Some Remarks on the Evolution of Bohm's Proposals for an Alternative to Standard Quantum Mechanics.

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(30 January 2010)

Abstract

In this paper an account of Bohm's own attitude to his early papers on an alternative interpretation to quantum theory is presented. He has made it clear that it was never his intention to return to a deterministic mechanical theory, rather it was an attempt to show that an alternative was possible and, in itself, may provide clues for further developments. From the earliest he felt that a more radical approach was needed, an approach that depended on his notion of structure process. In such an approach the particle must not be regarded as some *a priori* given immutable entity, but must take its dynamical properties from the environment in which it finds itself so that the notion of wholeness, so important to both Bohr and Bohm, could be a fundamental part of the description. It will be argued that the essential feature of his later ideas of the implicate order were already present in his early work in "Causality and Chance" written shortly after his original proposals.

1 Introduction

It is a great pleasure to contribute to this collection of essays dedicated to Paavo Pylkkänen. I am indebted to the many conversations I have had with him over the many years that we have known each other. He has introduced me to the world of professional philosophy, cognitive science and the exciting

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boundaries where mind meets matter, taking me on intellectual journeys into regions which, as a physicist, I would have remained blissfully unaware. For that I thank him. But I must also thank him for his persistent questioning of the proposals that Bohm made in his pursuit of an alternative approach to standard quantum mechanics, a question that is never far from many of our discussions.

Paavo has already published an excellent discussion of Bohm's later and more general ideas on wholeness and the implicate order that were designed to throw more light, not only on quantum phenomena, but also on the more difficult topic, the relation between mind and matter [1]. The aim of this paper is to review Bohm's earlier thinking about the conceptual problems presented by the quantum formalism and to overcome the difficulties he experienced in trying to understand Bohr's position, which has become known as the Copenhagen interpretation. I feel this discussion is needed because a great deal of confusion exists as to exactly what Bohm was striving towards when he first published his papers "A Suggested Interpretation of the Quantum Theory in Terms of Hidden Variables" [2].

Many discussions about Bohm's ideas in this area seem to concentrate on these two papers, assuming that they represented some kind of definitive theory and an accurate reflection of his later views. Bohm wrote these papers, not to present a definite alternative view of the quantum formalism, but merely to show that an alternative view, one that attributes definite properties to individual particles, was possible without radically changing the formalism and altering the predictions.

In chapter 4 of his book "Causality and Chance in Modern Physics" [3], published in 1958, he presented an appraisal of these earlier papers. On page 110, he writes

At this stage, as pointed out in Section 1, the author's principal purpose had not been to propose a definitive new theory, but was rather mainly to show, with the aid of a concrete example, that alternative interpretations of the quantum theory were in fact possible. Indeed, the theory in its original form, although completely consistent in a logical way, had many aspects which seemed quite artificial and unsatisfactory. Nevertheless, as artificial as some of these aspects were, it did seem that the theory could serve as a useful starting-point for further developments, which it was hoped could modify and enrich it sufficiently to remove these unsatisfactory features.

His own assessment of his early work seems to have been forgotten and,

in consequence, the work he published in later papers refining and making new proposals tends to have been completely ignored. In meeting some of his own earlier objections, he found it necessary to make radical new proposals about the deeper structure underlying quantum phenomena. It was this later development of his more radical ideas that was the subject of Paavo's book [1]. However the seeds of what eventually emerged were already implicit, and sometimes even explicit, in his book "Causality and Chance" [3]. It is the origins of these ideas that I want to discuss in this paper. Before doing this I want to highlight some of the ideas developed by others that have been wrongly attributed to him.

2 Non-Bohmian Mechanics.

The misunderstandings gathered pace after the unfortunate appearance of the term "Bohmian Mechanics" [BM], in a paper by Dürr, Goldstein and Zanghi [DGZ] [4]. The very term, "Bohmian Mechanics", lies at the root of much confusion concerning the direction that Bohm's own thinking took after he first published his two seminal papers in 1952 [2]¹. While I quite understand the wish to give credit to Bohm for his pioneering work, the linking of Bohm's name with the term 'mechanics' has led many to believe that Bohm himself was motivated to find a classical order based on a deterministic mechanics from which the quantum formalism would emerge. That was never his intention.

