Phase Space Description of Quantum Mechanics and Non-commutative Geometry: Wigner-Moyal and Bohm in a wider context

B. J. Hiley*

TPRU, Birkbeck, University of London, Malet Street, London WC1E 7HX.

[To appear in "Beyond the Quantum", eds Th.M. Nieuwenhuizen, V. Spicka, B. Mehmani, M. Jafar-Aghdami and A. Yu Khrennikov (World Scientific, 2007).]

Abstract

In this paper we approach the question of the existence of a x, p phase space in a new way. Rather than abandoning all hope of constructing such a phase-space for quantum phenomena, we take aspects from both the Wigner-Moyal and Bohm approaches and show that although there is no unique phase space we can form 'shadow' phase spaces. We then argue that this is a consequence of the non-commutative geometry defined by the operator algebra.

Keywords: Quantum Mechanics, Wigner-Moyal approach, Bohm model, Poisson deformation algebra, non-commutative geometry, shadow manifolds.

1 Introduction

Although Schrödinger's [1] original 'derivation' of his equation originates from a modification of the classical Hamilton-Jacobi equation, the relation between the standard formalism of quantum mechanics and classical mechanics has always been problematical and not without controversy. The success of the Hilbert space formalism and the interpretation of the wave function as a probability amplitude has had such predictive success that

*E-mail address b.hiley@bbk.ac.uk.

attempts to find a phase space description of quantum phenomenon have generally been regarded as futile.

In spite of this, physicists often turn to the Wigner-Moyal approach, treating it as a 'semi-classical' approach. Yet when Moyal's origin paper [2] is examined carefully [3] one finds there is nothing semi-classical about it. It turns out that what is called a 'quasi-distribution function' is in fact the density matrix expressed in terms of an $x - p$ variables. We will show that the x and p here are the mean values of a cell constructed in classical phase space, whose size we eventually integrate over. [4] They are not the position and momentum of the particle under consideration. It is important to note that x and p are not operators but ordinary commuting variables.

Fortunately Moyal's work has not been neglected by the mathematicians. They have shown that Moyal's results emerge from a deformed Poisson algebra with Planck's constant being used as the deformation parameter. This algebra reproduces all the expectation values of quantum mechanics exactly from a distribution function $\rho(x, p, t)$. This algebra has the added advantage that it contains classical mechanics in the limit of an expansion in terms of $\hbar$ to $O(\hbar)$.

There is of course another phase space approach and that is one that I have explored with David Bohm. This goes under the name de Broglie-Bohm although I prefer to call it the Bohm approach because it differs in some significant ways from the way de Broglie originally envisioned. Recently I was surprised to discover [3] that the defining set of equations for the Bohm interpretation are already contained in the appendices of Moyal's original paper [2]. It turns out that in the x-representation, the momentum variable p used in Bohm's model is just the marginal momentum $\overline{\overline{p}}$ in the Moyal approach. One of the task's of this paper is to bring out these connections and to show that at the heart of the phase space approach is the dynamics that depends on two equations, the Liouville equation and a new equation describing the time evolution of the phase. It is this equation that becomes the classical Hamilton-Jacobi equation in the expansion of the deformation algebra to $O(\hbar)$.

The second point I want to bring out is that if we exploit the Heisenberg matrix mechanics so that we have a completely algebraic description of quantum phenomena, we get exactly the operator analogues of the two equation mentioned in the last paragraph. The significance of this result is that we now have a purely algebraic description of the phase space and furthermore this description is non-commutative.

In this algebraic approach we are thus faced with a non-commutative structure. To understand the significance of this for geometry we must recall the work of Gel'fand [7]. He showed that if one has a commutative algebra of functions then we can either start with an underlying continuous manifold with an *a priori* topological and metric structure and then derive the algebra governing the field equations. This is the usual approach adopted in physics. Or one can start with the algebra and deduce the properties of the underlying manifold. Provided the algebra is commutative there is a dual relation between the field algebra and its underlying spatial support.

