A dependence model for pairs of commodity forward curves: application to the US natural gas and oil markets

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Why model the dependence between commodity forward curves

• Pricing and managing multi-commodity assets:
  → Gas-fired power plants, oil-indexed gas contracts, oil refineries
  = options on the spread between commodity forward curves

• Hedge funds develop strategies based on the spread between the futures prices of economically related commodities
What a dependence model should account for

• A dependence model should capture:
  - The local dependence structure
    • correlation between the daily comovements of the two forward curves
  - The global dependence structure:
    • long term relations between the futures prices
    • error-correction mechanisms ensuring the reversion to the long-term equilibrium
Review of related literature

• Error-correction models on particular points of one or several forward curves, e.g.,
  – N’guyen (2002), PhD thesis Université Paris Dauphine: cointegration between the futures prices of different metals on the LME
  – …

• Models on one forward curve, e.g.,
  – Schwartz & Smith (2000), Management Science
  – Geman & N’guyen (2005), Management Science

• No model (to our knowledge) for the joint evolution of two entire commodity forward curves
Approach

• Generic model for two commodity forward curves
  - capturing global and local dependence
  - parsimonious in terms of number of risk factors

• Application to the US gas and oil markets from 1999 to July 2005

• Contributions:
  - economic standpoint: better understanding of the links between the US gas and oil markets
  - statistic and financial modeling: new type of model for the evolution of two commodity forward curves
  → can be viewed as an extension of cointegration to forward curves
Outline

1. Economic relations between US oil and gas markets
2. Definition of global and local dependence structures
3. Empirical observation of the global dependence
4. A generic model for the joint evolution of two commodity forward curves
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## Demand side

Global available fuel switching potential of the US gas consumption = 5%

<table>
<thead>
<tr>
<th></th>
<th>% of US gas consumption</th>
<th>% of fuel switching potential</th>
<th>Contribution in the global fuel switching potential of the US gas consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry</td>
<td>35%</td>
<td>4%</td>
<td>29%</td>
</tr>
<tr>
<td>Power generation</td>
<td>19%</td>
<td>20%</td>
<td>71%</td>
</tr>
<tr>
<td>Residential/commercial</td>
<td>36%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Development of CCGT and rise of emission costs → the fuel switching potential is expected to drop in the future
Supply side: 2 opposite effects

1st effect: oil and gas are produced/refined/processed in the Gulf of Mexico...

2nd effect: natural gas is a co-product of oil:
rise in oil price → rise in oil and gas production → drop in gas prices
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Price trajectories from Jan 1997 to Aug 2006

Natural Gas

Crude Oil
Model for the daily deformations of a forward curve

\[
\frac{\Delta F(t, T)}{F(t, T)} = e^{-k(T-t)} \Delta X_t^e + \Delta Y_t^e
\]

\[
\Delta X_t^e = \underbrace{\alpha_t^{e,X} \Delta t}_\text{trend} + \underbrace{\sigma_t^{e,X} \Delta x_t^e}_\text{shock}
\]

\[
\Delta Y_t^e = \underbrace{\alpha_t^{e,Y} \Delta t}_\text{trend} + \underbrace{\sigma_t^{e,Y} \Delta y_t^e}_\text{shock}
\]

The forward curve is subject to:

- ✓ a long term shock that results in a global translation
- ✓ a short-term shock which fades away exponentially with time-to-delivery

Residual variance:

- ✓ Natural Gas 5.6%
- ✓ Oil 1.4%
Implied model for the shape of the forward curve

\[
\ln F^e(t, T) = \underbrace{Q_e(T)}_{\text{season}} + e^{-k_e(T-t)} \underbrace{\bar{X}_t^e}_{\text{slope}} + \underbrace{Y_t^e}_{\text{level}} + \ldots
\]

log futures price

- \( \bar{X}_t^e = X_0^e e^{-k_e t} + \sum_{s=0}^{t} \Delta X_s^e e^{-k_e (t-s)} \)
- \( \bar{X}_{t+\Delta t}^e = e^{-k_e \Delta t} \bar{X}_t^e + \Delta X_t^e \)

- \( Y_t^e = Y_0^e + \sum_{s=0}^{t} \Delta Y_s^e \)
- \( Y_{t+\Delta t}^e = Y_t^e + \Delta Y_t^e \)

- slope and level are derived from the daily-shocks:
  - the slope is a (stationary) mean-reverting process:
  - the level is a (non-stationary) random walk:
Global and local dependence structures

\[
\frac{\Delta F^e(t, T)}{F_e(t, T)} = e^{-k_e(T-t)} \Delta X^e_t + \Delta Y^e_t
\]

\[
\ln F^e(t, T) = Q_e(T) + e^{-k_e(T-t)} \bar{X}^e_t + Y^e_t + \ldots
\]

Global dependence structure  
Local dependence structure

\[
\begin{pmatrix}
\bar{X}^e_t \\
Y^e_t
\end{pmatrix}
\leftrightarrow
\begin{pmatrix}
\bar{X}^{e'}_t \\
Y^{e'}_t
\end{pmatrix}
\]

\[
\begin{pmatrix}
\Delta X^e_t \\
\Delta Y^e_t
\end{pmatrix}
\leftrightarrow
\begin{pmatrix}
\Delta X^{e'}_t \\
\Delta Y^{e'}_t
\end{pmatrix}
\]
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Levels

