Non-commutative Geometry, the Bohm Interpretation and the Mind-Matter Relationship*.

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Abstract.

It is argued that in order to address the mind/matter relationship, we will have to radically change the conceptual structure normally assumed in physics. Rather than fields and/or particles-in-interaction described in the traditional Cartesian order based a local evolution in spacetime, we need to introduce a more general notion of process described by a non-commutative algebra. This will have radical implications for both for physical processes and for geometry. By showing how the Bohm interpretation of quantum mechanics can be understood within a non-commutative structure, we can give a much clearer meaning to the implicate order introduced by Bohm. It is through this implicate order that mind and matter can be seen as different aspects of the same general process.

1. Introduction.

The aim of this talk is provide a general framework in which the relation of ordinary matter to ordinary mind can be discussed[1]. I will not address any details concerning the structure of the complex of neurons or of the electro-chemical processes occurring in the brain, vital though these details are. Rather I will try to provide a general framework in which we can eventually explain how the physical-chemical-electrical properties of the brain can give rise to thoughts, feelings and ultimately consciousness.

I do not believe that today's physics is rich enough to handle these questions and it will be necessary to develop new concepts before we can really begin to explore this relationship adequately. It has been argued that classical physics will provide all the answers we need. I do not share this position. Nor does Stapp (1993) who writes "Classical physics strives to exclude the observer from physics and succeeds. On the other hand quantum mechanics strives to exclude the observer and fails". The first part of this quotation is undoubtedly correct and therefore classical physics excludes the very thing that we are hoping to understand.

On the other hand I do not share Stapp's belief that quantum mechanics already contains sufficient structure to answer the deep questions. My position here does not stem only from my study of the Bohm interpretation (Bohm and Hiley 1987 and 1993). It also

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[1] By 'ordinary mind' I specifically exclude the so-called 'paranormal'.

follows from my study of the Copenhagen view. Unfortunately the Bohm interpretation is misunderstood and misrepresented in the literature. It does not stand diametrically opposite Bohr's views. It actually shares some of its radical conclusions. For example, it clearly shows that there is an essential element of participation involved, a notion that has already been shown by Wheeler (1991) to be a feature of the standard interpretation. Bohr saw it slightly more radically when he remarked that basic to quantum theory is the impossibility of making a sharp distinction between the observed system and the means of observation. At this stage participation does not necessarily involves the human observer, but it certainly involves the observing apparatus. The extension to the brain/mind interface seems very inviting.

However this brings us to one of the main difficulties in trying to discuss whether quantum theory will help us to understand the mind/matter relationship. Quantum mechanics itself is plagued by problems of interpretation. Why else do we have all these other interpretations, viz. the standard interpretation, the Copenhagen interpretation, the statistical interpretation, the many-worlds interpretation, the Bohm-de Broglie interpretation, the Bohm ontological interpretation, the consistent histories interpretation, the transactional interpretation, the many minds interpretation, the modular interpretation.......

This list is by no means exhaustive! Clearly such a profusion of interpretations can only lead to the conclusion that there is something very wrong somewhere. I feel that there is a deep and fundamental problem in the framework into which we can try to fit quantum processes.

Notice that amongst the main properties demanded of an explanation in classical physics is that it is deterministic, continuous and local in space-time. In quantum theory at the particle level we find indeterminism, quantum jumps and non-locality in space-time and we are perplexed. However at the level of the wave function we restore determinism, continuity and locality in space-time, through the Schrödinger equation, the Dirac equation, and the Klein-Gordon equation (Dirac 1973). This means we have implicitly given the wave function ontological status by considering it to be the most complete description of the *state* of the system.

We as physicists are happy with this even though it leaves us with all the problems of interpretation at the particle level. We are happy that we have found a way of describing quantum phenomena without being forced to give up the classical paradigm. But by keeping this paradigm, which I will for convenience call the Cartesian order, we have continued to separate the observed from the means of observation. Thus we must necessarily maintain the sharp separation between mind and matter, between res cogitans and res extensia (Hiley 1997). By retaining this classical order, we have made it very difficult to see how physics is ever going to explain what I call the 'ouch factor'.

Before I go on to discuss how we can change this classical paradigm, I must say a little about the way I came to this position through my collaboration with David Bohm. Bohm started effectively with the question "In quantum mechanics, can we keep the notion of a particle with its simultaneously well-defined position and momentum and always talk about particles following trajectories?" This of course was denied by the conventional wisdom of the day. Indeed as far back as 1936 Norbet Weiner (1936) wrote,

> One might suppose that it is still possible to maintain that a particle such as an electron has a definite momentum and a definite position, whether we can measure them simultaneously or not, and that there are precise laws of motion into which this position and momentum can enter. *Von Neumann has shown that this is not the case, and the indeterminacy of the world is genuine and fundamental* [my italics]. There are no clear-cut laws of motion which enable us to predict the momentum and position of the world at future times in any precise way in terms of any observable data whatever at the present time.

