - v(t, z) - ye^{-\alpha(T-t)} \frac{\partial v}{\partial z} \right] f_J(y)dy.$$
Note that the coefficients of $L_z$ only depend on $t$ and are bounded due to the moment condition (5.2).

Since $S_T = F(T, T)$, the option price $V(t, x)$ in (5.10) may be expressed in terms of $z$ as

$$u(t, z) = e^{-r(T-t)}E[H(\exp(Z_T^t^z))]$$

(A.2)

$$\equiv e^{-r(T-t)}E[h(Z_T^t^z)].$$

(A.3)

If $h$ is Lipschitz, Proposition 5.3 in Pham [16] yields that the PIDE

$$\left\{ \begin{array}{l}
\frac{\partial u}{\partial t} + L_z u - ru = 0, \quad (t, z) \in [0, T) \times \mathbb{R}, \\
u(T, z) = h(z), \quad z \in \mathbb{R},
\end{array} \right.$$ (A.4)

has a unique solution $u \in C^{1,2}([0, T) \times \mathbb{R}) \cap C^0(\mathbb{R})$ given by (A.3).

Next we show that the above result also holds for prices of futures contracts, which have unbounded payoffs given by $h(z) = e^z - K$. Inserting this payoff into (A.3) gives $u(t, z) = e^{-r(T-t)}(e^z - K)$. This function is $C^{1,2}$ and trivially solves (A.4). Furthermore, Theorem 3.3.1 in Lions and Bensoussan [2] ensures the uniqueness of this solution. By the linearity of $L_z$, (A.3) is the unique solution to (A.4) for payoffs $h$ which are linear combinations of futures contract prices and derivatives with Lipschitz payoffs. This includes call options by the put-call parity.

Finally we wish to switch back to the original state variable $x$. By (5.8) it is related to $z$ through the relation

$$z = f(T) + xe^{-\alpha(T-t)} - \frac{\sigma \lambda}{\alpha} (1 - e^{-\alpha(T-t)}) + \frac{\sigma^2}{4\alpha} (1 - e^{-2\alpha(T-t)})$$

$$+ \lambda_f \int_t^T \mathbb{E}[\exp(J e^{-\alpha(T-s)})]ds - (1 + Je^{-\alpha(T-s)})]ds$$

$$\equiv g(t, x).$$

Since $u \in C^{1,2}$, we can apply the chain rule to $V(t, x) = u(t, g(t, x))$, which retrieves the PIDE (5.13). It also follows that this PIDE also has a unique solution provided that $h$ is linear combination of futures contracts and derivatives with Lipschitz payoffs.

Department of Economics, Göteborg University, P.O Box 600, S-405 30 Göteborg, Sweden

Email address: mats.kjaer@economics.gu.se

\footnote{$C^0(\mathbb{R})$ is the space of continuous functions on $\mathbb{R}$.}