Quantum Reality Unveiled Through Process and the Implicate Order.

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Abstract

In this paper we discuss the possibility of understanding quantum phenomena such as interference and quantum non-locality (i.e. Bohr's notion of wholeness) by assuming an underlying notion of activity or process rather than the conventional approach which uses fields-in-interaction. By exploiting some ideas of Grassmann, we show how Clifford algebras essential to relativistic electron physics can be generated from first principles. Using Bohm's notion of implicate order, we show how his notion of unfolding leads to what appears to be Heisenberg's equation of motion, the algebraic equivalent to the Schrödinger equation. We briefly discuss what this whole approach means for our wider view of reality.

1 Introduction

Surely anyone who first experiences some of the explanations of quantum phenomena cannot but feel like the little boy who sees the "Emperor's new clothes" for the first time! The professionals have accommodated the more exotic views in the realisation that it is the abstract formalism which actually carries them along. Without that we have very little to work with, but once this formalism is mastered we find a comprehensive agreement with many physical situations. For example it provides an account of the stability of matter, the energy levels of atoms and molecules, interference effects

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produced by atoms and molecules. and many, many other phenomena, some so totally different from what one should expect from our experience of the classical world that it can sometimes stretch credibility. Nevertheless the mathematics works.

What makes it difficult for those wanting an intuitive understanding of quantum phenomena is that the algorithm is an abstract mathematical structure, namely separable Hilbert space. This has been rigourously studied in great detail because, as we have said above, without it there is no explanation of quantum phenomena. But can a physical theory be replaced by such an abstract mathematical structure? Even the professionals admit that we must relate these abstract symbols to the physical properties of the system that are subject to experimental investigation and this is where the problems begin.

Let me illustrate some of these problems with a simple example. We first note that we associate physical observables with certain operators, called 'observables'. It is then argued that the eigenvalues of these operators correspond to the numbers we find in experiments designed to reveal these observables. Thus we have a well defined method of associating elements of the mathematics with physical properties. For example the energy levels of an atom are characterised by the eigenvalues obtained from the energy operator by using the Schrödinger equation. However we must be very careful in understanding the seeming innocuous word *property.* To highlight the problem consider the following example taken from a toy quantum world.

I played a lot of cricket in my youth. To play this game we need to find a red sphere, the ball that is used in the game. However in this toy universe we need glasses to see the objects; furthermore we need one type of glasses to see colours and a *different type* of glasses to see shapes. So I first put on the colour sorting glasses and pick out a set of red objects from a collection of red and green objects. I then replace the glasses with the shape discriminating pair and from the red sub-set I pick out the spheres leaving, hopefully, the cubes behind. Good I now have a set of red spheres, just what I need for a game of quantum cricket or have we?

You should know in this universe the operators associated with colour do not commute with the operators associated with shapes. A child of this universe will immediately say, "Wait a moment, how do you know you that your chosen set actually contains only red spheres?" This is a very strange question for a classically mind sane person. OK, we can check by putting the colour discriminating spectacles back on. To our horror we discover that half this set are now green! This is certainly not what we expect of the word "property" in the classical world. Surely the sphere should always remain the same colour regardless of which aspect you choose to look at?

This is one of the problems we have to face and it is essentially the feature that underlies quantum non-locality. Clearly the means of observing something changes things and this suggests that the human observer might have an active role to play in Nature. But does this mean that some form of subjectivity is creeping into our science? That some thing *we* do changes the properties of a system? Even though this might be true, it doesn't matter which person sets up the experiments, the results turn out to be the same. Bernard d'Espagnat (2006) suggests that in order to emphasise the objectivity of the measuring process, we distinguish between two types of objectivity, *strong* objectivity and *weak* objectivity.

In a strong objective view, every system possesses all its properties, independently of whether we observe them or not. These properties are called *beables*. A measurement then simply reveals what is there without changing anything. Or at least if there is a disturbance this disturbance can either minimised or at least corrected for. In other words we can stand, as it were, outside the phenomena and intuitively get some understanding of how they are evolving. This is the view of the classical world.

In the weak objective view, a system can only be attributed those properties that can be revealed by a particular experimental set up. Such a view is also sometimes called an *instrumentalist* view. This type of view always leaves an ambiguity in one's mind, do we argue that the unknown properties still exist even though we can never know them in that particular situation or do we assume that these properties do not even exist so it is not clear what we are dealing with. In other words in the example I have given above we are left with the question, if I know I have a sphere how are we to think of the colour?

Traditionally there have been several positions adopted with regard to this question. Here I pick out just two.

1. As there appears to be nothing in the mathematics with which we can describe *all* the properties' so should we talk at all about the properties whose values we don't know? This situation arises because not all operators commute with each other and such operators cannot have simultaneous eigenvalues. However we can group operators into sets that commute with each other and we then have sets of complementary properties, not all of which can be sharp at a particular time. Clearly if we have no mathematical way to talk about a set of properties, should we talk about them at all? Thus we seem to be faced with a very limited type of explanation certainly more limited than the one we use in classical physics. This seems to arise because we are in some sense *participating* in Nature. To perform an experiment we

must participate, or at least our apparatus 'participates'. Thus we are no longer standing outside looking in, rather we are actually inside looking out.

