Inertia in Taylor Rules

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Abstract

The inertia found in econometric estimates of interest rate rules is a continuing puzzle. Many reasons for it have been offered, though unsatisfactorily, and the issue remains open. In the empirical literature on interest rate rules, inertia in setting interest rates is typically modeled by specifying a Taylor rule with the lagged policy rate on the right hand side. We argue that inertia in the policy rule may simply reﬂect the inertia in the economy itself, since optimal rules typically inherit the inertia present in the model of the economy. Our hypothesis receives some support from US data. Hence we agree with Rudebusch (2002) that monetary inertia is, at least partly, an illusion, but for different reasons.

JEL Classiﬁcation: E52, E58

Keywords: Monetary Policy, Interest Rate Rules, Taylor rule, Interest Rate Smoothing, Monetary Policy Inertia, Predictability of Interest Rates, Term Structure, Expectations Hypothesis

1 Introduction

There is a conventional view that central banks adjust interest rates gradually in response to macroeconomic developments. The empirical evidence on the behaviour of central banks in the last two decades has

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been summarized as an inertial Taylor (1993) rule, where the nominal interest rate adjusts only partially to inflation and the output gap, as there is an interest rate smoothing component. A typical formulation has the policy rate responding to its own lagged value as well as a measure of the output gap and the inflation rate, such as the following:

$$i_t = \rho i_{t-1} + (1 - \rho) (\mu_\pi \pi_t + \mu_y y_t)$$

Here $i_t$ is some sort of nominal interest rate that is used as a policy instrument, $\pi_t$ is a measure of the inflation rate, and $y_t$ represents a measure of the output gap. The coefficient $\rho (\in [0, 1])$ is taken to represent the degree of inertia or interest-rate-smoothing. (The coefficients $\mu_\pi$ and $\mu_y$ are the usual long-run responses of the policy rate to inflation and the output gap.)

Numerous explanations for smoothing have been offered, but they all seem in some sense unsatisfactory. The main reason for the unsatisfactoriness is that Central Banks say they do not do it.

A list of popular explanations for the apparent gradualism includes the following:

- Financial stability. It is argued that by adjusting interest rates in small steps spread out over time, less pressure is put on the balance sheets of financial institutions which might otherwise be caught out by large unexpected changes.

- Financial markets may react adversely to frequent changes in the direction of movement of short-term interest rates (Goodfriend 1991). Frequent reversals may give the impression that the Central Bank is incompetent.

- Uncertainty about the structure of the macroeconomic model or about the values of its parameters

- Measurement errors in relevant data.

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1See for instance Clarida, Galì and Gertler (2000), who emphasize the empirical importance of including a lagged interest rate in a monetary policy rule for the United States. For a similar result for other industrial countries see Clarida, Galì and Gertler (1998).

2Reviews of this literature are provided by Cukierman (1992), Goodhart (1996), Walsh (2003), Sack and Wieland (2000).

• The linkage between future monetary policy and aggregate demand can be exploited by central banks in order to stabilize the economy optimally. When the current state of the economy is affected by expectations of future inflation (and other variables) it may be optimal to adjust the interest rate with some inertia.\textsuperscript{4}

• It may be desirable to choose a central banker with an explicit interest rate smoothing objective, in a regime in which policy is delegated to a central banker who pursues policy in a discretionary (i.e., non-precommitted) manner.\textsuperscript{5}

While many scholars accept that the apparent inertia is real, Rudebusch (2002) argues that it is an illusion. Since the coefficient of the lagged policy rate in empirical analyses frequently turns out to be large and highly significant, interest rates should be highly predictable.\textsuperscript{6} However, on the basis of data on yield curves, he argues that they are not. He suggests that empirical Taylor rules may be misspecified and that what looks like inertia may actually be caused by serially correlated shocks.\textsuperscript{7} English, Nelson and Sack (2003) show that it is possible to test directly the null hypothesis of serial correlated errors against the alternative of partial adjustment; but they are unable to reject the presence of either of them. Söderlind, Söderström and Vredin (2002) take up the question of predictability, and find further evidence against the inertial Taylor rule. They argue that a high coefficient of the partial adjustment component is a necessary but not sufficient condition for having a highly predictable interest rate. Predictability depends also on the other variables, namely the output gap and inflation. They find that, while it is relatively easy to predict these, it is very difficult to predict interest rates. They conjecture that this might result from the omission of an unpredictable variable from the Taylor rule.

In this paper we try to reconcile monetary policy inertia with the low predictability of short-term interest rates by proposing a different inertial Taylor rule than the one usually considered in the literature. We argue that the apparent inertia might arise from the the central

\textsuperscript{4}See Woodford (1999).

\textsuperscript{5}See Woodford (2003a). The previous two arguments for the optimality of monetary inertia considered in the text do not presume a central bank’s loss function trading off objectives related to macroeconomic stability with an interest rate smoothing objective (usually interpreted as a financial stability motive).

\textsuperscript{6}In the empirical literature the estimated coefficient for the lagged policy rate is ranging from .7 to .9. See Rudebusch (2002) for a review of the estimates found in the literature.

