

A dependence model for pairs of commodity forward curves: application to the US natural gas and oil markets

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Why model the dependence between commodity forward curves

- Pricing and managing multi-commodity assets:
 - Gas-fired power plants, oil-indexed gas contracts, oil refineries
 - = options on the spread between commodity forward curves
- Hedge funds develop strategies based on the spread between the futures prices of economically related commodities

What a dependence model should account for

- A dependence model should capture:
 - The local dependence structure
 - correlation between the daily comovements of the two forward curves
 - The global dependence structure:
 - long term relations between the futures prices
 - error-correction mechanisms ensuring the reversion to the long-term equilibrium

Review of related literature

- Error-correction models on particular points of one or several forward curves, e.g.,
 - Alexander (1999) in Managing Energy Price Risk : cointegration between oil spot and front-month prices on the NYMEX
 - N'guyen (2002), *PhD thesis Université Paris Dauphine* : cointegration between the futures prices of different metals on the LME
 - ...
- Models on one forward curve, e.g.,
 - Schwartz & Smith (2000), *Management Science*
 - Geman & N'guyen (2005), *Management Science*
- No model (to our knowledge) for the joint evolution of two **entire commodity forward curves**

Approach

- Generic model for two commodity forward curves
 - capturing global and local dependence
 - parsimonious in terms of number of risk factors
- Application to the US gas and oil markets from 1999 to July 2005
- Contributions:
 - economic standpoint: better understanding of the links between the US gas and oil markets
 - statistic and financial modeling: new type of model for the evolution of two commodity forward curves
 - can be viewed as an **extension of cointegration to forward curves**

Outline

1. Economic relations between US oil and gas markets
2. Definition of global and local dependence structures
3. Empirical observation of the global dependence
4. A generic model for the joint evolution of two commodity forward curves

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Demand side

Global available fuel switching potential of the US gas consumption = 5%

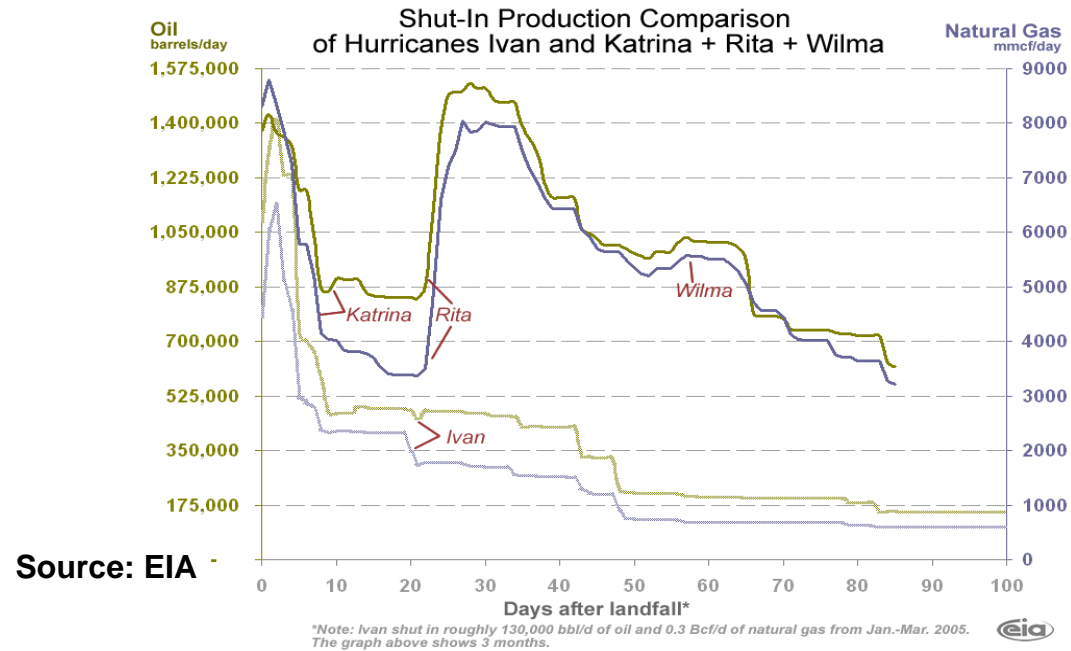
	%of US gas consumption	%of fuel switching potential	Contribution in the global fuel switching potential of the US gas consumption
Industry	35%	4%	29%
Power generation	19%	20%	71%
Residential/commercial	36%	0%	0%

Development of CCGT and rise of emission costs

→ the fuel switching potential is expected to drop in the future

Supply side: 2 opposite effects

1st effect: oil and gas are produced/refined/processed in the Gulf of Mexico...



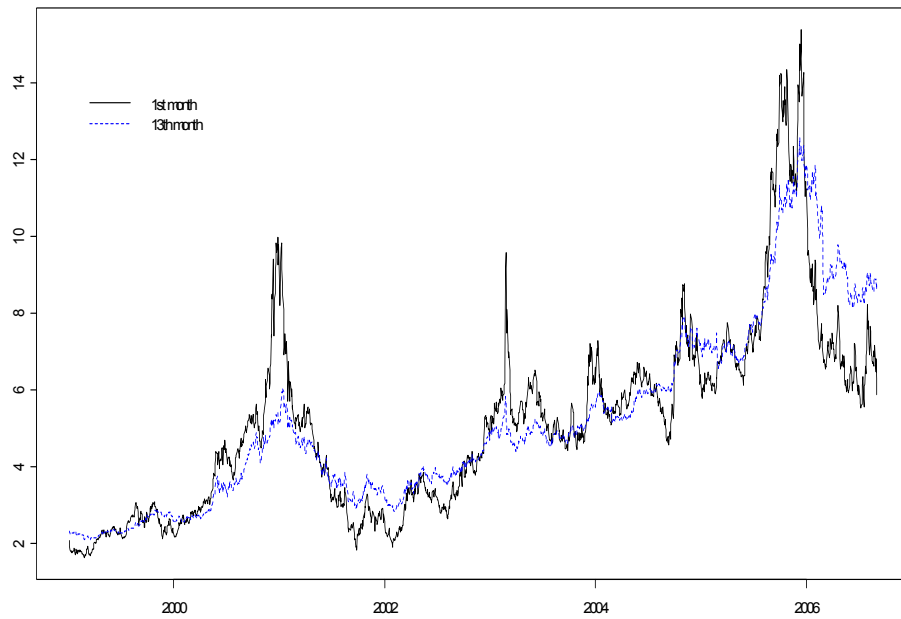
2nd effect: natural gas is a co-product of oil:

