Towards a Dynamics of Moments: The Role of Algebraic Deformation and Inequivalent Vacuum States.

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[Published in Correlations ed. K. G. Bowden, Proc. ANPA 23, 104-134, 2001]

Abstract.

We continue our investigations of the algebra of process by discussing a general evolution in terms of 'moments', $[T_1, T_2]$, a notion that arises naturally in our approach. Firstly we show how our work is related to the iterant algebra of Kauffman and a possible connection to the incidence algebra of Raptis and Zapatrin. Then by considering the limit as the duration of the moment approaches the infinitesimal, we obtain a pair of dynamical equations, one expressed in terms of a commutator and the other which is expressed in terms of an anti-commutator. These two real equations are equivalent to the Schrödinger equation. If these moments are then described in purely algebraic terms we make contact with the quantum field approach of Umezawa. This enables us to discuss quantum processes at finite temperatures within one formalism. This in turn enables us to express our dynamical evolution in a way that is equivalent to a movement between inequivalent vacuum states. Thus we are able to make contact with the discussion on the thermodynamic origin of time.

Introduction.

I want to start by facing directly a very basic question "What is a quantum object?" That's easy surely? An electron is a quantum object! Those simple words hide a perplexing riddle that takes us far from the comfort of our classical world. Let us venture into a quantum world with a simple analogy, originally proposed by Weyl. Take 'shape' and 'colour' as quantum operators that do not commute. To make this world simple suppose there are only two shapes that are the 'eigenvalues' of the 'shape' operator and only two colours that are 'eigenvalues' of the 'colour' operator—sphere, cube, red, and blue.

I want to collect together an ensemble of red spheres. In this world I have to use an instrument to measure colour and another incompatible instrument to measure shape. I decide first to collect together spheres and discard all the cubes. I then decide to collect together those spheres that the colour-measuring device classifies as red. I am done. I have an ensemble of red spheres. So what is the problem? Just recheck that the objects in the ensemble are in fact spheres. We check and find that half are now cubes! No *either/or* in this world. No *and/and* either!

Let us look closer at this world and follow a suggestion of Eddington (1958) that the elements of existence can be described by idempotents. The eigenvalues of an idempotent is 1 or 0, existence or non-existence. In symbols

$$E^2 = E, \qquad \text{with} \quad \lambda_e = 1 \text{ or } 0.$$

This is fine if all idempotents commute. Existence is absolute. However in quantum theory, idempotents (projection operators) do not commute.

$$[E_a, E_b] \neq 0$$

What then of existence?

$$\text{Either } e_a \text{ or } e_b, \qquad \text{never} \qquad e_a \text{ and } e_b$$

Existence, non-existence and in between? Clearly no world of classical objects.

What now is the position of reductionism? It won't work because we cannot start with some set of basic building blocks. We cannot separate objects into ensembles with well-defined properties. How can we build stable structures if we can't do that? And when cube is blue, can we rely on it still being a cube as we try to build a structure of blue cubes?

No structures at all? How can this be? Quantum mechanics was introduced to explain stable structures. Without quantum mechanics there is no stability of matter! Without quantum mechanics there would be no atom as we know it. No crystalline structures, no DNA, and no classical world. But our observations start from the classical world. We are the DNA unfolded! We probe the quantum world from our classical world, so naturally we insist on reductionism. We strive to find the elementary objects, the quarks, the strings, the loops and the M-branes from which we try to reconstruct the world.

Surely we are starting from the wrong premise. Parker-Rhodes (1981) must be right, so too is Lou Kauffman (1982)! We should start with the *whole* and then make distinctions. Within these distinctions we can make finer distinctions and so on. These provide us with an order in the world, but an order that starts with us looking *out*. We are not God-like looking *in*. Should we think of these distinctions as passive marks or are we going to allow for the fact that we are part of the process of making these distinctions? Are we participators? Wholeness implies that we and our instruments are inside the whole process, yet our current theories start with the assumption that we and our instruments are outside our cosmos and we are struggling to get back in!

At this stage we must pause. The mere thought of "putting ourselves back into it" traps us into thinking that there is something independent and separate to be put back in. We should never be *out of it* in the first place! Now I hear alarm bells ringing. "He is going to suggest that we must put subjectivity back into our science whereas we know that the whole success of science has been to keep the subject out!" That is true of classical physics, but quantum physics says we must at least put our measuring instruments back into the system.

As Bohr constantly reminds us there is no separation between the system and its means of observation. He emphasises that this fundamental inseparability arises as a direct consequence of what he called "the indivisibility of the quantum of action". After warning us of the dangers of using phrases like 'disturbing the phenomena by observation' and 'creating physical attributes to atomic objects by measurements' he gives an even clearer statement of his position. He writes, "I advocate the application of the word phenomenon exclusively to refer to the observations obtained under specified circumstances, *including an account of the whole experimental arrangement*." Because of the meaning Bohr attaches to the word 'phenomenon', he insists that analysis into parts is *in principle* excluded.

However Bohr (1961) himself as the observer, is still outside. He claims to be a *detached* observer. No pandering to subjectivity here. But the question that fascinates me is "How did he become detached?" Let me spell out problem. I am assuming that the universe did come into being from some form of quantum fluctuation along the lines that is currently assumed. The exact details as to whether this takes the form of a unique occurrence or in the form of a multiverse, or yet something else is of no significance for my argument here. Any quantum birth must have evolved into our classical world and the question is what are the essential properties of this evolution for the emergence of a classical world to take place.

Bohm and I have already given a description of how this could happen in the context of the Bohm approach (Bohm and Hiley 1993), but there we already start half-way along the road when we single out the particle. In this paper I want to look at the problem from a very different perspective. This perspective does not allow me to start with particles. It does not let me use the popular story of decoherence. That is fine if a classical world already exists. There decoherence plays a vital role. But I see no way of making sense of classical ideas starting from the notion of an indivisible unity that is the baby universe.

Activity and Process.

I want to start from the flow of experiences we encounter from the time we leave our collective intellectual womb. As Lou Kauffman (1982) stresses, the primitive perception is *distinction.* We perceive differences, make distinctions and build an order. We do this through relationships. We relate different differences. We perceive similarities in these differences and then look for the differences in these similarities and so on. In this way we construct a hierarchy of order and structure in the manner detailed by Bohm (1965) in his long forgotten paper *Space, Time and the Quantum Theory understood in Terms of Discrete structure Process*.

But the differences of what? Just difference! We experience a flux of sensations, which we must order if we are to make sense of our world. We focus on the invariant features in that flux. What is inside, what is outside, what is left, what is right and so on. More generally what is *A*, what is *not-A*. But the distinction *A*/*not-A* is not necessarily absolute in a world of process. In a different flux of perceptions, *B* and *not-B* may become a distinction. In this context it may not be possible to make the distinction between *A* and *not-A.* The processes are ontologically and epistemologically incompatible so that even distinction becomes a relative concept. Ultimately we could reach some domain when the distinction becomes absolute in that domain. Thus emerges the classical world with its absolute and stable distinctions. But note that this ordering does not only apply to the material world. It also applies to the world of thought. Here it is quite clear that the observer, the *I* of my mind, is part and parcel of the overall structure of the same mind. It is here that we have direct experience with the notion of wholeness. It is also here that we have direct experience of flux, activity and process philosophically highlighted by Fichte and Schelling.

