A quantitative approach to carbon price risk management

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joint work with M. Fehr
Global warming

- is evident now
- explained by human activity
- challenges policy makers

**Problem:**
How to reduce emissions in a cost effective manner?

**Answer:**
By introduction of marketable environmental securities.

**Example:** European Trading Scheme (ETS), mandatory market for CO$_2$-allowances (carbon), since 2005
In this talk

- regulatory framework of ETS
- mechanism behind carbon price formation
- identification of carbon price drivers
- pricing of financial products related carbon
- quantitative aspects of market design
Kyoto protocol (1997)

- Annex I = developed countries (not green), are set
- Cat = 5.2 % below 1990 level, to reach in average in 2008 – 2012
- ETS is introduced by EU (blue), to reach Kyoto targets
European Trading Scheme (ETS)

- deputes the liability for to the emission sources
- by mandatory participation of 12,000 installations (power plans, industrial users responsible for 45% of the entire EU carbon emission)
- installations have to cover their carbon emission yearly by EU-allowances (EUAs), which yearly allocated by corresponding governments according to the National Allocation Plan (NAP)

Periods in ETS


• within each, allowances are valid for compliance independently of the allocation year
• at the end of each period, a fee is to pay for each CO₂–ton which is not covered (in 2008, one EUA from 2005-2008 is also charged)
Periods in ETS

installations reduce potential penalty payment by

- costly abatement strategies
- allowance trading
Work to do

understand price of EUA, to value options on carbon

- investments in energy business have to take into account this risk
- each GHG-reduction project is a long position in appropriate (real) option on carbon prices, estimate this risk to attract capital for green projects
EUA spot price from EEX.

Price drop occurred while carbon emission data became public, showing that the overall market position is long.
EUA’s price dynamics \((A_t)^{T}_{t=0}\)

- financial asset \(\rightarrow\) no-arbitrage modeling
- terminal value, \(T = \) end of 2012
  - either market has fulfilled the reduction \(A_T = 0\)
  - or market failed on the reduction \(A_T =\) penalty

  this is **not usefull** since the interesting part is **missing**

  the link to

- price evolution of energy-related commodities
- behavior of carbon-consuming factors 
  (weather, climate)
How to link EUA to fuel and climate

**Idea:** for given game rules, exploit the mechanism of price formation with above uncertainty sources

**Approach:** market model, where agents

- are set caps by allocated initial credits
- averse potential penalty payments
- trade allowances
- apply costly abatement strategies to carbon emission (prime example is the fuel switch, the replacing coal by gas in electricity production)
Model ingredients

determine \((A_t)^{T}_{t=0}\) (EUAs futures price)
given
- \((\Omega, \mathcal{F}, P, (\mathcal{F}_t)^{T}_{t=0})\) filtered space
- \(\pi \in [0, \infty[\) penalty for each ton not covered by EUA
- \(i = 1, \ldots, N\) agents with
  - \(\Gamma^i_t\) (\(\mathcal{F}_T\)-measurable) agent’s \(i\) carbon demand
    (total emission in the business as usual scenario less initial credit, \(\Gamma^i > 0, \Gamma^i < 0\) possible)
  - \((E^i_t)^{T-1}_{t=0}\) fuel switch price process
Model ingredients

strategies of agents \( i = 1, \ldots, N \)

- \( \theta^i = (\theta^i_t)_{t=0}^{T} \) carbon trading, giving at \( T \)

\[
\sum_{t=0}^{T-1} \theta^i_t (A_{t+1} - A_t) - \theta^i_T A_T
\]

futures position change of phys. position

- \( \xi^i = (\xi^i_t)_{t=0}^{T-1} \) switch policy, \([0, \lambda^i]\)-valued, gives at \( T \)

total reduction \( \sum_{t=0}^{T-1} \xi^i_t \) at costs \( \sum_{t=0}^{T-1} E_t \xi^i_t \)
Each agent $i$ manages by $(\theta^i, \xi^i)$ the own exposure

$$
\sum_{t=0}^{T-1} \theta^i_t (A_{t+1} - A_t) - \theta^i_T A_T - \pi (\Gamma^i - \sum_{t=0}^{T-1} \xi^i_t - \theta^i_T)^+ - \sum_{t=0}^{T-1} \xi^i_t \varepsilon^i_t 
= I^{\theta^i, \xi^i, i}(A)
$$

as follows:

given $A = (A_t)_{t=0}^T$, agents $i = 1, \ldots, N$ selects

$$(\theta^i(A), \xi^i(A)) = \text{argmax} \left( (\theta^i, \xi^i) \mapsto E(I^{\theta^i, \xi^i, i}(A)) \right)_{i = 1, \ldots, N}$$
In reality, markets are in equilibrium thus, a realistic carbon price process $A^* = (A_t^*)_{t=0}^T$ is characterized by: all futures positions sum up to zero:

$$\sum_{i=1}^{N} \theta_t(A^*) = 0 \quad t = 0, \ldots, T - 1.$$ 

the changes in physical positions sum up to zero:

$$\sum_{i=1}^{N} \theta_T(A^*) = 0$$

We determine equilibrium EUA prices. Thereby the social-optimality is crucial.
Social-optimality

in equilibrium, agents apply overall switching policy
\[ \xi^* = \left( \xi^i(A^*) \right)_{i=1}^N \]
which is optimal for the entire market, maximizing \( \xi \mapsto E(G(\xi)) \) with

\[
-\pi \left( \sum_{i=1}^N \Gamma^i \right) - \sum_{i=1}^N \sum_{t=0}^{T-1} \xi_t^i + \sum_{i=1}^N \sum_{t=1}^{T-1} \xi_t^i \xi_t^i = G(\xi)
\]

which is quasi-social costs of compliance
(quasi, since penalty is not lost for the society)
From quasi-social optimality,
under some assumptions, equilibrium carbon price is determined as

1. solve the global optimal control problem

\[
(G) \quad \xi^* = \text{argmax}(\xi \mapsto E(G(\xi)))
\]

2. determine the EUA price as marginal contribution of the allowance to lower the expected penalty payment

\[
A_t^* = \pi E(1_{\{\Gamma - \Pi(\xi^*) \geq 0\}} | F_t) = -\frac{\partial}{\partial x} E(\pi(\Gamma - \Pi(\xi^*) - x)^+ | F_t)
\]
Equilibrium strategies are

1. \( (\theta^i_t = 0)_{t=0}^{T-1} \) since trading martingale does not pay

2. \( \theta^i_T = \Gamma^i - \sum_{t=0}^{T-1} \xi^*_t - \frac{(\Gamma - \Pi(\xi^*))}{N} \) /N

agent’s need

markets need

ideal change in physical position is \( \Gamma^i - \sum_{t=0}^{T-1} \xi^*_t \). However the position change differs from the desirable by \( (\Gamma - \Pi(\xi^*))/N \) which is OK, since

if the market is short \( \Gamma - \Pi(\xi^*) \geq 0 \) then \( A^*_T = \pi \)

if the market is long \( \Gamma - \Pi(\xi^*) < 0 \) then \( A^*_T = 0 \).
More insights

- trading allowances is not important, it suffices to adjust the final position
- applying fuel switching is very important, here the EUA price is essential. It turns out that \((A_t^*)_{t=0}^T\) is the exercise boundary of the local control problem. Meaning that following the rule

\[
A_t^* > \mathcal{E}_t^i \implies \text{apply fuel switching}
\]

\[
A_t^* < \mathcal{E}_t^i \implies \text{do not apply fuel switching}
\]

yields \((\xi_t^*)_{t=0}^{T-1}\) which solves \((\mathbf{I}(A^*))\).
To see the exercise boundary

a re-parameterization helps

introduce virtual trading \((\vartheta^i_t)_{t=0}^T\) as

\[
\vartheta^i_t = \theta^i_t + \sum_{s=0}^{t} \xi^i_s, \quad t = 0, \ldots, T
\]

then

\[
\sum_{t=0}^{T-1} \vartheta^i_t (A_{t+1} - A_t) - \vartheta^i_T A_T - \pi (\Gamma^i - \vartheta^i_T)^+ + \sum_{t=0}^{T-1} \xi^i_t (A_t - \mathcal{E}_t^i)
\]

\[
=: I^{\vartheta^i, \xi^i, i}(A) = I^{\theta^i, \xi^i, i}(A)
\]
In this new parameterization

equilibrium is characterized by such $A^*$ where the solution to

$$\bar{I}(A) \ (v^i(A), \xi^i(A)) = \text{argmax} \left((\theta^i, \xi^i) \mapsto E(\bar{I}^\theta^i, \xi^i, i(A))\right)$$

satisfies

$$\sum_{i=1}^{N} v^i_t(A^*) = \sum_{i=1}^{N} \sum_{s=0}^{t} \xi^i_s(A^*) \quad t = 0, \ldots, T.$$
Formal results

introduce process spaces

\[ \mathcal{L}_0 := \{(l_t)_{t=0}^{T-1} : \text{all real-valued process}\} \]
\[ \mathcal{L}_1 := \{(l_t)_{t=0}^{T-1} \in \mathcal{L}_0 : E(|l_t|) < \infty\} \]
\[ \mathcal{U}_i := \{\xi_t^{i})_{t=0}^{T-1} \in \mathcal{L}_0 : [0, \lambda_i]-\text{valued}\} \]
\[ \mathcal{U} := \times_{i=1}^{N} \mathcal{U}_i \]

assume that

\[ \Gamma^i \text{ is integrable for each } i = 1, \ldots, N \]
\[ P(\Gamma | \mathcal{F}_{T-1}) \text{ possesses no point masses} \]
Existence of the equilibrium

