Valuing and Hedging LNG Contracts
A Derivative Pricing Approach
Robert Doubble – BP Oil International Commodities 2007
The LNG Market – 1
Industry Context

• The first commercial LNG trades took place in 1964 between Algeria and Europe, export of North American gas to Japan followed in 1969

• In the 1990’s the industry expanded with Australian, Arab Gulf (AG), African and North American production serving new markets, eg Korea

• Most of this gas was contracted on inflexible Long Term Agreements (LTAs) with pricing formulae indexed to oil

• By 2000 global liquefaction capacity had grown to 115 mTpa with imports totalling 110 mTpa

• This close match in production and imports was a direct result of project sanctions for LNG infrastructure only being granted once all future production had been sold on LTAs
Post 2000, with increased liquidity in the US and European markets the Atlantic Basin initiated more flexible contract structures. These would allow regional arbitrages and optimisation across a global LNG market. In 2005 imports were estimated to be 140 mTpa vs a production capacity of 145 mTpa. LNG is changing from a niche, high cost activity focussed on specific markets to a core feature of the global gas balance. Demand is expected to grow by ~30% pa over the next 10 years. LNG currently accounts for ~8% of gas demand.
The LNG Market – 3
The Outlook Today

• Today the global LNG market can be divided into two distinct regions, the Atlantic Basin (AB) and the Pacific Basin (PB)
• Demand is strongest in countries surrounding the PB with Japan being the largest individual market
• Growth is expected to be strongest in the regions that surround the AB
• LNG markets are still dominated by LTAs but this is changing as more flexibility is built into the markets
• The emergence of the US as a major market has seen contracts evolve to allow a greater degree of diversion flexibility across the AB, creating opportunities for regional gas-on-gas arbitrage and optimisation
• The industry is heavily influenced by the regulatory environment and competition to secure market access is fierce
• Successful players will be those willing to offer non-traditional contractual arrangements
BP’s LNG Trading Activities
History and Overview

• BP entered the LNG industry in 1997 through upstream positions in the AG and then Australia
• The mergers with Amoco and Arco led to positions in the Americas, Africa and Asia
• The Traded LNG Business Unit within BP is mandated to operate and trade around a ‘web’ of supply, end-markets and ships with the aim of creating incremental value
• BP currently optimises:
  - Equity purchase agreements
  - Merchant supply contracts
  - Shipping contracts
  - Delivery obligations to third-parties
  - Regasification capacity agreements
BP’s LNG Portfolio
Diversion Value and Trading Strategy

• BP has a significant portfolio of LNG purchase and sales agreements residing in its trading book
• BP’s shipping fleet allows LNG to be transported from source to any market where regasification facilities exist
• Diversion flexibility offers value since vessels can sail to the most profitable markets
• LNG contracts often recognise destination flexibility and embed pricing clauses to capture this value
• We will consider a ‘toy’ contract that embeds some typical price clauses that feature in many modern LNG agreements
• We will apply a derivative pricing approach to value the contract and identify the correct hedging strategy
• We will see that due to the contractual complexities the appropriate trading strategy can be non-trivial to determine
A ‘Toy’ LNG Purchase Agreement

Contract Outline

• We consider an LNG contract that allows the holder to buy cargoes at regular time intervals
• The base case is for the LNG to be resold in the US market
• The holder typically has the right to divert the vessel to the UK, continental Europe or Asia if it is profitable to do so
• Purchase and sales prices are a function of the cargo destination
• The vessel destination is nominated in M-1, where month M is the physical delivery month
• As a starting point we will:
  − Only consider sales in the US and UK markets
  − Assume that each exercise decision is not influenced by earlier decisions
• This allows us to decompose the contract into a strip of options
• For this presentation we consider only a single option in the strip
Vessel Sails to the US - 1
Contract Purchase Price

• In this case the purchase price is defined to be:

\[ P_{\text{base}} + \zeta \cdot D_{\text{us}} \quad [\$/\text{mmbtu}] \]

where:

\( P_{\text{base}} \) is a US reference (base) price
\( D_{\text{us}} \) is a US Destination Premium (DP)
\( \zeta \) is a scalar with a typical range of \( 0 \leq \zeta \leq 1 \)

• The \( P_{\text{base}} \) price is defined to be:

\[ P_{\text{base}} = f(\text{HH}) - R_{\text{base}} - S_{\text{Cbase}} \quad [\$/\text{mmbtu}] \]

where:

\( f(\text{HH}) \) is a Henry Hub linked price
\( R_{\text{base}} \) is a reference (base) re-gas cost
\( S_{\text{Cbase}} \) is a associated (base) reference shipping cost
Vessel Sails to the US – 2
Contract Purchase Price

• The $DP_{us}$ is defined to be:

$$DP_{us} = \text{Max}[ADP_{us} – P_{base}; 0]$$

where:

$$ADP_{us} = g(\text{index}) – RG_{\text{prem}} – SC_{\text{prem}}$$

and:

$g(\text{index})$ is a function of a US gas price Index (which need not be HH)

$RG_{\text{prem}}$ is a regas cost for delivery into a ‘premium’ US location

$SC_{\text{prem}}$ is the shipping cost for delivery into a ‘premium’ US location

• The costs $RG_{\text{base}}$ and $RG_{\text{prem}}$ are (typically) functions of a HH-linked price
If the vessel sails to the UK then the purchase price is:

\[ P_{\text{base}} + \xi \cdot DP_{\text{uk}} \]  

[$/\text{mmbtu}]

where:

\[ DP_{\text{uk}} = \text{Max}[ADP_{\text{uk}} - P_{\text{base}} ; 0] \]

\[ \xi \] is a scalar with a typical range of \( 0 \leq \xi \leq 1 \)

\[ ADP_{\text{uk}} = h(\text{NBP, X}) - RG_{\text{uk}} - SC_{\text{uk}} \]

and:

\[ h(\text{NBP, X}) \] is a price linked to the UK NBP gas and FX markets

\[ RG_{\text{uk}} \] is a UK regas cost

\[ SC_{\text{uk}} \] is a UK shipping cost
For ease of exposition I assume the following for the toy contract:

\[ P_{\text{base}} = \alpha(t).\text{Avg}_{\text{us}}[\text{HH}_i] + \beta(t) \]$/mmbtu$

\[ \text{ADP}_{\text{us}} = \gamma(t).\text{Avg}_{\text{us}}[\text{HH}_i] + \delta(t) \]$/mmbtu$

\[ \text{US sales price} = \phi(t).\text{Avg}_{\text{us}}[\text{HH}_i] + \eta(t) \]$/mmbtu$

\[ \text{ADP}_{\text{uk}} = \varphi(t).X(t).\text{Avg}_{\text{uk}}[\text{NBP}_i] + \kappa(t) \]$/mmbtu$

\[ \text{UK sales price} = \lambda(t).X(t).\text{Avg}_{\text{uk}}[\text{NBP}_i] + \mu(t) \]$/mmbtu$

The coefficients \( \alpha, \gamma, \phi, \varphi \) and \( \lambda \), and the \( \beta, \delta, \eta, \kappa \) and \( \mu \) are contract and market specific, they also typically display seasonal variation.

Here \( \text{Avg}_{\text{us}}[.] \) and \( \text{Avg}_{\text{uk}}[.] \) refer to price averages specific to the US and UK markets and are contract dependent.

The prices \( \text{Avg}_{\text{us}}[\text{HH}_i], \text{Avg}_{\text{uk}}[\text{NBP}_i] \) and \( X(t) \) will be defined rigorously later.
Decomposing the Problem - 1
Vessel Sails to the US

• LNG purchase price:
  \[ P_{\text{base}} + \zeta \cdot D_{\text{us}} \]
  \[ = \alpha \cdot \text{Avg}_{\text{us}}[H(t_i)] + \beta + \zeta \cdot \text{Max}[(\gamma - \alpha) \cdot \text{Avg}_{\text{us}}[H(t_i)] + \delta - \beta ; 0] \]

• LNG sales price:
  \[ \phi \cdot \text{Avg}_{\text{us}}[H(t_i)] + \eta \]

where:
- \( H(t_i) \) is the closing price at time \( t_i \) for the NYMEX HH contract corresponding to the physical delivery of gas in month \( M \)
- \( \text{Avg}_{\text{us}}[H(t_i)] \) is the average of the closing prices of the month \( M \) HH contract where the averaging window is contractually defined

• Here we have dropped the time arguments for the coefficients \( \alpha, \gamma, \phi, \varphi \) and \( \lambda \), and the \( \beta, \delta, \eta, \kappa \) and \( \mu \) to aid clarity
Decomposing the Problem - 2
Vessel Sails to the UK