But even here it is easy to slip back into the categories of objects being the primary, forgetting that these objects take their form from the very activity that is thinking. I cannot capture this point better than Eddington (1958) did when he wrote,

> Causation bridges the gap in space and time, but the physical event at the seat of sensation (provisionally identified with an electrical disturbance of a neural terminal) is not the *cause* of the sensation; it *is* the sensation. More precisely, the physical event is the structural concept of that which the sensation is the general concept.

Or perhaps we should use the school of continental philosophers like Fichte (1994) who wrote,

> For the same reason, no real being, *no subsistence or continuing existence*, pertains to the intellect; for such being is the result of a process of interaction, and nothing yet exists or is assumed to be present with which the intellect could be posited to interact. Idealism considers the intellect to be a kind of doing and absolutely nothing more. One should not even call it an *active subject*, for such an appellation suggests the presence of something that continues to exist and in which an activity inheres.

Idealism? Probably much too far for physicists, but the emphasis on activity *per se* and *not* the activity of a thing is the message to take. Neither idealism nor scientific materialism, but something new.

How can we hope to begin a description of such a general scheme? Start with Grassmannn (1995). In the process of thought we ask the question "Is the new thought distinct from the old thought, or is it one continuous and developing activity? We find it easier to 'hold' onto our description as the old, T_1, and the new, T_2. But are they separate? Clearly not! The old thought has the potentiality of the new thought, while the new thought has the trace of the old thought. They are aspects of one continuing process. They take their form from the underlying process that *is* thought. Each has a complex structure of yet more distinctions, so that each T can be thought of as the tip of an 'iceberg' of activity.

In order to symbolise this basic indivisibility, we follow Grassmann (1995) and Kauffman (1980, 1987) who enclose the relationship in a square bracket, $[T_1, T_2]$[1]. Relationship is a start but not enough in itself. Our task then is to order these relationships into a multiplex of structure. To do that we need some set of rules on how to put these relationships together.

In my paper on *The Algebra of Process* (Hiley 1995) I tentatively suggested two rules of combination. Firstly a multiplication rule that I am told defines a Brandt groupoid. Secondly I introduce a rule for addition. These two binary relations, of course, define an algebra. Our defining relations are

(1) $[kA,kB] = k[A,B]$ Strength of process.

(2) $[A,B] = -[B,A]$ Process directed.

(3) $[A,B][B,C] = [A,C]$ Order of succession.

(4) $[A,B] + [C,D] = [A+C,B+D]$ Order of coexistence.

(5) $[A,[B,C]] = [A,B,C] = [[A,B],C]$ Associativity.

Notice $[A,B][C,D]$ is NOT defined.

From these rules I showed how the quaternions and indeed how a general orthogonal Clifford algebra emerges from this structure (Hiley 1995, 2002a).

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[1] NB this is not a Lie bracket.

These ideas have been put on a firmer mathematical footing and indeed have been taken much further by the excellent and detailed work of Arleta Griffor (2000, 2001) reported in earlier conferences. I can recommend these papers for those interested in the beautiful mathematical structure implicit in these ideas.

I don't want to develop these ideas here as I have done this elsewhere (Hiley 1995). Rather I want to relate them to a structure introduced by Lou Kauffman (1980), which he called the iterant algebra. To explain the ideas lying behind his work I will start with the plane and divide it into two, the 'inside' and the 'outside'. Now introduce the activity of 'crossing the boundary' and denote the activity of crossing from inside to outside by $[I, O]$, while the crossing from outside to inside is denoted by $[O, I]$. Here I and O are simply symbols denoting 'inside' and 'outside'. This is the primary distinction.

Kauffman then introduces a product defined by

$$[A, B]*[C, D] = [AC, BD] \qquad (1)$$

and shows that one can also use this relationship to generate the quaternions. Thus we have two structures with two different products producing the same algebra. But are they so different? When $B = C$ we have

$$[A, B]*[B, D] = [AB, BD] = B[A, D] \qquad (2)$$

Thus the products can be brought closer together. In fact product (3) above is simply an equivalence class of the Kauffman product. But notice product (3) is undefined when
$B \neq C$ which is what makes it a Brandt groupoid.

To see how the quaternions arise in Kauffman's approach we need to introduce a transformation T defined by

$$T([A, B]) = W*[A, B] \tag{3}$$

where W can be some suitable pair C, D. We then need to introduce three transformations, p, q and r defined by

$$p*[A, B] = [A,-B]$$
$$q*[A ,B] = [-A, B]$$
$$r*[A, B] = [B, A]$$

Then it is not difficult to show that

$$ir \Leftrightarrow \mathbf{i} \quad ip \Leftrightarrow \boldsymbol{j} \quad pr \Leftrightarrow \mathbf{k} \tag{4}$$

where **i**, **j**, **k** are the quaternions and $i = \sqrt{-1}$ (See Kauffman 1982). If we use the physicist's language these can be written in the form

$$r \Leftrightarrow \sigma_x \quad p \Leftrightarrow \sigma_z \quad ipr \Leftrightarrow \sigma_y. \tag{5}$$

where σ are the Pauli spin operators.

Now I want to go deeper into the general structure of Clifford algebras. Schönberg (1958) and Fernandes (1996) have shown us how to build any orthogonal Clifford algebra from a pair of dual Grassmann algebras whose generators satisfy the relationship

$$\left[a_i, a_j^{\dagger}\right] = g_{ij} \quad \left[a_i, a_j\right] = 0 = \left[a_i^{\dagger}, a_j^{\dagger}\right] \tag{6}$$

These will be recognised as vector fermionic 'annihilation' and 'creation' operators[2]. Notice these are *vector* operators and not the *spinor* operators used in particle physics. Using these we find

$$\sigma_x = a + a^\dagger \quad \text{and} \quad \sigma_y = a - a^\dagger \qquad (7)$$

Now it is interesting to ask what these operators do when they operate on an iterant pair. In fact it is straightforward to show

$$a*[A,B] = [B,0]$$

$$a^\dagger*[A,B] = [0,A]$$

Thus we see that here the annihilation operator a destroys the inside and puts the outside inside. While the creation operator $a^\dagger$ destroys the outside and puts the inside outside!

We can actually carry this further and ask what action the algebraic spinors (minimal left ideals) have on an iterant pair. It is not difficult to show that we have two algebraic spinors given by

$$\psi_{L1} = aa^\dagger + a^\dagger \qquad \text{and} \qquad \psi_{L2} = a + a^\dagger a. \qquad (8)$$

This is in contrast to the spinor used by physicists who have only one spinor in this case. One can show that this single spinor is an equivalence class of the algebraic spinor when it is projected onto a Hilbert space (see Bratteli and Robinson 1979). The projection means that we have lost the possibility of exploiting the additional structure offered by the algebraic spinors. Notice that these spinors are themselves part of the algebra. The whole thrust of my argument is that we must exploit the properties of the algebra and not confine ourselves to Hilbert space. When we do this we can continue with our idea that the elements of our algebra describe activity or process. Then

[2] I have suggested that these operators create and annihilate extensions.

in the case of the iterant algebra the spinor itself must produce some change in [A, B]. What is this change? Again this is easy to answer because

$$\psi_{L1}*[A, B] = (aa^{\dagger}+a^{\dagger})*[A,B] = [A, A] \qquad (9)$$

$$\psi_{L2}*[A, B] = (a+a^{\dagger}a)*[A, B] = [B, B] \qquad (10)$$

This means that one type of spinor, ψ_{L1}, destroys the outside and puts a copy of the inside outside, while the other spinor, ψ_{L2}, destroys the inside and puts a copy of the outside inside! In other words these operators remove the original distinction, but in different ways. More importantly from the point of view that we are exploring here is that we see how the algebraic spinor itself is active in producing a specific change in the overall process.

