Area Yield Futures and Futures Options: Risk Management and Hedging.

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Abstract

• Imagine there exist markets for yield futures contracts as well as ordinary price futures contracts.

• Intuitively one would think that a combined use of yield futures contracts and price futures contracts ought to provide a reasonable strategy for insuring revenue.

• In the paper this idea is made precise. It is shown that revenue can be secured in by a combined replication of these two contracts.
• The relevant dynamic strategy is characterized. It depends only on *observable* price information in these two separate markets, not on "unobservables", like parameters in utility functions of the agents involved.

• The identified dynamic strategy is, under certain conditions, equivalent to optimal revenue insurance. Only market risk is considered.

• The ability to trade continuously can not be dispensed with. These results should of relevance, since markets for crop yield futures and options have been established.
Introduction

• Area yield crop insurance, where the index is based on average yield in a given geographical area, has been offered in India, Brazil, Canada and the USA.

• Parametric insurance (e.g., rainfall insurance) has been proposed in Canada, India and Mexico. A livestock mortality index has been recently designed to cover herders against livestock losses in Mongolia.

• In 1995, the Chicago Board of Trade (CBOT) launched its Crop Yield Insurance (CYI) Futures and Options contracts, but at the moment there is no trade in these contracts. Another example is Nord Pool (The Nordic Power Exchange).

• Crop Yield Insurance contracts are designed to provide a hedge for crop yield risk.
In the following we abstract from production costs, and assume zero local price basis (i.e., local cash price equals futures price) and zero yield basis (i.e., individual farm yield equals index yield).

This is to say, we only address market risk, not idiosyncratic risk. We also ignore asymmetric information.

There is a large literature on non-market based risk management and insurance of crop yield, using mostly a one period, expected utility framework.

Yield contracts have been analyzed from the perspective of hedging, using a mean variance approach by Vukina, Li and Holthausen 1996. Minimizing the variance of revenue was the objective in Li and Vukina 1998.
The model

- Consider two futures markets, one where yield futures options are traded, and one where standard price futures options are traded.

- The quantity index $q(t)$ at time $t$ is measured in bushels per acre.

- The spot price $p(t)$ at time $t$ is measured in $ per bushel.

- The revenue $R(t) = q(t)p(t)$.

- A yield futures contract specifies that the payoff of $q(T)$ bushels per acre at time $T$, has market price at time $t < T$ given by

\[ F_t^q = E_t^Q (q(T) \cdot c). \]  

(1)
• We could alternatively consider an option on the futures index. (This is outlined in the paper.)

• The constant $c$ signifies a conversion factor measured in $\text{\$ per bushel}$, so that the futures price is measured in $\text{\$ per acre}$.

• For example, for the Iowa Corn Yield Insurance Futures (ticker symbol CA) the unit of trading is the Iowa yield estimate times $\text{\$100}$ (e.g., a yield of 140.3 bushels per acre gives a contract value of $\text{\$14,030}$).

• We set this conversion factor equal to 1 without loss of generality.
• Similarly an ordinary futures option contract on price is given by
\[ F_t^p = E_t^Q(p(T)) \]
measured in $ per bushel.

• The linear pricing rule of quantity futures implied by the expression (1) is, of course, far from obvious.

• In addition to the usual frictions in ordinary futures markets, like no short sale possibilities of the crop, an additional difficulty arises here, since the index \( q \) is not a traded asset.
• In Aase 2004 this is resolved by considering the quantity $s = pq$ and identifying $s$ as a spot price process.

• Based on the price processes $s$ and $p$ a no-arbitrage model is constructed as permitted by financial theory, where $s$ is identified as the spot price of a leasing contract of agrarian land for the crop in the particular region of consideration.

• This solves, at least in theory, the pricing problem of these contracts.

• In practice the hedging resulting from this use of the different markets may not be entirely accurate, but then one should perhaps have in mind that the only “perfect hedge” is found in a Japanese garden.
Turning to the dynamics of the two processes $p$ and $q$, we assume that the process $q$ for quantity and $p$ for price are both defined as follows:

$$dq(t) = \mu_q(t)dt + \sigma_q(t)dB(t)$$  \hspace{1cm} (2)

Similarly

$$dp(t) = \mu_p(t)dt + \sigma_p(t)dB(t),$$  \hspace{1cm} (3)

where $B(t) = (B_1(t), B_2(t))$ is a standard two dimensional Brownian motion process.
The main result

• Consider the product contracts of the form \( R(t) = p(t)q(t) \).

• We want to investigate whether we can lock in a prespecified “revenue” \( R(t) \) at any time \( t \) prior to the expiration time \( T \) by dynamically trading in the two separate futures options markets described above.

• To this end imagine first that a separate market for this type of “revenue” were available. The futures price of this contract we denote by \( F_t^{q,p} \), and it must be given as follows under our assumptions:

\[
F_t^{q,p} = E_t^Q \{ q(T) \cdot p(T) \}, \quad 0 \leq t \leq T.
\] (4)
• Notice that this can be written
\[ E^Q_t \{ q(T) \cdot p(T) - F^*_t \cdot p \} = 0, \quad 0 \leq t \leq T, \] 
the usual starting point for analyzing futures contracts.