Indeed the content of his book "Quantum Theory" published in 1951 [6], which gives an exhaustive account of the orthodox view of the theory, already sows the seeds of how radical a change Bohm thinks is needed in order to begin to understand the structure that underlies the quantum formalism. In that book he sees the need to go beyond mechanical ideas. In the section headed 'The need for a nonmechanical description', he writes,

....the entire universe must, in a very accurate level, be regarded as a single indivisible unit in which separate parts appear as

¹This remark should not be taken as a criticism of what Dürr *et al* [4] do in their paper. It is a perfectly legitimate and worthwhile enterprise to see how far a mechanical model will take us. Although this was not Bohm's intention, his original papers [2] left us with the problem of justifying the identification of the classical action with the phase of the wave function in the so called 'guidance condition' $\mathbf{p} = \nabla S$. Dürr *et al* claimed to have derived this relation from first principles and thus removed this difficulty. Bohm's proposal was that it should simply be taken to be a 'subsidiary condition' [5] until the emergence of a deeper theory.

idealisations permissible only on a classical level of accuracy of the description. This means that the view of the world as being analogous to a huge machine, the predominant view from the sixteenth to nineteenth century, is now shown to be only approximately correct. The underlying structure of matter, however, is not mechanical [7].

In a footnote to this quote he writes "This means that the term 'quantum mechanics' is very much a misnomer. It should, perhaps, be called 'quantum nonmechanics'."

It cannot be argued that this was his position before he published his '52 papers because he takes up this 'nonmechanics' theme again in his book "Causality and Chance" [3] which was written later in 1956-7. This treatise was an attempt to put his '52 work in a more general context. Chapter 5 of this book contains a detailed criticism of the attempt to use classical mechanism in the quantum domain and could legitimately be sub-titled "Against Mechanism". One does not even have to read the content to get a flavour of his arguments as the section headings speak for themselves. For example, Section 3 is entitled "Criticism of the Philosophy of Mechanism", while section 4 reads "A point of view that goes beyond mechanism".

2.1 Some Common Misapprehensions.

One of the ideas that has been given prominence in the discussions of BM is the emphasis placed on the notion of a 'particle', almost to the total exclusion of any wave-like properties. Thus in many discussions, it is only the particle position and its momentum, defined by $\mathbf{p} = \nabla S$, that is given significance. The exclusive use of particle properties leads to many problems when applied to various experiment situations, which often lead to the conclusion that BM is actually 'wrong'. Yet how can a theory that uses the quantum formalism, without adding any new content, disagree with the predictions of the standard approach? [8].

In many of these discussions, I find that there is no significance given to the quantum potential, yet for Bohm this was a key element in gaining insights into what could underlie the quantum formalism. It was his deeper analysis of this aspect of the approach that convinced him that the theory could not be mechanical. Rather it was organic in the sense of Whitehead. Namely, that it was the whole that determined the properties of the individual particles and their relationship, not the other way round. For Bohm the universal interconnectedness of things could no longer be questioned and although this may be accepted even in the Newtonian paradigm, he felt this interconnectedness could not ultimately be reduced to nothing more than an *interaction* between some *a priori* given fundamental entities which compose the system [9].

For Bohm there was no such thing as *the* fundamental particle, the 'ultimon'. At any stage of the analysis there are quasi-stable, semi-autonomous features that could be related, ordered and welded into a coherent structure which we could call a theory. However these quasi-stable features, the 'particles' say, of the theory took their properties from the total process itself. We have the classic example of this type of feature in the appearance of phonons in a lattice of a solid. These phonons, although treated as the particles of the theory, do not exist outside the lattice. The lattice itself can be thought of as composed of much more stable particles, the atoms, but ultimately the atoms themselves could be thought of as quasi-stable, semi-autonimous entities of a deeper level. Of course in many cases we can treat the atom as an entity independent of its background. This works for a large variety of processes until we come to effects like quantum interference. Are we right to continue to think of the atom in this situation to be independent of the experimental background in which we are examining its properties? Bohr clearly thought not when he wrote,

the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear. [10]