\textsuperscript{7}See also Lansing (2002) for a theoretical support of the ‘illusion of monetary inertia’ hypothesis, based on real-time estimation of trend output.
bank’s pursuing an optimal rule (or something of a similar character – an ‘optimal-ish’ rule) for interest rates, which therefore inherits the inertia in the economy itself. If the evolution of the output gap and inflation depend on their own lagged values, the optimal rule for the interest rate will typically do so too. We will argue that, for a given coefficient of partial adjustment, our alternative specification implies lower predictability of the interest rate than that implied by the standard specification of the inertial Taylor rule. In our empirical analysis we find support for the alternative specification against the standard specification. Moreover, in the alternative specification, the estimated coefficient of partial adjustment is below 0.5, which is lower than is usually found in the literature.

The structure of the paper is as follows. In section 2 we consider a simple empirical macro-economic model, along the lines of Svensson (1997), and derive the optimal interest rate rule for the central bank. We show that under certain conditions, this may be a simple rule that looks rather like an inertial Taylor rule. Section 3 discusses our empirical findings based on this alternative inertial Taylor rule. Section 4 makes some concluding observations and address future research.

2 A simple framework

2.1 The model

Here we use a simple framework for examining the optimal interest rate rule for a central bank, which is an extended version of the model used by Svensson (1997).\(^8\) He argues that, even if there is no explicit role for private agents’ expectations, the model has many similarities with more elaborate models used by central banks.\(^9\)

Consider the following model\(^{10}\)

\[
\pi_{t+1} = \alpha_1 y_t + (1 - \alpha_2) \pi_t + \alpha_2 \pi_{t-1} + \epsilon_{t+1},
\]

\(^8\)In the literature, Svensson’s (1997) model has been extended in several directions: for examining nominal income targeting (Ball 1999); for studying the implications of monetary policy for the yield curve (Ellingsen and Söderström 2001; Eijffinger, Schaling and Verhagen 2000); for examining model uncertainty, interest rate smoothing and interest rate stabilization – i.e. for studying the optimality of a more gradual adjustment of the monetary instrument (Svensson 1999). Moreover, Rudebusch and Svensson (1999) provide empirical estimates for a model similar to Svensson (1997) and use a calibrated version of the model in order to evaluate a large number of interest rate rules.

\(^9\)See for instance the discussions in Rudebusch and Svensson (1999) and Rudebusch (2001).

\(^{10}\)We have used the same notation as in Svensson (1997).
and

\[ y_{t+1} = \beta_1 y_t - \beta_2 \left( i_t - E_t\pi_{t+1} \right) + \beta_3 y_{t-1} + \eta_{t+1}, \]  

where \( \pi_t \) is the inflation rate, \( y_t \) is the output gap, \( i_t \) is the nominal repo rate, i.e. the monetary instrument of the central bank, and \( \epsilon_t, \eta_t \) are i.i.d. shocks.\(^{11}\) All the variables are considered as deviations from their long-run average levels, which are normalized to zero for simplicity.

After substituting \( E_t\pi_{t+1} \) with the expectation of expression (1), expression (2) becomes:

\[ y_{t+1} = \beta_1 y_t - \beta_2 i_t + \beta_3 y_{t-1} + \beta_4 \pi_t + \beta_5 \pi_{t-1} + \eta_{t+1}, \]  

with

\[ \beta_1 \equiv \beta'_1 + \beta_2 \alpha_1; \]
\[ \beta_4 \equiv \beta_2 (1 - \alpha_2); \]
\[ \beta_5 \equiv \beta_2 \alpha_2. \]

The coefficients in (1) and (3) are all assumed to be positive, with \( 0 < \alpha_2 < 1 \). Equations (1) and (3) coincide with those considered in Svensson (1997) (equations 6.4 and 6.5 in his text) when \( \alpha_2 = \beta_3 = 0.\(^{12}\) The restriction that the sum of the lag coefficients of inflation in (1) equals 1 is consistent with the empirical evidence.\(^{13}\) An important feature of this model is the presence of lags in the transmission of monetary policy. In particular, the repo rate affects output with a one-period lag, while affects inflation with a two-period lag. This feature is broadly consistent with the "stylized facts" of the impact of monetary policy on output and inflation.

Finally, suppose that monetary policy is conducted by a central bank with the following period loss function

\[ L(\pi_t, y_t) = \frac{1}{2} \left[ \pi_t^2 + \lambda y_t^2 \right], \]

\(^{11}\)See Svensson (1997) for the details on the model and in particular for the implications of substituting the long-term nominal rate with the repo rate.

\(^{12}\)We have assumed that the coefficient of one-period lagged inflation in (1) is less than 1, instead of equal to it. In this we differ from Svensson. McCallum (1997) has shown that when the coefficient is equal to 1 we may have problems of instability of nominal income rules that would not arise if expectations of current or future inflation were included in the model considered. See also Rudebusch (2002) and Jensen (2002) for further analyses of the performance of nominal income rules for monetary policy when a forward-looking price-setting behaviour is explicitly included in the analytical framework.

\(^{13}\)See for instance Rudebusch and Svensson (1999) for a test of this restriction in a model similar to the one considered here.
where $\lambda > 0$ is the relative weight on output stabilization. The intertemporal loss function is

$$E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L (\pi_\tau, y_\tau).$$

(6)

The central bank minimizes the above intertemporal loss function by choosing a sequence of current and future repo rates $\{i_\tau\}_{\tau=t}^{\infty}$.

