Stochastic Volatility and Seasonality in Commodity Futures and Options: The Case of Soybeans

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Contributions and results

• Model and estimation of the seasonal patterns in commodity prices and volatilities using both time-series characteristics as well as information reflected in prices on soybean derivatives (i.e. an integrated time-series study)

Related research

Integrated time-series studies of commodity prices:


Option pricing/estimation of affine asset pricing models:


Analysis of the theory of storage:

Model

State-variables:

P: spot commodity price.
δ: convenience yield.
v: volatility.

dP_t = P_t [(r - \delta_t) dt + e^{\nu(t)} \sqrt{v_t} dW_{1,t}^Q]

d\delta_t = (\alpha(t) - \beta \delta_t) dt + e^{\nu(t)} \sigma_\delta \sqrt{v_t} dW_{2,t}^Q

dv_t = (\theta - \kappa v_t) dt + \sigma_v \sqrt{v_t} dW_{3,t}^Q

\alpha(t) = \alpha_0 + \sum_{k=1}^{2} (\alpha_k \cos (2\pi kt) + \alpha_k^* \sin (2\pi kt))

\nu(t) = \sum_{k=1}^{2} (\nu_k \cos (2\pi kt) + \nu_k^* \sin (2\pi kt))
Pricing results

Futures prices:

\[ F(P, \delta, v, t; T) = Pe^{A(t;T)+B(t;T)v+D(t;T)} \delta \]

Option prices:

\[ C(F, v, t) = e^{-r(\tau-t)} [FP_1 - KP_2] \]

where

\[ P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left[ \frac{e^{-i\phi \ln[K]} f_j(x, v, t; \phi)}{i\phi} \right] d\phi , \ j = 1, 2 \]
Estimation

State-vector at $t_n$: $X_n = (\log P_{tn}, \delta_{tn}, v_{tn})$

Observation-vector at $t_n$: $Y_n$

Log-likelihood function:

$$l(Y_0, Y_1, \ldots, Y_N; \psi, \Gamma) = \sum_{n=1}^{N} \left[ \log p(X_n|X_{n-1}) - \log \left| \frac{\partial f(1,n)}{\partial X} \right| ight]$$

$$- \frac{m}{2} \log(2\pi) - \frac{1}{2} \log |\Gamma| - \frac{1}{2} \epsilon'_n \Gamma^{-1} \epsilon_n \right].$$
Empirical results

1. Soybean futures and options data

2. Parameter estimates – and implications for...
   - stochastic volatility and seasonality
   - convenience yields, theory of storage, and timing options
   - inverse leverage effect
Table 1: Summary statistics for soybean futures, 1984-1999.

<table>
<thead>
<tr>
<th>Contracts</th>
<th>Number of observations</th>
<th>Days to maturity</th>
<th>Settlement prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>6024</td>
<td>209.53</td>
<td>627.49</td>
</tr>
</tbody>
</table>

**Grouped into time to maturity:**

1. Maturity 753 28.60 622.86 97.13
2. Maturity 753 81.75 624.82 95.24
3. Maturity 753 132.52 626.84 92.24
4. Maturity 753 183.57 627.88 87.92
5. Maturity 753 234.00 628.75 83.81
6. Maturity 753 284.36 628.99 80.09
7. Maturity 753 337.77 629.18 76.38
8. Maturity 753 393.63 630.59 72.68

**Grouped into expiration months:**

<table>
<thead>
<tr>
<th>Month</th>
<th>Number</th>
<th>Days to maturity</th>
<th>Settlement prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>893</td>
<td>215.46</td>
<td>622.51</td>
</tr>
<tr>
<td>March</td>
<td>877</td>
<td>210.84</td>
<td>628.02</td>
</tr>
<tr>
<td>May</td>
<td>872</td>
<td>212.17</td>
<td>634.03</td>
</tr>
<tr>
<td>July</td>
<td>883</td>
<td>214.26</td>
<td>638.37</td>
</tr>
<tr>
<td>August</td>
<td>812</td>
<td>197.62</td>
<td>632.63</td>
</tr>
<tr>
<td>September</td>
<td>808</td>
<td>197.84</td>
<td>621.02</td>
</tr>
<tr>
<td>November</td>
<td>879</td>
<td>216.55</td>
<td>615.77</td>
</tr>
</tbody>
</table>

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Figure 1: Time series of soybean futures prices, 1984-2006.
Table 2: Summary statistics for soybean call options, 1984-1999.

<table>
<thead>
<tr>
<th>Contracts</th>
<th>Number of observations</th>
<th>Days to maturity</th>
<th>Implied volatilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td>All</td>
<td>3765</td>
<td>71.95</td>
<td>19.99</td>
</tr>
</tbody>
</table>

Grouped into time to maturity and moneyness:

1. Maturity – ITM*  | 753                    | 40.50            | 18.88 | 6.53
1. Maturity – ATM*  | 753                    | 40.50            | 19.61 | 6.99
1. Maturity – OTM*  | 753                    | 40.50            | 21.58 | 7.19
2. Maturity – ATM*  | 753                    | 93.71            | 19.81 | 6.38
3. Maturity – ATM*  | 753                    | 144.69           | 20.06 | 5.51

Grouped into expiration months:

<table>
<thead>
<tr>
<th>Month</th>
<th>Number of observations</th>
<th>Days to maturity</th>
<th>Implied volatilities</th>
</tr>
</thead>
</table>
| January          | 548                    | 66.65            | 18.13 | 4.72
| March            | 636                    | 77.32            | 17.13 | 3.72
| May              | 645                    | 78.49            | 16.71 | 4.17
| July             | 631                    | 81.50            | 19.40 | 4.84
| August           | 431                    | 71.85            | 25.17 | 8.40
| September        | 374                    | 59.04            | 26.28 | 8.76
| November         | 500                    | 60.17            | 21.47 | 5.74
Figure 2: Time series of implied volatilities on soybean options, 1984-2006.
Parameter estimates

\[ dP_t = P_t \left[(r - \delta_t) \, dt + e^{\nu(t)} \sqrt{v_t} \, dW_{1,t}^Q \right] \]
\[ d\delta_t = (\alpha(t) - \beta \delta_t) \, dt + e^{\nu(t)} \sigma_\delta \sqrt{v_t} \, dW_{2,t}^Q \]
\[ dv_t = (\theta - \kappa v_t) \, dt + \sigma_v \sqrt{v_t} \, dW_{3,t}^Q \]

Volatility: \hspace{1cm} \kappa: 2.2708 \hspace{1cm} \theta: 0.1542 \hspace{1cm} \sigma_v: 0.7414 \\
(0.2654) \hspace{1cm} (0.0085) \hspace{1cm} (0.0299)

Convenience yield: \hspace{1cm} \beta: 0.8145 \hspace{1cm} \sigma_\delta: 0.9933 \hspace{1cm} \alpha_0: 0.0612 \\
(0.2100) \hspace{1cm} (0.1345) \hspace{1cm} (0.0086)

Correlations: \hspace{1cm} \rho_{12}: 0.3979 \hspace{1cm} \rho_{13}: 0.4078 \hspace{1cm} \rho_{23}: -0.0784 \\
(0.0110) \hspace{1cm} (0.0461) \hspace{1cm} (0.0772)

Risk premia: \hspace{1cm} \lambda_P: -0.6274 \hspace{1cm} \lambda_\delta: -0.1282 \hspace{1cm} \lambda_v: -3.7829 \\
(1.3001) \hspace{1cm} (1.2286) \hspace{1cm} (1.5610)

* Standard errors in parentheses.
Figure 3: Seasonal functions. 1: volatility. 2: convenience yield.