We had to wait until the sixties for Bell (1987a) to point out exactly why the von Neumann theorem was limited as were the subsequent improvements of Gleason (1957) and Kochen and Specker (1967). These limitations do not apply to the Bohm approach. They were based on attributing to the particle eigenvalues of all possible operators simultaneous, together with the assumption of non-invasive measurement. The Bohm approach does not make such an assumption and treats measurement as non-invasive.

Let us now show how all this comes about. Following Bohm (1952), we substitute $\Psi = R\exp[iS/\hbar]$ into the Schrödinger equation. By splitting this equation into its real and imaginary parts, we obtained two equations, one showed that probability is conserved, while the other could be interpreted as providing, as Weiner (1936) put it, a "precise law of motion into which this position and momentum can enter". Solving this second equation we can calculate sets of trajectories for each quantum situation. All of the relevant details appear in the literature and references can be found in Bohm and Hiley (1993) and in Holland (1993).

This second equation can be written in the form

$$m\left(\frac{d^2x}{dt^2}\right) = \quad [V+Q] \qquad \text{where} \quad Q = -\frac{\hbar^2}{2m}\frac{{}^2R}{R} \tag{1}$$

These equations give the impression that we have returned to a classical account of quantum phenomena. Nothing could be further from the truth. Not only is there no return to Newtonian mechanics, but no form of mechanics can be sustained (Bohm 1957). Central to understanding how the Bohm interpretation is the appearance of the quantum potential, Q. It is not *ad hoc* as suggested by Heisenberg (1959) but emerges directly from the Schrödinger equation and without it, energy would not be conserved.

The quantum potential does not have the usual properties expected from a classical potential. It does not arise from an external source; it does not fall off with distance. It seems to indicate a new quality of internal energy and more importantly from our point of view, it give rise to the notion of participation, non-separation and non-locality. At the deeper level it arises because, as Bohr often stressed, it is not possible to make a sharp separation between the observing instrument and the quantum process while the interaction is taking place. The Bohm approach makes a *logical* distinction between the two but then the quantum potential links them together again so that they are *actually* not separate. It is this factor that gives rise to context dependence, and to the irreducible feature of participation between relevant features of the environment in the evolution of the system itself. It was this factor was not incorporated into by the no-go theorems discussed above.

The appearance non-separability and non-locality in the Bohm approach led Bell (1987b) indirectly to his famous inequalities. Of course, non-locality is not a feature that fits comfortably within the mechanical paradigm, but it was this feature that led Bohm to the conclusion that his approach was NOT mechanical. More details can be found in Bohm and Hiley (1993).

Our conclusion from our detailed investigations into these questions in the Bohm approach and in our review of other approaches to quantum mechanics led us to the conclusion that the Cartesian order could no longer be used to explain quantum processes. Indeed even in a model that re-writes the Schrödinger equation in a form that apparently brings it closer to the equations of classical mechanics contains non-separability, participation and non-locality. What is needed is a radically new order in which to understand quantum phenomena. Bohm (1980) suggested that this new order would be based on process. He called this new order the implicate order. I will not discuss Bohm's own justification for his proposals, but would rather approach the subject from a different point of view, which suggests that novel structure actually lies beyond space-time. This structure is rich enough to discuss the relation between mind and matter.

2. Is spacetime primary?

I first came across this possibility from a lecture by Geoffrey Chew (1960). He pointed out that there is no necessity to start an explanation of quantum processes in space-time. Complementarity shows that we could either start in space-time, or we could have started in the energy-momentum plane, but we can never start with *both together*. This is actually an old idea stressed by Bohr (1925) in the early days of quantum mechanics. He writes

> I am quite prepared that the view we proposed (Bohr-Kramers-Slater theory) on the independence of the quantum process in widely-separated atoms should turn out to be incorrect….. the Ramsauer's results on the penetration of slow electrons through atoms, presents difficulties for our ordinary space-time description of nature similar to those presented by a simultaneous understanding of interference phenomena and a coupling through radiation of the changes of state of widely-separated atoms. I believe that these difficulties so thoroughly rule out the retention of the ordinary space-time description of phenomena…

Chew (1960) brought this idea out in a new and striking way by drawing attention to the S-matrix approach to high-energy processes. Here the energy-momentum plane is taken as basic so that we can exploit strict energy and momentum conservation. But then the role of the spacetime manifold has to be *derived* since it can no longer be regarded as basic. This brings us to the question of the role of space-time itself. Why is it regarded as primary and basic?

When we come to consider the problems of quantising gravity while retaining general relativity, we face the following dilemma. As is well known in general relativity the gravitational potential is identified with the metric tensor. Now in any quantum field theory, the fields themselves are subject to quantum fluctuations. Thus the quantised gravitational field would imply fluctuations in the field and since the gravitational potential reflects the metric properties of the space, the space-time itself must be fluctuating. But what then is meant by a fluctuating space-time?