### 2.2 Optimal interest rate rule

In solving the optimization problem we use a convenient simplification. In the expression (3) of output the choice of $i_\tau$ affects $y_{t+1}$, but $y_t, y_{t-1}, \pi_t$ and $\pi_{t-1}$ are all predetermined. Thus we can write

$$y_{t+1} = \Delta_t + \eta_{t+1},$$

(7)

with

$$\Delta_t \equiv \beta_1 y_t - \beta_2 i_t + \beta_3 y_{t-1} + \beta_4 \pi_t + \beta_5 \pi_{t-1}. $$

(8)

As observed above, the repo rate affects inflation with a two-period lag. This can be seen by rewriting the expression (1) for inflation in the following way

$$\pi_{t+2} = \alpha_1 \Delta_t + (1 - \alpha_2) \pi_{t+1} + \alpha_2 \pi_t + \alpha_1 \eta_{t+1} + \epsilon_{t+2},$$

(9)

where we have considered inflation at time $t+2$ and inserted expression (7). We can treat $\Delta_t$ as the control variable. Using dynamic programming, we can derive the optimal rule as the solution to the following problem

$$V (E_t \pi_{t+1}, \pi_t) = \min_{\Delta_t} E_t \left\{ \frac{1}{2} \left[ \pi_{t+1}^2 + \lambda y_{t+1}^2 \right] + \delta V (E_{t+1} \pi_{t+2}, \pi_{t+1}) \right\},$$

subject to (7) and (9). The value function $V (E_t \pi_{t+1}, \pi_t)$ will be quadratic and in the present case, where constant terms are absent, it can be expressed without loss of generality as

$$V (E_t \pi_{t+1}, \pi_t) = \frac{1}{2} \gamma_1 \pi_{t+1}^2 + \gamma_2 \pi_{t+1} \pi_t + \frac{1}{2} \gamma_3 \pi_t^2 + k,$$

(11)

where the coefficients $\gamma_1, \gamma_2$ and $\gamma_3$ need to be determined. The remaining constant $k$ is a function of the variances of the shocks.

Here we have two state variables and one control variable. In general, the optimization problem cannot be solved analytically by means
of dynamic programming if there is more than one state variable. In the simpler case with only one state variable, considered by Svensson, it is possible to get an analytical solution for the optimization problem. Nevertheless, we can make a qualitative assessment of the form of the optimal rule. Svensson has shown that in the simpler case considered by him the optimal rule takes the form of the Taylor (1993) rule
\[ i_t = \phi_1 \pi_t + \phi_2 y_t, \]
with \( \phi_1 > 1 \) and \( \phi_2 > 0 \). What emerges in the present case?

The first order condition with respect to \( \Delta_t \) is given by
\[ E_t y_{t+1} = -\frac{\alpha_1 \delta}{\lambda} (\gamma_1 E_t \pi_{t+2} + \gamma_2 E_t \pi_{t+1}), \quad (12) \]
where we have used (11).

The optimal interest rate can be derived by substituting (1) in (12) and using (3) to yield
\[ i_t = \alpha_2 \left[ (1 + C) \pi_{t-1} + \frac{\beta_3}{\alpha_2 \beta_2} y_{t-1} \right] + \]
\[ (1 - \alpha_2) \left[ (1 + A) \pi_t + \left( \frac{\beta_1}{(1 - \alpha_2) \beta_2} + B \right) y_t \right], \quad (13) \]
with
\[ A \equiv \delta \alpha_1 \gamma_1 + \gamma_2 \frac{(1 - \alpha_2)}{(1 - \alpha_2) \beta_2 (\lambda + \delta \alpha_2 \gamma_1)}, \quad (14) \]
\[ B \equiv \delta \alpha_1 \gamma_1 + \gamma_2 \frac{(1 - \alpha_2)}{(1 - \alpha_2) \beta_2 (\lambda + \delta \alpha_2 \gamma_1)}; \]
\[ C \equiv \delta \alpha_1 \gamma_1 + \frac{(1 - \alpha_2)}{\beta_2 (\lambda + \delta \alpha_2 \gamma_1)}. \]

In general, in a problem of this type, the optimal feedback rule can be represented as a linear function of the state variables, here \( E_t \pi_{t+1} \) and \( \pi_t \). So we could represent the rule for \( \Delta_t \) as \( \Delta_t = f_1 E_t \pi_{t+1} + f_2 \pi_t \). Since \( E_t \pi_{t+1} \) can be represented as a function of current values and the first lag of the output gap and inflation, when we solve for the interest rate, the policy rule also emerges as a linear function of the same variables. It would be useful to be able to sign the parameters in the feedback rule (14). Since the value function is a positive definite quadratic form, it must be the case that \( \gamma_1 > 0, \gamma_3 > 0 \), and \( \gamma_1 \gamma_3 - \gamma_2^2 > 0 \), but it is not possible to sign \( \gamma_2 \). If the coefficients on the right-hand side variables in (13) are all positive, and if the ratios of coefficients on the current variables (and) are the same as the ratios of coefficients on lagged
variables (and), then the policy rule may have the form of a moving average of a simple Taylor rule. That is, (13) can be written as

\[ i_t = \alpha_2 \left[ \mu_3 \pi_{t-1} + \mu_4 y_{t-1} \right] + \left( 1 - \alpha_2 \right) \left[ \mu_1 \pi_t + \mu_2 y_t \right], \tag{15} \]

with \( \mu_1 = (1 + A), \mu_2 = \left( \frac{\beta_1}{(1 - \alpha_2) \beta_2} + B \right), \mu_3 = (1 + C), \) and \( \mu_4 = \frac{\beta_3}{\alpha_2 \beta_2}. \)