2. On the other hand, we could still try to argue that systems do have all their properties and the reason why we are forced in this direction because we have assumed that the eigenvalues of the observables *alone* describe the properties of the quantum system. In the classical world we don't use the language of 'observables' therefore how can we be sure that all properties must only be described by the eigenvalues of these observables?

2 Clues from the Bohm Model.

This is the question raised by the Bohm interpretation (Bohm and Hiley 1993). Let us give up the idea that all the properties must *always* correspond to the eigenvalues of operators. Why don't we simply assume that experiments reveal one set of properties, those that correspond to the eigenvalues to the observables while the complementary set do not correspond to the eigenvalues of their 'observables? They could be described by other variables which have traditionally been called *hidden variables.* I personally do not like that phrase for two reasons. Firstly because we already use a hidden variable in the conventional interpretation, namely the *wave function.* Secondly, the phrase invokes some form of mysterious new variables that no one has seen before but that is not what happens in the Bohm approach.

What Bohm (1952) did was to show how to retain a description of all the usual properties of a classical world and yet remain completely within the conventional quantum formalism. He achieved this in a remarkably simple fashion: he simply split the Schrödinger equation, usually expressed in terms of complex numbers, into its two real components under a polar decomposition of the wave function. Except for one term, one of these equations looked remarkably similar to one well known known in classical mechanics, namely, the Hamilton-Jacobi equation. The quantum equation takes the form

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0 \tag{1}$$

where

$$Q = -\frac{\hbar}{2m}\frac{\nabla^2 R}{R}$$

If we simply identify ∇S with the beable momentum, p_B, the Bohm momentum, then the quantum Hamilton-Jacobi (1) is simply an expression for the conservation of energy provided we allow for a new quality of energy, which has traditionally been inappropriately called the ‘quantum potential’ Q. In this case the Bohm momentum is not the momentum eigenvalue of the operator $\hat{P}$.

What Bohm then assumes is that the particle behaves as if it had simultaneous values of (x, p_B), the beables of the particle. Then since the Bohm momentum is a well defined function of x and t we can integrate $p = \nabla S$ to find a trajectory, actually we find an ensemble of trajectories which becomes identical to the classical trajectories when Q is negligible. Thus we can assume the particle actually travels down one of these trajectories and in this way we provide an intuitive way to understand, for example, the interference produced by a beam of particle incident on two slits. (see for example Philippidis, Dewdney and Hiley 1979 and Gull, Lasenby and Doran 1993). All this means that we have found a way of talking about a particle with a simultaneously determined position and momentum even though we, as observers, do not know the value of both of these variables simultaneously.

In the example of the cricket ball, it would be like saying the set of red objects comprise 50% spheres and 50% cubes. If you pick one of them to play cricket you only have a 50% chance of being able to play ‘real’ cricket. This is where the uncertainty enters the Bohm model. If you had chosen a sphere you could have a good game, but if you had chosen a cube then the fun and games would start! It all depends on what you pick, but you have no control on what you choose. That is simply a random variable. It is not that the quantum world has lost its determinism, it is that we have ultimately no control over the quantum world.

Thus in this view a quantum dynamical system evolves in a deterministic way following a well defined trajectory, but we have no control over which specific trajectory it follows. In this sense our reality is ‘veiled’ as d’Espagnat (2003, 2006) has put it. But of course being veiled, we can not be sure that the quantum system is actually behaving the way the Bohm model predicts. It could be that the classical world emerges from a very different ‘quantum’ world, a world which is rooted in, say, process as Whitehead (1957) has suggested. These possibilities are open for exploration and speculation. The concern I always had for the standard interpretation was the implication that there was no other way. This was particularly strongly voiced by Bohr (1961) and Heisenberg (1958) where Bohm’s proposals were dismissed in very strange ways. Rather than following a rational argument, Heisenberg dismisses the idea by simply quoting Bohr’s, “We may hope that it (the hope for new experimental evidence) will later turn out that sometimes 2 x 2 = 5, for this would be of great advantages for our finances”.

I do not want to get side-tracked into debating these arguments here, I just want to illustrate the type of objection that has been used against the Bohm model. I personally am unconvinced by them. In fact the more I explored the Bohm model the more clearer becomes the radical nature of the novel features appearing in quantum phenomena.

3 The Appearance of Quantum Non-Locality.

Perhaps the most unexpected was the appearance of non-locality. Although this notion was already present in the standard approach as noted in the classic paper of Einstein, Podolksy and Rosen (1935), the argument seems to be about additional elements of reality rather than non-locality which is not even mentioned. It was something that Einstein (1948) totally rejected. Apart from his well known remark about "spooky at a distance" he also stresses that without these elements of reality quantum phenomena "imply the hypothesis of action at a distance, an hypothesis which is unacceptable." One exception was Schrödinger (1936) who made it very clear that central to the argument was non-locality, but in spite of this clear message, non-locaity was not pursued in the literature of the time and the debate focussed on the disputed existence of *hidden variables*.