The third problem in assuming that space-time is fundamental arises from the appearance of quantum non-locality. If space-time is taken as primary, then, *ipso facto*, locality is absolute. Indeed the space-time manifold dominates classical physics because it has locality built into it right at the beginning. If we retain the space-time manifold, then quantum non-locality sits very uncomfortable in such a structure.

Could it be that our insistence on taking a given space-time as basic is at fault? Could space-time merely be an appearance, a feature that has to be abstracted from some deeper structure, a structure where space-time itself is not taken as basic? If this were the case, then it would be establishing locality that would present the problem. Could it be that locality itself is merely a relationship? This relationship dominates the macroscopic world, but it would not be universally valid at the quantum level. Yes there is relativity, but does that theory apply to the level of a single photon or only to a statistical ensemble of photons?

The first suggestive example of showing how locality could be a relationship appears in the hologram. Here a picture of an object is recorded as an interference pattern. The image of the original object can be re-created by using an appropriate light source. If the hologram is now torn in half and the light passed through this half, we again see the *whole* object, albeit with some loss of overall definition. Clearly the local regions of the original object are mapped into the whole of the photograph, so that locality is being carried in a non-local way. Thus locality here is clearly carried as a relationship. Can idea be generalised?

Suppose locality is a relationship, could it be that quantum phenomena are in some sense beyond space-time and are merely projected into space-time by our macroscopic instruments? In other words, could quantum processes be evolving in some more general space, which for convenience we call simply 'pre-space'. This pre-space (Hiley 1991, Hiley and Monk 1993) would then give rise to Wheeler's (1980) pre-geometry. In this view, the space-time of the classical world would be some statistical approximation and not all quantum processes can be projected into this space without producing the familiar paradoxes, including non-separability and non-locality. In classical physics everything is local so that a single space-time can provide a contradiction free description.

If we adopt this radical view, we can see that it is not necessary to insist on the Cartesian division between res extensa and res cogitans. Matter actually has its origins in a deeper structure, a structure where space-time and hence extension is not primary. If such an approach were viable then matter and mind need no longer be separated by space-time constraints as illustrated in the picture below.

Cartesian cut

Res extensa	Res cogitans
Locality, Continuity & Determinism	Nonlocality, Jumps & Indeterministic!!
IN SPACE-TIME	NOT IN SPACE-TIME

This is fundamentally the wrong view. Something new is needed, and this new order must not take space-time as basic and fundamental.

3. Non-commutative algebras.

The question I now want to turn to is how are we going to implement this general programme mathematically? I want to suggest the answers lie in the non-commutatity of the quantum formalism. To bring this point out, consider a particle in motion. In classical mechanics both the position, x, and the momentum, p, are known, so that we can construct a phase space in which we can track the particle as it follows a specific trajectory (see figure 1)

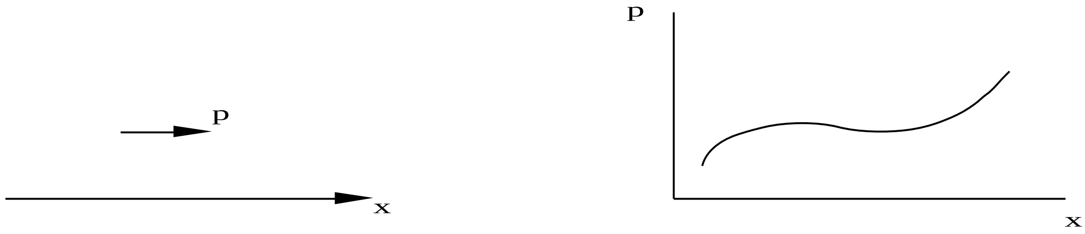

Figure 1. Classical phase space.

In quantum mechanics, non-commutativity implies that we can either have exact information about x through the eigenvalues of the position operator, X, or we can have exact information about the momentum eigenvalues, p, of the momentum operator, P. We can not have both together because $[X, P] = i\hbar$. Thus we have either

$$S_1 X S_1^{-1} = X_{dia} = \begin{pmatrix} x_1 & & & \\ & x_2 & & \\ & & . & \\ & & & . \end{pmatrix}$$

x_1	x_2	x_3	x_4	....

Or

$$S_2 P S_2^{-1} = P_{dia} = \begin{pmatrix} p_1 & & & \\ & p_2 & & \\ & & . & \\ & & & . \end{pmatrix}$$

p_1	p_2	p_3	p_4	....

Thus we cannot construct an exact phase space as we can in classical mechanics. All of this is, of course, well known, but I want to take it further. To do this I have to take you back to the end of last century, to the work of Hamilton (1967), Grassmann (1995) and Clifford (1878). In this period in the development of mathematics, the study of the properties of algebras was in a fairly primitive stage and there was an energetic discussion as to the metaphysical significance of algebra in general.