rise in oil price → rise in oil and gas production → drop in gas prices

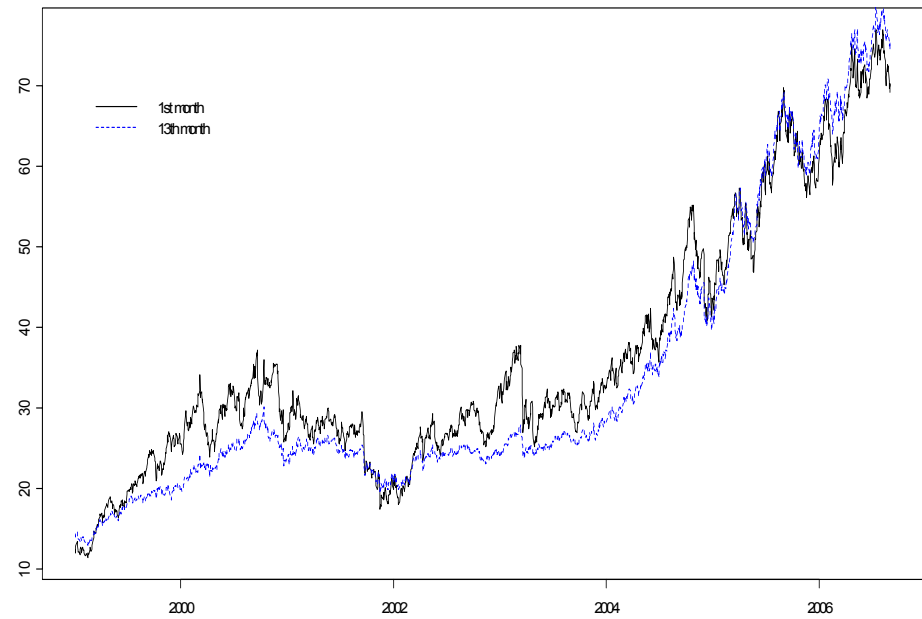
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Price trajectories from jan 1997 to Aug 2006

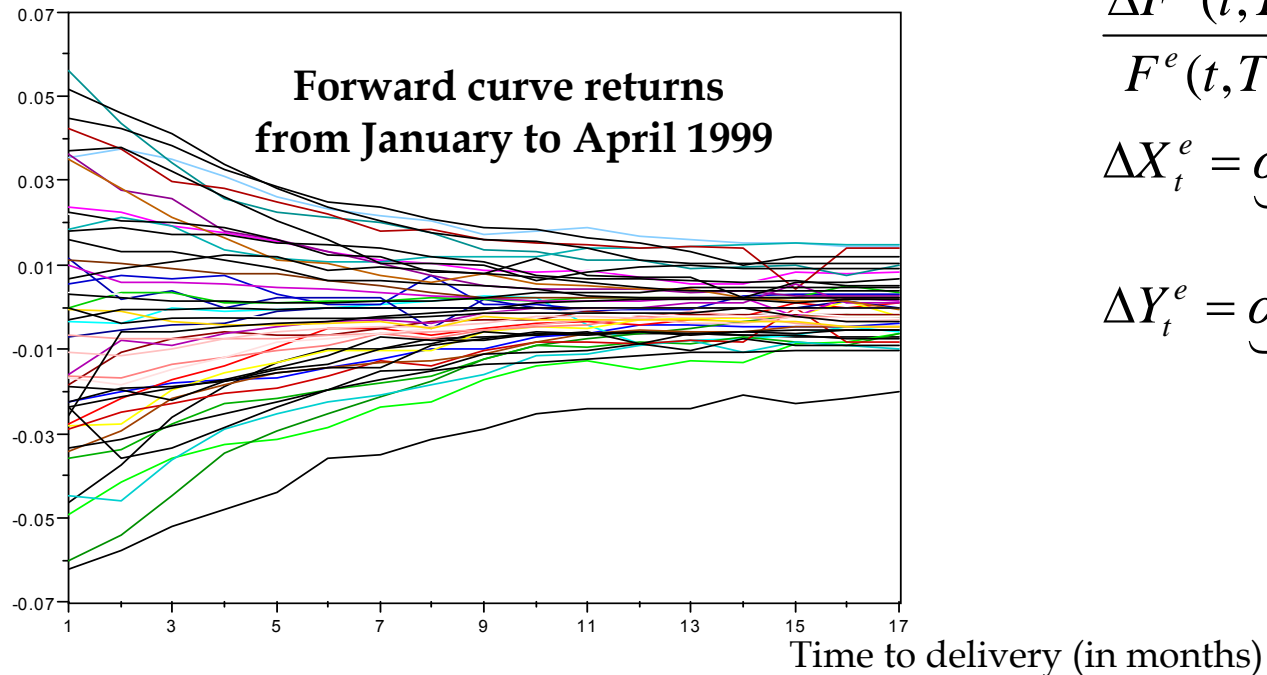


Natural Gas



Crude Oil

Model for the daily deformations of a forward curve



$$\frac{\Delta F^e(t, T)}{F^e(t, T)} = e^{-k_e(T-t)} \Delta X_t^e + \Delta Y_t^e$$

$$\Delta X_t^e = \underbrace{\alpha_t^{e, X} \Delta t}_{trend} + \underbrace{\sigma_t^{e, X} \Delta x_t^e}_{shock}$$

$$\Delta Y_t^e = \underbrace{\alpha_t^{e, Y} \Delta t}_{trend} + \underbrace{\sigma_t^{e, Y} \Delta y_t^e}_{shock}$$

