Statistics: Theory and Practice

Examples 4

1. Let $X$ be the time it takes, starting from midnight, for a total of 3 buses to leave Trafalgar Square. It is assumed that $X$ has the following probability density function

$$f_X(x) = \begin{cases} 
108x^2e^{-6x} & x > 0 \\
0 & \text{o.w.}
\end{cases}$$

(a) Find the moment generating function, $M_X(t)$, of $X$ (for $t$ in a suitably defined range).
(b) Compute $M_X^{(3)}(t)$ and hence find $E[X^3]$.

2. Suppose that the joint density function of $X$ and $Y$ is given by

$$f_{(X,Y)}(x, y) = \begin{cases} 
\frac{1}{2} & 0 < x < y < 2 \\
0 & \text{o.w.}
\end{cases}$$

(a) Find the marginal probability density functions of $X$ and $Y$.
(b) Are $X$ and $Y$ independent? Justify your answer.