1. Consider a continuous time Markov chain with state space \{0, 1, 2, \ldots, K\}, where \(K\) is some positive integer, and the following transition rates.

<table>
<thead>
<tr>
<th>transition</th>
<th>rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i \rightarrow i + 1)</td>
<td>(\lambda(K-i)) for (0 \leq i \leq K-1)</td>
</tr>
<tr>
<td>(i \rightarrow i - 1)</td>
<td>(\mu i) for (1 \leq i \leq K)</td>
</tr>
</tbody>
</table>

Show that the equilibrium distribution is binomial.

2. Consider a continuous time Markov chain model for a single-server queueing system, in which the customer arrival rate is \(\lambda\) and the service rate is \(\mu\). Assume that there is a waiting room with finite capacity, so that the number of customers in the system (including any customer being served) can never exceed \(K\), and that a customer who arrives to find the queue full (i.e., the system size equal to \(K\)) does not join the queue and is lost to the system.

   (i) Write down the state space and transition rates for the model.

   (ii) Find the equilibrium distribution, distinguishing between the cases \(\lambda \neq \mu\) and \(\lambda = \mu\).

      Deduce that in equilibrium the probability that an arriving customer finds the queue full and is lost to the system is given by

      \[
      \frac{1 - (\mu/\lambda)}{1 - (\mu/\lambda)^{K+1}} \quad \lambda \neq \mu
      \]

      and by

      \[
      \frac{1}{K+1} \quad \lambda = \mu.
      \]

3. Consider the use of Laplace transforms as an alternative method of solving the forward equations for the Poisson process with rate \(\lambda\). Let \(p^*_n(s)\) denote the Laplace transform of \(p_n(t)\), \(n \geq 0\). From the forward equations for the \(p_n(t)\), deduce recurrence relations satisfied by the \(p^*_n(s)\) and hence show that

   \[
   p^*_n(s) = \frac{\lambda^n}{(\lambda + s)^{n+1}} \quad n \geq 0.
   \]

   Check that this is the Laplace transform of

   \[
   p_n(t) \equiv e^{-\lambda t} \frac{(\lambda t)^n}{n!}.
   \]