Examples 6

1. A computing manufacturer constructs floppy disks so that 20 of them are in each packet. A packet is considered to be \textit{unacceptable} if it contains 5 or more defective disks. Each disk has a probability of 0.05 of being defective, independently of all other disks.
   (a) Let the r.v. $Y$ be the number of defective disks in a randomly selected packet. What is the distribution of $Y$?
   (b) Find the exact probability that a randomly selected packet is \textit{acceptable}.
   (c) By using the Poisson approximation to the Binomial distribution, find the \textit{approximate} probability that a randomly selected packet is \textit{acceptable}.

2. Let $X_1, X_2, \ldots, X_n$ be a random sample of size $n = 40$, with population mean $\mu = 6.53$, and population variance $\sigma^2 = 10$.
   By using the \textit{Central Limit Theorem}, find the approximate probability that $\sum_{i=1}^{40} X_i$ lies between 245 and 255.

3. An observed random sample $x_1, x_2, \ldots, x_{10}$ of size 10 is assumed to come from the $N(\mu, \sigma^2)$ distribution. It is found that
   \[ \sum_{i=1}^{10} x_i = 6, \quad \sum_{i=1}^{10} x_i^2 = 24. \]
   Find a 95\% equal tails C.I. for the mean $\mu$, given that
   (a) the variance is known, and assumed to be equal to the observed sample variance,
   (b) the variance is unknown.