(i) Given fuel switching prices $\mathcal{E} \in \mathcal{L}_1^N$, there exists a solution $\xi^* \in \mathcal{U}$ to the optimal control problem

$$E(G(\xi^*)) = \sup_{\xi \in \mathcal{U}} E(G(\xi)).$$

(ii) For $\xi^* \in \mathcal{U}$ market equilibrium $(A^*, \vartheta^*, \xi^*)$ is

$$A_t^* = \pi E(1\{\Gamma - \Pi(\xi^*) \geq 0\} \mid \mathcal{F}_t) \quad \text{for } t = 0, \ldots, T$$

$$\vartheta^*_t = \sum_{s=0}^t \xi^*_s \quad \text{for all } i = 1, \ldots, N, t = 0, \ldots, T,$$

$$\vartheta^*_{T} = \Gamma^i - \sum_{s=0}^{T-1} \xi^*_s - (\Gamma - \Pi(\xi^*)) / N.$$
Proof idea

(1) Banach-Alaoglu

(2) Optimality of $\vartheta^*$ for $\bar{I}(A)$: straightforward

(3) Optimality of $\xi^{i*}$ for $\bar{I}(A)$: Change the strategy $\xi^*$ only at time $t$ only for agent $i$ from $\xi^*_t$ to $\lambda \in [0, \lambda^i]$ giving $\xi(\lambda, i) \in \mathcal{U}$. Then we have

$$E(G(\xi^*)|\mathcal{F}_t) - E(G(\xi(\lambda, i))|\mathcal{F}_t) \geq 0$$

since otherwise we could improve $\xi^*$ by $1_M \xi_s + 1_{\Omega \setminus M} \xi^*_s$

with $M := \{E(G(\xi^*)|\mathcal{F}_t) < E(G(\xi(\lambda, i))|\mathcal{F}_t)\}$. 
Optimality of $\xi^{i*}$ individual optimization

now discuss the difference $G(\xi^*) - G(\xi(\lambda, i))$

$-(\xi_t^* - \lambda)\mathcal{E}_t - \pi \left( (\Gamma - \Pi(\xi^*))^+ - (\Gamma - \Pi(\xi^*) + (\xi_t^{i*} - \lambda))^+ \right)$

conditioned on $\mathcal{F}_t$, divided by $|\xi_t^{i*} - \lambda|$, this gives

$0 \leq -\frac{\xi_t^{i*}(\omega) - \lambda}{|\xi_t^{i*}(\omega) - \lambda|} \mathcal{E}_t(\omega)$

$-\pi E\left( \frac{(\Gamma - \Pi(\xi^*))^+ - (\Gamma - \Pi(\xi^*) + (\xi_t^{i*}(\omega) - \lambda))^+}{|\xi_t^{i*}(\omega) - \lambda|} \right) |\mathcal{F}_t)(\omega)$

for all $\omega \in \tilde{\Omega}$ with $\lambda \neq \xi_t^{i*}(\omega)$
Discuss left limit

For $\xi_t^*(\omega) \in ]0, \lambda_i]$, the left limit $\lambda \uparrow \xi_t^*(\omega) \in ]0, \lambda_i]$ with

$$-\mathcal{E}_t^i + \pi E \left( \frac{1_{\{\Gamma - \Pi(\xi^*) \geq 0\} | \mathcal{F}_t\} (\omega)}{A_t^* (\omega)} \right) \geq 0$$

giving

$$\{ \xi_t^* \in ]0, \lambda^i] \} \subseteq \{ A_t^* - \mathcal{E}_t^i \geq 0 \} \iff \{ A_t^* - \mathcal{E}_t^i < 0 \} \subseteq \{ \xi_t^* = 0 \}$$
Discuss right limit

For $\xi^*_t(\omega) \in [0, \lambda_i[$, the right limit $\lambda \downarrow \xi^*_t(\omega) \in ]0, \lambda_i]$ with

$$E_{it}^i - \pi E \left( \frac{1_{\{\Gamma - \Pi(\xi^*) > 0\}} \mid \mathcal{F}_t}{A^*_t(\omega)} \right)(\omega) \geq 0$$

giving

$$\{\xi^*_t \in [0, \lambda^*_i]\} \subseteq \{A^*_t - E^i_t \leq 0\} \iff \{A^*_t - E^i_t > 0\} \subseteq \{\xi^*_t = \lambda^*_i\}$$
Fuel switch price process