The forward curve is subject to:

- ✓ a long term shock that results in a global translation
- ✓ a short-term shock which fades away exponentially with time-to-delivery

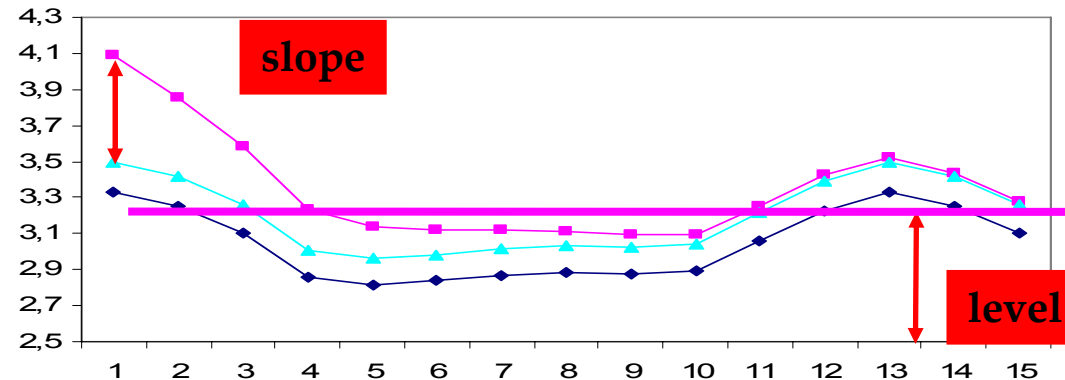
Residual variance:

- ✓ Natural Gas 5.6%
- ✓ Oil 1.4%

Implied model for the shape of the forward curve

$$\ln F^e(t, T) = \underbrace{Q_e(T)}_{\text{season}} + e^{-k_e(T-t)} \underbrace{\bar{X}_t^e}_{\text{slope}} + \underbrace{Y_t^e}_{\text{level}} + \dots$$

log futures price



Time to delivery (in months)

- slope and level are derived from the daily-shocks:

the slope is a (stationary) mean-reverting process:

$$\bar{X}_t^e = \bar{X}_0^e e^{-k_e t} + \sum_{s=0}^t \Delta X_s^e e^{-k_e(t-s)}$$

$$\bar{X}_{t+\Delta t}^e = e^{-k_e \Delta t} \bar{X}_t^e + \Delta X_t^e$$

the level is a (non-stationary) random walk:

$$Y_t^e = Y_0^e + \sum_{s=0}^t \Delta Y_s^e$$

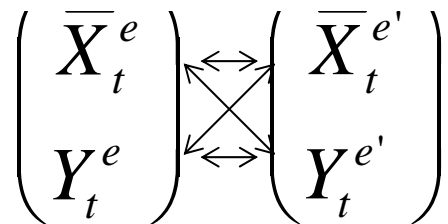
$$Y_{t+\Delta t}^e = Y_t^e + \Delta Y_t^e$$

Global and local dependence structures

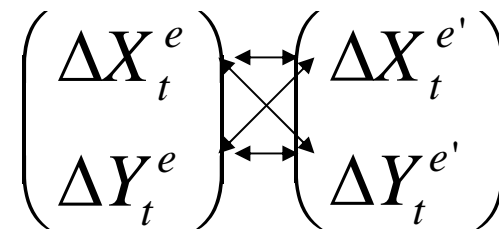
$$\frac{\Delta F^e(t, T)}{F_e(t, T)} = e^{-k_e(T-t)} \Delta X_t^e + \Delta Y_t^e$$

$$\ln F^e(t, T) = \underbrace{Q_e(T)}_{\text{season}} + e^{-k_e(T-t)} \underbrace{\bar{X}_t^e}_{\text{slope}} + \underbrace{Y_t^e}_{\text{level}} + \dots$$