Lets take this a little further. Consider the following relationship

$$a*[A, 0] = [0, 0] \qquad (11)$$

This should be compared with the physicist's definition of a vacuum state,

$$a|0\rangle = a \quad = 0 \qquad (12)$$

where we have introduced the notation used by Finkelstein (1997). Thus equation (12) shows that [A, 0] acts like the vacuum state in physics. Furthermore

$$a^{\dagger}*[0,B] = [0,0] \quad \text{should be compared with } a^{\dagger} \quad = 0$$

Finkelstein calls, , the plenum. Thus we see that [0, B] acts like the plenum. Thus we see that here the vacuum state is not empty. Internally it has content but externally it is empty. For the plenum it is the other way round, so unlike Parmenides we can have 'movement' from outside to inside!

We can take this a bit further by recalling that we can write the projector onto the vacuum as $V = |0\rangle\langle 0|$, then we have $aV = 0$. If the projector onto the plenum is $P = |\infty\rangle\langle\infty|$, we find $a^{\dagger}P = 0$.

All of this suggests that we could write $[A, B]$ as $|A\rangle\langle B|$. By making this identification we can bring out the relationship of our work to that of Zapatrin (2000) and of Raptis and Zapatrin (2001) who developed an approach through the incident algebra. In this structure the product rule is written in the form

$$|A\rangle\langle B|C\rangle\langle D| = |A\rangle\langle B|C\rangle\langle D| = \delta_{BC}|A\rangle\langle D| \qquad (13)$$

Again this multiplication rule is essentially rule (3), the order of succession above. But there is a major difference. When $B \neq C$ the product to zero, whereas we leave it undefined at this level. Since there is a close similarity between these three different structures it would be worth studying this relationship in some detail. However I will not discuss this relationship here as I want to take the rest of my time exploiting the ideas that are open to us when we look at process in terms I have been trying to develop above.

The Intersection of the Past with the Future.

I want to look more deeply into the structure based on relationships like $[T_1, T_2]$ by, as it were, 'getting inside' the connection between T_1 and T_2. Remember I am focusing on process or flux and I am symbolising *becoming* by $[T_1, T_2]$. Ultimately I want to think of these relationships as ordered structure defining what we have previously called 'pre-space'. (See Bohm 1986 and Hiley 1991.) In other words these relationships are not to be thought of as occurring in space-time, but rather space-time is to eventually be abstracted from this pre-space. This is a radical suggestion so let me try to develop my thinking more slowly.

Conventionally physics is always assumed to unfold in space-time, and furthermore the evolution is always assumed to be from point to point. In other words physics always tries to talk about time development *at an instant*. Any change always involves the limiting process $\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$. But before taking the limit it looks as if we were taking a point in the past (x_1, t_1) and relating it to a point in the future (x_2, t_2), that is relating what *was* to what *will be*. But we try to hide the significance of that by going to the limit $t_2 - t_1 \to 0$ when we interpret the change to take place at an instant, t. Yet curiously the instant is a set of measure zero sandwiched between the infinity of that which has passed and the infinity of that which is not yet. This is fine for evolution of point-like entities but not for the evolution of structures.

When we come to quantum mechanics it is not positions that develop in time but wave functions and wave functions like the Pauli spinor can be treated as an ideal in the algebra (See Hiley 2002b) which, as we have seen above, are actions involving 'separate' regions. Recall Heisenberg's (1925) original suggestion that $x(t)$, the position of the electron in the atom, must be replaced by

$$X_q(t) = \sum_a R(n, n-a) \exp[i\nu(n, n-a)t]$$

The exponent ensures that the Ritz combination rule of atomic spectra can be satisfied, namely

$$\upsilon(n, n-a) + \upsilon(n-a, n-a-b) = \upsilon(n, n-a-b).$$

This result in needed when we form variables like $X_q(t)^2$ which appear in the discussion of a quantum oscillator. Heisenberg then proposed that the amplitudes combine as

$$R(n, n-b) = \sum_a R(n, n-a) R(n-a, n-b)$$

This is immediately recognised as the rule for matrix multiplication. It is when this matrix idea is extended to momentum we find the need for the Heisenberg commutation relations. Notice here we are talking about transitions between stationary states, one being characterised by quantum number n and the other characterised by $(n - a)$. Thus we are talking about transitions between one state and another, that is between structures defining what has been to what will be.

I will continue in the same theme by recalling Feynman's classic paper where he sets out his thinking that led to his 'sum over paths' approach (Feynman 1948). There he starts by dividing space-time into two regions R' and R''. R' consists of a region of space occupied by the wave function before time t', while R'' is the region occupied by the wave function after time t'', i.e $t' < t''$. Then he suggested that we should regard the wave function in region R' as contain information coming from the 'past', while the conjugate wave function in the region R'' representing information coming from the 'future'[3]. The possible present is then the intersection between the two, which is simply represented by the transition probability amplitude $\psi(R'')$ú$\psi(R')$. But what I want to discuss here is ú$\psi(R')$ $\psi(R'')$ú This is where all the action is!

Before taking up this point I would like to call attention to a similar notion introduced by Stuart Kauffman (1996) in his discussion of biological evolution, again an evolution of structure. He talks about the evolution of biological structures from their present form into the 'adjacent possible'. This means that only certain forms can develop out of the past. Thus not only does the future form contain a trace of the past, but it is also constrained by what is 'immediately' possible. So any development is governed by the *tension between the persistence of the past, and an anticipation of the future.*

[3] This is essentially the same idea that led to the notion of the anti-particle 'going backward in time', but here we are not considering exotic anti-matter.

What I would now like to do is to build this notion into a dynamics. Somehow we have to relate the past to the future, not in a completely deterministic way, but in a way that constrains the possible future development. My basic notion is thus an extended structure in both space and time. I have elsewhere called it a 'moment' (see Hiley and Fernandes 1997). In spatial terms it is fundamentally non-local; it is also 'non-local' in *time*. I see this as an a-local concept, which has extension in time. It is a kind of 'extenson' or 'duron'. We can think of this as a necessary consequence of the energy-time uncertainty principle. For a process with a given energy cannot be described as unfolding at an instant except in some approximation.

How then are we to discuss the dynamics of process, which depends on this notion of a moment? I will start in the simplest possible way by proposing that the basic dynamical function will involve two times. Thus we will discuss the time development of two-point functions of the form $[A(t_1), B(t_2)]$ and show that we capture the usual equations of motion in the limit $t_1 \rightarrow t_2$.

Fortunately even in classical physics two-point functions abound. They are implicit in all variational principles that form the basis of modern physics. For example in his classic work on optics, Hamilton (1967) recognising the importance of Fermat's least-time principle, and suggested that both optics and classical mechanics could be united into a common formalism by introducing a two-point characteristic function, $\Omega(A, B)$. Following on from Hamilton's work Synge (1964), in his unique approach to general relativity, proposed that a two-point function, which he called the world function, lies at the heart of general relativity[4]. Can we exploit these two-point functions to develop a new way of looking at dynamics?