Changes fuel in electricity production from coal to gas

- emits less carbon per MWh electricity
- causes costs per ton of saved carbon

Popular fuel switch hard coal $\leftrightarrow$ CCGT turbine

causes costs at time $t$ in EURO per Ton CO$_2$

$$
\mathcal{E}_t = \frac{h_g G_t - h_c C_t}{e_g - e_c} \quad \text{for all } t = 0, \ldots, T - 1
$$

- $h_g, h_c$ heating rates $e_g, e_c$ emission rates
- $G_t, C_t$ spot prices for gas and coal

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EUA versus fuel switching price

no correlation?

\[ e^i_g = 0.388 \frac{t_{CO_2}}{MWh_{el}} \quad e^i_c = 0.897 \frac{t_{CO_2}}{MWh_{el}} \quad \text{(CCGT)} \]

NBP gas, McCloskey’s Coal price index
Fuel switching price $\mathcal{E}(t) = P(t) + X(t)$

\[ P(t) = a + bt + \sum_{j=0}^{2} c_j \cos(2\pi \varphi_j t + l_j) \]

<table>
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<th>$a$</th>
<th>$b$</th>
<th>$c_0$</th>
<th>$\varphi_0$</th>
<th>$l_0$</th>
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<td>7.62</td>
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<td>0.55</td>
<td>2</td>
<td>1.14</td>
<td>1.11</td>
<td>3</td>
<td>3.24</td>
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\[ dX(t) = \gamma(\alpha - X(t))dt + \sigma dW(t) \]

<table>
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<tr>
<th></th>
<th>$\gamma$</th>
<th>$\alpha$</th>
<th>$\sigma$</th>
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<tr>
<td>Value</td>
<td>31.82</td>
<td>-0.12</td>
<td>68.24</td>
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(time unit = year)
Parameter fit for $\mathcal{E}(t) = P(t) + X(t)$
Solution to \( G(\xi^*) = \sup_{\xi \in \mathcal{U}} G(\xi) \)

given \((\mathcal{E}_t)_{t \in [0,T]}\), put \((\Gamma(t) := m + \nu W'(t)))_{t \in [0,T]}\), apply

time and space discretization giving trinomial forest,
solve optimal control problem by backward induction

identify \((A_t^*)_{t=0}^T\) as the exercise boundary
What are carbon price drivers?

long-term fuel switch prices $E(\sum_{t=0}^{T-1} \epsilon_t)/T \leftarrow \alpha$

long-term fuel switch price variance $\sigma^2/(2\gamma) \leftarrow \sigma$

recent fuel switch price $\leftarrow \epsilon_t$

expected allowance demand $E(\Gamma|\mathcal{F}_t) \leftarrow \delta_t$
What are carbon price drivers?

long-term fuel switch prices \( E(\sum_{t=0}^{T-1} \varepsilon_t)/T \) ← \( \alpha \) \ YES

long-term fuel switch price variance \( \sigma^2/(2\gamma) \) ← \( \sigma \) ?

recent fuel switch price ← \( \varepsilon_t \) ?

expected allowance demand \( E(\Gamma | F_0) \) ← \( \delta_t \) \ YES
The impact of $\alpha$ and $\sigma$

Here $2005 \sim \{0, \ldots T\}$. On the left: the impact of $\alpha$ and $\sigma$ on allowance price, through long-term fuel switch price mean $E(\sum_{t=0}^{T-1} \epsilon_t)/T$ and variance $\sigma^2/(2\gamma)$ respectively. On the right: the impact of $\nu$ and $E(\Gamma)/(\lambda T)$.
The impact of $\mathcal{E}_t$ and $\delta_t = \frac{E(\Gamma|\mathcal{F}_t) - \sum_{s=0}^{t} \xi_s}{\lambda(T-t)}$

Here $2005 \sim \{0, \ldots T\}$. The dependence of allowance price on $\delta_t$ and $\mathcal{E}_t$ for different times (right: $t = t_1$ beginning of March, left: $t = t_2$ beginning of September

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Emission to cap indicator published by Point Carbon reflects the expected allowance demand.
The probability of non-compliance (left) and initial allowance price (right) depending on penalty size and fuel switch demand. If demand is above 50%, then decrease in non-compliance probability is expensive.
Expected payoff of a European call plotted against $\mathcal{E}_t$ and $\delta_t$

with strike price 35 EURO and maturity at the beginning of October at $t = t_1$ beginning of March (left), $t = t_2$ beginning of September, (right)
Conclusion

• carbon price depends on two factors
  – demand for carbon emission
    temperature, rainfall, power plant outages
  – costs of abatement strategies
    in short term fuel switch, in long term other emission reduction projects

• their impact is non-trivial, effected through solution of an optimal control problem

• quantitative understanding is possible and essential for both, regulators and risk managers