To give a flavour of the attitudes of the time, consider the title of Hamilton's (1837) lecture "The Metaphysics of Mathematics-Algebra as Pure Time", a title that one would expect from someone on the fringe, and not someone at the centre of things! In that lecture he wrote:

> In algebra relations are between successive states of some changing thing or thought. In other words algebra is not about material process but something more general that could be applied to both matter and mind.

Grassmann (1995) takes this further. He argues that mathematics is about thought, not material reality. It is about relationships of form, not relationships of content. Mathematics is to do with ordering forms created in thought. Thus since thoughts are not located in space-time, mathematics is not necessarily about material things in space-time.

Now thought is about becoming, how one thought evolves into an other. Is a new thought independent of the old thought; is the old thought independent of the new thought? Surely the old thought contains the potentiality of the new thought and the new thought contains a trace of the old thought. There is no separation. Thinking is about becoming not being. Being is a relative invariant in the overall process of becoming. The basic ingredient must therefore be activity or process and this process is described by the elements of an algebra.

4. The algebra of process.

The main novel feature of the mathematics of quantum theory lies in the non-commutative structure of its algebra of operators. If we regard the eigenvalues as labelling the properties of things, non-commutativity does not seem to make any sense. For example, objects should have position and momentum simultaneously. Objects should have an *x*-component and a *y*-component of angular momentum simultaneously, but in quantum theory they do not.

There no room for non-commutativity in the classical world of objects. Yet the classical world does actually contain lots of non-commutativity. Try taking a cup from a cupboard before opening the door! Try rotating an object through a 90° rotation about the *x*-axis and then 90° about the *y*-axis. Repeat by first rotating about the *y*-axis and then about the *x*-axis. You end up with different configurations. In other words the classical world contains plenty of examples of non-commutativity, but it is always associated with activity or process.

With this in mind let us look more closely at what the algebra of quantum theory implies when we apply it to simple situations. To highlight the problem let me illustrate the difficulty if something like colour and shape were described by non-commuting operators[2]. To make things even simpler let us suppose there are only two colours red and blue, and there are just two shapes, spheres and cubes. Furthermore we cannot view these properties directly but need some instrument to determine the colour and another to determine the shape.

Suppose further that in this example our observables are represented by the non-commuting operators C and S, with $[C, S] \neq 0$. Our objects must be described by wave functions, Ψ_R for red, Ψ_B for blue, Φ_S for sphere and Φ_C for cube.

Now let us try to collect together a set of red spheres. First we measure the colour and collect all the reds together in one group, separating them from the blues. Take the red set and find out which of these red objects are spheres. Thus we can collect a set of objects that were red according to the first measurement and spheres according to the second measurement[3].

We might be tempted to conclude that we now have a collection of red spheres, but we had better check that they are all still red! When we check this, we find half of our spheres have changed colour and are now blue! This result follows from the fact that the 'observables' do not commute. If $[C, S] = 0$ then re-measuring the colour of the objects would still all be red. It is only in this case that we can divide our objects into four unique sets; sets of red spheres, sets of red cubes, sets of red cubes and sets of blue cubes. In the world of non-commutativity this is something you cannot do. You cannot display every property in one 'picture'.

This example shows very clearly what we are up against in quantum theory. The central question is how do we understand this situation. We can follow Bohr (1961) and argue that it is simply a fact that must be understood in terms of the principle of complementarity.

As an alternative you could try to maintain the assumption that colour and shape are still properties but that the measurement process itself changes the complementary variables in some new way. Measurement simply makes manifest one particular partial view of nature and it is not possible to make manifest *all* aspects of reality in one single universal 'picture'. This is the view expressed by Bohm's implicate order (Bohm 1980).

At the deeper level, the order is not explicit, its is implicit and the structure of this implicate order is captured by the algebra. Our measurement merely displays one particular aspect, which we call the explicate order. Different measurements produce different explicate orders. Thus in the example above, colour would be one explicate order and shape another. At the deepest level the process has neither colour nor shape. These features arise only in relation to the process of manifestation. Each process forms a totality and our attempts to describe this requires us to divide the process into text and context to make it meaningful.

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[2] Here we will effectively replace spin by colour and direction by shape.

[3] Remember we have two different pieces of apparatus, one to measure colour and the other to measure shape.

5. Consequences for phase space.

Let us now move on to consider how it might be possible to discuss the general evolution of such a process. I have already discussed a number of these ideas in some detail elsewhere (Hiley 1991, Hiley and Fernandes 1997). Here I simply want to illustrate what I have in mind without going into detail again, much of which can be found in the above papers.

All process starts with a law of succession, how one process evolves globally into another. This law is expressed through a rule of multiplication. Succession must be complemented with coexistence, giving rise to the law of coexistence. This is represented by addition thus defining an algebraic structure. We can add one further feature, namely, the process must be made manifest relative to a context. Different contexts are then determined by different representations.