Global dependence structure



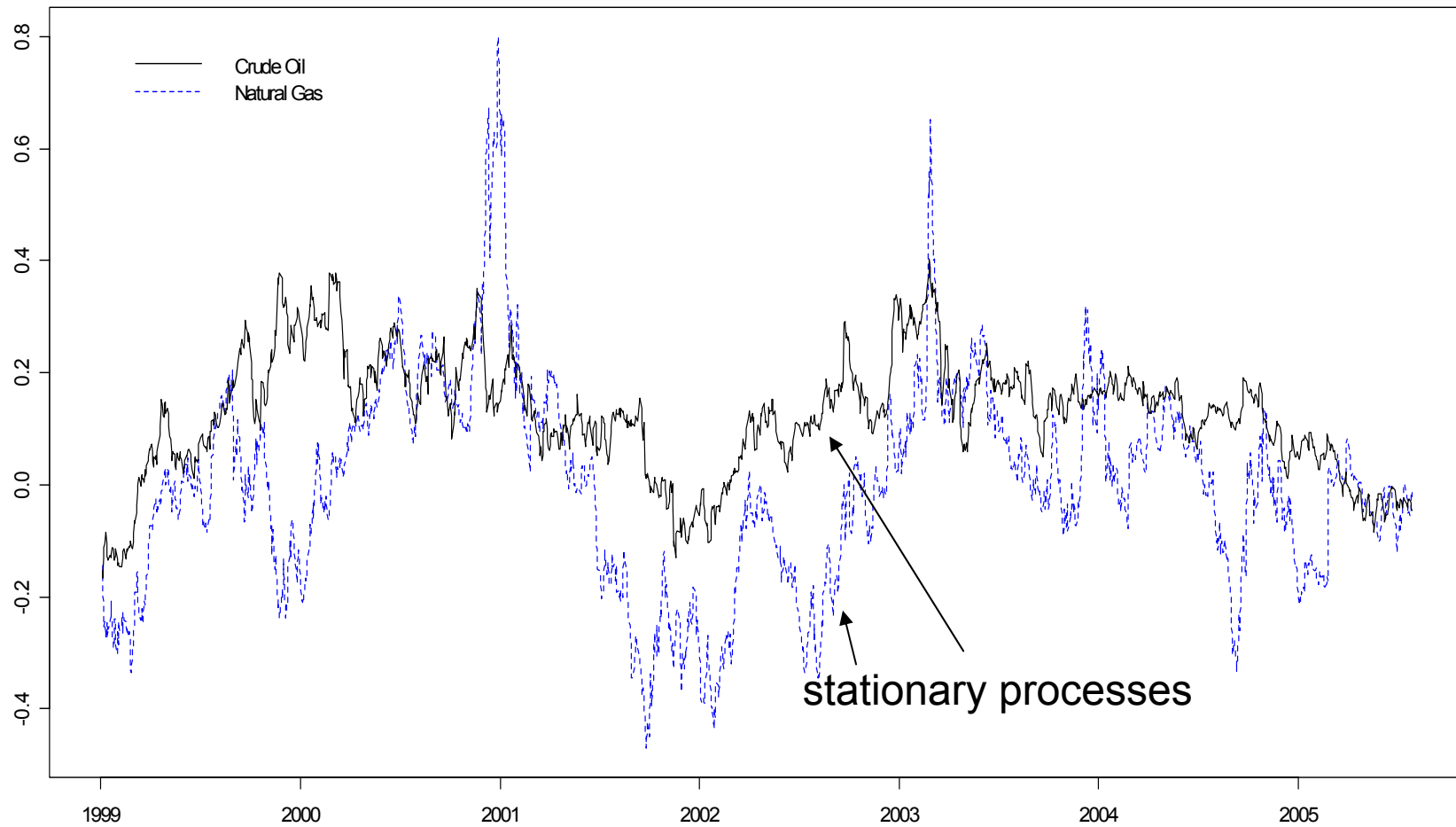
Local dependence structure



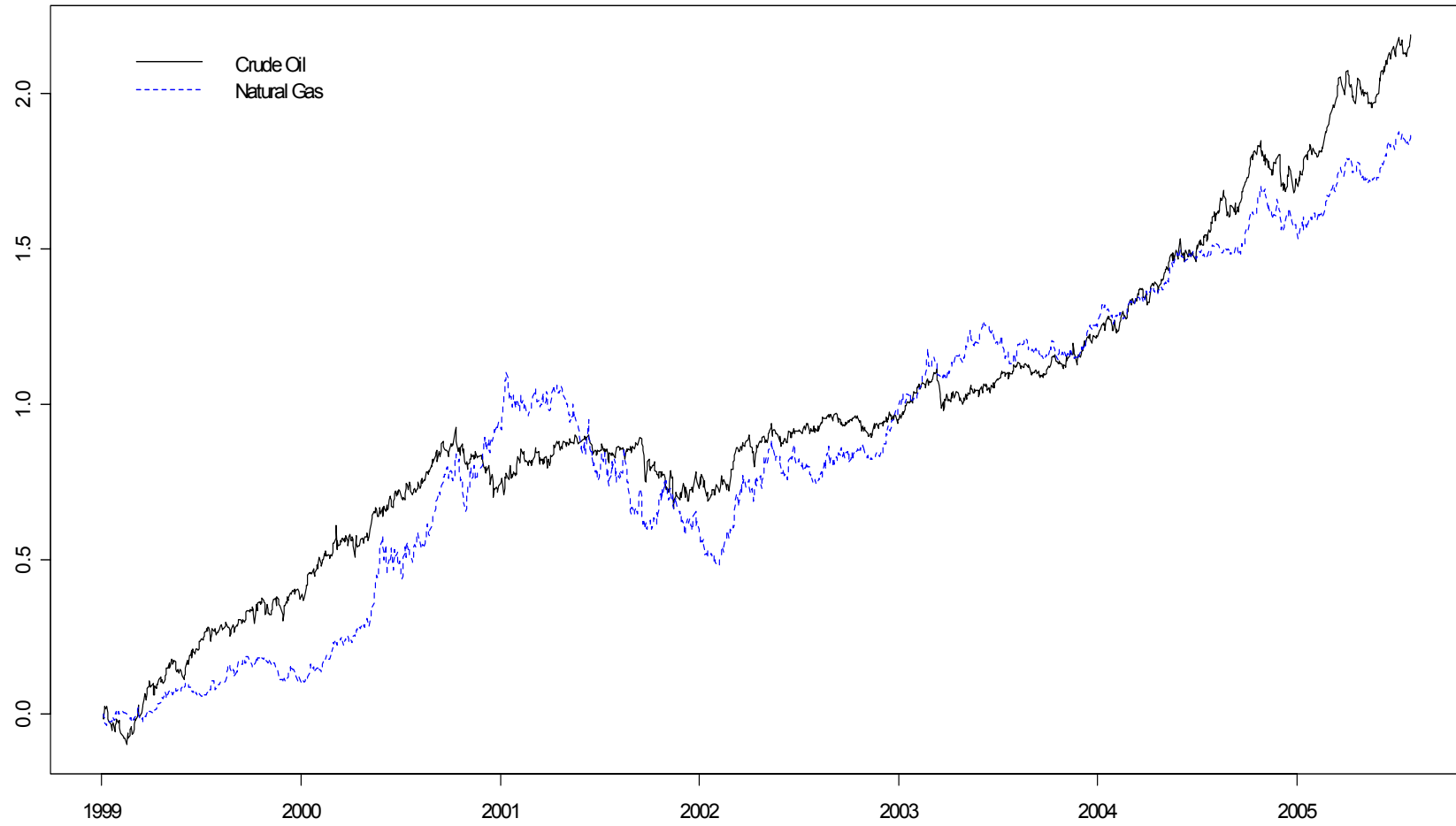