If the pattern of coefficient were such that \( \mu_1 / \mu_2 = \mu_3 / \mu_4 \) then the actual rule could be thought of as a moving average of a simple rule \( \overline{r}_t = \mu_1 \pi_t + \mu_2 y_t. \)

### 2.3 Simple rules

During the past decade, research on monetary policy design has focused on simple rules – among which Taylor’s (1993) rule is a prominent example – as opposed to more complicated or fully optimal rules.\(^{14}\) As Woodford (2003b, p. 507) argues, a rationale for this choice can be found in the greater transparency provided by simple rules, which may increase central bankers’ accountability and commitment to the rule.\(^{15}\)

Typically this literature has focused on simple rules based on two or three parameters (and variables) which are optimized for the given preferences. For example Rudebusch and Svensson (1999) estimate a model similar to that presented here, with more lagged variables and an interest rate smoothing argument added in the loss function. They derive the optimal policy rule, which looks more complicated than ours, numerically. They also use the model to evaluate a large number of simple rules.

Two findings of this literature are that simple rules perform nearly as well as fully optimal rules and that simple rules are more robust than more complicated rules if the model is misspecified. In this vein, we can simplify the rule derived above (13) and write it as

\[ i_t = \rho \pi_{t-1} + (1 - \rho) \overline{r}_t, \tag{16} \]

with

\[ \overline{r}_t = \mu_1 \pi_t + \mu_2 y_t, \tag{17} \]

and \( 0 < \rho < 1. \)

In the empirical literature the standard inertial Taylor rule takes instead the following form

\(^{14}\)For a review of this literature see for example Williams (2003).

\(^{15}\)See Svensson (2003) for a discussion of the problems associated to using judgments in monetary policy based on simple instrument rules or targeting rules.
\[ i_t = \rho i_{t-1} + (1 - \rho) \tilde{i}_t, \] 

(18)

with \( \tilde{i}_t \) equal to (17) or to a forward-looking version of (17) with future expected inflation. The term \( \tilde{i}_t \) is usually interpreted as an operating target for the policy rate.

The crucial difference of (16) with respect to (18) is that the inertial component is proportional to the lagged operating target, instead of the lagged interest rate. Hence, our alternative specification of the inertial policy rule implies that the central bank gradually adjusts the operating target for the policy rate.\(^{16}\)

In our framework, substituting the lagged operating target \((\tilde{i}_{t-1})\) with the lagged actual interest rate \((i_{t-1})\) in the simple rule would improve its approximation to the optimal rule only if the lagged interest rate appeared in the optimal rule. This only happens if there is an interest rate smoothing objective in the central bank’s loss function. There are circumstances under which this objective might appear. For example, Woodford (2003a) has shown that it may be optimal to delegate monetary policy to a central bank that has an objective function with an interest rate smoothing motive when the private sector is forward-looking. However, while there are real-world examples of institutional arrangements that penalize central banks for not achieving given inflation targets, there is less evidence of them being penalized for changing interest rates. References to financial stability are typically very general and do not necessarily imply an interest rate smoothing objective.\(^{17}\)

Sack (2000, pp. 230-231) provides a further argument against an explicit interest rate smoothing objective:

“To describe this behaviour, which has been referred to as gradualism, many empirical studies of monetary policy incorporate an explicit interest-rate smoothing incentive in the objective function of the Fed. However, introducing this argument has little justification beyond matching the data. Furthermore, the above statistics provide evidence of gradualism only if the Fed would otherwise choose a random-walk policy in the absence of an interest-rate smoothing objective. Therefore, while establishing that the funds rate is not a random walk, these statistics do not necessarily provide evidence of gradualism in monetary policy.”

\(^{16}\)See Woodford (2003b, p. 96) for a discussion of interest rate rules with partial adjustment on lagged operating target.

\(^{17}\)See for example Goodfriend (1987).
Thus we prefer to leave open the question of whether or not the Central Bank has a smoothing objective, and to inquire how far we can explain the behaviour of the policy rate without invoking it. We test empirically for alternative specifications of simple rules which do not necessarily include the lagged interest rate and instead derive some degree of inertia from the dynamic structure of the economy.

3 Empirical evidence

3.1 Estimation of inflation and output equations

In order to gain some insights into the parameters of the inflation and output equations used in the previous theoretical analysis we have first estimated the following empirical model based on Rudebusch and Svensson (1999):

\[ \pi_t = \kappa_1 \pi_{t-1} + \kappa_2 y_{t-1} + \kappa_3 \pi_{t-2} + \omega_t, \] (19)

and

\[ y_t = \kappa_1 y_{t-1} + \kappa_2 y_{t-2} + \kappa_3 \left( \tilde{i}_{t-1} - \tilde{\pi}_{t-1} \right) + \psi_t, \] (20)

where the variables were de-meaned prior to estimation. The data used here are ex post revised quarterly data. Inflation is defined using the GPD-chain weighted price index \( P_t \), with \( \pi_t = 400 \cdot (\ln P_t - \ln P_{t-1}) \). The output gap is defined as the percentage difference between actual real GDP \( Q_t \) and potential output \( Q^* \) estimated by the Congressional Budget Office. The interest rate \( i_t \) is the quarterly average of the Fed Funds rate.\(^{18}\) The data are illustrated in Figures 1, 2, and 3. In the text we do not discuss the stationarity or otherwise of the data. A note at the end of the appendix summarises some simple checks.