Indeed it wasn't until the Bohm model appeared that the debate about the real issue emerged. As we have seen the Bohm model was about attributing all properties to a quantum system so that particles could be given exact simultaneous values for position and momentum. As positions of particles were sharp in this approach the issue of non-locality could not be avoided. Indeed it was the Bohm model that led Bell (1964) to think about these issues which resulted in his famous inequalities. Subsequent experiments confirmed that non-locality does exist in systems described by entangled states. Of course this notion of non-separability had already been implicitly recognised by Bohr himself (1935) but while denying that there was a mechanical force between the two separated particles, there was nevertheless an "influence on the very conditions which define the possible types of predictions regarding the future behaviour of the system." I never understood what the word 'influence' meant here, but in the Bohm model this influence is provided by the quantum potential.

Bohr never uses the terms "non-locality" or "non-separability"; he preferred the term "wholeness". For example he writes (Bohr 1961a)

> This discovery revealed in atomic processes a feature of wholeness quite foreign to the mechanical conception of nature, and

> made it evident that the classical physical theories are idealizations valid only in the description of phenomena in the analysis of which all actions are sufficiently large to permit the neglect of the quantum.

I have always felt that wholeness was key to understanding quantum phenomena. In this I agree with Bohr, but how do we bring out these ideas clearly?

In this regard the Bohm model has served its purpose. It has shown that it is possible to lift the veil of reality, but has it been lifted it far enough? Both Bohm and I felt that it had not. Something more radical was involved. Our exploration of the Bohm model over the question of non-locality did not reveal any mechanism for the transfer of information from one system to another. Rather it seemed as if the quantum potential had simply 'locked together' the particles described by entangled states. Indeed as we have already pointed out, it does not take much imagination to realise that the quantum potential is exactly what is needed to give mathematical form to Bohr's somewhat vague notion of *wholeness*.

What the Bohm model actually shows is that although there is no *physical* separation between the particles, we make a *logical* separation by attributing to them individual positions and momenta. However we find that when we do that the two particles are locked together by the quantum potential. In other words, when the Bohm model is carefully analysed, it is found that it is not a return to classical picture, but a very different picture emerges, one which puts wholeness centre stage with the classical picture only emerging when we can neglect the quantum of action as Bohr's remarks above make clear.

As we are so immersed in reductionism, it is very difficult to know what the notion of wholeness actually means. Put simply wholeness implies that the properties of the individual parts are determined by the whole, rather than the parts determining the whole. If we put wholeness centre stage then Nature at its very core is *organic* and by using the term *organic*, I am using it in the same spirit as Whitehead (1939).

> The concrete enduring entities are organisms, so that the plan of the whole influences the very characters of the various subordinate organisms which enter into it.

Clearly this is contrary to the normal reductionist view in which it is in the complex relations of individual building blocks that new properties are assumed to emerge. Instead, in this view the atoms, molecules, fields and ultimately space-time itself arise from activity, process. By starting from this more basic position I hope we can lift the veil of reality further.

4 The Basic Process or Activity

Are the notions I am proposing too far removed from physics as we know it? After all I am asking us to seriously consider whether space-time itself is a statistical feature of a deeper underlying process. Why do we need to consider such a notion?

To begin to see the necessity of making this change, consider a different problem, namely, the problem of quantising gravity, a problem that presents many conceptual difficulties (See Isham 1987). When a field is quantised (such as the electromagnetic field), it is subjected to fluctuations. If general relativity is a correct theory of gravity then we know that the metric of space-time plays the role of the gravitational potential. If the fields fluctuate, the metric must fluctuate. But the metric is intimately related to the geometry of space-time. It enables us to define angle, length, curvature, etc. In consequence, if the metric fluctuates, the geometric property of space-time will also fluctuate. What does it mean to have a fluctuating space-time?

Normally such a question is thought only to be relevant at distances of the order of 10^{-24} cm. I want to suggest that it is not only at those distances that we will experience the consequences of these fluctuations. We have actually changed the very foundations upon which our theories depend and in this approach the very notion of space-time is being called into question. Space-time itself with its local relations and Lorentz invariance are all assumed to be statistical features of the underlying process so why shouldn't the non-local features also appear at a macroscopic level, reflecting this deeper structure. In other words, this pre-space is not merely a curiosity manifesting itself at distances of the order of the Planck length, but also has much more immediate consequences at the macroscopic level.

I am not alone in in suggesting we start from this position. Penrose (1972) writes:

> I wish merely to point out the lack of firm foundation for assigning any physical reality to the conventional continuum concept.......Space-time theory would be expected to arise out of some more primitive theory.

John Wheeler (1978) puts it much more dramatically.

> It is NOT

Day One: Geometry

Day Two: Quantum Physics .

But

Day One: The Quantum Principle

Day Two: Geometry.

But if we deny that space-time is a fundamental descriptive feature, where do we start to develop a new theory?