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[4] In modern parlance these functions are the generating functions of the symplectomorphisms in classical mechanics (see de Gosson 2001).

I want to pick up on a very small aspect of this well-known work and show how ideas already implicit in that structure are relevant to what I have in mind. As is well known use of the variational principle produces the classical Hamilton-Jacobi equation (see Goldstein 1950). This emerges by considering a variation of the initial point *A* of the trajectory. Standard theory shows that by varying the initial point *A* we can obtain the relations

$$\frac{\partial S}{\partial x_1} = p(x_1) \qquad \frac{\partial S}{\partial t_1} + H(t_1) = 0 \tag{14}$$

where we have written $H(t_1) = H\left(x_1(t_1), \frac{\partial S}{\partial x_1}(t_1)\right)$ for convenience and we have replaced the world function Ω by the classical action function *S*.

What is not so well known is that if we vary the final point *B*, we find another pair of equations

$$\frac{\partial S}{\partial x_2} = -p(x_2) \qquad \frac{\partial S}{\partial t_2} - H(t_2) = 0 \tag{15}$$

Here the second Hamilton-Jacobi equation formally becomes the same by writing $t_2 = -t_1$. Could this be taken to signify a wave coming from the 'future' and fit into the general scheme I am developing here?

Leaving that speculation aside, let us see how we can formally exploit these two Hamilton-Jacobi equations. Consider a pair of points with co-ordinates (x_1, t_1) and (x_2, t_2) in configuration space. The world function (generalised action) for this pair can be written as $S(x_1, x_2, t_1, t_2)$

We will find it more convenient to use 'sums' and 'differences' rather than the co-ordinates themselves. Thus we change to co-ordinates $(X, \Delta x, T, \Delta t)$ where

$$X = \frac{x_1 + x_2}{2} \qquad T = \frac{t_1 + t_2}{2} \qquad \text{and} \qquad \Delta x = x_2 - x_1 \qquad \Delta t = t_2 - t_1$$

so that the generalised action becomes

$$S(x_1, x_2, t_1, t_2) = S(X, \Delta x, T, \Delta t)$$

Then equations (14) and (15) can be replaced by

$$\frac{\partial S}{\partial X} = \Delta p \qquad \frac{\partial S}{\partial T} = \left[H(t_2) - H(t_1)\right] \tag{16}$$

$$\frac{\partial S}{\partial \Delta x} = P \qquad \frac{\partial S}{\partial \Delta t} = \tfrac{1}{2}\left[H(t_2) + H(t_1)\right] \tag{17}$$

In order to see the meaning of the two equations let us make a Legendre transformation

$$K(X, P, T, E) = P\Delta x + E\Delta t - S(X, \Delta x, T, \Delta t) \tag{18}$$

so that

$$\frac{\partial S}{\partial T} = -\frac{\partial K}{\partial T} \qquad \frac{\partial S}{\partial \Delta t} = E$$

A general background discussion to these ideas can be found in Bohm and Hiley (1981).

Then when we go to the limit as $\Delta t \to 0$, we define

$$\lim_{\Delta t \to 0} \frac{\partial S}{\partial T} = -\left[H(t_2) - H(t_1)\right] \qquad \frac{\partial S}{\partial T} + \frac{\partial H}{\partial P}\Delta p + \frac{\partial H}{\partial X}\Delta x \approx 0 \tag{19}$$

But

$$\Delta p = -\frac{\partial K}{\partial X} \qquad \Delta x = \frac{\partial K}{\partial P}$$

Then equation (19) becomes

$$\frac{\partial K}{\partial T} + \{K, H\} = 0 \qquad (20)$$

where {.} is the Poisson bracket so that equation (20) becomes the classical equation of motion for the dynamical variable *K*. Indeed when this K is identified with the probability distribution this is nothing more than the Liouville equation. The second equation (16) becomes

$$\frac{\partial S}{\partial \Delta t} = \tfrac{1}{2}\left[H(t_2) + H(t_1)\right] \qquad \frac{\partial K}{\partial \Delta t} = E \qquad (21)$$

which is simply the equation for the total energy of the system.

In order to anticipate the approach the quantum mechanics I will describe in the next section, I need to introduce the notion of a generalised Poisson bracket defined by

$$\{\ \} = \frac{\partial}{\partial X}\frac{\partial}{\partial \Delta p} - \frac{\partial}{\partial \Delta p}\frac{\partial}{\partial X} + \frac{\partial}{\partial \Delta x}\frac{\partial}{\partial P} - \frac{\partial}{\partial P}\frac{\partial}{\partial \Delta x}$$

so that we find the following relationships

$$\{X, \Delta p\} = \{\Delta x, P\} = 1$$

$$\{X, P\} = \{\Delta x, \Delta p\} = \{X, \Delta x\} = \{P, \Delta p\} = 0 \qquad (22)$$

This suggests another pair of brackets of the form

$$\{T, H(t_2) - H(t_1)\} = \{\Delta t, H(t_2) + H(t_1)\} = 1$$

If we were to introduce the quantity $L(t_1, t_2) = H(t_2) - H(t_1)$ we have the classical correspondence to the Liouville operator introduced by Prigogine (1980). We will discuss this connection further when we generalise these results to the quantum domain.

Quantum Pasts and Futures.

Now let me return to the quantum domain and consider Feynman's suggestion mentioned earlier in more detail. Introduce a world function defined by

$$\hat{\rho}(t_1,t_2)=|\psi(t_1)\rangle\langle\psi(t_2)| \tag{24}$$

I use the symbol $\hat{\rho}(t_1,t_2)$ because I am dealing essentially with a generalised density operator. Then let us form

$$\frac{\partial}{\partial T}\left(|\psi(t_1)\rangle\langle\psi(t_2)|\right)=\left(\frac{\partial}{\partial t_1}|\psi(t_1)\rangle\right)\langle\psi(t_2)|+|\psi(t_1)\rangle\left(\frac{\partial}{\partial t_2}|\psi(t_2)\rangle\right) \tag{25}$$

Since Feynman has already derived the Schrödinger equation from these considerations we can substitute the two equations

$$i\frac{\partial}{\partial t_1}|\psi(t_1)\rangle=\hat{H}_1|\psi(t_1)\rangle \qquad \text{and} \qquad -i\frac{\partial}{\partial t_2}\langle\psi(t_2)|=\langle\psi(t_2)|\hat{H}_2$$

into equation (25) and we find

$$i\frac{\partial\hat{\rho}(t_1,t_2)}{\partial T}+\hat{\rho}(t_1,t_2)\hat{H}_2-\hat{H}_1\hat{\rho}(t_1,t_2)=0 \tag{26}$$

If we now take the limit as $\Delta t\to 0$ when $T\to t$, we find

$$i\frac{\partial\hat{\rho}}{\partial t}+\left[\hat{\rho},\hat{H}\right]_-=0 \tag{27}$$

Here $\hat{\rho}$ has become the usual density operator for the pure state $|\psi(t)\rangle$. This equation is then recognised as the Liouville equation.