In quantum mechanics what we do have is a field algebra defined in terms of the operators. Notice further that all the symmetries of the system are carried by the operators. What we do not have is the properties of the manifold, i.e. the phase space that can carry this algebraic structure. Can we obtain the properties of this manifold from the algebra? This is where we hit a snag. If we have a non-commutative structure then there is no single, unique underlying manifold. However one can find shadow manifolds [7]. You can construct these shadow manifolds by projecting the algebra into a sub-space. Doing this is equivalent to projecting into a representation of Hilbert space. What we will show is that the Bohm approach picks out one of these shadow manifolds by choosing the x-representation. This particular choice was a historical contingency, Bohm could have chosen another representation.

What one also finds is that the algebraic time dependent equations do not contain the quantum potential explicitly. Recall it is this strange potential that carries the non-local implications for entangled particles. However the potential re-appears when we project onto the shadow manifold. Thus it is not an intrinsic property of the quantum system but only arises from the projection. In this sense it shares a property with the gravitational potential which arises when the geodesics of curved space-time are projected into a flat space-time. The big difference is that gravitation is universal whereas each quantum potential only applies to its own group of entangled particles. Particles not involved in this entanglement do not see the quantum potential of this particular group.

2 The Moyal approach

Rather than using Moyal's starting point, let us follow Takasabsi [8] and introduce a two-point quantum density operator $\rho(x_1, x_2, t)$ defined by

$$\rho(x_1, x_2, t) = \psi(x_1, t)\psi^*(x_2, t) = \frac{1}{2\pi}\int \phi(p_1, t)\phi^*(p_2, t)e^{-\frac{i}{\hbar}(x_2p_2 - x_1p_1)}dx_1dx_2 \quad (1)$$

Then let us change co-ordinates, introducing $X = (x_2 + x_1)/2;\ \theta = x_2 - x_1$ and $P = (p_2 + p_1)/2;\ \tau = p_2 - p_1$. Substituting we find

$$\rho(X, \theta, t) = \frac{1}{2\pi} \int \phi^*(P - \hbar\tau/2) e^{\frac{iX\tau}{\hbar}} \phi(P + \hbar\tau/2) e^{\frac{iP\theta}{\hbar}} dP d\tau \tag{2}$$

which we can write in the form

$$\rho(X, \theta, t) = \int f(X, P, t) e^{\frac{iP\theta}{\hbar}} dP \tag{3}$$

Thus we see that $f(X, P, t)$, which is conventionally treated as a quasi-classical distribution function in the Moyal approach, is actually the Fourier transform of the quantum density matrix. Notice further that the variables X and P are the mean values of a cell in phase space, not the co-ordinates of the single particle we are considering.

Now to link up with quantum mechanics we replace $\hat{A}(\hat{X}, \hat{P})$ by $A(X, P)$ so that mean values can be found from

$$\langle \hat{A} \rangle = \int A(X, P) f(X, P, t) dX dP \tag{4}$$

Here we have written

$$A(X, P) = \frac{1}{(2\pi)^2} \int \psi^*(X - \hbar\theta/2) \hat{A}(\hat{X}, \hat{P}) e^{\frac{iP\theta}{\hbar}} \psi(X + \hbar\theta/2, t) d\theta \tag{5}$$

It should be stressed that no approximations have been made in this derivation and consequently we get exactly the expectation values calculated from the standard formalism. However the expression (4) looks like the classical method to derive expectation values if we assume $f(X, P, t)$ is a distribution function. But notice it merely *looks* as if we have a classical theory but it is not a classical theory because $f(X, P, t)$ is the density matrix so it is not surprising to find it takes negative values [9]. Nevertheless we get the correct expectation values, thus the approach is not an approximation to the quantum formalism, it is exact.

3 Relation to the Bohm approach

In Appendix 1 of Moyal's original paper [2], a marginal expectation value for the momentum is defined by

$$\rho(X)\overline{\overline{p}} = \int P f(X, P, t) dP \tag{6}$$

which for a wave function that can be written as $\psi(X) = \rho^{1/2}(X)e^{\frac{iS(X)}{\hbar}}$ gives

$$\overline{\overline{p}}(X) = \frac{\partial S}{\partial X} \tag{7}$$

This is just the expression that Bohm uses for his momentum. But there is more.