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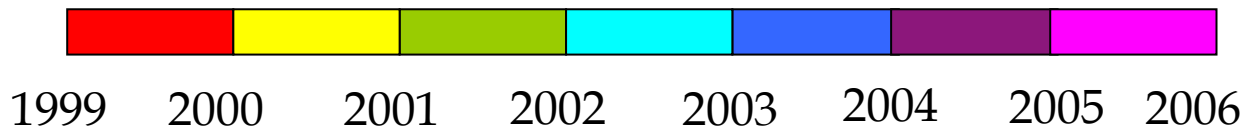
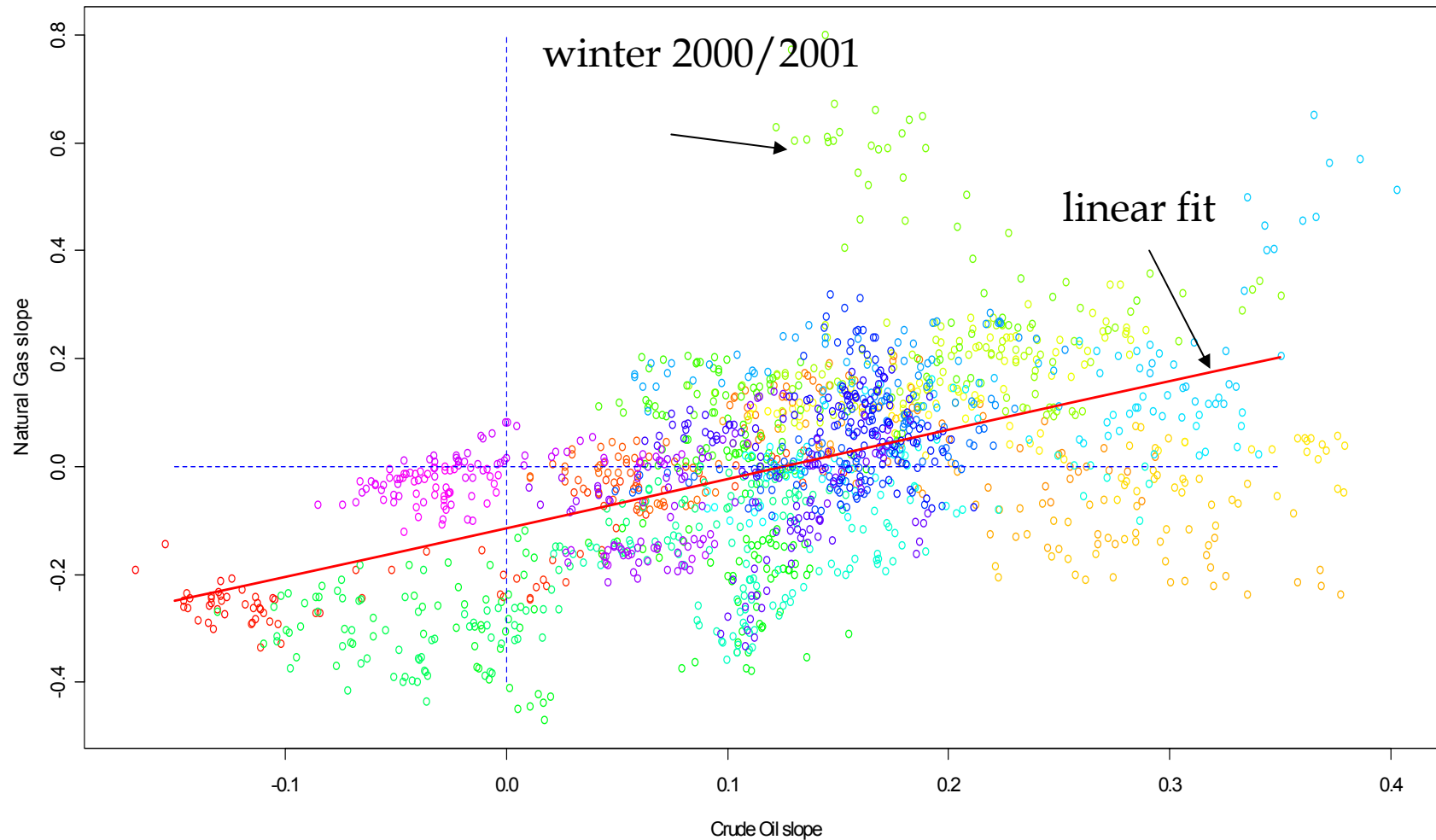
