Examples 7

1. Suppose that $X_1, X_2, \ldots, X_5 \sim N(\mu_X, \sigma_X^2)$ and $Y_1, Y_2, \ldots, Y_8 \sim N(\mu_Y, \sigma_Y^2)$ are independent random samples, with the following observed statistics:

\[
\sum_{i=1}^{5} x_i = 42.3, \quad \sum_{i=1}^{5} x_i^2 = 527.3, \quad \sum_{j=1}^{8} y_j = 46.2, \quad \sum_{j=1}^{8} y_j^2 = 783.7
\]

Find
(a) 99% C.I.’s for $\mu_X$ and $\mu_Y$
(b) a 95% C.I. for $\sigma_X^2$/$\sigma_Y^2$
(c) assuming $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, a 90% C.I. for $\mu_X - \mu_Y$.

2. Let $X$ and $Y$ equal the number of days of utilization by a user of a Brand ’X’ toothbrush and by a user of a Brand ’Y’ toothbrush respectively. It is assumed that $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$.
A random sample for brand ’X’ yielded the following observations:

\[
67 \quad 56 \quad 61 \quad 51 \quad 65 \quad 61 \quad 80
\]

and for brand ’Y’:

\[
55 \quad 64 \quad 56 \quad 53 \quad 48 \quad 62 \quad 56 \quad 53 \quad 56 \quad 64
\]

It is assumed that the two samples are independent.
(a) Suggest, with justification, a point estimate for $\mu_X - \mu_Y$.
(b) Assuming that the variability in the usage of the two brands are the same, find a 95% confidence interval for $\mu_X - \mu_Y$.

3. A team of doctors were interested in finding out the effect that a mother’s diet during pregnancy could have on the birth weight of their child.
Two diets were explored: Diet $F$ and Diet $G$.
Observed random samples of birth weights (in k.g.) for children from mothers using $F$ and $G$ were:

\[
1.39 \quad 3.06 \quad 3.12 \quad 2.32 \quad 2.63 \quad 3.23 \quad 3.80 \quad 3.23
\]

and

\[
3.00 \quad 3.71 \quad 2.78 \quad 3.12 \quad 2.75 \quad 3.37 \quad 3.37 \quad 3.65
\]

respectively. It is assumed that the samples are independent.
(a) With justification, suggest a point estimate for the ratio of the variances of birth weights resulting from the two diets.
(b) Assuming that the underlying distributions of the random samples are Normal, construct a 98% interval estimate for the ratio of the variances of birth weights resulting from the two diets.