VALUATION OF ELECTRICITY FORWARD CONTRACTS: THE ROLE OF DEMAND AND CAPACITY

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COMMODITIES 2007
Commodities Finance Center, Birkbeck
January 2007
COMMODITY FORWARD PRICING LITERATURE

   
   Gibson & Schwartz, 1990; Schwartz, 1997

   Problem: Electricity is NON-STORABLE

2. Long-Term & Short-Term Model (Schwartz & Smith, MS, 2000)
   
   Spot price = long-term state variable + short-term variable

   Application to electricity:
   
   Lucía & Schwartz (2000); Cartea & Figueroa (2005); Villaplana (2003) …

3. “Equilibrium Model”: Spot price linked to fundamental economic variables
   
GOAL

• DEVELOP A MODEL FOR THE VALUATION OF ELECTRICITY FUTURES (AND OTHER DERIVATIVES).
  – CAPTURES THE ROLE OF “DEMAND” AND “SUPPLY”
  – CLOSED FORM, allow to extract risk-neutral parameters from traded contracts.

• TO COMPLEMENT/EXTEND THEORETICAL RESULTS AND EMPIRICAL EVIDENCE ON THE BEHAVIOR OF RISK PREMIUM IN POWER MARKETS. Economic determinants of risk premium:

\[ RP_t \equiv F(t,T,S) - E_t^P(S_T) \]
MODEL

PRESENTS A FRAMEWORK FOR THE ANALYSIS OF THE EFFECT OF "SUPPLY" AND "DEMAND" CONDITIONS ON DERIVATIVE PRICES

\[
\text{ELECTRICITY SPOT PRICE}_t = \varphi (\text{DEMAND}_t, \text{SUPPLY}_t )
\]

⇒ \(\varphi (\bullet)???

⇒ “Supply”? Stochastic processes for demand and supply?
State Variables: Demand / Supply

- Supply: Available generation capacity
- Demand

Both are random variables that affect the behavior of spot prices and as a consequence risk premium

Some preliminary empirical evidence:
   PJM, UK, NordPool

**Spot Price**

**Ratio Load / Capacity**
NORDPOOL: RELATIONSHIP BETWEEN PRICE AND DEMAND
(weekly data 2000-2003)
NORDPOOL: Price – Demand – Supply (Hydro Reservoirs)
Hydro Reservoirs

Price
UK: Log Price and Log Generator Available Margin, 2006
ELECTRICITY SPOT PRICE$_t = \varphi (\text{DEMAND}_t, \text{GENERATION CAPACITY}_t )$

$\Rightarrow \varphi (\bullet)$

**CONTRAINTS:**

- **EMPIRICAL OBSERVATION**
- **VALUATION OF FINANCIAL ASSETS:**


  $\Rightarrow \text{Log-spot price linear (or quadratic) function of state variables}$

$\Rightarrow \text{HOW DO WE MODEL STATE VARIABLES??}$

AFFINE (JUMP-DIFFUSIONS) PROCESSES
\[ \Rightarrow \phi (\bullet) ??? \]


1-period model; \( N_P \) power producers;

- **Power production cost function:**
  \[ TC_i = F + \frac{a}{c} (Q_{P_i})^c \]

- **Profit:**
  \[ \Pi_{P_i} = P^W Q^W_{P_i} + P^F Q^F_{P_i} - F - \frac{a}{c} (Q_{P_i})^c \]

- **Clearing Condition Spot Market:**
  \[ N_P \cdot Q^W_{P_i} = Q^D \]

- **Equilibrium Price:**
  \[ P^W = a \left( \frac{Q^D}{N_P} \right)^{c-1} \]
Therefore for a given generic cost function

\[ TC_i = F + f(Q_{P_i}) \rightarrow \Pi_{P_i} = P^W \cdot Q^W_{P_i} - F - f(Q_{P_i}) \]

\[ \max_{Q^w_{P_i}} \Pi_{P_i} \Rightarrow P^W - f'(Q^w_{P_i}) = 0 \Rightarrow Q^W_{P_i} = f'(P^W)^{-1} \]

And the resulting clearing condition and equilibrium price are:

\[ N_P \cdot Q^W_{P_i} = Q^D \Rightarrow N_P \cdot f'(P^W)^{-1} = Q^D \]

\[ \Rightarrow P^W = f'\left(\frac{Q^D}{N_P}\right) \]
We take a **general** $f$ and we allow $N_p$ to be **stochastic**.

