Solutions 0

1. (a) $A \cap (B \cup C \cup D)^c = A \cap (B^c \cap C^c \cap D^c)$.  
(b) $(A \cup B \cup C \cup D)^c = A^c \cap B^c \cap C^c \cap D^c$.  
(c) $A \cap B \cap C \cap D$.  
(d) $A^c \cup B^c \cup C^c \cup D^c = (A \cap B \cap C \cap D)^c$.  
(e) $(A \cap B \cap C) \cup (A \cap C \cap D) \cup (B \cap C \cap D) \cup (A \cap B \cap D)$.  
(f) $(A^c \cap B \cap C \cap D) \cup (A \cap B^c \cap C \cap D) \cup (A \cap B \cap C \cap D^c)$.  
(g) $(A \cap B^c \cap C^c \cap D^c) \cup (A^c \cap B \cap C^c \cap D^c) \cup (A^c \cap B^c \cap C \cap D^c) \cup (A^c \cap B^c \cap C^c \cap D)$.  
(h) $(A \cap B^c \cap D^c) \cup (A^c \cap B \cap D^c)$.

2. Let $A$ be the event that the randomly selected student uses Shotmail, and let $B$ be the event that the student uses Payserve. Then

$\mathbb{P}(A) = 0.5, \quad \mathbb{P}(B) = 0.3, \quad \mathbb{P}(A \cap B) = 0.25$

(a) Need to find $\mathbb{P}(A \cup B)$. In fact,

$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B) = 0.5 + 0.3 - 0.25 = 0.55$.

Hence, statement is TRUE.

(b) $\mathbb{P}((A \cap B)^c) = 1 - \mathbb{P}(A \cap B) = 1 - 0.25 = 0.75$. TRUE.

(c) $\mathbb{P}((A \cup B)^c) = 1 - \mathbb{P}(A \cup B) = 1 - 0.55 = 0.45$. FALSE.

(d) $\mathbb{P}((A \cup B) \setminus (A \cap B)) = \mathbb{P}(A \cup B) - \mathbb{P}(A \cap B) = 0.55 - 0.25 = 0.3$, where the 1st equality follows from the fact that $A \cap B \subseteq A \cup B$. FALSE.

3. (a) Set of outcomes whose sum is divisible by 4:

$A = \{(1, 3), (2, 2), (2, 6), (3, 1), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6)\}$.

$\mathbb{P}(A) = \frac{\#A}{36} = \frac{9}{36} = \frac{1}{4}$.

(b) Set of outcomes in which a 3 turns up exactly once:

$B = \{(3, 1), (3, 2), (3, 4), (3, 5), (3, 6), (1, 3), (2, 3), (4, 3), (5, 3), (6, 3)\}$.

$\mathbb{P}(B) = \frac{10}{36} = \frac{5}{18}$.

(c) Set of outcomes whose sum is 5:

$C = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$.
\[ \mathbb{P}(C) = \frac{4}{36} = \frac{1}{9}. \]

(d) Set of outcomes for which both numbers are even:
\[ D = \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}. \]
\[ \mathbb{P}(D) = \frac{9}{36} = \frac{1}{4}. \]

4.
Let \( A \) be the event that the sum is 4.
Let \( B \) be the event that the sum is \( \geq 4 \).
\[ \mathbb{P}(A) = \mathbb{P}(\{(1, 3), (2, 2), (3, 1)\}) = \frac{3}{25}. \]
\[ \mathbb{P}(B) = \mathbb{P}(\Omega \setminus \{(1, 1), (1, 2), (2, 1)\}) = \mathbb{P}(\Omega) - \mathbb{P}(\{(1, 1), (1, 2), (2, 1)\}) = 1 - \frac{3}{25} = \frac{22}{25} > 0. \]
\[ A \cap B = \{(1, 3), (2, 2), (3, 1)\}. \]
\[ \mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{3/25}{22/25} = \frac{3}{22}. \]

5.
\( A \) and \( B \) independent \( \Rightarrow \mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B). \)
(a) \[ \mathbb{P}(A^c \cap B) = \mathbb{P}(B \setminus (A \cap B)) = \mathbb{P}(B) - \mathbb{P}(A \cap B) = \mathbb{P}(B) - \mathbb{P}(A)\mathbb{P}(B), \]
where the last equality follows by the independence of \( A \) and \( B \); hence \[ \mathbb{P}(A^c \cap B) = (1 - \mathbb{P}(A))\mathbb{P}(B) = \mathbb{P}(A^c)\mathbb{P}(B). \]
(b) \[ \mathbb{P}(A^c \cap B^c) = \mathbb{P}((A \cup B)^c), \]
by De-Morgan’s Laws, which is equal to
\[ 1 - \mathbb{P}(A \cup B) = 1 - \{\mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)\} = 1 + \mathbb{P}(A \cap B) - \mathbb{P}(A) - \mathbb{P}(B), \]
which, by independence of \( A \) and \( B \), is equal to
\[ 1 + \mathbb{P}(A)\mathbb{P}(B) - \mathbb{P}(A) - \mathbb{P}(B) = (1 - \mathbb{P}(A))(1 - \mathbb{P}(B)) = \mathbb{P}(A^c)\mathbb{P}(B^c). \]