I have already suggested the motion of process or activity but these notions may be too abstract for many. Let me try to make this suggestion a little more palatable, by asking the question: "Where is the 'substance' of matter?" Is it in the atom? The answer is clearly "no". The atoms are made of protons, neutrons and electrons. Is it then in the protons and neutrons? Again "no", because these particles are made of quarks and gluons. Is it in the quark? We can always hope it is, but my feeling is that these entities will be shown to be composed of "preons", a word that has already been used in this connection. But we need not go down that road to see that there is no ultimon. A quark and an antiquark can annihilate each other to produce photons and the photon is hardly what we need to explain the solidity of macroscopic matter such as, for example, tables and chairs. Thus we see the attempt to attribute the stability of the table to some ultimate "solid" entity is misguided. We have, in Whitehead's words, a "fallacy of misplaced concreteness".

However let us remember that one of the first reasons for introducing the quantum theory was to explain the stability of matter, a feature that classical mechanics failed to do. So let me propose that there is no ultimate "solid" material substance from which matter is constructed, there is only "energy" or perhaps we should use a more neutral term such as "activity" or "process". This is implicitly what most physicists assume when they use field theory. But field theory depends on continuity and local connection. As I have remarked above, it is the local continuum from which I want to breakaway.

I am suggesting that what underlies all material structure and form is the notion of activity, movement or process. I will use the term "process" as part of my minimum vocabulary to stand for pure activity or flux and regard all matter as being semi-autonomous, quasi-local invariant features in this back ground of continual change. Bohm preferred to call this fundamental form "movement" and he called the background from which all physical phenomena arose the "holomovement".(See Hiley 1991 for an extensive discussion of this notion in the present context). I have since learnt that the word "movement" invariably invokes the response "movement of what?" But in our terms, movement or process cannot be further analysed. It is simply a primitive descriptive form from which all else follows. Here it replaces the term "field" as a primitive descriptive form of present day physics. Thus in our approach, the continuity of substance, either particle or field, is being replaced by the continuity of form within process.

5 Grassmann's contribution.

While I was thinking about the possibilities of developing a description of physical processes in terms of process, activity or movement, my attention was drawn to Grassmann's (1894) own account of how he was led to what we now call a Grassmann algebra. (See Lewis (1977) for a discussion of Grassmann's work).

To begin with he made, what for me seemed an outrageous suggestion when I first encountered it, namely, that mathematics was about *thought* not about material reality. Mathematics studies relationships in thought not a relationships of content, but a relationship of forms within which the content of thought is carried. Mathematics is to do with ordering forms created in thought and is therefore of thought. Now thoughts are clearly not located in space-time. They cannot be co-ordinated within a Cartesian frame. They are "outside" of space.

Thought is about *becoming*, how one thought becomes another. It is not about *being*. Being is a relative invariant or stability in the overall process of becoming. What I would like to suggest is that there is a new general principle lying behind Grassmann's ideas, namely, "Being is the outward manifestation of becoming".

Bohm and I briefly exploited this principle in the last chapter of our book *The Undivided Universe* (1993). There we argued that in the Bohm interpretation, the classical level is to be regarded as the relatively stable manifest level (literally that which can be held in the hand or in thought), while the quantum level is the subtle level that is revealed in the manifest level.

We also showed how similar arguments hold for thought. Thought is always revealed in thought. One aspect of thought becomes manifest and stable through constant re-enforcement, either by repetition or learning. New and more subtle thoughts are then revealed in these aspects of thought, which in turn may became stable and form the basis for revealing yet more subtle thoughts. In his way a hierarchy of complex thought structures can be built up into a multiplex of structure process.

What I want to suggest in this paper is that both material process and thought can be treated by the same set of categories and hence by the same mathematics. They appear to be very different, thoughts being ephemeral, whereas matter is more permanent. For me it is a question of relative stability and that stability in the case of material process is compounded to produce the appearance of permanence to us. I want to argue that if we can find a common language then it will be possible to remove the Cartesian barrier between them and we will have the possibility of a deeper investigation into the relation between matter and mind.

6 The Algebra of Process.

How are we to build up such a mathematical structure? I believe that Grassmann has already begun to show us how this might be achieved in terms of an algebra which we now find very useful for some purposes in physics, but the full possibilities have been lost because Grassmanns original motivation has been forgotten. With this loss the exploitation of its potentially rich structure has been stifled.

To begin the discussion, let us ask how one thought becomes another? Is the new thought independent of the old or is there some essential dependence? The answer to the first part of the question is clearly “no” because the old thought contains the potentiality of the new thought, while the new thought contains a trace of the old.