• LNG purchase price:

\[ P_{\text{base}} + \xi \cdot DP_{\text{uk}} = \alpha \cdot \text{Avg}_\text{us}[H(t_i)] + \beta + \xi \cdot \text{Max}[\varphi \cdot X(t_s) \cdot \text{Avg}_{\text{uk}}[l(t_i)] - \alpha \cdot \text{Avg}_{\text{us}}[H(t_i)] + \kappa - \beta ; 0] \]

where:

- \( l(t_i) \) is the closing price at time \( t_i \) of the IPE gas contract for physical delivery in month M at the UK NBP
- \( \text{Avg}_{\text{uk}}[l(t_i)] \) is the average of the closing prices of the month M contract where the averaging window is contractually defined
- \( X(t_s) \) is the spot USD/GBP FX price at time \( t_s \)

• LNG sales price:

\[ \lambda \cdot X(t_{\text{uk}}) \cdot \text{Avg}_{\text{uk}}[l(t_i)] + \mu \]

• Here we are assuming that the GBP cashflows are converted to USD at times \( t_s \) and \( t_{\text{uk}} \)
Vessel Nomination Time Line
IPE, HH and FX Pricing Windows

where:

t_{exp} is the vessel destination nomination date

t_{1},...,t_{N} are the dates of the N closing prices featuring in the IPE window

t_{H} is the final date for the NYMEX price averaging window (featuring M fixes)

t_{s} is the cash settlement date with the LNG supplier

t_{uk} is the date on which BP receives payment for a UK bound cargo

t_{us} is the date on which BP receives payment for a US bound cargo

1 I use the term ‘fix’ or ‘fixing’ to indicate a date on which an IPE, HH or FX price pertinent to the cargo pricing is no-longer floating and instead is ‘known’ by the market participants
Trader Exercise Decision
US vs UK Cashflows

- At $t_{\text{exp}}$ in month M-1 the trader nominates the vessel destination
- In general neither $\text{Avg}_{\text{US}}[H(t_i)]$ nor $\text{Avg}_{\text{UK}}[I(t_i)]$ is known with certainty at $t_{\text{exp}}$
- The traders nomination decision is governed by the market prices at time $t_{\text{exp}}$ of two floating cashflows
- We introduce the notation:
  - $S_{\text{HH}}(t)$ is the price at time $t$ of a swap on the price average $\text{Avg}_{\text{US}}[H(t_i)]$
  - $S_{\text{NBP}}(t)$ is the price at time $t$ of a swap on the IPE price average $\text{Avg}_{\text{UK}}[I(t_i)]$
- The trader compares the market prices of the two net cashflows that would result if the vessel sailed to the US vs the UK
- In a derivative pricing framework the higher of the two values dictates the nomination decision

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2 More precisely $S_{\text{HH}}(t)$ and $S_{\text{NBP}}(t)$ are the prices at time $t$ of Balance-Of-The-Window swaps on the part of the respective US and UK averaging windows not ‘priced-out’ (fixed) at time $t_{\text{exp}}$

In addition I assume that the swaps $S_{\text{HH}}(t)$ and $S_{\text{NBP}}(t)$ cash settle at times $t_{\text{us}}$ and $t_{\text{uk}}$, respectively
• **US Net Cashflow:**

\[ V_{us}[\text{US sales price}] - V_0[\text{US purchase price}] - V_0.F_{us} \]

• This cashflow has a market price at time \( t_{exp} \) of:

\[ V_{us}.Z_{us}.[\phi.(M^{-1}\sum H(t_i)+\omega.S_{HH}(t_{exp})) + \eta] - V_0.Z_s.[\alpha.(M^{-1}\sum H(t_i)+\omega.S_{HH}(t_{exp})) + \beta + F_{us}] \]

\[ + \zeta.C_{us}(S_{HH}(t_{exp}), t_{exp}, t_s) \]

where:

- \( V_0 \) is the LNG volume purchased by BP
- \( V_{us} \) is the LNG volume delivered to the US terminal
- \( F_{us} \) is a freight cost in USD per mmbtu purchased
- \( Z_s \) scales time \( t_s \) money to time \( t_{exp} \) money
- \( Z_{us} \) scales time \( t_{us} \) money to time \( t_{exp} \) money
- \( C_{us}(.) \) is the value at time \( t_{exp} \) of a derivative instrument specific to US delivery
- \( \omega = (M-m).M^{-1} \) and \( m \) is the number of HH closing prices fixed at time \( t_{exp} \)
• Here \( C_{us}(S_{HH}(t_{exp}), t_{exp}, t_s) \) is the price at time \( t_{exp} \) of a derivative with a terminal pay-off at time \( t_s \) of:

\[
C_{us}({H(t_i)}, t_s, t_s) = \max([\gamma - \alpha].\text{Avg}_{us}[H(t_i)] + \delta - \beta; 0]
\]

• Note that the final form of the terminal pay-off function above is dependant on the values of \( \alpha \), \( \beta \), \( \gamma \) and \( \delta \).

• The derivative provides upside to the seller should the US premium price prove higher than the base price.

• If the vessel should sail to the US then the holder is short this derivative.
Market Price of UK Cashflow - 1

- **UK Net Cashflow:**
  
  \[ V_{uk} \cdot [\text{UK sales price}] - V_0 \cdot [\text{UK purchase price}] - V_0 \cdot F_{uk} \]

- **This cashflow has a market price at time** \( t_{exp} \) **of**
  
  \[ = Z_{uk} \cdot V_{uk} \cdot [\lambda \cdot X(t_{exp},t_{uk}) \cdot \{N^{-1} \Sigma I(t_i) + \theta \cdot S_{\text{NBP}}(t_{exp})\} + \mu] - \]

  \[ V_0 \cdot [Z_s \cdot [\alpha \cdot \{M^{-1} \Sigma H(t_i) + \omega \cdot S_{\text{HH}}(t_{exp})\} + \beta + F_{uk}] + \xi \cdot C_{uk}(X(t_{exp},t_s),S_{\text{NBP}}(t_{exp}),S_{\text{HH}}(t_{exp}),t_{exp},t_{s})] \]

where:

- \( V_{uk} \) is the volume of gas delivered to the UK
- \( F_{uk} \) is a freight cost in USD per mmbtu purchased
- \( X(t_{exp},t_{uk}) \) is a forward FX price at time \( t_{exp} \) for delivery at time \( t_{uk} \)
- \( X(t_{exp},t_s) \) is a forward FX price at time \( t_{exp} \) for delivery at time \( t_s \)
- \( Z_{uk} \) scales time \( t_{uk} \) money to time \( t_{exp} \) money
- \( C_{uk} \) is the value at time \( t_{exp} \) of a derivative instrument specific to UK delivery
- \( \Sigma I(t_i) \) is the sum of IPE m closing prices published at time \( t_{exp} \)
- \( \theta = (N-n) \cdot N^{-1} \) and \( n \) is the number of IPE closing prices fixed at time \( t_{exp} \)
Market Price of UK Cashflow – 2
Embedded UK Destination Derivative

• Here $C_{uk}(X(t_{exp}, t_s), S_{NBP}(t_{exp}), S_{HH}(t_{exp}), t_{exp}, t_s)$ is the value at $t_{exp}$ of a
  derivative with a terminal pay-off function at time $t_s$ of:

$$C_{uk}(X(t_s), \{I(t_i)\}, \{H(t_i)\}, t_s, t_s) = \text{Max}[\varphi . X(t_s) . \text{Avg}_{uk}[I(t_i)] - \alpha . \text{Avg}_{us}[H(t_i)] + \kappa - \beta ; 0]$$

• Again the precise functional form of the pay-off function is dependant on
  the values of $\varphi$, $\alpha$, $\kappa$ and $\beta$

• The purpose of this derivative is to provide upside to the seller should
  the NBP price prove higher then the base price

• If the vessel should sail to the UK then the holder is short this derivative
LNG Contract Value – 1
Terminal Pay-Off Function