In order to show how a quantum phase space can be constructed we need the symplectic Clifford algebra (Hiley and Monk 1998, Crumeyrolle 1990). This algebra contains the necessary structure to build a quantum phase space. The points of this space can be constructed from the idempotents of the algebra. An idempotent is defined through the relation $P^2 = P$, that is a point is a process, which under the law of succession, transforms into itself. Thus points themselves are not static concepts, but part of the underlying process.

The algebra itself contains elements that enable one idempotent to be translated into another, so that the algebra contains within itself, its own translation operator. This whole structure enables us to define a space, which we will call the *x*-space. This is illustrated in the figure below.

● ● ● . . . . . . ● ●

x_1 x_2 x_3 x_{n-1} x_n

x-space

But this is not the only space that is contained in the algebra. An appropriate inner automorphism produces the *p*-space.

● ● ● . . . . . . ● ●

p_1 p_2 p_3 p_{n-1} p_n

p-space

However the *p*-space can only be 'constructed' at the expense of 'destroying' the *x*-space. Thus the basic underlying process itself is such that it is not possible to construct simultaneously a phase space in which both x and p are sharply defined. This is of course exactly what the theorems of von Neumann (1955) and Gleason (1957) are about.

Notice we are not regarding the lack of precision as being due to an 'uncertainty' as if everything is actually certain, but that we, as observers, are uncertain as to the precise values because of some 'ham-fisted' use of apparatus. We are arguing that the process itself is such that it is not possible in principle to define x and p together because simultaneous x and p does not have a meaning.

The basic underlying assumption of this general approach is that the ontology is based on process, a process that cannot be described explicitly. It can only be described implicitly, hence the terminology 'implicate' order. This implicate order is a structure of relationships, and this order of structures is described by an algebra, the algebra of process (Hiley 1995). Here the implicate order is not some woolly metaphysical construction, it is a precise description of the underlying process, mathematically expressed in terms of a non-commuting algebra. This process only allows partial views because nature is basically participatory.

To put it more strongly, it is not that we as observers who participate in nature, *but that nature participates in nature.* Thus the observer is not something special. The cosmos does not need observers to function and evolve. Observation is simply a particular example of general notion of transformation in which the observed and observing processes fuse in an indivisible and irreducible way. Bohr (1961) talked about it as "the indivisibility of the quantum of action". It is not that we can never separate objects from the observing process, we can once the interaction has ceased. But during the interaction the individual becomes an intrinsic part of the whole process, and becomes transformed in the process. This is how the example of the colours and shapes outlined above can be understood. The colour measuring process can transform the shape and the shape measuring process can transform the colour.

6. The evolution of process.

The notions discussed in the previous section are very different from what we are used to but fortunately Bohm found a very illuminating metaphor through which to illustrate some of the key features. Indeed the metaphor has the advantage of suggesting how we may describe the evolution of process mathematically without the need of a space-time manifold.

Consider a hollow outer cylinder containing an inner cylinder that can be rotated relative to the outer cylinder. Glycerine is poured between the cylinders as shown in figure 2. Then a spot of dye is introduced into the glycerine at some suitable point. If the inner cylinder is rotated, the dye disappears. There is nothing remarkable about that, but if the inner cylinder is rotated in the opposite direction, the spot re-appears. (There is some diffusion if this is carried out in a real experiment, but the diffusion is actually small and can be ignored for the purposes of the metaphor.)

In the spirit of the implicate order, we can regard the 'order' of the spot to be enfolded in the glycerine, so that it becomes 'implicit' in the glycerine. It can be made manifest again by rotating the inner cylinder in the opposite direction. We could imagine a series of dots enfolded at different times and at different neighbouring positions. As the process is 'unfolded' a succession of spots are re-manifested giving the appearance of something moving through the glycerine. This gives the appearance of a particle following a trajectory although there is no particle, there is just a process of enfolding and unfolding.

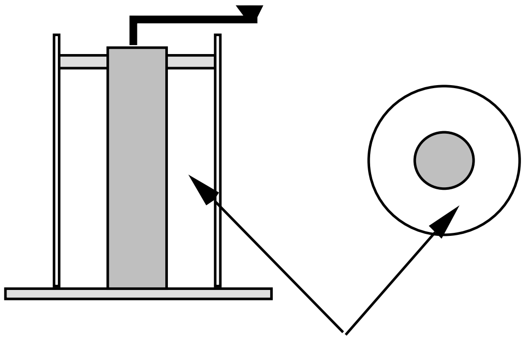

Figure 2. The Unmixing Experiment.

The physical process lying behind the glycerine illustration has a clear classical explanation at the atomic level. That is not the point. What the metaphor is intended to do is to bring out the fact that if the basic process was activity *per se*, then the 'track' left in, say, a bubble chamber could be explained by such an unfolding and enfolding process. Thus rather than the track being seen as the continuous movement of some material object, it can be regarded as the continuity of a quasi-stable form, evolving within the unfolding process.