Let us follow Grassmann and regard P_1 and P_2 as the opposite poles of an indivisible process of a thought. To emphasise the indivisibility of this process, Grassmann wrote the mathematical expression for this process between a pair of braces as $[P_1P_2]$ which we can represent in the form of a diagram The braces and the arrow emphasise that P_1 and P_2 cannot be

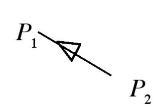

Figure 1: The 1-simplex

separated. When applied to space, these braces were called extensives by

Grassmann. For more complex structures, we can generalise these basic processes to:

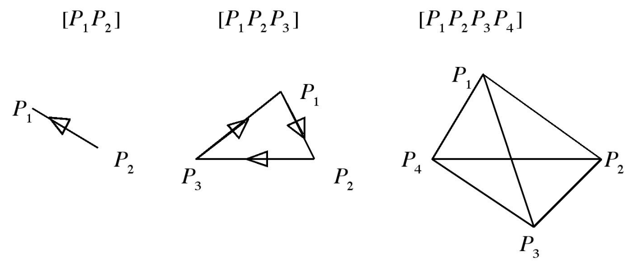

Figure 2: The 1-, 2- and 3-simplex

In this way we have a field of extensives from which we can construct a multiplex of relations of thought, process, activity or movement. The sum total of all such relations constitutes the holomovement.

For Grassmann, space was then a particular realisation of the general notion of process. Each point of space is a distinctive form in the continuous generation of distinctive forms, one following the other. It is not a sequence of independent points but each successive point is the opposite pole of its immediate predecessors so that points become essentially related in a dynamic way. Thus space in this view cannot be a static receptacle for matter. It is a dynamic, flowing structure.

Process forms an algebra over the real field in the following sense:

- multiplication by a real scalar denotes the strength of the process.
- the process is assumed to be oriented. Thus $[P_1P_2] = -[P_2P_1]$.
- the addition of two processes produces a new process. A mechanical analogy of this is the motion that arises when two harmonic oscillations at right angles are combined. It is well-known that these produce an elliptical motion when the phases are adjusted appropriately. This addition process can be regarded as an expression of the order of co-existence.
- to complete the algebra there is a inner multiplication of processes defined by $[P_1P_2][P_2P_3] = [P_1P_3]$

This can be regarded as the order of succession. Interestingly the mathematical structure generated is a groupiod (Brown 1987).

7 The Clifford Algebra of Process

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Let me now illustrate briefly how this algebra can carry the directional properties of space within it, without the need to introduce a co-ordinate system. I will only discuss the ideas that lead to what I call the directional calculus.

I start by assuming there are three basic movements, which corresponds to the fact that space has three dimensions. The process can be easily generalised to higher dimensional spaces with different metrics.

Let these three movements be $[P_0P_1], [P_0P_2], [P_0P_3]$. I then want to describe movements that take me from $[P_0P_1]$ to $[P_0P_2]$, from $[P_0P_1]$ to $[P_0P_3]$ and from $[P_0P_2]$ to $[P_0P_3]$. This means I need a set of movements $[P_0P_1P_0P_2], [P_0P_1P_0P_3]$ and $[P_0P_2P_0P_3]$. At this stage the notation is looking a bit clumsy so it will be simplified by writing the six basic movements as $[a], [b], [c], [ab], [ac], [bc]$.

We now use the order of succession to establish

$$\begin{aligned}
[ab][bc] &= [ac] \\
[ac][cb] &= [ab] \\
[ba][ac] &= [bc]
\end{aligned}$$

where the rule for the product (contracting) is self evident. There exist in the algebra, three two-sided units, $[aa], [bb]$, and $[cc]$. For simplicity we will replace these elements by the unit element 1. This can be justified by the following results $[aa][ab] = [ab]$ and $[ba][aa] = [ba]; [bb][ba] = [ba]$, etc.

Now

$$\begin{aligned}
[ab][ab] &= -[ab][ba] = -[aa] = -1 \\
[ac][ac] &= -[ac][ca] = -[cc] = -1 \\
[bc][bc] &= -[bc][cb] = -[bb] = -1
\end{aligned}$$

There is the possibility of forming $[abc]$. This gives

$$\begin{aligned}
&[abc][abc] = -[abc][acb] = [abc][cab] = [ab][ab] = -1 \\
&[abc][cb] = [ab][b] = [a], \\
&\textit{etc.}
\end{aligned}$$

Thus the algebra closes on itself and it is straight forward to show that the algebra is isomorphic to the Clifford algebra $C(2)$ which is called the Pauli-Clifford algebra in Frescura and Hiley (1980).

The significance of this algebra is that it carries the rotational symmetries and this is the reason I call it the directional calculus. The movements $[ab], [ac]$ and $[bc]$ generate the Lie algebra of SO(3), the group of ordinary rotations. For good measure this structure contains the spinor as a linear sub-space in the algebra, which shows that the spinors arises naturally from an algebra of movements. The background to all of this has been discussed in Frescura and Hiley (1984)

The generalisation to include translations has been carried out but the formulation of the problem is not as straight forward as we have to deal with an infinite dimensional algebra (see Frescura and Hiley 1984 and Hiley and Monk 1993). It would be inappropriate to discuss this structure here. I will merely remark that our approach leads to the Heisenberg algebra, strongly suggesting that our overall approach is directly relevant to quantum theory.