In Appendix 4, Moyal considers the transport equation for the probability density ρ and the momentum $\overline{\overline{p}}$. These are found from the time development equation for $f(X, P, t)$ which is derived by Moyal in the form

$$\frac{\partial f(X,P,t)}{\partial t} = H(X,P)\left[\frac{2}{\hbar}\sin\frac{\hbar}{2}\left(\frac{\overleftarrow{\partial}}{\partial X}\frac{\overrightarrow{\partial}}{\partial P} - \frac{\overleftarrow{\partial}}{\partial P}\frac{\overrightarrow{\partial}}{\partial X}\right)\right]f(X,P,t) \tag{8}$$

where the arrows indicate the function which must be differentiated. Let us now consider a particle in three-space and choose the Hamiltonian $H(\mathbf{X}, \mathbf{P}) = \frac{1}{2m}\mathbf{P}^2 + V(\mathbf{X}, t)$. Integrating equation (8) over $\overline{\overline{\mathbf{p}}}$ we find

$$\frac{\partial \rho}{\partial t} + \sum_i \frac{\partial}{\partial X_i}\left(\frac{\rho}{m}\frac{\partial S}{\partial X_i}\right) = 0 \tag{9}$$

which is just the conservation of probability equation used in the Bohm model.

Let us now work out the transport equation for $\overline{\overline{p}}$. This is obtained by multiplying equation (8) by P_k and integrating over P_k to give

$$\frac{\partial}{\partial t}(\rho\overline{\overline{P}}) + \sum_i \frac{\partial}{\partial X_i}\left(\rho \overline{\overline{p_k \frac{\partial H}{\partial X_i}}}\right) + \rho\overline{\overline{\frac{\partial H}{\partial X_k}}} = 0 \tag{10}$$

If we now substitute from equation (7) and use

$$\overline{\overline{P^2}} - \left(\overline{\overline{P}}\right)^2 = -\frac{\hbar^2}{4}\frac{\partial^2 \rho}{\partial X^2} \tag{11}$$

we find

$$\frac{\partial}{\partial X_k}\left[\frac{\partial S}{\partial t} + \overline{\overline{H}} - \frac{\hbar^2}{8m\rho}\sum_i \frac{\partial^2 \rho}{\partial X_i^2}\right] = 0 \tag{12}$$

If we write $\rho = R^2$, equation (12) is equivalent to

$$\frac{\partial S}{\partial t} + \frac{1}{2m}(\nabla S)^2 + V - \frac{\hbar^2}{2m}\nabla^2 R/R = 0 \tag{13}$$

which is just the quantum Hamilton-Jacobi used as a basis for the Bohm approach [?],[10]. Using $\rho = R^2$ in equation (11) gives

$$\overline{\overline{P_i P_k}} - \overline{\overline{P_i P_k}} = \frac{\hbar^2}{2}\frac{\partial}{\partial X_k^2}\left[\frac{\partial^2 R}{\partial X_i^2}/R\right] \tag{14}$$

Thus we see that the quantum potential $Q = \frac{\partial^2 R}{\partial X_i^2}/R$ arises from the dispersion of the momentum when we use the position representation.

All of this gives a different insight into the the Bohm model showing that it is an intrinsic part of the quantum formalism even when we approach the formalism from the Moyal point of view.

4 The star-product and its properties

The algebra behind the Moyal structure is known as the Poisson deformation algebra (See Chari and Pressley [11]), an algebra based on new star-product defined by

$$A(x,p)\star B(x,p) := A(x,p)\exp\left[\frac{i\hbar}{2}\left(\frac{\overleftarrow{\partial}}{\partial X}\frac{\overrightarrow{\partial}}{\partial P} - \frac{\overleftarrow{\partial}}{\partial P}\frac{\overrightarrow{\partial}}{\partial X}\right)\right]B(x,p) \tag{15}$$

This Moyal product is non-commutative and we can from two brackets that play a key role in the Wigner-Moyal approach. The first is the anti-symmetric Moyal bracket defined by

$$\{A(x,p), B(x,p)\}_{MB} = \frac{A\star B - B\star A}{i\hbar} \tag{16}$$

and the other is the symmetric Baker bracket [12]

$$\{A(x,p), B(x,p)\}_{BB} = \frac{A\star B + B\star A}{2} \tag{17}$$

These brackets are the analogues of the commutator and the anti-commutator, better known in mathematical circles as the Jordon product. We can write these brackets as

$$\{A,B\}_{MB} = A(x,p)\left[\frac{2}{\hbar}\sin\frac{\hbar}{2}\left(\frac{\overleftarrow{\partial}}{\partial x}\frac{\overrightarrow{\partial}}{\partial p} - \frac{\overleftarrow{\partial}}{\partial p}\frac{\overrightarrow{\partial}}{\partial x}\right)\right]B(x,p) \tag{18}$$

and

$$\{A,B\}_{BB} = A(x,p)\left[\cos\frac{\hbar}{2}\left(\frac{\overleftarrow{\partial}}{\partial x}\frac{\overrightarrow{\partial}}{\partial p} - \frac{\overleftarrow{\partial}}{\partial p}\frac{\overrightarrow{\partial}}{\partial x}\right)\right]B(x,p) \tag{19}$$

