1. Consider the continuous time Markov chain model for a queueing system with two servers \((c = 2)\), arrival rate \(\lambda\) and service rate \(\mu\) per server.

(i) Write down the instantaneous transition rates for this model.
Define the traffic intensity \(\rho\) in terms of the parameters \(\lambda\) and \(\mu\) and state the condition that must be satisfied for an equilibrium distribution to exist.

(ii) Assuming the condition of part (i), find the equilibrium distribution.
Deduce that the Erlang formula for the probability that in equilibrium an arriving customer is delayed reduces to \(2\rho^2/(1 + \rho)\).

(iii) If at any given time point both servers are busy, what is the distribution of the length of time until the next completion of a service time?

(iv) Assuming a first come first served queueing discipline, prove that in equilibrium an arriving customer’s waiting time, i.e., the length of time until he starts being served, is a mixture of a discrete probability atom at zero and an exponential distribution with parameter \(2\mu - \lambda\).