- Mathematically the value of this contract \( V(t_{\text{exp}}) \) at \( t_{\text{exp}} \) is:

\[
V(t_{\text{exp}}) = \text{Max}\{\text{Market price US cashflow} \ ; \ \{\text{Market price UK cashflow}\}\}
\]

\[
V(t_{\text{exp}}) = \text{Max}\{M^{-1}\sum H(t_i) + \omega S_{HH}(t_{\text{exp}})\} \cdot [\phi V_{us} Z_{us} - \alpha V_0 Z_s] + \eta V_{us} Z_{us} - V_0 Z_s (\beta + F_{us})
\]

\[
- \zeta V_0 C_{us} (S_{HH}(t_{\text{exp}}), t_{\text{exp}}, t_s)
\]

\[
Z_{uk} V_{uk} [\lambda X(t_{\text{exp}}, t_{uk}) \cdot \{N^{-1}\sum I(t_i) + \theta S_{NBP}(t_{\text{exp}})\} + \mu]
\]

\[
- V_0 Z_s \{\alpha \cdot M^{-1}\sum H(t_i) + \omega S_{HH}(t_{\text{exp}})\} + \beta + F_{uk}
\]

\[
- \xi V_0 C_{uk} (X(t_{\text{exp}}, t_s), S_{NBP}(t_{\text{exp}}), S_{HH}(t_{\text{exp}}), t_{\text{exp}}, t_s)
\]

- We rewrite this expression as:

\[
V(t_{\text{exp}}) = V_1(t_{\text{exp}}) + V_2(t_{\text{exp}})
\]
LNG Contract Value – 2
Terminal Pay-Off Function

- Where:

\[ V_1(t_{exp}) = \text{Max}[Z_{uk}\cdot V_{uk}\cdot \{\lambda\cdot X(t_{exp}, t_{uk})\cdot \{N-1^\sum t_i\} + \theta\cdot S_{NBP}(t_{exp})\} + \mu}\]

- \phi\cdot V_{us}\cdot Z_{us}\cdot \{M-1^\sum H(t_i) + \omega\cdot S_{HH}(t_{exp})\} + V_0\cdot Z_s\cdot (F_{us}-F_{uk}) - \eta\cdot V_{us}\cdot Z_{us}\]

- \zeta\cdot V_0\cdot C_{uk}(X(t_{exp}, t_s), S_{NBP}(t_{exp}), S_{HH}(t_{exp}), t_{exp}, t_s)\]

\[ V_2(t_{exp}) = \{M-1^\sum H(t_i) + \omega\cdot S_{HH}(t_{exp})\}\cdot [\phi\cdot V_{us}\cdot Z_{us} - \alpha\cdot V_0\cdot Z_s] + \eta\cdot V_{us}\cdot Z_{us} - V_0\cdot Z_s\cdot [\beta + F_{us}] \]

- \zeta\cdot V_0\cdot C_{us}(S_{HH}(t_{exp}), t_{exp}, t_s)\]

- \(V_2(t)\) is the value at time \(t\) of the base case, ie US delivery
- \(V_1(t)\) is the value at time \(t\) of the option to divert the vessel to the UK
Pricing the Contract – 1
The Choice of Numeraire and Probability Measure

• Select the USD Money Market Account (MMA) $M^d(t)$ as the numeraire:

$$M^d(t) = e^{\int_0^t r^d(u) \, du}$$

where:

- $r^d(t)$ is the domestic (USD) overnight deposit rate at time $t$
- $\tilde{P}^d$ is the Risk Neutral (RN) measure for this numeraire

• In this toy example our model will be driven by a 3-dimensional Brownian Motion (BM) $\tilde{W}^d(t)$ where:

$$\tilde{W}^d(t) = \left(\tilde{W}_1^d(t), \tilde{W}_2^d(t), \tilde{W}_3^d(t)\right)$$

defined on a probability space $\left(\Omega, F, \tilde{P}^d \right)$

• The $\tilde{W}_i^d(t)$ are independent BMs
• For this presentation we will assume GBM price-processes
• Under the measure $\tilde{P}^d$ the price process for the HH futures contract is:

$$\frac{dH(t)}{H(t)} = \sigma_1(t).d\tilde{W}_t^d(t)$$

where:

- $H(t)$ is the price at time $t$ of the month $M$ delivery NYMEX HH contract
- $\sigma_1(t)$ is a time-dependent ‘local’ volatility for the month $M$ contract

• Note, to value a strip of these options in a single framework we would:
  - Explicitly model the term structure by using a volatility function of the type $\sigma(t,T)$
  - Employ a multifactor price process
Pricing the Contract – 3
The FX Price Process