Expanding in powers of $\hbar$ to order $\hbar^2$ we find

$$\{A, B\}_{MB} = \{A, B\}_{PB} + O(\hbar^2) \approx \left[\frac{\partial A}{\partial x}\frac{\partial B}{\partial p} - \frac{\partial A}{\partial p}\frac{\partial B}{\partial x}\right] \tag{20}$$

where the bracket on the RHS is the Poisson bracket. Similarly expanding (17) we find

$$\{A, B\}_{BB} = AB + O(\hbar^2) \tag{21}$$

Thus we see that the Moyal approach has a very natural classical limit. There is no need to appeal to decoherence to reach the classical limit. Processes with high values of action naturally behave classically.

5 Time dependent equations in the Moyal approach

Let us now examine the time evolution from a different point of view from that used by Moyal [2]. We take as our starting point the work of Fairlie and Manogue [13] and write down the analogues to the Schrödinger equation and its complex conjugate. These are

$$H \star f = \frac{i}{2\pi}\int d\theta e^{-\frac{ip\theta}{\hbar}}\left[\psi^*(x - \hbar\theta/2, t)\frac{\partial\psi(x + \hbar\theta/2, t)}{\partial t}\right] \tag{22}$$

and

$$f \star H = \frac{i}{2\pi}\int d\theta e^{-\frac{i\theta p}{\hbar}}\left[\frac{\partial\psi^*(x - \hbar\theta/2, t)}{\partial t}\psi(x + \hbar\theta/2, t)\right] \tag{23}$$

If we take the difference between these last two equations we find

$$\frac{\partial f}{\partial t} + \{f, H\}_{MB} = 0 \tag{24}$$

which is just equation (8). In the limit of $O\hbar^2$, this equation becomes the usual classical Liouville equation

$$\frac{\partial f}{\partial t} + \{f, H\}_{PB} = 0 \tag{25}$$

where we find the Poisson bracket replaces the Moyal bracket.

We can now explore the result of adding the two equations. We now obtain the equation

$$\{H,f\}_{BB}=H\star f+f\star H=$$
$$\frac{i}{2\pi}\int d\theta e^{-\frac{i\eta p}{\hbar}}\left[\psi^*(x-\hbar\theta/2,t)\frac{\partial\psi(x+\hbar\theta/2,t)}{\partial t}\right]$$
$$-\frac{i}{2\pi}\int d\theta e^{-\frac{i\theta p}{\hbar}}\left[-\frac{\partial\psi^*(x-\hbar\theta/2,t)}{\partial t}\psi(x+\hbar\theta/2,t)\right] \quad (26)$$

If we try to simplify by writing $\psi=Re^{iS/\hbar}$ we find the RHS of (26) can be written as

$$\left[\psi^*\frac{\partial\psi}{\partial t}-\frac{\partial\psi^*}{\partial t}\psi\right]$$
$$=\left[\frac{1}{R(x+\hbar\theta/2)}\frac{\partial R(x+\hbar\theta/2)}{\partial t}-\frac{1}{R(x-\hbar\theta/2)}\frac{\partial R(x-\hbar\theta/2)}{\partial t}\right]\psi^*\psi$$
$$+\frac{i}{\hbar}\left[\frac{1}{S(x+\hbar\theta/2)}\frac{\partial S(x+\hbar\theta/2)}{\partial t}-\frac{1}{S(x-\hbar\theta/2)}\frac{\partial S(x-\hbar\theta/2)}{\partial t}\right]\psi^*\psi \quad (27)$$

This still looks a very messy result but if we go to the limit $O(\hbar^2)$, we find

$$\left[\psi^*\frac{\partial\psi}{\partial t}-\frac{\partial\psi^*}{\partial t}\psi\right]\rightarrow-\frac{\partial S}{\partial t}f+O(\hbar^2)=0 \quad (28)$$

and since the Baker bracket becomes the product, we have

$$\frac{\partial S}{\partial t}+H=0 \quad (29)$$

which is just the classical Hamilton-Jacobi equation. Thus we see that the Poisson deformation algebra contains both the classical Liouville equation and the classical Hamilton-Jacobi equations as classical limiting equations of the Schrödinger equation and its dual.