In Table 1 we report Ordinary Least Squares estimates of the above two equations over the period 1961 Q1 - 2004 Q2, with robust standard errors for the inflation equation. Following Rudebusch and Svensson the equations were estimated individually. In the output equation \( \tilde{i}_t = (1/4) \sum_{j=0}^3 i_{t-j} \) and \( \tilde{\pi}_t = (1/4) \sum_{j=0}^3 \pi_{t-j} \). The inflation equation is somewhat simpler than that estimated by Rudebusch and Svensson. According to the Wald test, the null hypothesis that \( \kappa_3 = (1 - \kappa_1) \) has a \( p \)-value of .15, therefore we have imposed this restriction in the reported estimates.

Table 1 Inflation and Output Equations with ex post revised data

\(^{18}\)While real GDP and the GPD-chain weighted price index were taken from FRED of the Federal Reserve of San Louis, the (effective) Fed Funds rate was taken from Datastream.
Inflation & Output

<table>
<thead>
<tr>
<th>Inflation</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_{\pi 1}$</td>
<td>0.72 (7.80)</td>
</tr>
<tr>
<td>$\kappa_{\pi 2}$</td>
<td>0.09 (3.35)</td>
</tr>
<tr>
<td>$\kappa_{\pi 3}$</td>
<td>$-0.06$ (-2.12)</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.81</td>
</tr>
<tr>
<td>SE</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Notes: Ordinary Least Squares estimates. T statistics in parentheses. $\bar{R}^2$ and standard errors (SE) of residuals also reported. For the inflation equation T-statistics are based on heteroskedasticity- and serial correlation-corrected standard errors (Newey and West, 1987). Variables are de-meaned before estimation. Sample period 1961Q1 – 2004Q4.

Despite the simplicity of this model, it has remarkably good statistical properties, a fact on which Lars Svensson and Glenn Rudebusch have
commented in several papers, including Svensson and Rudebusch (1999), Rudebusch (2001), Rudebusch (2002) and Rudebusch (2005). It is remarkably stable over the estimation period despite the policy changes that have taken place. The model is backward-looking and includes no meaningful forward-looking rational-expectations element. Therefore it is subject to the ‘Lucas critique’ that its parameters are not policy-invariant and it does not provide a reliable basis for evaluating alternative policy rules. However, Rudebusch (2005) shows that the empirical significance of the Lucas critique for this model is very small. Taking some widely-used estimated models that allow for forward-looking behaviour, and applying a number of alternative policy rules which cover the range of policies pursued from the early 1960s to the late 1990s, the parameters of the reduced form model that emerges are affected only very slightly by the policy changes. In view of these findings we use this simple model with some confidence.

3.2 Optimal Rules from the Estimated Macro Model
In the model above, equations (1) and (2), we insert parameter values from our estimates in Table 1, as follows:

$$\pi_{t+1} = 0.09y_t + 0.72\pi_t + 0.28\pi_{t-1}$$

$$y_{t+1} = 1.19y_t - 0.06(i_t - E_t\pi_{t+1}) - 0.27y_{t-1}$$

We then compute some optimal interest rate rules, taking as the objective function a slightly more general one than in (5) and (6). Here we allow for a smoothing objective:

$$L(\pi_t, y_t) = \frac{1}{2} [\pi_t^2 + \lambda y_t^2 + S(i_t - i_{t-1})^2]$$

The rules that emerge for various parameter values are shown in Tables 2 and 3. The entries in the tables are coefficients on each of the variables shown at the top of each column.
Table 2
Rule for interest rate $i_t$
With a high weight on output stabilization

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$S$</th>
<th>$\pi_t$</th>
<th>$\pi_{t-1}$</th>
<th>$y_t$</th>
<th>$y_{t-1}$</th>
<th>$i_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>12.39</td>
<td>3.56</td>
<td>21.71</td>
<td>-4.50</td>
<td>0.0</td>
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<tr>
<td>1</td>
<td>0.01</td>
<td>4.31</td>
<td>1.24</td>
<td>7.51</td>
<td>-1.80</td>
<td>0.28</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>2.50</td>
<td>0.72</td>
<td>4.24</td>
<td>-1.06</td>
<td>0.42</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>1.94</td>
<td>0.56</td>
<td>3.25</td>
<td>-0.82</td>
<td>0.48</td>
</tr>
<tr>
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<td>0.5</td>
<td>1.07</td>
<td>0.31</td>
<td>1.71</td>
<td>-0.44</td>
<td>0.60</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.83</td>
<td>0.24</td>
<td>1.29</td>
<td>-0.33</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Table 3
With a low weight on output stabilization