• Equation (5) implies that if the futures price \( F^*_t \cdot p \) is agreed upon at time \( t \), then no money changes hands when the futures position is initiated.

• Recall the main features of a simple futures contract on, say, price. For the holder of one long contract, the payoff at expiration is
\[ \int_t^T 1 \cdot dF_s = F_T - F_t = p_T - F_t \] 
by the principle of convergence in the futures market, where \( F_t \) is the futures price of one contract at time \( t \).
• If an agent holds $\theta_s$ futures contracts at time $s$ in the time interval $(t, T]$, the resettlement gain at time $T$ from this strategy would similarly be
\[
\int_t^T \theta_s dF_s. \tag{7}
\]

• Consider a strategy that holds $F^q_s$ futures options on $p_T$, and $F^p_s$ futures options on $q_T$ at each time $s$ between $t$ and $T$.

• The resettlement gain from this strategy is given by
\[
\int_t^T F^q_s \, dF^p_s + \int_t^T F^p_s \, dF^q_s. \tag{8}
\]

• Using stochastic integration by parts, this can be written
\[
= q_T p_T - (F^q_t F^p_t + \int_t^T dF^q_s \, dF^p_s). \tag{9}
\]
• Returning to the basic equation (5), the starting point for analyzing futures contracts, consider the equality

\[ E_t^Q (q_T p_T - (F_t^q F_t^p + \int_t^T dF_s^q dF_s^p)) = 0. \]  

(10)

• If this is true, it would mean that the dynamic strategy given in (8) is equivalent to a futures contract on the product \( p_T q_T \) with the associated futures price

\[ F_t^{q,p} = E_t^Q (F_t^q F_t^p + \int_t^T dF_s^q dF_s^p). \]  

(11)

• We can demonstrate that the equality (10) indeed holds true under mild conditions.

• We assume the futures price processes \( F_t^p \) and \( F_t^q \) can both be written as smooth functions \( a(p_t, t) \) and \( b(q_t, t) \) respectively.
• Denote by \( a_p(p, t) \) the partial derivative of the function \( a(p, t) \) with respect to its first argument, and similarly for \( b_q(q, t) \).

**Theorem 1** Consider the resettlement gain from the strategy given in (8). This strategy is equivalent to a futures price directly on revenue \( p_T q_T \) with associated futures price given in (11), which can also be written

\[
F_{t}^{q \cdot p} = F_{t}^{q} F_{t}^{p} + E_{t}^{Q} \left( \int_{t}^{T} dF_{s}^{q} dF_{s}^{p} \right) 
= F_{t}^{q} F_{t}^{p} + E_{t}^{Q} \left( \int_{t}^{T} a_p(p_{s}, s)(\sigma_p(s) \cdot \sigma_q(s))b_q(q_{s}, s) \, ds \right). 
\]

• In the special case of zero correlation rate between yield and price, i.e., \( \sigma_p(s) \cdot \sigma_q(s) = 0 \) for all \( s \in (t, T] \), this strategy is equivalent to a futures contract on the product \( p_T q_T \) with futures price \( F_{t}^{q \cdot p} = F_{t}^{q} F_{t}^{p} \).
• **Proof:** According to (5) we have to show that

\[
E^Q_t\left(q_T p_T - (F^q_t F^p_t + \int_t^T dF^q_s dF^p_s)\right) = 0,
\tag{14}
\]

and this follows, since

\[
E^Q_t\left(\int_t^T F^q_s dF^p_s + \int_t^T F^p_s dF^q_s\right) = 0,
\tag{15}
\]

by standard properties of stochastic integrals.

• Thus we get the conclusion from the expression for the futures price in equation (4), the fact that $F^q_t$ and $F^p_t$ are both $\mathcal{F}_t$-measurable, and from the representations for the stochastic processes $a(p_t, t)$ and $b(q_t, t)$. 
• The above results show that there is no need for a specialized futures market of, say, revenue \( R = pq \) for someone who has access to the two separate markets for price and yield contracts.

• One can then, at least in principle, achieve exactly the same results in terms of risk management by simultaneous, dynamic trade in these two markets. Since a dynamic strategy is then needed, needless to say, we here abstract from transactions costs.

• In the situation where the correlation \( \sigma_{p,q}(s) := \sigma_p(s) \cdot \sigma_q(s) = 0 \) for all \( s \in (t, T] \), the corresponding futures price becomes particularly simple, namely the product of the corresponding futures prices \( F_t^q \) and \( F_t^p \).
Suppose, on the other hand, that this correlation is positive. The last term in the futures price formula is accordingly positive, which raises the futures price.

This seems reasonable due to the increased risk this situation represents compared to the one with a zero covariance function: If the harvest is poor, the price is also low on the average, both contributing to a smaller revenue.