We can think about describing such a process in the following way. Suppose we consider two successive moments described by the explicit orders $e(\tau_1)$ and $e'(\tau_2)$, where τ is some parameter[4]. Let the unfolding process be described by M_1 and the enfolding process be described by M_2. Then if we use the law of succession we have the two processes $e(\tau_1)M_1$ and $M_2e'(\tau_2)$. But the continuity of form demands that

$$e(\tau_1)M_1 = M_2 e'(\tau_2) \qquad\qquad \tau_2 > \tau_1.$$

Thus the enfolding and unfolding movement is an automorphism of the algebra since

$$e'(\tau_2) = M_2^{-1} e(\tau_1) M_1 \tag{2}$$

If we now assume for simplicity that

$$M_1 = M_2 = M; \qquad\qquad M = \exp[-iH(\tau_2 - \tau_1)] \tag{3}$$

then for small τ we find

[4] Here we can regard τ as an unfolding parameter.

$$e' = (1 + iH\Delta\tau)e(1 - iH\Delta\tau)$$

so that

$$i\frac{\Delta e}{\Delta\tau} = [H, e], \tag{4}$$

which is the same form as the Heisenberg equation of motion if we identify the unfolding parameter as time. Thus our general ideas have led us to an equation of motion that is identical to the one basic to quantum mechanics.

If we now write $e = AB$, then equation (4) becomes

$$i\left(\frac{\Delta A}{\Delta\tau}\right)B + iA\left(\frac{\Delta B}{\Delta\tau}\right) = (HA)B - A(BH).$$

The form of this equation suggests the possibility that the whole process is can be described by a pair of equations

$$i\left(\frac{\Delta A}{\Delta\tau}\right) = HA \qquad \text{and} \qquad -i\left(\frac{\Delta B}{\Delta\tau}\right) = BH$$

If we identify A with Ψ and B with Ψ^* we see that these equations have the same form as the Schrödinger equation and its conjugate. There is an important difference however, A and B are *operators*, not wave functions, so that the equations are in the algebra itself.

7. Schrödinger equation in operator form.

In a recent paper Brown and Hiley (2000) have shown formally how to obtain the Schrödinger equation in terms of operators from within the quantum algebra itself. We will not repeat the details here but will merely note the results.

We first considered the wave operator $\Psi(A)$, where Ψ is some function of the operator A. If we again use the polar form for the wave operator, we can arrive at the two equations

$$i\frac{d\rho}{dt} + [\rho, H]_- = 0 \tag{5}$$

and

$$\rho\frac{dS}{dt} + \tfrac{1}{2}[\rho, H]_+ = 0 \tag{6}$$

where $\rho = \Psi^*(A)\ \Psi(A)$, is the density operator[5]. Equation (5) is an expression for the conservation of probability, while equation (6) is an expression of the conservation of energy when the energy is well defined in the quantum system.

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[5] It is essential to include the projector onto the standard ket . For details see Brown and Hiley (2000).

In passing I wish to point out the close relation of our results to those exploited in the Bohm approach to quantum theory. If we express (5) and (6) in the *x*-representation we arrive at the original Bohm theory. It is straight forward to show that they become

$$\frac{\partial \rho}{\partial t} + \quad {}_r . j = 0 \qquad \text{(Conservation of probability)} \quad (7)$$

and

$$\frac{\partial S}{\partial t} + \frac{\left(\quad {}_r S\right)^2}{2m} + Q(\boldsymbol{r},t) + V(\boldsymbol{r},t) = 0 \qquad \text{(Conservation of energy)} \qquad (8)$$

It can be shown that equation (8) is equivalent to equation (1). Now we see that this extra energy, $Q(\boldsymbol{r}, t)$ is needed to conserve energy because what we have called p is only the real part of p since

$$ {}_r S = \quad \left[\Psi^*(\boldsymbol{r},t)P\Psi(\boldsymbol{r},t)\right] = p_{Bohm} \qquad (9)$$

It is through this identification that the streamlines of the probability current given by equation (7) can be regarded as particle trajectories with p_{Bohm} being the momentum of the particles at any point on the trajectory. These are the usual Bohm trajectories discussed in section 1.

Before going on to explain the meaning of these ideas in the present context, we should also note that (5) and (6) in the *p*-representation become

$$\frac{\partial \rho}{\partial t} + \quad {}_p j = 0 \qquad (10)$$

$$\frac{\partial S}{\partial t} + \frac{p^2}{2m} + Q(p,t) + V(\quad {}_p S,t) = 0 \qquad (11)$$

Once again we have a quantum potential, but it no longer is a simple expression since it now depends on the form of the classical potential. In the momentum representation the momentum is the observable momentum, but the 'beable' position is now given by

$$ {}_p S = \quad \left[\Phi^*(p,t)X\Phi(p,t)\right] = x_B \qquad (12)$$

Once again we see that we need the quantum potential energy because the classical potential energy is determined by $V(x_B)$ and not by $V(x)$ where x is the observable position. Again, the streamlines of the probability current appearing in equation (10) give the particle trajectories in this representation.