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$S$</th>
<th>$\pi_t$</th>
<th>$\pi_{t-1}$</th>
<th>$y_t$</th>
<th>$y_{t-1}$</th>
<th>$i_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0</td>
<td>36.66</td>
<td>10.15</td>
<td>23.83</td>
<td>-4.50</td>
<td>0.0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.01</td>
<td>5.30</td>
<td>1.50</td>
<td>4.36</td>
<td>-1.02</td>
<td>0.42</td>
</tr>
<tr>
<td>0.1</td>
<td>0.05</td>
<td>2.79</td>
<td>0.79</td>
<td>2.50</td>
<td>-0.60</td>
<td>0.54</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>2.11</td>
<td>0.60</td>
<td>1.95</td>
<td>-0.48</td>
<td>0.59</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>1.09</td>
<td>0.31</td>
<td>1.11</td>
<td>-0.27</td>
<td>0.68</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>0.83</td>
<td>0.24</td>
<td>0.87</td>
<td>-0.22</td>
<td>0.71</td>
</tr>
</tbody>
</table>

In Table 2 a relatively high weight is given to output deviations ($\lambda = 1$). Here output and inflation deviations are equally weighted. Results are provided for values of $S$ (the weight on interest rate changes) ranging from 0 to 1. We find, as many have done before, that if a zero weight is given to smoothing, the responses to inflation and output deviations are very aggressive. The coefficients are most un-Taylor-like! The coefficients on current and lagged output gaps sum to 19.53 and those on inflation sum to 46.81, far from the 0.5 and 1.5 found by Taylor! However, introducing a small degree of smoothing ($S$ lying between 0.1 and 0.5) reduces the short run coefficients to more Taylor-like levels. However, we do not find here one of the key requirements of our hypothesis that the ratio of the coefficients on $\pi_t$ and $y_t$ should equal the ratio of the coefficients on $\pi_{t-1}$ and $y_{t-1}$. Thus it does not seem likely that the optimal rule can be written as a moving average of an operating target like $\pi$ as in equation (16). Whatever the weight on smoothing, the parameter on $y_{t-1}$ is always negative while the other three are positive.

The aggressive responses result from the very weak effect that the interest rate has on the output gap, compounded with the weak effect of the output gap on inflation. The strong positive response of the policy rate to the current output gap and the negative response to the lagged
output gap probably reflect the pattern of lagged effects in the output gap equation: the output gap responds more than one-for-one to its own once-lagged value, and responds negatively to its own twice-lagged value. Meanwhile inflation responds positively but less than one-for-one to its own once-lagged value and responds positively but less to its own twice lagged value.

When a lower weight on output stabilization is chosen, as in Table 3 where \( \lambda = 0.1 \), the general character of the results is unchanged. Now the policy response to inflation is even more aggressive, but some degree of smoothing does a great deal to reduce the coefficients to Taylor-like levels. The pattern of signs of the policy response to output gaps and inflation is as before.

Despite the finding that, in the terms used in (15) above, \( \mu_1/\mu_2 \neq \mu_3/\mu_4 \), for this estimated model, we nevertheless go on in the next section to explore directly estimated policy rules.

### 3.3 Estimates of Alternative Policy Rules

We now turn to estimating alternative forms of policy rule, the standard inertial Taylor rule

\[
i_t = \rho i_{t-1} + (1 - \rho) \bar{\pi}_t + \xi_t, \tag{21}\]

and the alternative inertial Taylor rule

\[
i_t = \rho \bar{\pi}_{t-1} + (1 - \rho) \bar{\pi}_t + \xi_t, \tag{22}\]

with

\[
\bar{\pi}_t = \mu + \mu_\pi \bar{\pi}_{t-1} + \mu_y y_t, \tag{23}\]

and \( 0 < \rho < 1 \). \( \xi_t \) is an i.i.d. error term. Following Taylor (1993) and Rudebusch (2002a) the policy rate is assumed to react to the average inflation rate over four quarters, \( \bar{\pi}_t \).

We allow for serial correlation in the errors in these two equations. As Rudebusch (2002a) argues, a partial adjustment model and a model with serially correlated shocks can be nearly observationally equivalent. However English, Nelson and Sack (2003) find that both play an important role in describing the behaviour of the federal funds rate when they allow for both of these hypotheses in the estimation of the policy rule. The omission of a persistent, serially correlated variable that influences monetary policy could yield the spurious appearance of partial adjustment in the estimated rule. We assume that the shock \( \xi_t \) follows an AR(1) process:
\[ \xi_t = \theta \xi_{t-1} + \varepsilon_t. \quad (24) \]

The combination of rule (21) with (24) yields the following expression for the first difference of the interest rate:

\[ \Delta i_t = (1 - \rho) \Delta \tilde{i}_t - (1 - \rho)(1 - \theta)(i_{t-1} - \tilde{i}_{t-1}) + \rho \theta \Delta i_{t-1} + \varepsilon_t. \quad (25) \]

This expression corresponds to that used by English, Nelson and Sack (2003). The combination of rule (22) with (24) yields the following expression for the first difference of the interest rate:

\[ \Delta i_t = (1 - \rho) \Delta \tilde{i}_t - (1 - \theta)(i_{t-1} - \tilde{i}_{t-1}) + \rho \theta \Delta i_{t-1} + \varepsilon_t. \quad (26) \]

Nonlinear Least Squares estimates of (25) and (26) are reported in tables 4 and 5, for the period 1987 Q4 - 2004 Q2, and for two subsamples of it. The point estimates of \( \rho \) and \( \theta \) are both highly significant for all rules, suggesting that both partial adjustment and serially correlated errors are present. The coefficients on the output gap and inflation are largely consistent with other estimates from the literature, with a significant coefficient on the output gap and a coefficient on inflation greater than one. Moreover, both rules appear to fit the data relatively well.