Now let us consider

$$2\frac{\partial}{\partial \Delta t}\left(|\psi(t_1)\rangle\langle\psi(t_2)|\right) = -\left(\frac{\partial}{\partial t_1}|\psi(t_1)\rangle\right)\langle\psi(t_2)| + |\psi(t_1)\rangle\left(\frac{\partial}{\partial t_2}\langle\psi(t_2)|\right)$$

So that by using the two Schrödinger equations we find

$$2i\frac{\partial}{\partial \Delta t}\hat{\rho}(t_1,t_2) + \hat{\rho}(t_1,t_2)\hat{H}_2 + \hat{H}_1\hat{\rho}(t_1,t_2) = 0 \qquad (28)$$

Here we cannot go directly to the limit because of the appearance of Δt in the denominator. There are two ways to proceed. Firstly let me use the approach that directly links with the classical Legendre transformation (18). Take the Wigner transformation defined by

$$\hat{\rho}(T,\Delta t) = \int \hat{\rho}_E(E,T)\exp[iE\Delta t]dE \qquad (29)$$

Then

$$i\frac{\partial}{\partial \Delta t}\hat{\rho}(t_1,t_2) = i\frac{\partial}{\partial \Delta t}\hat{\rho}(T,\Delta t) = -\int E\hat{\rho}_E(E,T)\exp[iE\Delta t]dE = -\bar{E}$$

where we can regard $\bar{E}$ as the mean energy over the interval Δt. Thus we can finally write equation (28) as

$$2\bar{E} = \left[\hat{\rho},\hat{H}\right]_+ \qquad (30)$$

This is an expression of the conservation of energy equation.

Collecting together the main results so far we find

$$i\frac{\partial\hat{\rho}}{\partial t} + \left[\hat{\rho},\hat{H}\right]_- = 0 \qquad \Leftrightarrow \qquad \frac{\partial K}{\partial T} + \{K,H\} = 0$$

and

$$2\bar{E} = \left[\hat{\rho},\hat{H}\right]_+ \qquad \Leftrightarrow \qquad E = \frac{\partial K}{\partial \Delta t}$$

Again if K is the classical analogue of the density operator then we would have a correspondence between the classical 'Liouville' equation (20) and the quantum Liouville equation (27). In turn the quantum energy equation (30) then corresponds to the classical energy equation (21). Thus we have a clear correspondence between the classical and the quantum levels.

There is a second way to approach the limit $\Delta t \to 0$ that does not involve a Wigner transformation and perhaps produces a cleaner result. In fact this method produces exactly the same results that are derived in Brown and Hiley (2000), but from a different standpoint. The emphasis there was to do everything in the algebra itself and was probably too abstract for most physicists. This means replacing $|\psi(t_1)\rangle$ by an element of a minimal left ideal, $\hat{\Psi}_L(t_1)$, of a form similar to that of the spinor used in section 2. In the same way we can replace $\langle\psi(t_2)|$ by an element of a right ideal, $\hat{\Psi}_R(t_2)$. Then we write these algebraic elements in polar form

$$\hat{\Psi}_L(t_1) = \hat{R}(t_1)\exp[i\hat{S}(t_1)] \qquad \text{and} \qquad \hat{\Psi}_R(t_2) = \exp[-i\hat{S}(t_2)]\hat{R}(t_2)$$

Here we have put 'hats' on the $\hat{\Psi}$, $\hat{R}$ and $\hat{S}$ to emphasise that they are elements of the algebra (ie. *operators*) and *not* elements of a Hilbert space. Then

$$2\frac{\partial}{\partial \Delta t}\hat{\rho}(t_1,t_2) = \left[-\frac{\partial \hat{R}(t_1)}{\partial t_1}\hat{R}(t_2) + \hat{R}(t_1)\frac{\partial \hat{R}(t_2)}{\partial t_2} - i\hat{R}(t_1)\hat{R}(t_2)\left[\frac{\partial \hat{S}(t_1)}{\partial t_1} + \frac{\partial \hat{S}(t_2)}{\partial t_2}\right]\right] \times \exp\left[-i\left(\hat{S}(t_2) - \hat{S}(t_1)\right)\right] \qquad (31)$$

where we have assumed that $\hat{R}$ and $\hat{S}$ commute. Then when we go to the limit $\Delta t \to 0$ with $T \to t$ we find

$$\lim_{\Delta t \to 0} 2\frac{\partial}{\partial \Delta t}\hat{\rho}(t_1,t_2) = -i\hat{R}^2\frac{\partial \hat{S}}{\partial t} \qquad (32)$$

Thus equation (28) then becomes

$$2\hat{R}^2\frac{\partial\hat{S}}{\partial t}+\left[\hat{\rho},\hat{H}\right]_+=0 \qquad (33)$$

This equation is identical to the anti-commutator equation (11) derived in Brown and Hiley (2000). A yet different derivation of this equation will also be found in Hiley (2002b). The reason why I have re-derived this equation in different ways is because I have not seen this equation written down in this form in the literature and I wanted to make sure it was mathematically sound.

In Brown and Hiley (2000) we showed that there were two important consequences following from this equation. Firstly we showed that the Berry phase and the Aharonov-Bohm effect followed immediately from this equation in a very simple way. Secondly we used this quantum equation to see where the quantum potential introduced by Bohm emerges from what is essentially the Heisenberg picture (see also Hiley 2002a). We found that this potential only appeared as a result of *projecting* the algebraic elements onto a representation space. This led us to speculate that all the 'action' of quantum phenomena takes place in the algebra itself, in the pre-space.

It is well-known that we cannot display quantum processes in a phase space because we are using a non-commutative structure. Following on from the work of Gel'fand this means that there is no unique manifold underlying the algebra. You have to rely on shadow manifolds, which are constructed by projections. In this projection we get distortions like those found in maps produced by a Mercator's projection. Therefore it is not surprising to find it necessary to introduce forces to account for the predicted behaviour in the shadow manifold. This is exactly how the gravitation force is manifested in general relativity (For a more detailed discussion see Hiley 2002a)

Bi-Algebras and super-algebras.

In this final section I want to connect this work with the proposals made by Umezawa (1993) in his discussions of thermal quantum field theory. The aim here is to find a common formalism in which both quantum and thermal effects can be incorporated. Unlike the work presented here, Umezawa uses Hilbert space and shows that if we double the Hilbert space then the thermal state can also be represented by a single vector in this double space. For example, in more familiar notation, the thermal wave function can be written in the form

$$|\Omega(\beta)\rangle = Z^{-1/2} \sum \exp[-\beta E_n / 2] |\psi_n\rangle \quad |\psi_n\rangle \tag{34}$$

Here $\beta = 1/kT$ and $|\psi_n\rangle$ are the energy eigenkets. Z is the partition function. The ensemble average of some quantum operator A would then be given by

$$\langle\Omega(\beta)|A|\Omega(\beta)\rangle = Tr(\rho A)$$

where ρ is the thermal density operator. The more usual form of the density operator is

$$\rho = \exp[-H\beta]$$

These results show the essential relation between the two approaches.

Those familiar with algebraic quantum field theory will recognise that this is essentially the GNS construction (Emch 1972 and Hiley 2002b). However the Umezawa approach proceeds by doubling the number field elements so that an algebraic theory would have double the algebra. This means that we would introduce a pair of annihilation and creation operators for each degree of freedom, $\{a, a^\dagger, \tilde{a}, \tilde{a}^\dagger\}$

Now let us connect this with what I am doing here. So far I have essentially been dealing with a two-time quantum theory. This has been straightforward to deal with because time is a parameter and not an operator in the quantum domain. If I were in a classical theory I would simply generalise the two-time theory by considering two points in phase space at different times co-ordinated by $\{x_1, p_1, t_1, x_2, p_2, t_2\}$. This would lead me to a structure encompassing the generalised Poisson brackets defined in equation (22).