The appearance of two sets of trajectories might at first sight be surprising. The Bohm approach, particularly under the guise of "Bohmian mechanics" has insisted on attributing absolute reality to the particle evolving in a phase space. Bohm himself long abandoned that position. In Bohm and Hiley (1993) we were very careful to present the trajectories in the sense that if we assumed these were particle trajectories, then no inconsistency would arise. In fact not only was the approach consistent, but it was also free of many of the quantum puzzles such as the cat paradox, the measurement problem and so on. At no point did we insist that the particle view corresponded to what actually did take place. We did not insist that our description provided a mechanical approach to quantum processes. I thought we made that clear throughout the book, but it seems that we failed to get the message across. Again I thought the last chapter, which was a very brief summary of the ideas that I am developing in this paper, would have given a strong message that we did not believe the simplistic mechanical approach was viable.

For us the ontology is the notion of activity or process that was described by the algebraic structure of quantum formalism. This can be understood in terms of the implicate order, which in turn, finds it observable consequences in explicate orders.

As we have remarked earlier it is not possible to describe quantum processes in terms of a classical phase space because x and p cannot be defined simultaneously. This is a consequence of the non-commutative structure of the formalism. All we are able to do is to construct shadow phase spaces, each one being an explicate order defined by the context in which it is displayed.

Thus in the example we have discussed above we have two shadow phase spaces, one based on the x-representation, the other based on the p-representation. These are shown in figure below. Each, although different from the classical point of view, is necessary for a full representation of the quantum process. The non-commutativity of the underlying process produces an *ontological complementarity*. This must be contrasted to Bohr's epistemological complementarity.

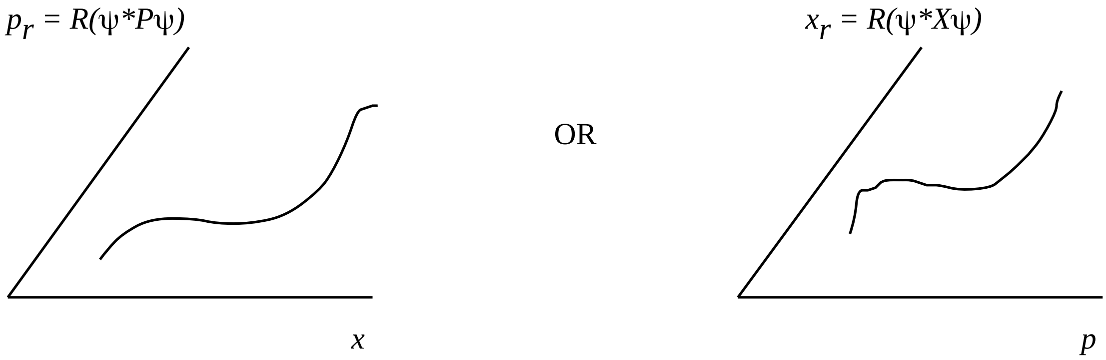

Figure 3. Shadow Manifolds.

Our shadow manifolds are not mutually exclusive, they are complementary and are a consequence of the participatory process of manifestation, or, more conventionally, of observation. Within this description the quantum potential plays the role of an internal energy necessary because of the way we are constructing our shadow manifolds. This potential is totally unlike any classical potential. It has features more akin to a self-organising potential. Indeed this self-organisation occurs in response to the environment in which the quantum process finds itself. In fact we have argued elsewhere that expressing the process in a shadow manifold determines an information dynamics (Hiley 1999)

8. The mind-matter relationship.

What I have tried to argue above is that for quantum processes space-time is not the basic manifold in which quantum processes evolve. The basic process unfolds in this pre-space, which is not subject to the Cartesian division, res extensa-res cogitans. What I want to suggest that it is in this pre-space that mind and matter appear as different aspects of the same underlying process.

Thus mind and matter are united through mutual participation in which separation is not possible. They are two aspects of an indivisible totality, the implicate order. Aspects of this whole activity involve the process of thinking, feeling, desire etc. The dance of the neurons is only the outward material manifestation of these processes. These physical processes are merely an explicate order in which one aspect of the overall process is projected. By restricting our discussions to the electrochemical process of the brain, we miss the deeper implicate order which contains our experience of the physical and mental worlds. But even in using those words, it must not thought that there are two sides, mind and matter. Mind and matter are but different projections from this deeper implicate order where such a division does not exist.

We experience this implicate order directly when we try to explain to others how we feel or think. The words we use are only signifiers that seem to float on a sea of inner energy. We struggle for words to try to capture what is implicit in our thinking. But the meaning is not merely in the words, it is in the context in which those words are used. The context is often implicit and as we try to clarify this context, another, yet deeper context is assumed. But we can never make any complex set of ideas totally explicit. What we do is to try to create in the reader the implicate structure that we feel within ourselves.