8 The Multiplex and Neighbourhood.

Let us now return to consider structures that are not necessarily tied to space-time. For simplicity let us confine our attention to structures that can be built only out of 0-dimension simplexes, $\sigma(0)$ (points) and 1-dimension simplexes $\sigma(1)$ (lines). Since we do not insist on continuity, any structure in the multiplex is constructed in terms of chains, i,e.,

$$C(0) = \sum_i a_i \sigma_i^{(0)}$$
$$C(1) = \sum_j b_j \sigma_j^{(1)}$$

where we will assume the coefficients a_i and b_j are taken over the reals or complex numbers

To those unfamiliar with the mathematics, it might be useful to have a simple example of a chain. One example of a 0-dim chain is a newspaper photograph. The 0-dim simplexes are the spots of print, while the real weights ai are the blackness of the dots. Notice this description does not locate the positions of the dots. To do that we need to introduce the notion of a neighbourhood. This requires us to ask what is on the boundary of what. The mathematical term that contains information about the neighbourhoods is called an incident matrix. This can be defined through the relations

$$B\sigma_i^{(1)} = \sum_j {}^{(1)}\eta_i^j \sigma_j^{(0)} \tag{2}$$

Here B is the boundary operator and ${}^{(1)}\eta_i^j$ is the incident matrix.

Since we have no absolute neighbourhood relation, we can define different neighbourhood relations. Here again an illustration might help to understand the rich possibilities that this descriptive form may have. In one of his experiments on the development of concepts in young children, Piaget and Inhelder (1967) describes how when children are asked to draw a map of the local area around their home and are asked to position the playground, school, ice cream shop, dentist etc. The children put the ice cream shop and the playground close to home, but will place the dentist and the school far from home, regardless of their actual physical distance from their home. The children are using a neighbourhood relation which is to do with pleasure and not actual distance.

Thus by generalising the notion of neighbourhood, it is possible to have many different orders on the same set of points depending on what is taken to be the relevant criterion for the notion of neighbourhood in a particular context. In this way our description becomes context dependent and not absolute. This is very important both for thought and quantum theory.

In thought the importance of context is very obvious. How often do people complain that their meaning has been distorted by taking quotations out of context? The importance of context in quantum theory has only recently begun to emerge. However in the Bohm interpretation context dependence becomes crucial. Indeed the famous von Neumann "no hidden variable" theorem (von Neumann 1955) only goes through if it is assumed that physical processes are independent of context. Exploration of an ontology for quantum phenomena was held up for a long time before the full significance of context was appreciated.

The new approach that I am suggesting has another interesting feature. It may be possible to have many different orders on the same set of points, or it may even be possible to define a different set of points, i.e., 0-dimensional simplexes, since the points themselves are to be regarded as particular movements i.e., of a movement into itself. Thus it may be possible to abstract many different orders from the same underlying process. To put it another way, the holomovement contains many possible orders not all of which can be made explicit at the same time.

This general order has been called the "implicate order" by Bohm (1980). The choice of one particular set of neighbourhood relations enables one to make one particular order manifest over some other. Any order that can be made manifest is called an "explicate order". So we have emerging from this approach a new set of ideas which fit the categories that Bohm was developing. It is in terms of these categories that we can give order to both physical and mental processes.

One feature of this new description is that it is not always possible to make manifest all orders together at one time. This is an important new idea that takes us beyond what I have called the Cartesian order where it is assumed that it is always possible to account for all physical processes on one level, namely, in space-time (Hiley 1997). The new order removes the primacy of space-time allowing other important orders to be given equal importance. Thus in quantum mechanics, the use of complementarity is now seen as a necessity arising out of the very nature of physical processes, rather than being a limitation on our ability to account for quantum processes.

Mathematically this new idea can be expressed through what we have called an "exploding " transformation. This is brought about by considering a structure built out of a set of basic simplexes $\sigma_i^{(0)}$ and $\sigma_j^{(1)}$ and then transforming the structure to one built out of a different set of basic simplexes $\Sigma_i^{(0)}$ and $\Sigma_j^{(1)}$. For simplicity we will call these basic simplexes "frames". Suppose these frames are related through the relations

$$\sigma_i^{(0)} = \sum_j a_{ji}\Sigma_j^{(0)} \qquad \text{and} \qquad \sigma_i^{(1)} = \sum_j \beta_{ji}\Sigma_j^{(1)}$$

We may now ask how the neighbourhood relations are related under such a transformation. Suppose we have

$$B\sigma_i^{(1)} = \sum_j {}^{(1)}\eta_i^j \sigma_j^{(0)} \qquad \text{and} \qquad B\Sigma_i^{(1)} = \sum_j {}^{(1)}\eta'^j_i \Sigma_j^{(0)}$$

and then ask how the incidence matrices are related. We have

$$\begin{aligned} B\sigma^{(1)}i = B(\sum_j \beta_i^j \Sigma_j^{(1)}) &= \sum_{j,k} a_k^i \, {}^{(1)}\eta'^j_k \Sigma_j^{(0)} \\ = \sum_l {}^{(1)}\eta_i^l \sigma_l^{(0)} &= \sum_l {}^{(1)}\eta_i^l a_l^n \Sigma_n^{(0)} \end{aligned}$$

So that

$${}^{(1)}\eta_i^l = a_i^k \, {}^{(1)}\eta'^k_j (a_l^j)^{-1}$$

Or in matrix form

$$\boldsymbol{\eta} = \boldsymbol{\alpha}\boldsymbol{\eta}'\boldsymbol{\alpha}^{-1}$$

This is the exploding transformation so called because the original structure can look quite different after such a transformation. Furthermore what is local in one frame need not be not local in any other. Mathematically these transformations are similarity transforms or automorphisms. In the algebraic approach we have tried to exploit these automorphisms. (See Hiley and Monk 1994).