Interestingly, the degree of inertia implied by the alternative inertial Taylor rule is systematically lower than that implied by the standard specification, with an estimated coefficient of partial adjustment \( \rho \) for the whole sample of .60 against one of .77. Meanwhile, the coefficient \( \theta \) is systematically higher in the case of the alternative specification than in the standard specification. However, we have not tested whether these differences are significant statistically.

Thus, as in English, Nelson and Sack (2003), the empirical evidence suggests that specifications (25) and (26) of the policy rules perform no worse than the more usual specifications (21) and (22). The alternative specification suggests less monetary inertia but much greater importance of serially correlated errors than does the standard specification.

Table 4 Standard inertial Taylor Rule with ex post revised data
3.4 More general models

The two models – the standard rule and our revised rule – have been presented as two alternatives. However, they can both be represented as special cases of more general relations. The least restrictive is an unrestricted linear model involving lags of the change and level of the interest rate, and current and lagged values of the changes in the output.

Table 5  Alternative Inertial Taylor Rule with ex post revised data

<table>
<thead>
<tr>
<th></th>
<th>1987Q4-1993Q4</th>
<th>1987Q4-2001Q2</th>
<th>1987Q4-2004Q2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>0.15</td>
<td>1.10</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.94)</td>
<td>(0.89)</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>2.31</td>
<td>1.85</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>(7.12)</td>
<td>(4.31)</td>
<td>(2.41)</td>
</tr>
<tr>
<td>$\bar{\mu}_y$</td>
<td>0.92</td>
<td>0.77</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>(5.61)</td>
<td>(3.94)</td>
<td>(3.49)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.51</td>
<td>0.61</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>(7.58)</td>
<td>(7.34)</td>
<td>(6.49)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.34</td>
<td>0.80</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>(2.09)</td>
<td>(5.52)</td>
<td>(5.41)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.99</td>
<td>.097</td>
<td>0.98</td>
</tr>
<tr>
<td>SE</td>
<td>0.26</td>
<td>0.31</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Note: Nonlinear least squares estimates. T-statistics in parentheses based on standard errors corrected for heteroskedasticity and serial correlation (Newey and West, 1987). $R^2$ and standard errors (SE) of residuals are reported for the level of the funds rate.

Table 5  Alternative Inertial Taylor Rule with ex post revised data

<table>
<thead>
<tr>
<th></th>
<th>1987Q4-1993Q4</th>
<th>1987Q4-2001Q2</th>
<th>1987Q4-2004Q2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>0.41</td>
<td>1.70</td>
<td>−4.08</td>
</tr>
<tr>
<td></td>
<td>(0.41)</td>
<td>(1.07)</td>
<td>(−0.17)</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>2.15</td>
<td>1.40</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>(9.69)</td>
<td>(5.04)</td>
<td>(3.50)</td>
</tr>
<tr>
<td>$\bar{\mu}_y$</td>
<td>0.78</td>
<td>0.65</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>(5.66)</td>
<td>(4.56)</td>
<td>(4.56)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.48</td>
<td>0.59</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>(6.62)</td>
<td>(6.49)</td>
<td>(8.64)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.70</td>
<td>0.94</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>(6.18)</td>
<td>(17.29)</td>
<td>(26.68)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.99</td>
<td>.096</td>
<td>0.98</td>
</tr>
<tr>
<td>SE</td>
<td>0.28</td>
<td>0.35</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Note: Nonlinear least squares estimates. T-statistics in parentheses based on standard errors corrected for heteroskedasticity and serial correlation (Newey and West, 1987). $R^2$ and standard errors (SE) of residuals are reported for the level of the funds rate.

3.4 More general models

The two models – the standard rule and our revised rule – have been presented as two alternatives. However, they can both be represented as special cases of more general relations. The least restrictive is an unrestricted linear model involving lags of the change and level of the interest rate, and current and lagged values of the changes in the output.
gap and inflation and their lagged levels:

\[
\Delta i_t = c_0 + c_1 \Delta i_{t-1} + c_2 \Delta \pi_{t-1} + c_3 \Delta y_t + c_4 \Delta \pi_{t-1} + c_5 \Delta \pi_{t-1} + c_6 \Delta y_{t-1} + c_7 \Delta y_{t-1} + c_8 \Delta \pi_{t-1} + u_t
\]

A set of restrictions that brings this closer to the Taylor rules above is to assume that the output gap and inflation rate enter through some sort of target interest rate \(i_t\), which is a linear combination of the output gap and the inflation rate. The necessary restrictions are:

\[
c_3/c_4 = c_5/c_6 = c_7/c_8.
\]

When these hold, the change in the policy rate can be written as

\[
\Delta i_t = c_1' \Delta i_{t-1} + c_2' (i_{t-1} - \overline{i}_{t-1}) + c_3' \Delta \pi_t + c_4' \Delta \pi_{t-1} + u_t
\]

with

\[
\overline{i}_t = \mu_0 + \mu_\pi \pi_t + \mu_y y_t
\]

This might be termed a semi-restricted model.