Let us leave this question for the moment and return to the much maligned quantum potential. One of the assumptions behind BM is that the Schrödinger equation still holds. What Bohm did initially was to examine the real and imaginary parts of this equation under polar decomposition of the wave function. This is a standard procedure in mathematics and, as is well known, this decomposition does not alter the content of the equation. It is in this decomposition that the real part of the Schrödinger equation is found to be

$$\frac{\partial S}{\partial t} + (\nabla S)^2 / 2m + Q + V = 0. \tag{1}$$

where Q is given the name 'quantum potential'. This clearly is a manifest feature of the equation and must surely have some physical consequences. Bohm took its appearance seriously and began to explore its significance in his original papers [2]. Together we continued this exploration, the details of which can be found in our review paper [11], our book [12] and, in more general terms, in the book of Bohm and Peat [13]. What we concluded was that this potential enabled the global properties of quantum phenomena to be focussed on the particle aspect, but in doing this we must remember the 'particle' is not independent of the background. Furthermore it is the quantum potential that contains the effect of this background. This implies that the particle and quantum potential form an indivisible whole, which for the sake of simplicity, we can call a 'quantum blob' [14]. I will explain this feature in more detail later in this paper.

In the early nineties I became aware of the work of DGZ after the manuscript of our book "The Undivided Universe" [12] had been completed. On reading their paper "Bohmian Mechanics as the Foundation of Quantum Mechanics" [15], I began to realise that DGZ were not discussing the ideas that Bohm himself had been developing, but were proposing significantly different theory. This soon becomes very apparent when they dismiss the quantum potential as not being "fundamental". But they actually say more. It is "ad hoc", "misleading", and "not natural". They emphatically reject its relevance by writing, "there is neither the need nor room for any further axioms involving the quantum potential" [16].

Clearly BM is not the approach that Bohm originally proposed, nor is it the theory that our group at Birkbeck worked on with Bohm for three decades. I find the rejection of the quantum potential very puzzling, particularly in view of the beautiful work de Gosson [17] who shows that the quantum potential is an essential part of the mathematical structure that arises in symplectic geometry, a feature shared by both classical and quantum mechanics. Is the quantum potential ignored because it contains all the non-local features that are anathemer to those who desire to hold onto a mechanistic interpretation? Or do people still cling to the logical positivists belief, long since discredited, that we must only talk about is directly observable like the particle position?

3 Bohm's Motivation for his Original Proposals.

Let us put aside the speculations about the motives of others and return to consider what Bohm was trying to achieve in his '52 papers. To make absolutely clear his position, we must recall the background in which his proposals were put forward. The prevailing consensus at the time was that the only possible interpretation of the quantum formalism was Bohr's Copenhagen interpretation. von Neumann [18] had shown, it was claimed, that there was no alternative. What von Neumann actually claimed was that it was not possible to find a deeper reality, usually associated with 'hidden variables', underlying quantum phenomena without producing results that would disagree with the results of the standard approach. Bohm had reached a similar conclusion in his own book [6], but from very different arguments.

He explained to me that after the book was completed and on deeper reflection, he became dissatisfied with the argument and finally reached a different conclusion. He later writes [19]

What I felt especially unsatisfactory [with the conventional approach] was the fact that the quantum theory had no place in it for an adequate notion of an independent actuality.

Bohm insisted that his '52 papers should be not taken as his dramatic conversion to a deterministic, mechanical viewpoint. He was merely trying to show that an alternative that attributed properties to an underlying reality was possible. He was not offering these proposals as the final definitive interpretation of the quantum formalism in the non-relativistic domain. There were too many curious features that needed careful thought. However, for starters, we could see how far we could go by assuming there was a definite localised entity that we could call a particle, to which we could attribute a precisely definable and continuously varying value for the position and the momentum together with the background energy Q. He then shows that one can, indeed, obtain a consistent interpretation that would reproduce all the results of the standard formalism.

Throughout the papers he stresses that his approach opens up possibilities of modifying the formalism in ways that could not be possible in the present formalism. For example, he suggested that there was a possibility of exploring a deeper structures that could lie below 10^{-13} cm. Clearly he was not presenting the ideas as being, in some sense, a definitive interpretation of the formalism, but using it to look for something deeper.

In our conversations he said he felt his proposals were somewhat *ad hoc* and not totally convincing as a physically intelligible interpretation. Even Bell acknowledges that "Bohm did not like it very much" [20]. One can find many quotes from his own papers over the years backing this claim. Even in our book, "The Undivided Universe" [12], although we presented new ideas that answered some of the earlier criticisms, nowhere did we claim that it was entirely free of any troubles. If we had thought it to be the 'final' theory, why did we devote the last chapter to describing a more radical approach?