9 The Unmixing Experiment.

A specific example of such a transformation was given by Bohm (1980). I would briefly like to recall this example to show how it fits into my argument.

Consider two concentric transparent cylinders that can rotate relative to each other. Between these cylinders there is some glycerine (See figure 1). If a spot of dye is placed in the glycerine and the inner cylinder rotated,

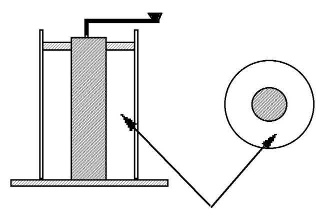

Figure 3: The unmixing Experiment

the spot of dye becomes smeared out and eventually disappears. There is nothing surprising about that, but what is surprising is that if we reverse the rotation, then the spot of dye reappears! This actually works in practice

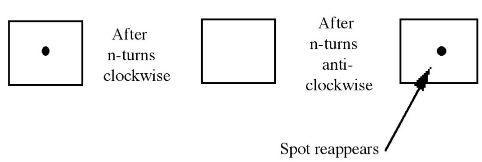

Figure 4: The reappearing spot.

and is easily explained in terms of the laminar flow of the glycerine under slow rotation. What we want to illustrate here is that in the "mixed" state, there does not seem to be any distinctive order present. Yet the order is, as it were, implicit in the liquid and our activity of unmixing i.e., unwinding, makes manifest the order that is implicit in the glycerine.

To carry the idea further, we can arrange to put in a series of spots of dye, displaced from one another in the glycerine. Place one spot at x_1 and then rotate the inner cylinder n_1 times. Then place another spot at x_2 and rotate the inner cylinder again a further n_2 times and so on repeating N times in all. If we were then to unwind the cylinder, we would see a series of spots apparently moving through the glycerine. If the spots were very close together we would have the impression of the movement of some kind of 'object' starting from position x_1 and terminating at position x_N (see figure 2). But no object has actually moved anywhere! There is simply an unfolding and then enfolding movement which creates a series of distinguished forms that are made manifest in the glycerine. So what we have taken to be the continuous movement of substance is actually a continuous unfoldment of form. Recalling my earlier remarks concerning the

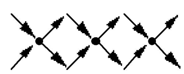

Figure 5: The morphology of stuff.

ephemeral nature of material processes, we can follow Whitehead (1957) and suggest that a quantum particle could be understood within the framework of these ideas. As Whitehead (1939) puts it "An actual entity is a process and is not describable in terms of the morphology of a stuff".

In this view, the cloud chamber photograph does not reveal a "solid" particle leaving a track. Rather it reveals the continual unfolding of process with droplets forming at the points where the process manifests itself. Since in this view the particle is no longer a point-like entity, the reason for quantum particle interference becomes easier to understand. When a particle encounters a pair of slits, the motion of the 'particle' is conditioned by the slits even though they are separated by a distance that is greater than any size that could be given to the particle. The slits act as an obstruction to the unfolding process, thus generating a set of motions that gives rise to the interference pattern. The Bohm trajectories referred to above can then be taken to be a representation of the average behaviour of these processes.

10 Evolution in the Implicate Order.

I would now like to show how we can arrive at an equation of unfoldment using the ideas of the last two sections. We assume that we have an explicate order symbolised by e. This could have considerable inner structure, but for simplicity we will simply use a single letter to describe it. We want to find an equation that will take us to a new explicate order e' as a result of unfolding movement.

Let us again use the order of succession to argue that the explicate order is enfolded via the expression eM_1. Here M_1 is an element of the algebra that describes the enfolding process. The next unfolded explicate order will be obtained from the expression M_2e'. Here M_2 is the process giving rise to the unfoldment. To express the continuity of form, we equate these two expressions to obtain

$$eM_1 = M_2e'.$$

or

$$e' = M_2^{-1}eM_1.$$

Thus the movement is an algebraic automophism, analogous to the transformation that we called the exploding transformation.

Let us now assume for simplicity that $M_1 = M_2 = M$, and we can write $M = \exp[iH\tau]$. Here H is some element of the algebra characterising the enfolding and τ is the enfolding parameter. For small τ we have

$$e' = (1 - iH\tau)e(1 + iH\tau).$$

so that

$$i\frac{(e' - e)}{\tau} = He - eH = [H, e].$$

Therefore in the limit as $\tau \to \infty$, we obtain

$$i\frac{de}{d\tau} = [H, e].$$

This equation has the same form as the Heisenberg equation of motion.