In the B&L framework given a generic cost function $f$ (non-observable) we obtain as a result the empirical observed (and estimated) function $\varphi(\bullet)$

An important point is we consider **generation capacity** as an stochastic variable.

The model may be seen as an “extension” of B&L, see also Ullrich (2006).

We present a new closed-form valuation formula (risk premium expression)
MAIN CHARACTERISTICS **DEMAND** PROCESS:

- **SEASONALITY** in the mean: $g(t)$
- **MEAN-REVERSION**
- **SEASONAL VOLATILITY**

\[
D_t = g(t) + X_t
\]
\[
dX = -k_x X dt + \sigma_{x}^{seas.} dZ_x
\]

\[
g(t) = B_0 + B_2 \cdot t + D_1 \cdot labora_t + C_1 \cdot \sin\left( (t + C_2) \cdot \frac{2\pi}{365} \right) + C_3 \cdot \sin\left( (t + C_4) \cdot \frac{4\pi}{365} \right)
\]
\[
\sigma_x^s = QS_1 + QS_2 \cdot spring_t + QS_3 \cdot fall_t + QS_4 \cdot summer_t
\]
| Parameter | **PJM (U.S.)** | | **NORDPOOL (Scandinavia)** | | | | | | **Constant Volatility** | **Seasonal Volatility** | **Constant Volatility** | **Seasonal Volatility** | | | | | | Model | Model | Model | Model | | | | | | Parameter | Coeff. | t-stat. | Coeff. | t-stat. | Coeff. | t-stat. | Coeff. | t-stat. | | | | | | | | | | | | **B0** | 26645,59 | 271,19 | 26904,66 | 123,11 | 9670,01 | 91,04 | 10101,38 | 103,45 | | | | | | | | | | | | **B1** | 0,598 | 0,02 | 0,544 | 25,76 | 0,802 | 47,99 | 0,790 | 42,95 | | | | | | | | | | | | **B2** | 2,59 | 0,35 | 2,05 | 7,00 | 2,97 | 21,68 | 2,26 | 17,69 | | | | | | | | | | | | **D1** | 4479,42 | 182,30 | 4374,31 | 27,35 | 4,68 | 0,07 | 11,17 | 0,17 | | | | | | | | | | | | **C1** | -4380,63 | 153,69 | 4063,42 | 157,76 | -3073,72 | -45,32 | -3011,19 | -52,52 | | | | | | | | | | | | **C2** | 65,70 | 2,82 | -3402,35 | -1636,51 | 257,64 | 214,54 | 258,91 | 225,81 | | | | | | | | | | | | **C3** | -4246,03 | 191,87 | -3974,79 | -22,81 | - | - | - | - | | | | | | | | | | | | **C4** | 113,02 | 1,13 | 660,71 | 719,38 | - | - | - | - | | | | | | | | | | | | **STDV** | 2404,42 | 182,30 | 797,43 | 48,65 | | | | | | | | | | | | **QS1** | 1939,42 | 30,36 | | 1023,62 | 21,21 | | | | | | | | | | | | **QS2** | -180,93 | -2,23 | | -259,82 | -4,53 | | | | | | | | | | | | **QS3** | 64,90 | 0,75 | | -174,81 | -2,92 | | | | | | | | | | | | **QS4** | 1708,53 | 9,05 | | -527,99 | -9,80 | | | | | | | | | | | | **LL** | -10879,26 | -10776,19 | | -8999,47 | -8944,78 | | | | | | | | | | | | **SC** | 21822,22 | 21637,32 | | 18062,07 | 17973,74 | | | | |
GENERATION CAPACITY VARIABLE:

- Mean- Reverting

- It could be seasonal (deterministic)

- We may allow the average level to change (deterministic or stochastic)

- We also allow the possibility of JUMPS: outages, imports constraints (transmission congestion).

\[ dc = k_c (\theta_c - c) dt + \sigma_c dZ_c + J_c (\eta_{J,c}) d\Pi(\lambda_c) \]
We present two models:

\( \varphi(\bullet) \) is the same in both models

Particular specification has been empirically compared against other possible specifications.