- Under measure $\tilde{P}^d$ the FX spot price process $X(t)=X(t,t)$ is:

$$
\frac{dX(t)}{X(t)} = [r^d(t) - r^f(t)]dt + \sigma_3(t)\left[a_{31}(t)d\tilde{W}_1^d(t) + a_{32}(t)d\tilde{W}_2^d(t) + a_{33}(t)d\tilde{W}_3^d(t)\right]
$$

where:

- $X(t)$ is the USD spot price of Foreign Currency (FC) (GBP) at time $t$
- $r^f(t)$ is the foreign (GBP) overnight deposit rate at time $t$
- $a_{3i}(t)$ are elements of a time-dependent Cholesky matrix

- The price process for a contract delivering GBP at time $T$ is:

$$
\frac{dX(t,T)}{X(t,T)} = \sigma_3(t)\left[a_{31}(t)d\tilde{W}_1^d(t) + a_{32}(t)d\tilde{W}_2^d(t) + a_{33}(t)d\tilde{W}_3^d(t)\right]
$$

where $X(t,T)$ is the forward price at time $t$ and $t \leq T$
Under the foreign RN measure $\tilde{P}^f$ the price-process for the month $M$ delivery IPE futures contract is:

$$\frac{dl(t)}{l(t)} = \sigma_2(t) \left[ a_{21}(t) d\tilde{W}_1^f(t) + a_{22}(t) d\tilde{W}_2^f(t) \right]$$

where:

$\tilde{W}^f(t)$ is a 3-dimensional BM under the measure $\tilde{P}^f$

$\tilde{P}^f$ is the RN measure associated with the foreign MMA numeraire

However we need to price the contract using a single measure, that associated with the DC MMA numeraire $M^d(t)$

We need to specify the IPE price process under the domestic measure $\tilde{P}^d$
Girsanov’s Theorem in Multiple Dimensions

- To change measure from \( \tilde{P}^f \) to \( \tilde{P}^d \) we (again) employ Girsanov’s Theorem which we state below.

- Define:

\[
Z(t) = \exp \left[ -\int_0^t \theta(u) \, dW(u) - \frac{1}{2} \int_0^t \|	heta(u)\|^2 \, du \right]
\]

\[
\tilde{W}(t) = W(t) + \int_0^t \theta(u) \, du
\]

where \( \theta(t) \) and \( W(t) \) are \( d \)-dimensional processes and \( W(t) \) is a BM.

- Then (subject to certain conditions) under the probability measure \( \tilde{P} \) given by:

\[
\tilde{P}(A) = \int_A Z(\omega) d\tilde{P}(\omega) \quad \text{for all } A \in \mathcal{F}
\]

the process \( \tilde{W}(t) \) is a \( d \)-dimensional BM.
• For the IPE price process previous the numeraire is the foreign MMA $M_f(t)$

• We now select $M_d(t).Q(t)$ as the numeraire, where $Q(t)$ is the FX spot price in units of FC per DC, ie $Q(t)=1/X(t)$

• Under the foreign measure $\tilde{P}^f$ one can show that:

$$\frac{d(M^d(t).Q(t))}{M^d(t).Q(t)} = r^f(t).dt - \sigma_3(t).[a_{31}(t).d\tilde{W}^f_1 + a_{32}(t).d\tilde{W}^f_2 + a_{33}(t).d\tilde{W}^f_3]$$

• This process has volatility vector:

$$\begin{align*}
    \mathbf{u}(t) &= (-a_{31}(t).\sigma_3(t), -a_{32}(t).\sigma_3(t), -a_{33}(t).\sigma_3(t))
\end{align*}$$
Pricing the Contract – 7
The IPE Process Under the Domestic Measure

• By application of Girsanov’s theorem it follows that:

\[
\tilde{P}^d(A) = \frac{1}{D^f(0).M^d(0).Q(0) A} \int D^f(T).M^d(T).Q(T) d\tilde{P}^f(\omega)
\]

and

\[
\tilde{W}_i^d(t) = \int_{0}^{t} a_{3i}.\sigma_3(u).du + \tilde{W}^f_i(t) \quad i=1,2,3
\]

• Using these results the IPE price process under the measure \( \tilde{P}^d \) is:

\[
\frac{dl(t)}{l(t)} = -[a_{21}(t).a_{31}(t) + a_{22}(t).a_{32}(t)]\sigma_2(t).\sigma_3(t).dt + \sigma_2(t).[a_{21}(t).d\tilde{W}_1^d(t) + a_{22}(t).d\tilde{W}_2^d(t)]
\]