4 Bohm's Own Criticism of his Early Ideas.

So what are the difficulties that Bohm, himself, perceived? These were set out in section 5 of chapter 4 of "Causality and Chance" [3]

Firstly there was the question of how to extend these ideas to the spin of the electron. Bohm, together with Schiller and Tiomno [21] soon proposed a model based on the notion of a spinning top using the Caley-Klien parameters and the Pauli equation. Although this has been investigated further by Dewdney, Holland and Kyprianidis [22], it is not without its own difficulties. Some of these difficulties have been removed in a recent paper by Hiley and Callaghan [23]

Secondly, although there have been attempts to extend these ideas to the relativistic electron, these have only been partially successful [12], [24], [25], [26]. In a recent paper, Hiley and Callaghan [27] have presented a complete treatment of the relativistic Dirac electron in which the fully relativistic quantum potential is presented.

In the same section of his book, Bohm draws attention to what he calls 'a serious problem' that confronts us when the theory is extended to deal with more than one particle. The problem with N particles is that the wave function is not in ordinary three-dimensional space, but instead, in an abstract 3N-dimensional configuration space. While of course this space is logically consistent, the concept of a wave in a 3N-dimensional space is far from physically obvious. At this stage Bohm simply regarded his proposals as an artifice that could be used provisionally until a better theory emerges "in which everything is expressed once more in ordinary three-dimensional space". This problem of configuration space was eventually resolved by introducing the notion of 'active information' [11].

However there remains a deeper problem as Bohm points out.

Finally, our model in which wave and particle are regarded as basically different entities, which interact in a way that is not essential to their modes of being, does not seem very plausible. The fact that wave and particle are never found separately suggests instead that they are both different aspects of some fundamentally new kind of entity which is likely to be quite different from a simple wave or a simple particle, but which leads to these two limiting manifestations as approximations that are valid under appropriate conditions [31].

It is this problem that radically changes the meaning of the term 'particle'.

5 The Way Forward.

In the last quotation of the previous section, we begin to see a germ of what was to lie ahead. The theory was not proposing the existence of a classical 'rock-like' particle, but rather a new kind of entity which is quite different from a classical particle. It is an entity that is a product of the interconnectivity of processes we see in nature. As we have remarked above, rather than think of interconnectivity as a product of the interaction of *a priori* given fundamental entities, it is the very interconnection that sustains the entities themselves, and defines their properties. Alter the background and the substructure of these interconnections and the entities themselves transform into new entities. His is not a world of immutable particles with well-defined properties interacting through mechanical forces. Something much more radical is involved. Bohm writes

A fundamental problem in scientific research is then to find what are the things that in a given context, and in a given set of conditions, are able to influence other things without themselves being significantly changed in their basic qualities, properties, and laws. These are, then, the things that are, within the domain under consideration, autonomous in their essential characteristics to an adequate degree of approximation. [32]

In the above Bohm talks about the 'wave' and the 'particle'. I would rather think of it in terms of energy, a localised kinetic energy and a global quantum potential energy. These are two manifestations of the same total interconnected process. Why do I say this?

Let us consider first two extreme situations. For the free particle, the total energy is all kinetic and there is no quantum potential energy. On the other hand for a particle in a box in a stationary state, we find no kinetic energy and all the energy is in the quantum potential. In between these two extreme cases, we find the energy split between these two forms of energy. We find this division typically appearing in interference phenomena [33]. In these situations we find the quantum potential energy depends on some global aspect, such as, for example, the position and widths of the two slits in a two-slit interference experiment. In contrast to quantum phenomena, the classical behaviour appears when all the energy is kinetic and the quantum potential energy vanishes. In this way the theory provides an explanation of why 'particles' behave differently in classical and quantum situations. What the theory does not tell us is why there is this schism in the energy in the first place. This is a question that a deeper theory must answer.

6 The Connection with Quantum Field Theory.

Now let us turn to conventional quantum field theory [QFT]. Here it is the field that captures the global aspects of the phenomena. To describe this energy in QFT, we use the energy momentum tensor, $T^{\mu\nu}$. If we consider the energy and momentum components, T^{00} and T^{j0} of this tensor, we find they are exactly what we call the Bohm energy and Bohm momentum density. The details can be found in Hiley and Callaghan [23] who show that

$$\rho P_B^j = T^{j0} \quad \Leftrightarrow \quad \mathbf{P}_B = \mathbf{\nabla} S.$$

and that

$$\rho E_B = T^{00} \quad \Leftrightarrow \quad E_B = -\partial_t S.$$

This generalises to both the Pauli [23] and Dirac [27] cases. A similar result has also been found by Horton and Dewdney for the Klein-Gordon equation [28], the scalar field [29] and the massive vector field [30].