MODEL A: Long-term is constant

MODEL B: Stochastic Long-term (Supply and Demand) Variables
MODEL A (Under Objective Probability Measure)

\[ P_t = C_t^\gamma \cdot \beta \cdot e^{\alpha \cdot D_t} \]

\[ D_t = g(t) + X_t \]

\[ dX = -k_x X dt + \sigma_x^{seas.} dZ_x \]

\[ dc = k_c (\theta_c - c) dt + \sigma_c dZ_c + J_c (\eta_{J,c}) d\Pi(\lambda_c) \]

\[ dZ_x dZ_c = \rho dt \]

\[ \gamma < 0; \alpha > 0 \]
Model A Under Risk-Neutral Probability Measure

\[ dX = k_x \left( \theta_x^* - X \right) dt + \sigma_x \text{seas} dZ_x^* \]
\[ dc = k_c \left( \theta_c^* - c \right) dt + \sigma_c dZ_c^* + J_c^* \left( \eta_{J,c}^* \right) d\Pi(\lambda_c) \]
\[ dZ_x^* dZ_c^* = \rho dt \]

where
\[ \theta_x^* = -\frac{\phi_x \cdot \sigma_x \text{seas}}{k_x} \]
and
\[ \theta_c^* = \theta_c - \frac{\phi_c \cdot \sigma_c}{k_c} \]

**ASSUMPTION:** Constant Market Price of Risk.

Can be relaxed as long as we maintain the AFJD structure.
Pricing Formula

(Forward Contracts; Characteristic Function)

State Variables: **Affine Jump-Diffusion Processes.**

\[
F(x, c, t, T) = \exp \left\{ f(T) + \gamma \cdot c_t \cdot e^{-k_c (T-t)} + \alpha \cdot x_t \cdot e^{-k_x (T-t)} + A(T - t) \right\}
\]

where

\[
A(T - t) = \theta_c^* \gamma \left( 1 - e^{-k_c \tau} \right) - \frac{\phi_x \cdot \sigma_{x_{seas}}}{k_x} \cdot \alpha \left( 1 - e^{-k_x \tau} \right) + \frac{(\sigma_{x_{seas}})^2 \alpha^2}{4k_x} \left( 1 - e^{-2k_x \tau} \right)
\]

\[
+ \frac{\sigma_c^2 \gamma^2}{4k_c} \left( 1 - e^{-2k_c \tau} \right) + \frac{\rho \sigma_x \sigma_c \gamma \alpha }{k_x + k_c} \left( 1 - e^{-(k_x + k_c) \tau} \right) + \frac{\lambda_c \ln \left( \frac{\gamma \eta_J^* e^{-k_c \tau}}{\gamma \eta_J^* - 1} \right)}{k_c}
\]
FORWARD RISK PREMIUM

\[ RP_t \equiv \ln F(t, T, P) - \ln E_t^P (P_T) = \]

\[ = -\gamma \frac{\phi_c \sigma_c}{k_c} (1 - e^{-k_c \tau}) - \alpha \frac{\phi_x \sigma_x^{seas}}{k_x} (1 - e^{-k_x \tau}) - \frac{\lambda_c}{k_c} J^{RP} \]

Forward Risk Premium will be higher (periods/markets) :

• Higher Volatility Demand

• Higher “Convexity Demand” and “Convexity Supply”:

• “Mkt. Price Demand risk” and “Mkt. Price Supply Risk”

Previous evidence PJM: summer months (Pirrong & Jermakyan (2002), Villaplana (2003), Bessembinder & Lemmon (JF, 2004),...
MODEL B (Under the Objective Probability Measure)

\[ P_t = C_t^\gamma \cdot \beta \cdot e^{\alpha \cdot D_t} \]

DEMAND

\[ D_t = s(t) + \xi_t^D + \chi_t^D \]
\[ d\xi_t^D = \mu_D dt + \sigma_{\xi} dZ_{\xi} \]
\[ d\chi_t^D = -k_{\chi}^D \chi_t^D dt + \sigma_{\chi}^D dZ_{\chi,D} \]