6 The operator formalism

If we examine the quantum Hamilton-Jacobi equation (8) which forms one of the defining equations of the Bohm approach we see that we obtain the classical limit by putting the quantum potential to zero. This suggests that there must be some relation between this equation and equation (26). To bring this connection out further let us recall that it is the operator equations that have a similar form to the classical equations. For example, Hamilton's equations of motion can be written in the form

$$\dot{x} = \{x, H\}_{PB} \qquad\qquad \dot{p} = \{p, H\}_{PB} \tag{30}$$

while the corresponding equation of motion in the Heisenberg picture are expressed in terms of the commutator, viz

$$i\hbar\dot{\hat{X}} = [\hat{X}, \hat{H}] \qquad\qquad i\hbar\dot{\hat{P}} = [\hat{P}, \hat{H}] \tag{31}$$

so it is necessary to look for operator equations which are of the same form as equations (24) and (28). These equations will be expressed in terms of the commutator and the anti-commutator. In fact these equations have been found in Brown and Hiley[14], and Hiley[3]. These equations are

$$i\hbar\frac{\partial\hat{\rho}}{\partial t} + \left[\hat{\rho}, \hat{H}\right]_- = 0 \qquad\qquad \hat{\rho}\frac{\partial\hat{S}}{\partial t} + \frac{1}{2}\left[\hat{\rho}, \hat{H}\right]_+ = 0 \tag{32}$$

Here $\hat{\rho}$ is the density operator and $\hat{S}$ is the phase operator. To obtain these equations the wave function have been replaced by an element of the left (right) ideal so that the wave function is replaced by a 'wave operator'[1].

The first equation in (32) is just the quantum Liouville equation giving rise to the conservation of probability equation. The second equation is the operator form of the equation involving the Baker bracket (17). It should be noticed that this equation should be the operator analogue of of the quantum Hamilton-Jacobi equation (13) but notice there is no explicit expression for the quantum potential. The quantum arises when we project the equation into a sub-space.

Suppose we chose the projection operator $P_x = |x\rangle\langle x|$ then the first equation, the quantum Liouville equation, becomes

$$\frac{\partial P}{\partial t} + \nabla\left(P\frac{\nabla S}{m}\right) = 0 \tag{33}$$

which is just the conservation of probability equation (9). The second equation becomes

$$\frac{\partial S}{\partial t} + \frac{1}{2m}(\nabla S)^2 + V - \frac{\hbar^2}{2mR}\nabla^2 R \tag{34}$$

This is just the quantum Hamilton-Jacobi equation (13) where the quantum potential appears explicitly.

[1]This wave operator is expressed in terms of a polar decomposition[3]

In section 3 we showed that these equations were obtained from a marginal distribution found by averaging over the momentum P. In the operator approach they are obtained by projecting into the x-representation. We could equally have taken the marginal distribution by integrating over the position X. It can be shown that this is equivalent to a projection into the p-representation. In this case we get a corresponding formalism in p-space including a quantum Hamilton-Jacobi equation with a corresponding quantum potential. Thus we have complete $x - p$ symmetry in the Bohm model, removing one of Heisenberg's original objections[15] to the model, namely, the apparent lack of this symmetry in Bohm's original approach[10].

Thus we have a complete correspondence between the equations of motion derived from the Poisson deformation algebra and those obtained from the operator algebra. Furthermore we see that there is an additional equation to the Liouville equation which appears to have received scant attention [13]

7 Conclusion

In this paper we have brought out some of the deeper connections between the Wigner-Moyal approach, the Bohm approach and the Heisenberg-type algebra of operators. These approaches are, in fact, different aspects of the same mathematical structure, each emphasising a different features of this structure.