• In general the drift term is now non-zero under the domestic measure

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Here we define \( D'(t) = M'(t)^{-1} \)
Pricing the Contract – 8
Application of The RN Pricing Formula

- We can now solve the SDEs to obtain the processes $H(t)$, $I(t)$ and $X(t,T)$ under the measure $P^d$
- We use the RN pricing formula to value the option for $t \leq t_{exp}$:

$$
\frac{V(t)}{M^d(t)} = \mathbb{E}^d \left[ \frac{V(t_{exp})}{M^d(t_{exp})} \right] = \mathbb{E}^d \left[ \frac{V_1(t_{exp})}{M^d(t_{exp})} \right] + \mathbb{E}^d \left[ \frac{V_2(t_{exp})}{M^d(t_{exp})} \right]
$$

$$
\frac{V(t)}{M^d(t)} = \frac{V_1(t)}{M^d(t)} + \frac{V_2(t)}{M^d(t)}
$$

where:

$$
\frac{V_1(t)}{M^d(t)} = \mathbb{E}^d \left[ \frac{V_1(t_{exp})}{M^d(t_{exp})} \right] \quad \text{and} \quad \frac{V_2(t)}{M^d(t)} = \mathbb{E}^d \left[ \frac{V_2(t_{exp})}{M^d(t_{exp})} \right]
$$

- We will now evaluate $V_1(t)$ and $V_2(t)$ individually
• We have:

\[
\frac{V_2(t)}{M^d(t)} = \mathbb{E}_d \left[ \frac{V_2(t_{\text{exp}})}{M^d(t_{\text{exp}})} \mid F(t) \right]
\]

• Given the price processes featured in the previous slides it follows immediately that the US base case value \(V_2(t)\) is:

\[
V_2(t) = Z_{\text{exp}} \cdot [\phi \cdot V_{us} \cdot Z_{us} - \alpha \cdot V_0 \cdot Z_s \cdot \{M^{-1} \sum H(t) + \omega \cdot S_{HH}(t_{\text{exp}})\} + Z_{\text{exp}} \cdot \{\eta \cdot V_{us} \cdot Z_{us} - V_0 \cdot Z_s \cdot [\beta + F_{us}]\} - \xi \cdot V_0 \cdot C_{us}(S_{HH}(t), t, t)]
\]

since the swap price \(S_{HH}(t)\) and the discounted derivative \(C_{us}(t)\) are martingales under the domestic RN measure \(\tilde{\mathbb{P}}^d\).

• Where we have set \(Z_{\text{exp}} = M^d(t)/M^d(t_{\text{exp}})\)
Pricing the Contract – 10
The UK Exercise Case $V_1(t)$

- We now consider $V_1(t_{exp})$:

$$V_1(t_{exp}) = \text{Max}\{Z_{uk}.V_{uk}.\{\lambda.X(t_{exp},t_{uk}).\{N^{-1}\Sigma I(t_i) + \theta.S_{NBP}(t_{exp})\} + \mu\}$$

- $- \phi.V_{us}.Z_{us}.\{M^{-1}\Sigma H(t_i) + \omega.S_{HH}(t_{exp})\} + V_0.Z_s.(F_{us}-F_{uk}) - \eta.V_{us}.Z_{us}$

- $- \zeta.V_0.C_{uk}(X(t_{exp},t_s), S_{NBP}(t_{exp}), S_{HH}(t_{exp}), t_{exp}, t_s)$

- $+ \zeta.V_0.C_{us}(S_{HH}(t_{exp}), t_{exp}, t_s); 0\}$

- $V_1(t_{exp})$ is the terminal pay-off function of a basket option with:
  - Quanto features (since it contains the FX prices $X(t_{exp},t_{uk})$ and $X(t_{exp},t_s)$)
  - Asian features (since it contains the terms $\Sigma I(t_i)$ and $\Sigma H(t_i)$)
  - Compound features (since it has embedded the options $C_{uk}$ and $C_{us}$)

- To calculate $V_1(t)$ for $t < t_{exp}$ we can use a combination of numerical simulation and analytic approximations
Pricing the Contract – 11
Pricing The Embedded Option $C_{us}$