SUPPLY

\[ C_t = \theta_t + \chi_t^C \]
\[ d\theta_t = \mu_\theta dt + \sigma_\theta dZ_\theta \]
\[ d\chi_t^C = -k_C^C \chi_t^C dt + \sigma_{\chi}^C dZ_{\chi,C} \]
MODEL UNDER THE RISK-NEUTRAL MEASURE

\[ P_t = C_t^{\gamma_c} \cdot \beta \cdot e^{\gamma_D \cdot D_t} \]
\[ \Rightarrow \ln P_t = [\ln \beta + \gamma_D \cdot g(t)] + \gamma_c \left( \theta^c_t + \chi^c_t \right) + \gamma_D \cdot (\xi^D_t + \chi^D_t) \]

\[ D_t = g(t) + \xi^D_t + \chi^D_t \]
\[ d\xi^D_t = \mu^*_D dt + \sigma_{\xi,D} dZ^*_\xi,D \]
\[ d\chi^D_t = -\left( k_D \chi^D + \phi_D \right) dt + \sigma_{\chi,D} dZ^*_\chi,D \]

\[ \ln C_t = c_t = \theta^c_t + \chi^c_t \]
\[ d\theta = \mu^*_\theta dt + \sigma_{\theta} dZ^*_\theta \]
\[ d\chi^c_t = -\left( k_c \chi^c + \phi_c \right) dt + \sigma_{\chi,c} dZ^*_\chi,c \]
\[ \ln F(t,T,X) = f(T) + A(\tau) + \gamma_c \cdot \theta + \gamma_c \cdot c_t \cdot e^{-k_c \cdot \tau} \]
\[ + \gamma_D \cdot \xi_t^D + \gamma_D \cdot \chi_t^D \cdot e^{-k_D \cdot \tau} \]

\[ A(\tau) = \mu_\theta \cdot \gamma_c \cdot \tau + \frac{\phi_c \gamma_c}{k_c} \left(1 - e^{-k_c \cdot \tau}\right) + \mu_\theta \cdot \gamma_D \cdot \tau + \frac{\phi_D \cdot \gamma_D}{k_D} \left(1 - e^{-k_D \cdot \tau}\right) \]
\[ + \frac{\sigma^2 \cdot \gamma_c^2}{2} \tau + \frac{\sigma^2 \cdot \gamma_c^2}{4k_c} \left(1 - e^{-2k_c \cdot \tau}\right) + \frac{\sigma^2 \cdot \gamma_D^2}{2} \tau + \frac{\sigma^2 \cdot \gamma_D^2}{4k_D} \left(1 - e^{-2k_D \cdot \tau}\right) \]
EMPIRICAL ANALYSIS: PJM market

We estimate MODEL A.

Daily Data:
Spot Prices, Demand, Available Capacity & 1-Month Forward Prices.

Period: 1999 – 2002

Step 1: Estimate the parameters under the empirical probability measure

Step 2: Extract risk-neutral parameters from Forward prices
(we allow market price of demand risk to be seasonal)
Model vs. Observed Forward Price, 2000 (in logs)

FITFUT
LPJMF1MON
EMPIRICAL RESULTS:

The model captures the observed pattern of 1-month forward prices. When we extract the risk premium component, we find it to be significant and being determined by economic risks. We find that the seasonal risk premium is clearly related to the economic determinants.

Risk premium seasonality is not only generated by demand volatility.

The result obtained by empirically estimating the proposed model corroborates the results by Bessembinder & Lemmon (JF, 2002) and those of Longstaff & Wang (JF, 2004), see also Pirrong & Jermakyan.

Economic determinants of the forward risk premium.
CONCLUSIONS

• General Framework that takes into account DEMAND and SUPPLY (“Generation Capacity”) VARIABLES. Relationship among Spot prices, States Variables and Derivative Prices.

• New Pricing Formula in Closed Form

• OUR MODEL IS A FIRST STEP ON THE ANALYSIS OF THE EFFECT OF DEMAND AND SUPPLY CONDITIONS ON DERIVATIVES PRICES

• PROVIDES INTUITION ON FORWARD RISK PREMIUM BEHAVIOR: “cross-market” and “time-series”.