Perhaps the clearest example of the role of this implicate order comes from listening to music. Listening is an active experience where we participate in the movement itself. We do not perceive a series of isolated notes. We hear new notes reverberating within the memory of the previous notes. This together with the anticipation of future notes constitutes an unbroken movement. What is apprehended, then, is an undivided state of flowing movement. We can argue that we directly perceive the implicate order because we become part of the total movement. We comprehend movement in terms of a series of inter-penetrating, intermingling elements of different degrees of enfoldment all present together.

To summarise then we have on the one hand mind where the content of thought is displayed in explicate orders, while the process of thinking, feeling, etc occurs through the activity of unfolding and enfolding in the implicate order. At the same time, the display of matter occurs in the explicate order, but its deeper quantum movement occurs through unfoldment and enfoldment in the implicate order. Thus the ground of both thought and matter is in the implicate order. Our task is to find an algebraic description of those aspects of this implicate order where mind and matter have their origins.

9. References.

J. S. Bell (1987a). *Speakable and Unspeakable in Quantum Mechanics*, pp. 1-13, Cambridge University Press, Cambridge.

J. S. Bell (1987b). *Speakable and Unspeakable in Quantum Mechanics,* pp.14-21, Cambridge University Press, Cambridge.

D. Bohm (1952). *Phys. Rev*., **85**, 166-179; **85**, 180-193.

D. Bohm (1953). *Phys. Rev*. **89**, 319-20.

D. Bohm (1957). *Causality and Chance in Modern Physics*, Routledge & Kegan Paul, London.

D. Bohm (1980). *Wholeness and the Implicate Order,* Routledge, London.

D. Bohm and B. J. Hiley (1987). *Phys. Reports*, **144**, 323-348.

D. Bohm and B. J. Hiley (1993). *The Undivided Universe: an Ontological Interpretation of Quantum Theory,* Routledge, London,

N. Bohr (1925). Letter to Geiger, 21 April 1925, Coll. Works, V, p. 353

N. Bohr (1961). *Atomic Physics and Human Knowledge*, Science Editions, New York,

M. R. Brown and B. J. Hiley (2000). quant-ph/0005026.

G. Chew (1963). *Science Progress*, **51**, 529-539.

W. K. Clifford (1878). *Am. J. Math.*, **1**, 350-358.

A. Crumeyrolle (1990). *Orthogonal and Symplectic Clifford Algebras*: *Spinor Structures*, Kluwer, Dordrecht.

P. A. M. Dirac (1973), in *The Physicist's Conception of Nature*, p. 10, ed., J. Mehra, Reidel, Dordrecht-Holland.

A. M. Gleason (1957). *J. Math. Mech*. **6**, 885-893.

H. Grassmann (1995). *A New Branch of Mathematics: the Ausdehnungslehre of 1844, and other works*, trans. by L. C. Kannenberg, Open Court, Chicago.

W. R. Hamilton (1837). *Trans. Roy. Irish Acad*., 17, 293-422.

W. R. Hamilton (1967). *Mathematical Papers*, Vol. 3, *Algebra*, Cambridge University Press, Cambridge,.

W. Heisenberg (1959). *Physics and Philosophy: the revolution in modern science*, p.116, George Allen &Unwin, London.

B. J. Hiley (1991). Vacuum or Holomovement, in *The Philosophy of Vacuum*, ed., S. Saunders and H. R. Brown, pp. 217-249, Clarendon Press, Oxford.

B. J. Hiley and N. A. M. Monk (1993). Mod. Phys. Lett., **A8,** 3225-33.

B. J. Hiley, The Algebra of Process (1995), in *Consciousness at the Crossroads of Cognative Science and Philosophy*, Maribor, Aug. 1994, pp. 52-67.

B. J. Hiley (1997). Quantum mechanics and the relationship between mind and matter, in *Brain, Mind and Physics*, 37-53, ed P. Pylkkänen et al, IOS Press, Helsinki.

B. J. Hiley and M. Fernandes (1997). Process and Time, in *Time, Temporality and Now*, ed., H. Atmanspacher and E. Ruhnau, 365-383, Springer-Verlag.

B. J. Hiley (1999). *Quantum theory, information dynamics and the relationship between mind and matter,* Proc. Int. School of Biophysics “Neuronal Coding of Perceptual Systems, Naples, Italy.

P. Holland (1993). *The Quantum Theory of Motion*, Cambridge University Press, Cambridge.
S. Kochen and E. P. Specker (1967). *J. Math. Mech*. **17**, 59-87.
H. P. Stapp (1993). *Mind, Matter and Quantum Mechanics*, Springer, Berlin.
J. von Neumann (1955). *Mathematical Foundations of Quantum Mechanics*, (Princeton University Press, Princeton.
J. A. Wheeler (1991). *At Home in the Universe*, p .286, AIP Press, New York.
J. A. Wheeler (1980). *Pregeometry: Motivations and Prospects*, in Proc. Symp. Loyola Uni., New Orleans, ed., A. R. Marlow, pp. 1-11, Academic Press, New York.
N. Wiener (1936). *Phil. of Sci*., **3**, 307.