When I move on to quantum theory I need now to consider the position and momentum as *operators* so I must base the theory on *pairs of algebraic* elements $\hat{x}_1, \hat{x}_2, \hat{p}_1, \text{and}\, \hat{p}_2$. Again I have added the 'hat' to emphasise that these are elements in the algebra (i.e. operators). In other words we are moving on to consider doubling up the algebra so that we are essentially dealing with bi-algebraic structures.

Let us continue the idea of regions of ambiguity linked by quantum process. Now there will be regions in the algebra that correspond to the regions *R*' and *R*'' so let us proceed quite naively by forming the set of operators

$$2\hat{X} = \hat{x}_1 \quad 1 + 1 \quad \hat{x}_2, \qquad \hat{\eta} = \hat{x}_1 \quad 1 - 1 \quad \hat{x}_2, \tag{35}$$

$$2\hat{P} = \hat{p}_1 \quad 1 + 1 \quad \hat{p}_2, \qquad \hat{\pi} = \hat{p}_1 \quad 1 - 1 \quad \hat{p}_2. \tag{36}$$

Then we find that the following commutator relations hold

$$\left[\hat{X}, \hat{\pi}\right] = \left[\hat{\eta}, \hat{P}\right] = i$$

and

$$\left[\hat{X}, \hat{P}\right] = \left[\hat{\eta}, \hat{\pi}\right] = \left[\hat{X}, \hat{\eta}\right] = \left[\hat{P}, \hat{\pi}\right] = 0 \tag{37}$$

These relations are the quantum analogues of the generalised Poisson brackets defined in equation (22). These results were already reported in

Bohm and Hiley (1981). What we have achieved so far is a formal correspondence between the classical and quantum structures based on the doubling of the variables. The important question is whether this structure will lead to new physics. I think it will but I will only outline two possibilities.

Both possibilities are connected with the introduction of irreversibility into quantum physics. The first suggestion we follow is that of Prigogine (1980). He suggested that we need a theory in which irreversibility plays a fundamental role directly in the dynamics itself. Then perhaps time could then be introduced, not as a parameter, but by an operator. It is the absence of such an operator that has handicapped my own attempts to find a fully algebraic description of physical processes.

In order to show how a time operator emerges from the work here, first note that we can write the quantum Liouville equation (27) in the form

$$i\frac{\partial \hat{\rho}_V}{\partial t} + \hat{L}\hat{\rho}_V = 0 \tag{38}$$

Here $\hat{\rho}_V$ is a vector equivalent of the density operator and $\hat{L} = \hat{H} \quad 1 - 1 \quad \hat{H}$. Thus we have simply re-expressed the Liouville equation in terms of the language of bi-algebras. The appearance of the super-operator enables us to introduced a time operator $\hat{T}$, defined through the relation

$$\left[\hat{T}, \hat{L}\right] = i \tag{39}$$

Note that this is the quantum version of the classical form presented by the first equation in (23).

Prigogine (1980) argues that this time operator, $\hat{T}$ represents the 'age' of the system. I don't want to discuss the reasons for this as I have already made some comments on it in Bohm and Hiley (1981) and in Hiley and Fernandes, (1997). A more general discussion of Prigogine's point of view will be found in George and Prigogine (1979), and in Prigogine (1980),

What I want to do now is to go on to the bi-algebraic generalisation of equation (30). This requires the introduction of the super-operator corresponding to the anti-commutator, which can be written in the form

$$\bar{E} = \left(\hat{H} \quad 1 + 1 \quad \hat{H}\right)\hat{\rho}_V = E_+\hat{\rho}_V \tag{40}$$

Such an operator was first introduced by George et al (1978) in their general discussion of dissipative processes. They, like us, regard this as an expression of the total energy of the system. This is only other discussion I have seen in non-relativistic quantum theory where the role the anti-commutator is taken to correspond to the energy of the system. The only reason why I want to mention this equation here is that it adds further legitimacy to equation (33). However I should point out that Fairlie and Manogue (1991) have discussed an analogous equation based on the cosine Moyal bracket introduced by Baker (1958).

I now want to move on a bit further and remark that not only can we introduce an 'age' operator $\hat{T}$, but we can also introduce a 'time difference' operator $\hat{\tau}$, the duron. This satisfies the commutator relations

$$\left[\hat{T},\hat{\varepsilon}\right] = \left[\hat{\tau},\hat{E}\right] = i$$

and

$$\left[\hat{T},\hat{E}\right] = \left[\hat{\tau},\hat{\varepsilon}\right] = \left[\hat{T},\hat{\tau}\right] = \left[\hat{E},\hat{\varepsilon}\right] = 0 \tag{41}$$

where we have written $\hat{\varepsilon}$ for $\hat{L}$ to bring out the symmetry. Hiley and Fernandes (1997) have already discussed these relationships in the context of finding operators for time. In particular they interpreted $\hat{\tau}$ as the mean time spent passing between two energy states.

Bi-algebras and the Bogoliubov transformations.

Before discussing the meaning of $\hat{\tau}$ in more detail let me return to my way of thinking about the bi-algebra. I have proposed that the evolution of a quantum process does not proceed at an instant of time at a point in space, but through the ambiguous region of phase space that I have called a 'moment'. We consider the relation between the two sides of this moment, describing one side as information coming from the past while the other side is to do with the possible developments for the future.

I have spoken at times rather dramatically about this latter feature as 'information coming from the future'. But such a way of talking is not that outrageous that it has not been suggested before. For example Cramer (1986) in his transactional interpretation of quantum mechanics uses the advanced potentials to carry information from the future. The transaction is a 'handshake' between emitter and the absorber participants of a quantum event. This notion, in turn, has a resonance with an earlier proposal of Lewis (1926) who has based his thinking on the following idea. In the rest frame of a photon time dilation suggests that there is no time lapse between emission and absorption and because of the length contraction, there is no distance between the emitter and absorber either. The light ray is a primary contact between the two ends of the process. These are both very radical ideas and unfortunately I have never known what to make of them so I tend to discuss the notion of a 'moment' hoping that Δt is small, but as these two examples show this may be a rather conservative view to adopt!

Recently was very happy to meet with Giuseppe Vitiello and to discuss some of his extremely interesting ideas on dissipative quantum systems. His ideas are, perhaps, even more conservative and therefore probably more reliable, yet they seem to fit into the overall scheme I am discussing here. His work is reported in a series of papers by Vitiello (1995), Celeghini, Rasetti and Vitiello (1992), Celeghini *et al* (1998) and Iorio and Vitiello (1995). I rely heavily on the mathematics contained in these papers.

They are interested in quantum dissipation, which they explore in terms of a pair of coupled dissipative oscillators, one emitting energy, the other absorbing energy. In terms of our two-sided evolution discussed above, we find one 'side' of the process is seen as representing the system while the other 'side' is seen as representing the environment, the latter acting as a sink for the dissipated energy.