If the corresponding correlation is negative, which is rather natural of this quantity, a farmer would typically not be willing to pay quite as much for this ”insurance coverage” as in the two other cases, confirmed by the equations (12)
Examples and Discussion

• The results of Theorem 1 do not depend on any specific assumptions about utility functions of the agents (except from some obvious axioms, like agents prefer more to less).

• The result is that a futures market for revenue can be obtained through the combination of the two markets for yield and price futures.

• There exists a dynamic replication strategy in quantity futures and price futures which is equivalent to a futures contract on revenue.

• Moreover, this strategy can be obtained directly from futures price information in these two separate markets.
• There are no parameters to estimate, no assumptions about the relative risk aversion, or the subjective interest rate, or anything like that. Thus this result ought to be of practical interest.

Example 1.

• The strategy \((-F^q, -F^p)\) duplicates exactly the payoff \((F_t^R - qT_p T)\) from one short “revenue” contract: At the initiation time \(t\) the futures prices \(F^q_t\) and \(F^p_t\) are both set such that no money changes hands.

• Instead of the proposed resettlement strategy consider the buy and hold strategy that sells \(F^p_t\) quantity contracts, priced at \(F^q_t\) at time \(t \leq T\), and holds this position until maturity, and sells \(F^q_t\) price contracts, priced at \(F^p_t\) at time \(t \leq T\), and holds this position till maturity as well.
The payoff at expiration for the hypothetical contract on revenue would be \((F_t^R - q_T p_T)\), for an agent selling one such “revenue” contract.

On the other hand, the combined contracts described above would yield the following payoff:

\[
(F_t^p - p_T)F_t^q + (F_t^q - q_T)F_t^p,
\]

where the first term is the payoff of \(F_t^q\) short futures contracts on price \(p\), and the second term is the corresponding payoff of \(F_t^p\) short contracts on quantity \(q\).

This latter sum can be seen to be equal to

\[
(F_t^R - q_T p_T) + (F_t^q - q_T)(F_t^p - p_T),
\]

in the situation where \(F_t^R = F_t^q F_t^p\), e.g., when the cross-correlation rate is zero.
• Since \( F_t^q (F_t^p) \) can be considered as an “economic forecast” of \( q_T (p_T) \) at time \( t \), the remainder term in (16) should be “small of second order” (it goes to zero faster than the first term in (16) as \( t \) approaches \( T \)), in which case this strategy may function reasonably close to a hypothetical futures market for revenue.

• Of course, this latter ideal market does not exist, so this simple arrangement of combining existing markets for quantity and price separately may be a reasonable substitute.

• The strategy described by our continuous resettlement strategy constitutes, on the other hand, a perfect substitute in this situation, as well as in the situation when the associated cross correlation is different from zero. \( \square \)
• Assuming no transaction costs, basis risk and correlation between yield and price, Mahul and Wright 2003 characterize the Pareto optimal indemnity payoff net of the premium for any risk averse agent, and risk neutral insurer.

• It is shown to be \( F_t^R - q_T p_T \). This argument requires risk neutral pricing, which in our model amounts to equating the risk adjusted probability measure \( Q \) and the given one \( P \).

• This really follows from standard (Pareto) optimal risk sharing theory.

• As a consequence of this, market prices are determined as
\[
F_t^R = E_t(p_T) E_t(q_T),
\]
and the optimal revenue insurance has payoff
\[
E_t(p_T) E_t(q_T) - q_T p_T.
\]
• From the relation in (16) it is seen that this payoff results, but in addition there is the remainder term mentioned above. The latter is caused by the *sell and hold* strategy.

• If the *dynamic* resettlement strategy is used instead, the correction term vanishes, and the optimal payoff is exactly achieved.

• This demonstrates a connection to optimal insurance coverage, showing that the Pareto optimal net indemnity payoff can be replicated by using separate yield and price futures contracts.
• In the zero cross correlation case of the above example, this also gives us the rare opportunity of finding an expression for the hedging error, the last term in the expression (16), the departure from the Pareto optimal contract.

• This departure is resulting from merely using the sell and hold strategy when the dynamic replication strategy is indeed optimal.

• The rewards from being able to trade continuously are here brought forward in an explicit way.
Conclusions

• We have presented a dynamic model for the analysis of futures contracts on quantity and futures contracts on price in separate markets for such contracts, in order to construct futures contracts on revenue. Only market risk is considered.

• Specifically, we have demonstrated how an agent can lock in a certain revenue by a combined trade in futures price and futures yield contracts, abstracting from production costs.

• This can be done perfectly if a certain dynamic strategy is used, identified in the paper. This strategy depends only on futures prices observed in the two different markets for price and yield futures, and not on the particular choice of model for the random dynamics.
• In consequence the result is independent of a complete market structure, and thus fairly robust. The identified dynamic strategy is, under certain conditions, equivalent to a Pareto optimal revenue insurance.

• Our results do not depend upon any specific assumptions about utility functions, relative risk aversions, subjective discount rates, or other model parameters.

• Provided one takes into account transactions costs in a manner that is customary when hedging derivatives, it should be possible to implement the main result in practice.