Statistics: Theory and Practice

Examples 6

Please do acquaint yourself with the following useful result:

\[
s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1}{n-1} \left\{ \sum_{i=1}^{n} x_i^2 - n\bar{x}^2 \right\}.
\]

1. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the $N(\mu, \sigma^2)$ distribution, where $\sigma^2$ is unknown.

   Construct a test at significance level $\alpha$ for
   
   $H_0 : \mu = \mu_0$ versus $H_1 : \mu > \mu_0$.

2. For business reasons, a certain watch battery is designed to last for 11.7 months. A random sample of nine watch-batteries were observed to have the following life-times (in months):

   \begin{align*}
   11.7 & \quad 12.2 & \quad 10.9 & \quad 11.4 & \quad 11.3 & \quad 12.0 & \quad 11.1 & \quad 10.7 & \quad 11.6.
   \end{align*}

   (a) By devising an appropriate test, determine whether these data are consistent with the intended life-time of 11.7 months.

   (b) Test the proposition that a life-time of 11.7 months is actually over optimistic at the 5% and 10% levels of significance?

3. A new type of wing design for a particular aircraft has been developed that is claimed to increase its top cruising speed. Tests were carried on aircraft of both the old and new designs (18 different aircraft), yielding the following cruising speeds (in knots).

   OLD WING:
   \begin{align*}
   416 & \quad 400 & \quad 426 & \quad 431 & \quad 432 & \quad 404 & \quad 398 & \quad 403 & \quad 422
   \end{align*}

   NEW WING:
   \begin{align*}
   426 & \quad 418 & \quad 424 & \quad 438 & \quad 440 & \quad 421 & \quad 412 & \quad 409 & \quad 427.
   \end{align*}

   Assuming that the data corresponding to the old and new wing designs constitute independent random samples, test the claim at the 10% level of significance.