In this model the degrees of freedom of the system are described by the set of annihilation operators $\{a_k\}$, while the environment is described by the set $\{\tilde{a}_k\}$. Thus there is a doubling of the mathematical structure. The extra field variables $\{\tilde{a}_k\}$ describing the 'environment' are a mirror image of the variables used to describe the system. Not only is a spatial mirror image but it is also a *'time-reversed* mirror image' as Vitiello (1996) puts it. So the 'environment sink' appears to be acting as if it were 'anticipating the future'.

Let us leave the imagery for the moment and move on to see how the ideas work out mathematically. For this we will need to introduce some more formalism. So far we have introduced elements of our bi-algebra by effectively defining two sets of co-products which we will now express formally as

$$\Delta_+\hat{A} = \hat{A} \quad 1 + 1 \quad \hat{A} \qquad \text{and} \qquad \Delta_-\hat{A} = \hat{A} \quad 1 - 1 \quad \hat{A} \tag{42}$$

We have then shown that when we go to the limit $\Delta t \to 0$, we produce two dynamical equations, namely,

$$i\frac{\partial\hat{\rho}_V}{\partial t} + \hat{L}\hat{\rho}_V = 0 \qquad \text{and} \qquad \lim_{\Delta t \to 0} i\left(\frac{\partial\hat{\rho}_V}{\partial \Delta t}\right) + \tfrac{1}{2}\hat{H}_+\hat{\rho}_V = 0 \tag{43}$$

But what do we make of the general co-products and the commutation relations listed in equations (35) – (37)? To explore these let us first make a Bargmann transformation from the Heisenberg algebra to the boson algebra of annihilation and creation operators. This will enable us to immediately relate our work to that of Vitiello (1995) and Celeghini *et al* (1998). Thus writing

$$a = \hat{x}_1 + i\hat{p}_1 \qquad \tilde{a} = \hat{x}_2 + i\hat{p}_2$$
$$a^\dagger = \hat{x}_1 - i\hat{p}_1 \qquad \tilde{a}^\dagger = \hat{x}_2 - i\hat{p}_2$$

We can immediately make contact with equation (34) by using the well-known generator of the Bogoliubov transformation

$$G = -i\left(a^\dagger \tilde{a}^\dagger - a\tilde{a}\right) \tag{44}$$

Then applying this to the vacuum state $|0,0\rangle$, we find a new vacuum state $|0(\theta)\rangle$ given by

$$|0(\theta)\rangle = \exp[i\theta G]|0,0\rangle = \sum_n c_n(\theta)|n\rangle \quad |n\rangle. \tag{45}$$

This means that by doubling the algebra we can immediately make contact with equation (34) provided we find the correct relationship between θ and β. Recall that in equation (34) β is proportional to the inverse of the temperature. Thus we have a way of combining thermodynamics and quantum mechanics in a single mathematical structure.

But let's go deeper and develop the boson bi-algebra by defining the following co-products based on equations (35) and (36),

$$\Delta_+ a = a \quad 1 + 1 \quad a = a + \tilde{a} \qquad \Delta_- a = a \quad 1 - 1 \quad a = a - \tilde{a} \tag{46}$$
$$\Delta_+ a^\dagger = a^\dagger \quad 1 + 1 \quad a^\dagger = a^\dagger + \tilde{a}^\dagger \qquad \Delta_- a^\dagger = a^\dagger \quad 1 - 1 \quad a^\dagger = a^\dagger - \tilde{a}^\dagger \tag{47}$$

We see immediately that these co-products are identical to those introduced by Celeghini *et al* (1998) but we can go further and form

$$A = \frac{1}{\sqrt{2}}(a + \tilde{a}) = \sqrt{2}\left(\hat{X} + i\hat{P}\right) \qquad \text{and} \qquad A^{\dagger} = \frac{1}{\sqrt{2}}\left(a^{\dagger} + \tilde{a}^{\dagger}\right) = \sqrt{2}\left(\hat{X} - i\hat{P}\right) \tag{48}$$

$$B = \frac{1}{\sqrt{2}}(a - \tilde{a}) = -\frac{1}{\sqrt{2}}(\hat{\eta} + i\hat{\pi}) \quad \text{and} \quad B^{\dagger} = \frac{1}{\sqrt{2}}\left(a^{\dagger} - \tilde{a}^{\dagger}\right) = -\frac{1}{\sqrt{2}}(\hat{\eta} - i\hat{\pi}) \tag{49}$$

These operators lie at the heart of their approach. In our approach we see that these operators have a very simple interpretation. They are simply the annihilation and creation operators of the mean position variables and the difference variables respectively. Thus

$$\hat{X} = \frac{1}{2\sqrt{2}}\left(A + A^{\dagger}\right) \quad \text{and} \quad \hat{P} = \frac{i}{2\sqrt{2}}\left(A - A^{\dagger}\right)$$

$$\hat{\eta} = -\frac{1}{\sqrt{2}}\left(B + B^{\dagger}\right) \quad \text{and} \quad \hat{\pi} = \frac{i}{\sqrt{2}}\left(B - B^{\dagger}\right)$$

In other words the operators A and B are the algebraic way of defining the ambiguous moments of in our algebraic phase space. They are the variables that we need to describe the unfolding process that forms the basis of our paper.

Now I want to follow Celeghini *et al* (1998) further and generalise our approach by deforming the bi-algebra. We do this by defining the co-product

$$\Delta_{+} a_q = a_q \quad q + q^{-1} \quad a_q \qquad \text{and} \qquad \Delta_{+} a^{\dagger}{}_q = a^{\dagger}{}_q \quad q + q^{-1} \quad a^{\dagger}{}_q \tag{50}$$

where we will write $q = e^{\theta}$ where θ is some parameter the physical meaning of which has yet to be determined. Then

$$A_q = \frac{\Delta a_q}{\sqrt{[2]_q}} = \frac{1}{\sqrt{[2]_q}}\left(e^{\theta} a + e^{-\theta}\tilde{a}\right) \qquad \text{and} \qquad B_q = \frac{1}{\sqrt{[2]_q}}\frac{\delta}{\delta\theta}\Delta a_q = \frac{1}{\sqrt{[2]_q}}\left(e^{\theta} a - e^{-\theta}\tilde{a}\right)$$

$$+\text{h.c.} \tag{51}$$

The A_q and B_q are then the deformed equivalents of equations (48) and (49). Notice also that

$$\Delta_{-}A_{\theta} = \frac{\delta}{\delta\theta}\Delta_{+}A_{\theta} \qquad \text{and} \qquad \Delta_{-}A = \lim_{\theta\to 0}\frac{\delta}{\delta\theta}\Delta_{+}A \tag{53}$$

so that the two sets of co-products defined in equations (46) and (47) are not independent. With these definitions it is not difficult to show that we can write

$$A(\theta) = \frac{1}{\sqrt{2}}\left(a(\theta) + \tilde{a}(\theta)\right) \qquad \text{and} \qquad B(\theta) = \frac{1}{\sqrt{2}}\left(a(\theta) - \tilde{a}(\theta)\right) \tag{54}$$

So that

$$a(\theta) = \frac{1}{\sqrt{2}}\left(A(\theta) + B(\theta)\right) = a\cosh\theta - \tilde{a}^{\dagger}\sinh\theta \tag{55}$$

and

$$\tilde{a}(\theta) = \frac{1}{\sqrt{2}}\left(A(\theta) - B(\theta)\right) = \tilde{a}\cosh\theta - a^{\dagger}\sinh\theta \tag{56}$$

This is immediately recognised as nothing but the Bogoliubov transformation from the set of annihilation and creation operators $\{a,\tilde{a}\}$ to a new set $\{a(\theta),\tilde{a}(\theta)\}$. This result justifies the use of the Bogoliubov generator given in equation (44), which was used to construct the GNS ket given in equation (45).