Thus the Bohm energy and momentum appears to be a part of standard QFT, so what are the essential differences between the two theories? QFT does not discuss the properties of $T^{\mu 0}$ itself. Rather it defines an energy-momentum P^{μ} via the equation

$$P^{\mu} = \int T^{\mu 0} d^3 x.$$

where the integral is taken over all space. In other words it defines a global energy-momentum and shows that this global energy is conserved.

On the one hand, Bohm considers the meaning of the local expression $T^{\mu 0}(x^{\rho})$ in the particular case when there is only one particle excitation present in the field. In other words the energy is below the threshold for particle creation. Then one makes the assumption that we can attribute the value of the energy-momentum components $T^{\mu 0}(x^{\rho})$ to a quasi-local, semiautonomous feature of the total energy. For convenience we can call this feature, the 'particle'. However this 'particle' is different from a classical particle because it does not carry all the energy. $T^{ij}(x^{\rho})$, remains to be accounted for. In the Bohm approach this energy is made manifest through the quantum potential and, as we have seen, it is this energy that is the source of all the quantum phenomena. This includes the quantum nonlocality and quantum teleportation [34], [35]. Since standard QFT considers only the global properties, it cannot possibly reveal any of these non-local features. Physicists are more familiar with the energy-momentum tensor, $T^{\mu\nu}(x^{\rho})$, when it arises in a fluid or in a crystal lattice. In these cases, the components $T^{ij}(x^{\rho})$ are understood as the local energy involved in stresses and strains in the medium. What is puzzling about the notion of a 'particle' in the quantum domain is that it seems to be behaving like an quasi-stable feature in some medium. Indeed this was a very early perception of some physicists and motivated the discussion of the possibility of a hydrodynamical model of a quantum phenomena. These ideas, in fact, go back to the suggestions of Madelung [36], which were used in a general theory by de Broglie [37] and applied in a relativistic context of the Dirac equation by Takabayasi [38].

Even Feynman [39] uses the idea of a "fluid" of electron pairs in his discussion of superconductivity. He derives, in fact, exactly the quantum Hamilton-Jacobi equation (1) and calls the quantum potential, simply the "quantum mechanical energy". However in a footnote he attributes the hydrodynamic ideas to Onsager, adding that "no one else ever understood why" this should be the case. Is this just a happy coincidence or is it a reflection of something deeper that is involved?

Certainly if we do not find more arguments to support the splitting the internal energy between kinetic and quantum potential, we would be inclined to dismiss this whole structure as fanciful. However there are two aspects of relativity that need to be remembered before coming to such a conclusion.

The first involves the difficulty that arises if we use the notion of a point particle in special relativity. Bohm argues that [40]

...it is not possible in relativity to obtain a consistent definition of an extended rigid body, because this would imply signals faster than light....physicists were driven to the notion of a particle that is an extensionless point, but, as is well known, this effort has not led to generally satisfactory results because of the infinite fields implied by point particles. Actually, relativity implies neither the point particles nor quasi-rigid bodies can be primary concepts. Rather, these have to be expressed in terms of events and processes.

In other words, we have to move away from the idea that we can regard the particle as a quasi-rigid body with some extension. Standard theory avoids this difficulty by regarding the particle as an excitation of the field. But the field itself produces its own well known singularities. For Bohm it is this particular difficulty that signaled the need to develop a radically new theory in terms of a discrete structure process [41]. He felt very strongly that the process philosophy that underlies these ideas should be accompanied by a

change of mathematical emphasis, introducing what he called the 'algebra of process' [42]. He was never able to complete this work and it was only after his passing that I was able to show one possible way to carry out this programme [43].

The second aspect to which I want to draw attention is the unusual properties the energy-momentum tensor of the gravitational field in general relativity. These properties are different from those of fields in flat spacetime. One of the first surprises that attracted my attention was a discussion of the energy-momentum tensor of gravitational waves by Bondi [44]. What clearly emerged from his work was that energy was not localizable. This was taken further by Penrose who reached the conclusion that "energy is a genuinely non-local quantity" [45].