- Embedded in $V_1(t_{exp})$ is the derivative $C_{us}$ with terminal pay-off function:

$$C_{us}({H(t_i)}, t_s, t_s) = \text{Max}[(\gamma - \alpha).\text{Avg}_{us}[H(t_i)] + \delta - \beta ; 0]$$

- Let us consider the scenario:

  $\gamma - \alpha < 0$
  $\delta - \beta > 0$

- In which case:

  $$C_{us}({H(t_i)}, t_s, t_s) = |\gamma - \alpha|.\text{Max}[K_{us} - \text{Avg}_{us}[H(t_i)] ; 0]$$

  where:

  $$K_{us} = (\delta - \beta) / |\gamma - \alpha|$$

- We can determine the value of $C_{us}$ at time $t_{exp}$ using an Asian option pricing function
Pricing the Contract – 12
Pricing The Embedded Option $C_{uk}$

- Embedded in $V_1(t_{exp})$ is the derivative $C_{uk}$ with terminal pay-off function:

$$C_{uk}(X(t_s), \{I(t_i)\}, \{H(t_i)\}, t_s, t_s) = \max[\varphi . X(t_s).\text{Avg}_{uk}[I(t_i)] - \alpha . \text{Avg}_{us}[H(t_i)] + \kappa - \beta ; 0]$$

- Let us consider the scenario:

  $\varphi > 0$

  $\alpha > 0$

  $\kappa - \beta < 0$

- In which case:

$$C_{uk}(X(t_s), \{I(t_i)\}, \{H(t_i)\}, t_s, t_s) = \max[\varphi . X(t_s).\text{Avg}_{uk}[I(t_i)] - \alpha . \text{Avg}_{us}[H(t_i)] - |\kappa - \beta| ; 0]$$

- One approach to valuing $C_{uk}$ at time $t_{exp}$ is to treat it as a spread option with the composite underlyings $\varphi . X(t_s).\text{Avg}_{uk}[I(t_i)]$ and $\alpha . \text{Avg}_{us}[H(t_i)]$, and to use a moment matching approximation.
Pricing the Contract – 13
Procedure for Calculating $V_1(t)$

1. For each simulation trajectory:
   - Generate the prices $\{I(t_i)\}$, $\{H(t_i)\}$ with $t_i \leq t_{\text{exp}}$, $S_{HH}(t_{\text{exp}})$, $S_{NBP}(t_{\text{exp}})$, $X(t_{\text{exp}}, t_s)$, $X(t_{\text{exp}}, t_{uk})$
   - Calculate the price at time $t_{\text{exp}}$ of $C_{us}$ using an Asian option pricing function
   - Use moment matching and a spread option pricer to compute $C_{uk}$ at time $t_{\text{exp}}$
   - Evaluate the terminal pay-off $V_1(t_{\text{exp}})^i$ for the i-th trajectory

2. The price $V_1(t)$ is estimated by:

$$V_1(t) = \frac{Z_{\text{exp}}}{N} \sum_{i=1}^{N} V_1(t_{\text{exp}})^i$$
The trader may choose to monetise the contract optionality, i.e. ‘synthetically’ sell the option, by trading dynamically in the underlyings.

This delta-hedging strategy comprises two components:

- The hedge strategy for the US destination base case
- The hedge strategy for the UK diversion option

Note that the base case value $V_2(t)$ may be a function of a non-linear instrument $C_{us}$ and hence the strategy may be dynamic.

We calculate the hedge positions using finite difference approximations.

The trader would:

- Hold positions in HH and IPE gas futures contracts, together with two FX forwards of differing delivery dates
- Adjust the paper positions according to the model at a frequency of her choosing, this would be influenced by her market view and transaction costs
• Typically an LNG contract obliges the holder to buy a series of cargoes, the holder is therefore long a strip of options
• The liquidity of paper markets imposes constraints on the number of options in the strip that can be hedged concurrently
• The gas volumes involved, (a cargo may be ~30 million therms), requires the Trader to be judicious in her decision to rebalance a position
• Regulatory requirements may mean that not all cargoes display the same degree of flexibility to divert
• As a consequence this type of activity may be restricted to cargoes corresponding to certain delivery months
• All sunk terminal regas costs must be included in the valuation
• The pricing methodology described here is best applied to the most liquid delivery months where the RN valuation paradigm is most appropriate
• Liquidity issues, physical and operational risk will all impact deal value