Unfolding through inequivalent representations?

Having put the formalism in place I now want to consider how all this leads to a radically new way of looking at the way quantum processes unfold in time. My ideas go back to the early eighties when David Bohm and I were discussing how we could think about the type of process underlying quantum phenomenon. Most of this work was unpublished essentially because I did not have an adequate understanding of the mathematics needed. However Bohm (1986) did publish some of the background relevant to the ideas I am developing here. There perhaps for the first time he makes a clear statement as to what we were thinking. I quote

> All these relationships (of moments of enfoldment) have to be understood primarily as being between the implicate "counterparts" of these explicate moments. That is to say, we no longer suppose that space-time is primarily an arena and that the laws describe necessary relationships in the development of events as they succeed each other in this arena. Rather, each law is a structure that interpenetrates and pervades the totality of the implicate order.

Implicit in this was the idea that space-time itself would emerge at some higher explicit level (Hiley 1991). All of this was easily dismissed as 'somewhat vague', but we did try to make it more specific by arguing that the inequivalent representations contained within quantum field theory would play a key role. However we could not see how to make the mathematics work.

In the general context of Bohm's ideas, the vacuum state should not be regarded as absolute and self-contained. Rather each vacuum state provides the basis for what we called an explicate order so that a set of inequivalent vacuum states could be thought of as providing an array of explicate orders, all embedded in the overall implicate order in which all movement is assumed to take place. The movement between inequivalent representations, between inequivalent vacuum states, is then regarded as a movement from one explicate order to another. It was the implicate order that enabled this transformation to take place as an unfolding of moments.

Within this structure we found the explanation as to why in a single Hilbert space formalism nothing *actually* happens. The inner automorphisms of the algebra of operators are simply a re-description of the *potentialities* of the process so that every unitary transformation becomes merely a re-expression of the order. In this sense everything is *potentiality*. But what about the actual occasions? This has been the continuing difficulty of the 'measurement problem'. Where do the *actual events* arise in the quantum formalism? First we should notice that in quantum field theory the vacuum kets $|0(\theta)\rangle$ belong to inequivalent representations of the boson algebra. Our suggestion is that not only is there a movement within each inequivalent representation but there is also another movement involved and this is the movement between inequivalent representations and thus between these inequivalent vacuum states. The key question is how this movement is described mathematically.

The answer appears to lie in the relationship between the two co-products described by equation (53) as Celeghini *et al* (1998) have already pointed out. It is this feature that allows us to discuss the movement between inequivalent representations. To explain this idea let us define

$$p_\theta = -i\frac{\delta}{\delta\theta} \qquad (57)$$

We can then think of p_θ as a conjugate momentum to the internal degree of freedom θ so that this momentum can be thought of as describing the movement between inequivalent Hilbert spaces. This identification becomes even more compelling once we realise that

$$-i\frac{\delta}{\delta\theta}a(\theta) = [G, a(\theta)] \qquad \text{and} \qquad -i\frac{\delta}{\delta\theta}\tilde{a}(\theta) = [G, \tilde{a}(\theta)] \qquad (58)$$

Here G is the generator of the Bogoliubov transformation given in equation (44). Indeed if we use the generator (44) then for a fixed value of $\bar{\theta}$ we have

$$\exp[i\bar{\theta}p_{\theta}]a(\theta) = \exp[i\bar{\theta}G]a(\theta)\exp[-i\bar{\theta}G] = a(\theta + \bar{\theta}) \tag{59}$$

which is equivalent to the transformation from $|0(\theta)\rangle \to |0(\theta + \bar{\theta})\rangle$. Furthermore and even more importantly from our point of view the movement is expressed in terms of an inner automorphism of the algebra[5].

Finally I want to turn my attention to the question of time. In the bi-algebra we have two time operators,

$$T = \Delta_{+}t = t \quad 1 + 1 \quad t \qquad \text{and} \qquad \tau = \Delta_{-}t = t \quad 1 - 1 \quad t \tag{60}$$

Since Δ_{+} and Δ_{-} are related, T and τ are not independent. If we regard T as being represented by θ then τ will take the form $-i\frac{\delta}{\delta\theta}$. The conjugate representation would involve taking τ to be represented by ϕ while T will take the form $i\frac{\delta}{\delta\phi}$ Here I am merely exploiting the analogy between the x- and the p-representations where the operators are $\left(x, -i\frac{\partial}{\partial x}\right)$ and $\left(p, i\frac{\partial}{\partial p}\right)$ respectively.

How are we to use this structure? When T is diagonal we remain within one of the inequivalent Hilbert spaces parameterised by θ. In that Hilbert space T now behaves as a parameter, which is proportional to the t that appears in the two equations (27) and (33) and hence in the Schrödinger equation. The system remains within this Hilbert space, getting older as it were but not actualising. All that is happening is that the potentialities are changing with time. Bohm (1987) calls T the implication parameter and regards it as a measure of the age of the system.

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[5] The inner automorphism is a way of expressing the enfolding and unfolding movement.

In this proposal an actual change comes about by a transformation to a different inequivalent representation or, in other words, to a new Hilbert space. Notice that during the transformation T is no longer diagonal implying that Schrödinger time is ambiguous during the transition process. Instead τ becomes diagonal meaning that the time between states is well defined. This would then tie in with the idea of Hiley and Fernandes (1997) mentioned above, where we regarded τ as a measure of the time between states. In the present paper it is a measure of times between inequivalent vacuum states. The fact that θ and its conjugate p_θ do not commute implies that transition between inequivalent states is not sharp and requires just the kind of ambiguity we have suggest accompanies the notion of a moment.

In some ways this ambiguity is necessary because quantum theory tells us that energy and time are complementary variables. So why do we insist on the evolution of a process with a definite energy occurring at a definite instant of time? Surely to have movement we must have this ambiguity in each moment of time. We must have a moment where *what has been* is separated from *what is yet to come*. We must exploit the tension between what has gone with what is to come. In emphasising this point I am proposing that not only does quantum theory contain spatial non-locality but that it also contains a 'non-locality' in time. That is several instants of time coexist in the same moment in the manner suggested by Bohm (1986).

I have yet to connect the parameter θ with the temperature. Superficially it is tempting to regard θ as the inverse of β, i.e. θ is proportional to the temperature. However I am reluctant to make this a definitive step at this stage because I am very aware of the idea of modular flow introduced by Rovelli (1993) and Connes and Rovelli (1994) which has some direct relevance to what I am discussing here. These papers have an extensive discussion on the thermodynamic origin of time. They have probed deeper into the mathematical structure implicit in the work I am discussing and have shown how the Tomita-Takesaki theorem provides this connection between time and the thermal evolution of a quantum system. There are clearly connections between this work and the tentative proposals I have outlined in my paper. There is much more to be said but this must be left for another publication.

Acknowledgements.

I would like to thank the members of the TPRU for their invaluable help in trying to straighten out the ideas expressed in this paper. In particular I would like to thank Melvin Brown for many stimulating discussions. I should also like to thank Keith Bowden for inviting me to present this talk at the ANPA 23 meeting.

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