It is very tempting to link the non-locality appearing in the gravitational energy with the quantum non-locality that manifests itself in the quantum potential. In fact Penrose has already made reference to this possibility and has suggested that this is where his twistor programme may help, although he feels more radically new ideas will ultimately be involved [46], [47].

Now twistor theory emerges from the conformal Clifford algebra and Bohm and I have already shown how this structure may be used to discuss the Dirac electron [48]. But this paper was written in the context of a deeper process philosophy that Bohm felt offered a possible approach. Indeed already in the last chapter of "Causality and Chance" [49] in the section headed "The process of becoming" he writes:-

Thus far, we have been discussing the properties and qualities of things mainly in so far as they may be abstracted from the processes in which things are always changing their properties and qualities and becoming other things. We shall now consider in more detail the characteristics of these processes which may be denoted by 'motion'.

The word 'motion' was later changed to 'movement' which, in turn, ultimately became the 'holomovement' [50]. Here Bohm's use of the notion of 'movement' is subtle. It is not the movement of objects, but is something more primitive and it is from this primitive movement that, not only objects, but space-time itself arises. I do not have the space to go into the subtle questions concerning Bohm's use of the term 'movement'. Here, fortunately, Paavo himself has presented an extensive discussion of what Bohm had in mind in his book [1] and I recommend the interested reader to that discussion. There Bohm's more general notions of the implicate and explicate orders are discussed in the context of other philosophies addressing the nature or reality.

One of the features that has been missing in this more general approach is a mathematics in which to ground the ideas. Bohm had already set out a very clear rational of the type of mathematics that he felt was worth exploring in his paper "Time, the Implicate Order and Pre-space" published in 1986 [42]. There he writes

My attitude is that the mathematics of the quantum theory deals primarily with the structure of the implicate pre-space and with how an explicate order of space and time emerges from it, rather than with movements of physical entities, such as particles and fields. (This is an extension of what is done in general relativity, which deals primarily with geometry and only secondarily with the entities described within this geometry.)

I have been continuing this work and have recently written a paper discussing the technical aspects of what is involved in carrying out this exploration [43]. What I have shown is that by using the categorical notion of process captured by groupoids, one is led fairly naturally to Clifford algebras, the mathematics that is central to the Pauli and Dirac theories, as well as twistor theory. Indeed I have shown how "shadow manifolds" can be projected out of these algebras, thus constructing space-times as explicate orders of the deeper structure process which Bohm refers to as the implicate order.

7 Conclusion.

I have argued here that Bohm's '52 papers [2] were not an attempt to find a deterministic, mechanistic account of the quantum formalism. He was well aware that if a clearer understanding of quantum phenomena was to be achieved, we need a radical new approach that would go deeper than the conventional view of what constitutes a quantum process. This deeper view would not take the notion of a particle as an *a priori* given structure. Rather the 'particle' would be some quasi-stable semiautonomous feature of this movement, taking its properties from the environment in which the movement is constrained. This gives a very different notion of a particle from that assumed in classical physics.

In the '60s when I had the privilege of working with Bohm, he was exploring many radical new ideas, and classical determinism could not have been further from his thoughts. Roger Penrose was at Birkbeck at that time and we had many discussions on how we could bring quantum mechanics and general relativity together in a single theory. Not surprisingly we reached different conclusions on how to proceed but what we did agree on was that a unification could not be achieved by sticking to the traditional view, namely, that physical processes can be adequately described in terms of particles/fields interacting in space-time. Rather there is a deeper underlying process from which not only do the particles and fields emerge, but this process is the source of space-time itself.

Not surprisingly this view has still to emerge, but what I wanted to do in this paper is to redress the view that prevails in much of the literature, namely, that Bohm was motivated by an overwhelming desire to return to a deterministic, mechanical view of the world. The fact that the main mathematical features of Bohm's approach, the Bohm momentum $\mathbf{P}_B = \nabla S$ and the Bohm energy $E_B = -\partial_t S$, have recently been shown to emerge more naturally from the low energy approximation of standard quantum field theory and that these features arise naturally in the context of Clifford algebras [43] should hopefully move the discussion on. This is a topic that Paavo and I have been debating recently and is a fitting note on which to end this dedication to him.

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