The Moyal algebra (Poisson deformation algebra) constructs a $X - P$ phase space where X and P are mean co-ordinates of a pair of points in classical phase space. In this sense it is essentially a bilocal model. This fits comfortably with the notion of quantum cells or 'quantum blobs' in phase space suggested by several authors including de Gosson [4]. The Bohm model appears in the Moyal algebra as a result of forming a marginal distribution which produces a mean momentum, $\overline{\overline{p}}$, which can be identified with the Bohm momentum. This marginal distribution produces what looks like a classical phase space provided it is supplemented with the quantum potential.

We also show how a purely algebraic approach, exploiting the algebra of operators, produces dynamical equations of motion in terms of operators which have the same form as those produced in the Moyal algebra. In this operator algebra approach, the Bohm model arises from a projection into an algebraic sub-space, which seem to be playing a role similar to the marginal distributions in the Moyal theory.

Finally I want to suggest that we can perhaps make more sense of all this if we adopt what I call the Gel'fand approach. Here rather than starting with an *a priori* given phase space manifold, we use the algebra of operators to abstract the underlying manifold. This works beautifully when the algebra is commutative but not when in is non-commutative. In this case there is no unique underlying manifold, but we are forced to introduce the concept of "shadow manifolds". This seems to be exactly what is happening when we project the algebraic form of the dynamical equations into a sub-algebra, the choice of the projection determining which shadow manifold is being chosen. All this supports the notion that the geometry underlying quantum mechanics is a non-commutative geometry [16]. It fits neatly into the philosophical framework of the implicate/explicate order introduced by Bohm [17].

References

[1] Schrödinger E., Quantization as an Eigenvalue Problem, *Ann. der Physik*,**79**,(1926), 361. English Translation in *Collective Papers on Wave Mechanics*, Blackie and Son, London, 1928.

[2] Moyal, J. E., Quantum Mechanics as a Statistical Theory, *Proc. Camb. Phil. Soc.* **45**, (1949), 99-123.

[3] Hiley, B. J., Phase Space Descriptions of Quantum Phenomena, Proc. Int. Conf. Quantum Theory, *Proc. Int. Conf. Quantum Theory: Reconsideration of Foundations*, **2**, 267-86, ed. Khrennikov, A., Växjö University Press, Växjö, Sweden, (2003).

[4] de Gosson, M., The optimal pure Gaussian state canonically associated to a Gaussian quantum state, *Phys. Lett.* **A 330**, 161-167, (2004).

[5] Bohm, D., and Hiley, B. J., An Ontological Basis for Quantum Theory: I - Non-relativistic Particle Systems, *Phys. Reports* **144**, (1987), 323-348.

[6] Bohm, D. and Hiley, B. J., *The Undivided Universe: an Ontological Interpretation of Quantum Theory*, Routledge, London 1993.

[7] J. Demaret, M. Heller and D. Lambert, Local and Global Properties of the world, *Foundations of Science*, **2**, (1997) 137-176.

[8] T. Takabayasi, *Prog. of Theor. Phys.* **11**, (1954),341-74,

[9] Feynman, R. P., Negative Probability, in *Quantum Implications: Essays in Honour of David Bohm*, Ed. Hiley, B. J. and Peat, F. D., pp. 235-248, Routledge & Kegan Paul, London 1987.

[10] Bohm, D., A Suggested Interpretation of the Quantum Theory in Terms of Hidden Variables, I, *Phys. Rev.*, **85**, 166-179, and II, **85**,180-193 (1952).

[11] Chari, V. and Pressley, A. *A Guide to Quantum Groups*, Cambridge, England, (1995).

[12] Baker, Jn. G. A., Formulation of Quantum Mechanics based on the Quasi-Probability Distribution Induced on Phase Space, *Phys. Rev.* **109**, (1958) 2198-2206.

[13] Farlie, D. B. and Manogue, C. A., The formulation of quantum mechanics in terms of phase space functions -the third equation, *J. Phys. A: Maths. Gen.* **24**, (1991) 3807-3815.

[14] Brown, M. R. and Hiley, B. J., 2000 Schrdinger revisited: an algebraic approach. quant-ph/0005026.

[15] W. Heisenberg, Physics and Philosophy: the revolution in modern science, George Allen and Unwin , London, 1958.

[16] Connes, A, Noncommutative Geometry, Academic Press, San Diego, 1990.

[17] D. Bohm, Wholeness and the Implicate Order, Routledge, London, 1980.