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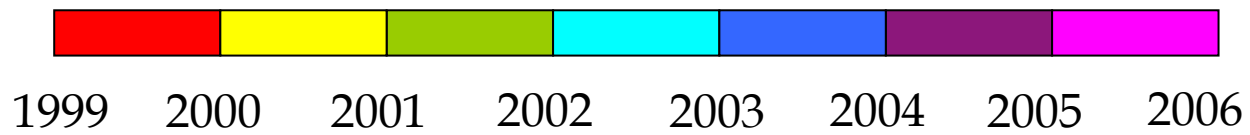
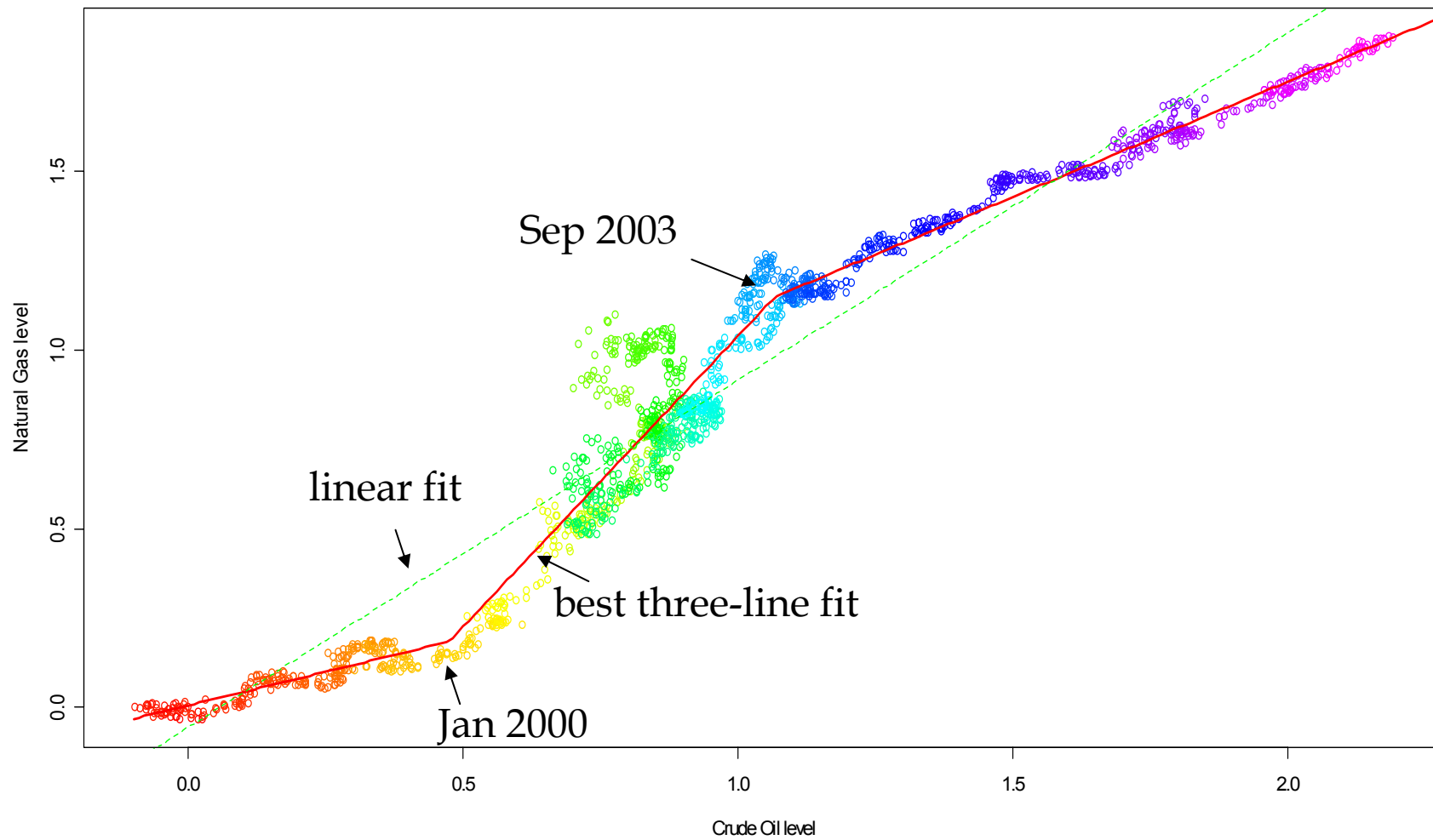
Slopes