If we represent the explicate order e by a matrix and assume it is factorable i.e., $e = \psi\phi$ we find

$$i\frac{d\psi}{d\tau}\phi + i\psi\frac{d\phi}{d\tau} = (H\psi)\phi - \psi(\phi H).$$

If we regard ψ and ϕ as independent, we can separate the equation into two

$$i\frac{d\psi}{d\phi} = H\psi \qquad \text{and} \qquad -i\frac{d\phi}{d\tau} = \phi H.$$

If H is identified with the Hamiltonian, then the first equation has the same form as the Schródinger equation. If ϕ is regarded as $\psi^{\dagger}$ then the second equation has the same form as the complex conjugate of the Schrödinger equation.

11 Conclusion.

In this paper I have tried to motivate a new way of looking at physical processes which is an attempt to further lift the veil of Bernard d'Espagnet's "Veiled Reality". Not only does this help to provide another perspective on quantum phenomena but I believe it also removes the sharp division between mind and matter. We began by showing that it is possible to explore new ways of describing material process that does not begin with an a priori given space-time continuum. By starting with the notion of activity or process, which we take as basic, it is possible to link up with some of the mathematics that is already used in algebraic geometry. In fact in the particular example we used, we were able to recover the Clifford algebra, implying that some aspects of the symmetries of space can be carried by the process itself, albeit in an implicit form. By extending these ideas to include Bohm's idea of the enfolding process, we were also able to construct an algebra similar to the Heisenberg algebra used in quantum theory.

The motivation for exploring this approach came from two different considerations. Firstly, it came from the problems of trying to understand what quantum mechanics seems to be saying about the nature of physical reality. Using our usual Cartesian framework we find that, rather than helping to clarify the physical order underlying quantum mechanics, we are led to the well-known paradoxes that make quantum theory so puzzling and often unacceptable to many. It seems to me that these difficulties will not be resolved by tinkering with the mathematics of present day quantum mechanics. What is called for is a radically new approach to quantum phenomena.

The second strand of my argument was inspired by the work of Grassmann who showed how by analysing an abstract notion of thought, one could be led to new mathematical structures. In other words, by regarding thought as an algebraic process, Grassmann was led to a new algebra which we now call the Grassmann algebra. The scope of this algebra has become rather limited by being grounded in space-time as, unfortunately, the original motivations have been largely forgotten. By reviving these ideas, I have been exploring whether the similarities between thought and quantum processes that I have tried to bring out in this paper could possibly lead to new ways of thinking about nature.

What this means is that if we can give up the assumption that space-time is absolutely necessary for describing physical processes, then it is possible to bring the two apparently separate domains of res extensa and res cogitans into one common domain. What I have tried to suggest here is that by using the notion of process and its description by an algebraic structure, we have the beginnings of a descriptive form that will enable us to explore the relation between mind and matter in new ways.

In order to discuss these ideas further we must use the general framework of the implicate order introduced by Bohm (1980). An important feature within this order is that it is not possible to make everything explicit at any given time. This feature is well illustrated in the unmixing experiment described above. Here when there are a series of spots folded into the glycerine, only one spot at a time can be made manifest. In order to make manifest another spot, the first spot must be enfolded back into the glycerine and so on. If we now generalise this idea and replace the spots by a series of complex structure-processes within the implicate order, then not all of these processes can be made manifest together. In other words, within the implicate order there exists the possibility of a whole series of non-compatible explicate orders, no one of them being more primary than any other.

This is to be contrasted with what I have called the Cartesian order where it is assumed that the whole of nature can be laid out in a unique space-time for our intellectual examination. Everything in the material world can be reduced to one level. Nothing more complicated is required. I feel this is the reason why it is such a shock when people first realise that quantum mechanics requires a principle of complimentarity. Here we are asked to look upon this as arising from the limitations of our ability to construct a unique description, this ambiguity having its roots in the uncertainty principle. But it is not merely an uncertainty; it is a new ontological principle that arises from the fact that it is not possible to explore complementary aspects of physical processes together. Within the Cartesian order, complimentarity seems totally alien and mysterious. There exists no structural reason as to why these incompatibilities exist. Within notion of the implicate order, a structural reason emerges and provides a new way of looking for explanations.

Finally I would like to emphasise that it is not only material processes that require this mutually exclusivity. Such ideas are well known in other areas of human activity. There are many examples in philosophy and psychology. To illustrate what I mean here, I will give the example used by Richards (1974, 1976). He raises the question: "Are there ways of asking 'what does this mean?' which destroys the possibility of an answer?" In other words can a particular way of investigating some statement make it impossible for us to understand the statement? In general terms what this means is that we have to find the apporpriate (explicate) order in which to understand the meaning of the statement. Context dependence is vital here as it is in quantum theory and this is ultimately a consequence of the holistic nature of all processes.

Such ideas cannot be accommodated within the Cartesian framework. If we embrace the notions of the implicate-explicate order proposed by Bohm, we have a new and more appropriate framework in which to describe and explore both material processes and mental processes.

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