- Crude Oil
- Natural Gas
The graph shows the relationship between Crude Oil level and Natural Gas level from Jan 2000 to Sep 2003. It includes a linear fit, a best three-line fit, and marked points for Jan 2000 and Sep 2003.
Residuals in the relation on the levels

-0.3 -0.2 -0.1 0.0 0.1 0.2 0.3 0.4

1999 2000 2001 2002 2003 2004 2005

residuals with the linear fit
residuals with the three-line fit

stationary residuals
non-stationary residuals

stationary residuals
Three stationary state variables for the prediction of the trends of the two curves

\[ \xi_t = \begin{pmatrix} \overline{X}_t^e \\ \overline{X}_t^{e'} \\ Y_t^e - f(Y_t^{e'}) \end{pmatrix} \]

Slope of energy e
Slope of energy e'
Deviations to the long-term relation on the levels
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A generic model for two commodity forward curves

\[
\frac{\Delta F^e(t, T)}{F^e(t, T)} = e^{-ke(T-t)} \Delta X_t^e + \Delta Y_t^e \\
\frac{\Delta F^{e'}(t, T)}{F^{e'}(t, T)} = e^{-ke'(T-t)} \Delta X_t^{e'} + \Delta Y_t^{e'}
\]

\[
\overline{X}_t^e = \overline{X}_0 e^{-k_e t} + \sum_{t=0}^{T} e^{-k_e (t-s)} \Delta X_s^e \\
Y_t^e = Y_0^e + \sum_{t=0}^{T} \Delta Y_t^e
\]

\[
\begin{pmatrix}
\Delta X_t^e \\
\Delta X_t^{e'} \\
\Delta Y_t^e \\
\Delta Y_t^{e'}
\end{pmatrix} =
\begin{pmatrix}
\mu^X,e \\
\mu^X,e' \\
\mu^Y,e \\
\mu^Y,e'
\end{pmatrix}
+ \Gamma
\begin{pmatrix}
\Delta X_{t-\Delta t}^e \\
\Delta X_{t-\Delta t}^{e'} \\
\Delta Y_{t-\Delta t}^e \\
\Delta Y_{t-\Delta t}^{e'}
\end{pmatrix}
+ \Pi
\begin{pmatrix}
\overline{X}_t^e \\
\overline{X}_t^{e'} \\
Y_t^e - f(Y_t^{e'})
\end{pmatrix}
+ \begin{pmatrix}
\sigma_t^{e,X} \eta_t^X,e \\
\sigma_t^{e',X} \eta_t^X,e' \\
\sigma_t^{e,Y} \eta_t^Y,e \\
\sigma_t^{e',Y} \eta_t^Y,e'
\end{pmatrix}
\]

\text{Error correction term associated to the global dependence structure}

\text{Random part of the local dependence structure}

\text{Predictable part of the local dependence structure}

\text{Constant drift}

\text{independent GARCH processes for the volatilities}
Results for US gas and oil markets: local dependence structure

- Negative causality runs from oil short-term shocks to gas shocks

- Volatilities:
  → Seasonal stochastic volatility for natural gas short-term shocks

- Standardized residuals:
  → Positive correlations (20-30 %) between all pairs of cross-energy shocks
  → Upward trend in the correlations between oil and gas short-term/long-term shocks

### Variance of gas short-term shocks

![Graph of variance of gas short-term shocks]

### Correlation gas/oil short-term shocks

![Graph of correlation gas/oil short-term shocks]
Results for US gas and oil markets: global dependence structure

\[ \Delta Y_t^{gas} = \left( \begin{array}{c} \overline{X}_t^{gas} \\ \overline{X}_t^{oil} \\ Y_t^{gas} - f(Y_t^{oil}) \end{array} \right) \]

Spread amplification effect
Spread attenuation effect

Gas is the leader for the slopes
Oil is the leader for the levels

1 and 2 broke in the period Jan 2004-July 2005
3 and 4 stable throughout the period
Results for US gas and oil markets: global dependence structure

• **Influence of the slopes on the shocks:**
  - Crude oil short-term shocks respond positively to the gas slope and negatively to the crude slope (spread correction effect)
  - Gas long-term shocks react positively to the gas slope and negatively to the crude slope (spread amplification effect)
  - Breaking of the two relations in the period Jan 2004-July 2006

• **Influence of the deviations to the long-term relation on the levels:**
  - Gas long-term shocks react negatively to an overvalued gas long-term price (spread correction effect)
  - Gas short-term shocks react negatively to an overvalued gas long-term price (spread correction effect)
  - Stability of these correction mechanisms throughout the period under study
Conclusion

• Contributions:
  – New dependence model for commodity forward curves:
    • Global and local dependence structures
  – Application to the US gas and oil markets:
    • Lead and lag properties between oil and gas
    • Upward trend of local correlations

• Areas of future research:
  – Inclusion of Hurricane Katrina
  – Improvements of the model (e.g., Markov regime switching…)
  – Application to other pairs of commodities (e.g., gas/electricity)
  – Application to spread option pricing/portfolio optimization for utilities and diversified funds