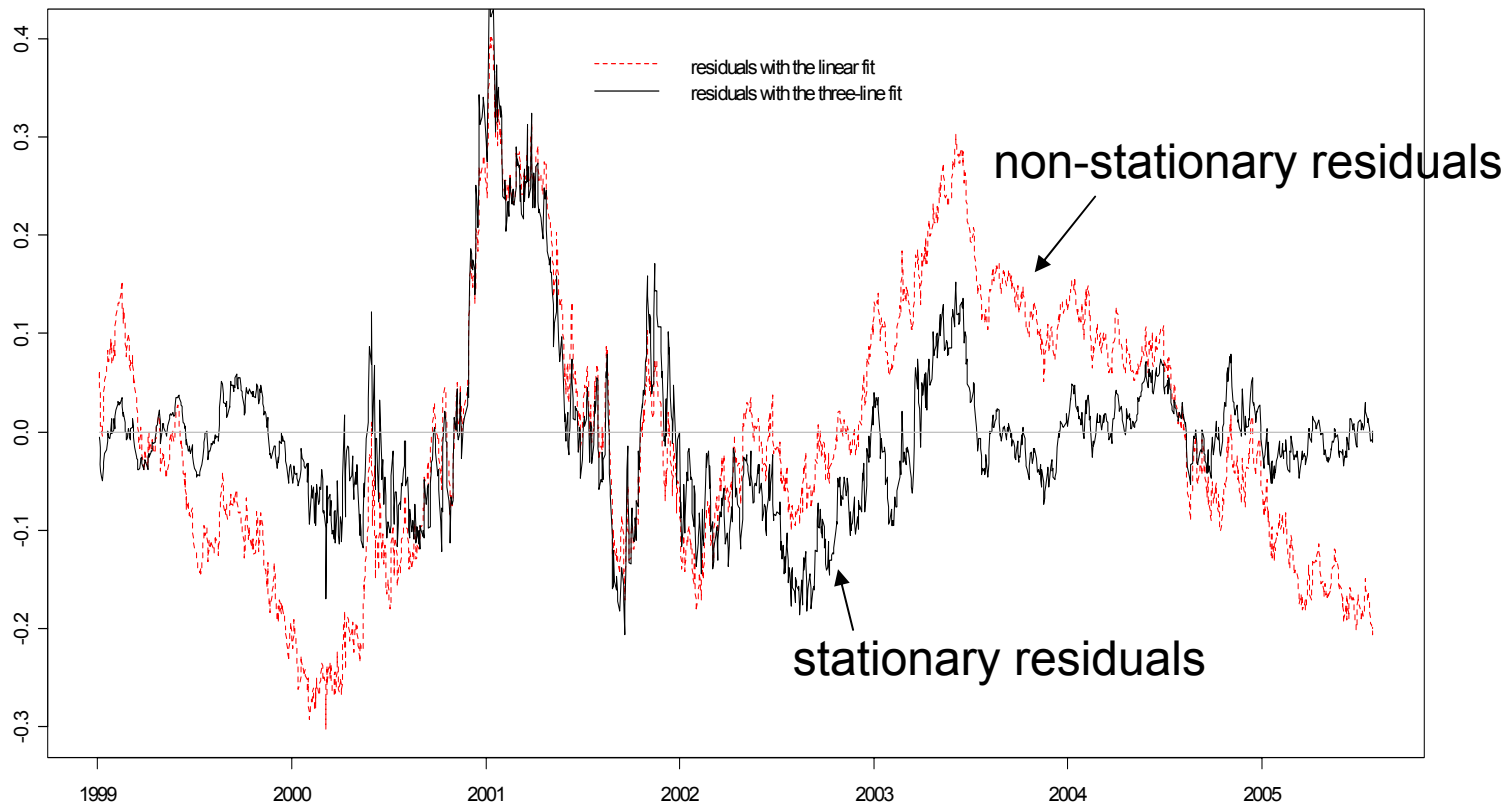
Levels







Residuals in the relation on the levels



Three stationary state variables for the prediction of the trends of the two curves

$$\xi_t = \begin{pmatrix} \bar{X}_t^e \\ \bar{X}_t^{e'} \\ Y_t^e - f(Y_t^{e'}) \end{pmatrix}$$

Slope of energy e

Slope of energy e'

Deviations to the long-term relation on the levels

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A generic model for two commodity forward curves

$$\frac{\Delta F^e(t, T)}{F^e(t, T)} = e^{-ke(T-t)} \Delta X_t^e + \Delta Y_t^e$$

$$\frac{\Delta F^{e'}(t, T)}{F^{e'}(t, T)} = e^{-ke'(T-t)} \Delta X_t^{e'} + \Delta Y_t^{e'}$$

$$\bar{X}_t^e = \bar{X}_0^e e^{-k_e t} + \sum_{s=0}^T e^{-k_e(t-s)} \Delta X_s^e \quad Y_t^e = Y_0^e + \sum_{t=0}^T \Delta Y_t^e$$

independent GARCH processes for the volatilities

$$\begin{pmatrix} \Delta X_t^e \\ \Delta X_t^{e'} \\ \Delta Y_t^e \\ \Delta Y_t^{e'} \end{pmatrix} = \begin{pmatrix} \mu^{X,e} \\ \mu^{X,e'} \\ \mu^{Y,e} \\ \mu^{Y,e'} \end{pmatrix} + \Gamma \begin{pmatrix} \Delta X_{t-\Delta t}^e \\ \Delta X_{t-\Delta t}^{e'} \\ \Delta Y_{t-\Delta t}^e \\ \Delta Y_{t-\Delta t}^{e'} \end{pmatrix} + \Pi \begin{pmatrix} \bar{X}_t^e \\ \bar{X}_t^{e'} \\ Y_t^e - f(Y_t^{e'}) \end{pmatrix} + \begin{pmatrix} \sigma_t^{e,X} \eta_t^{X,e} \\ \sigma_t^{e',X} \eta_t^{X,e'} \\ \sigma_t^{e,Y} \eta_t^{Y,e} \\ \sigma_t^{e',Y} \eta_t^{Y,e'} \end{pmatrix}$$

Constant drift

Error correction term associated to the global dependence structure

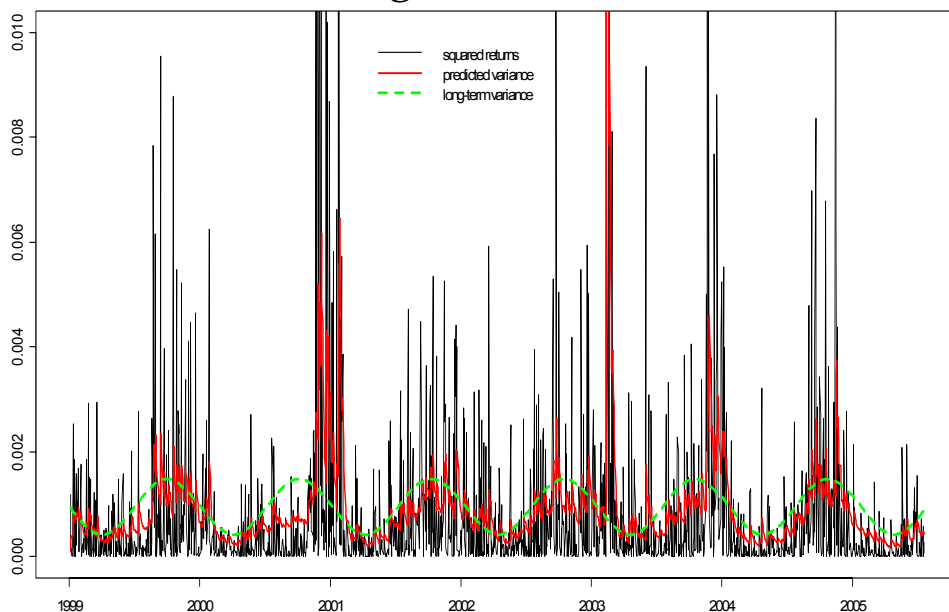
Predictible part of the local dependence structure

