Assignment 2

Deadline: Thursday, 8\textsuperscript{th} January, 2009

All questions are weighted equally. Total marks: [20]. Marks are shown in boxes [ ].

1. A roll of a biased die results in a '3' occurring with probability $\frac{1}{5}$. Let $X$ be the number of 3’s that occur in 50 independent rolls of the die.

   (a) What is the exact distribution of $X$? \[1\]
   (b) Using the Poisson approximation to the Binomial distribution, find (as precisely as possible) an approximate value for
   
   i) $\mathbb{P}(5 \leq X \leq 8)$ \[2\]
   ii) $\mathbb{P}(X \geq 13)$. \[2\]

2. Suppose that the joint probability density function (p.d.f.) of $X$ and $Y$ is given by

   $$f_{X,Y}(x,y) = \begin{cases} 
   2e^{-(x+y)} & 0 \leq x \leq y < \infty \\
   0 & \text{o.w.} 
   \end{cases} .$$

   (a) Find the marginal p.d.f of $X$. \[2\]
   (b) Find the conditional p.d.f. of $Y$ evaluated at $y$, given that $X = x$. What is the range of values that $Y$ can take when $X = x$? \[2\]
   (c) Are $X$ and $Y$ independent? Justify your answer. \[1\]

3. Suppose that the random variables $W_1, W_2, \ldots, W_n$ constitute a random sample of size $n = 32$ such that

   $$E[W_i] = 40 \quad \text{var}(W_i) = 8, \quad i = 1, \ldots, n.$$ 

   Using the central limit theorem, find (as precisely as possible), an approximate value for $\Pr(39.75 \leq \bar{W} \leq 41.25)$, where $\bar{W} = \frac{1}{n} \sum_{i=1}^{n} W_i$. \[5\]
4. A random sample of 150 students at a higher education institution for mature students was divided up into those who were on programmes emphasizing left-brain skills (e.g. chemistry, mathematics, physics) and right-brain skills (e.g. creative writing, music, performing arts). The students were also classified according to age-range. The data are presented below.

<table>
<thead>
<tr>
<th>Age-Range</th>
<th>Emphasis</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>LEFT-BRAIN</td>
</tr>
<tr>
<td>21-35</td>
<td>94</td>
</tr>
<tr>
<td>36-55</td>
<td>6</td>
</tr>
<tr>
<td>56 OR OVER</td>
<td>6</td>
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</tbody>
</table>

Is there any evidence in favour of a lack of independence between Age-Range and brain Emphasis of programme of study? Report your conclusions with reference to percentage points at both the 1% and 0.5% levels of significance.

**Important Note:**
- Please read the current version of the *Mathematics & Statistics Coursework Policy*. Copies can be obtained from the course website, or in hardcopy from the programme administrator.