B.Sc./Grad. Dip.: Probability Models and Time Series

Examples 1

1. Suppose that the random variables $X$ and $Y$ have joint probability density function (p.d.f.) given by

$$f_{(X,Y)}(x,y) = \begin{cases} 
  x + y & 0 < x < 1, \ 0 < y < 1 \\
  0 & \text{otherwise}
\end{cases}.$$

(a) State the range of $X$, i.e. $R_X$.
(b) For $x \in R_X$, find an explicit expression for $f_X(x)$.

2. Suppose that

$$p_{(X,Y)}(m,n) = \begin{cases} 
  e^{-(\lambda+\mu)\frac{\lambda^m\mu^n}{m!n!}} & m = 0,1,\ldots, n = 0,1,\ldots \\
  0 & \text{otherwise}
\end{cases}.$$

(a) State the ranges of $(X,Y)$, $X$ and $Y$, i.e. $R_{(X,Y)}$, $R_X$ and $R_Y$.
(b) Are $X$ and $Y$ independent? Justify your answer.

3. Suppose that the random variables $X$ and $Y$ have joint p.m.f. $p_{(X,Y)}(\cdot,\cdot)$ given by

$$p_{(X,Y)}(x,y) = \begin{cases} 
  0.4 & \text{for } (x,y) = (0,0) \\
  0.2 & \text{for } (x,y) = (0,1) \\
  0.1 & \text{for } (x,y) = (1,0) \\
  0.3 & \text{for } (x,y) = (1,1) \\
  0 & \text{otherwise}
\end{cases}.$$

(a) Find the values of $p_X(0)$, $p_X(1)$, $p_Y(0)$ and $p_Y(1)$.
(b) Are $X$ and $Y$ independent? Justify your answer.
(c) Show that $p_{Y|X}(0|0) = \frac{2}{3}$ and $p_{Y|X}(1|0) = \frac{1}{3}$.
(d) Find the value of $E[Y|X = 0]$. 