Random part of the local dependence structure

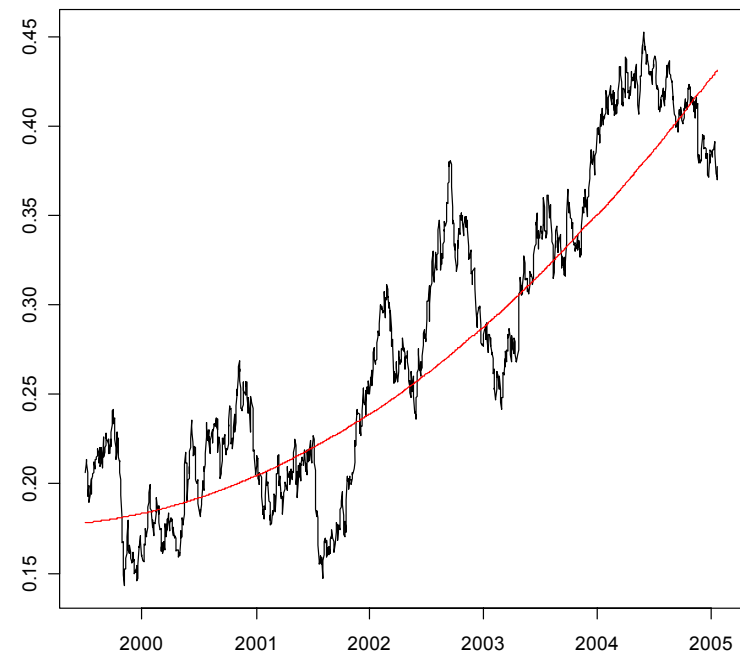
Results for US gas and oil markets: local dependence structure

- Negative causality runs from oil short-term shocks to gas shocks
 - Volatilities:
 - Seasonal stochastic volatility for natural gas short-term shocks
 - Standardized residuals:
 - Positive correlations (20-30 %) between all pairs of cross-energy shocks
 - Upward trend in the correlations between oil and gas short-term/long-term shocks

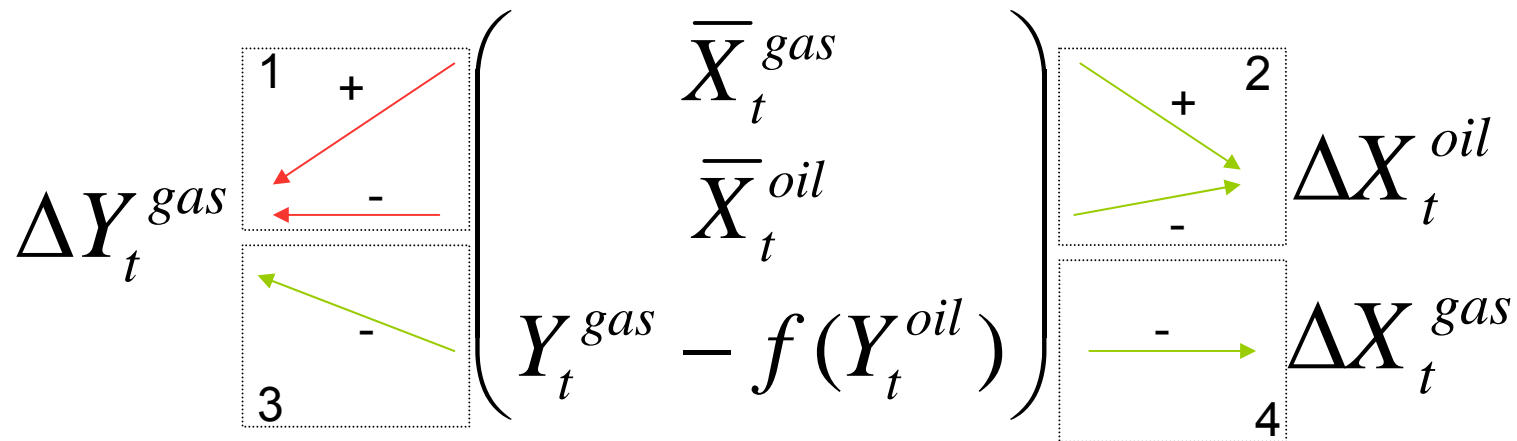
variance of gas short-term shocks



correlation gas/oil short-term shocks



Results for US gas and oil markets: global dependence structure



- Spread amplification effect
- Spread attenuation effect

Gas is the leader for the slopes
Oil is the leader for the levels

1 and 2 broke in the period Jan 2004-July 2005
3 and 4 stable throughout the period

Results for US gas and oil markets: global dependence structure

- Influence of the slopes on the shocks:
 - Crude oil short-term shocks respond positively to the gas slope and negatively to the crude slope (spread **correction** effect)
 - Gas long-term shocks react positively to the gas slope and negatively to the crude slope (spread **amplification** effect)
 - Breaking of the two relations in the period Jan 2004-July 2006
- Influence of the deviations to the long-term relation on the levels:
 - Gas long-term shocks react negatively to an overvalued gas long-term price (spread **correction** effect)
 - Gas short-term shocks react negatively to an overvalued gas long-term price (spread **correction** effect)
 - Stability of these correction mechanisms throughout the period under study

Conclusion

- Contributions:
 - New dependence model for commodity forward curves:
 - Global and local dependence structures
 - Application to the US gas and oil markets:
 - Lead and lag properties between oil and gas
 - Upward trend of local correlations
- Areas of future research:
 - Inclusion of Hurricane Katrina
 - Improvements of the model (e.g., Markov regime switching...)
 - Application to other pairs of commodities (e.g., gas/electricity)
 - Application to spread option pricing/portfolio optimization for utilities and diversified funds