To get another step closer to the models estimated above, we can impose the restriction that there is a common factor in the lag polynomials for \(i_t\) and \(\pi_t\) so that the model can be represented as having a first-order autoregressive error term. This might be termed the hybrid model, as it takes the form of a linear combination of the two models set out above. The restriction that is imposed on the semi-restricted model above is that

\[
\frac{c_3'}{1 - c_3'} = \frac{c_2'}{1 - c_3' - c_1' - c_4'} - \frac{c_4'}{c_1' + c_4'}
\]

and when this restriction is valid we can reduce the four parameters \(c_1', c_2', c_3', \) and \(c_4'\) to three, \(\rho, \phi, \) and \(\theta\), which satisfy

\[
(1 - \rho - \phi) = c_3', (1 - \phi)(1 - \theta) = c_2', \theta \phi = c_1', \text{and } \theta \rho = c_4'
\]

In terms of the levels of the interest rate the hybrid model gives:

\[
i_t = (1 - \rho - \phi)i_t + \rho \pi_{t-1} + \phi i_{t-1} + \xi_t
\]

\[
\overline{i}_t = \mu_0 + \mu_\pi \pi_t + \mu_y y_t
\]

\[
\xi_t = \theta \xi_{t-1} + \epsilon_t
\]

As an expression for the change in the policy rate, the hybrid model gives:

\[
\Delta i_t = (1 - \rho - \phi)\Delta i_t + (1 - \phi)(\overline{i}_{t-1} - i_{t-1}) + \theta \phi \Delta i_{t-1} + \theta \rho \Delta \pi_{t-1} + \epsilon_t
\]

\[18\]
The models set out above are special cases of this hybrid model. If we assume \( \rho = 0 \), we get the "standard" type of inertial Taylor Rule. If instead we assume \( \phi = 0 \), we get the moving average form of Taylor rule, in which there is no real, only apparent inertia. If we assume \( \theta = 0 \), we are assuming that the error term is not serially correlated.

Estimated over the sample period 1987Q4 to 2004Q2, the unrestricted, semi-restricted, and hybrid models show that the hybrid model is an acceptable simplification of the unrestricted model. The relevant summary statistics are reported in Table 6.

Table 6 Summary Statistics: Unrestricted, Semi-Restricted, and Hybrid Models

<table>
<thead>
<tr>
<th></th>
<th>Unrestricted</th>
<th>Semi-Restricted</th>
<th>Hybrid</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
<td>0.62</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>( \bar{R}^2 )</td>
<td>0.57</td>
<td>0.57</td>
<td>0.58</td>
</tr>
<tr>
<td>SE of Regression</td>
<td>0.31</td>
<td>0.31</td>
<td>0.310</td>
</tr>
<tr>
<td>Sum of squared residuals</td>
<td>5.65</td>
<td>5.86</td>
<td>5.86</td>
</tr>
<tr>
<td>Akaike</td>
<td>0.63</td>
<td>0.61</td>
<td>0.58</td>
</tr>
<tr>
<td>Schwartz</td>
<td>0.93</td>
<td>0.84</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Note: Sample period 1987Q4–2004Q2. \( R^2 \) measured for \( \Delta i_t \).
However, when further restrictions are imposed on the hybrid model, they prove to be rejected by the data. Both the ‘Normal Taylor’ and the ‘Alternative’ models are rejected against the alternative hypothesis of the hybrid model. Consequently, for this sample period, neither model suffices. There appear to be elements of both in the data. The most that can be claimed is that, while structural inertia (represented by our alternative model) plays some role in explaining interest rate movements, there still appears to be an element of the ‘inexplicable’ inertia remaining.

Table 7 Estimates of Various Models

<table>
<thead>
<tr>
<th></th>
<th>Hybrid Model</th>
<th>Normal Taylor ($\rho = 0$)</th>
<th>Alternative ($\phi = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.28 (0.10)</td>
<td>0</td>
<td>0.60 (0.10)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.48 (0.11)</td>
<td>0.71 (0.11)</td>
<td>0</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>0.92 (0.21)</td>
<td>0.93 (0.38)</td>
<td>0.67 (0.11)</td>
</tr>
<tr>
<td>$\mu_\pi$</td>
<td>1.15 (0.42)</td>
<td>1.65 (0.63)</td>
<td>1.10 (0.26)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.93 (0.06)</td>
<td>0.76 (0.14)</td>
<td>0.98 (0.03)</td>
</tr>
<tr>
<td>const</td>
<td>0.07 (0.10)</td>
<td>0.083 (0.09)</td>
<td>-0.04 (0.11)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.61</td>
<td>0.56</td>
<td>0.47</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.58</td>
<td>0.53</td>
<td>0.44</td>
</tr>
<tr>
<td>SE</td>
<td>0.310</td>
<td>0.325</td>
<td>0.36</td>
</tr>
<tr>
<td>Sum Squared Residuals</td>
<td>5.86</td>
<td>6.56</td>
<td>7.93</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-13.43</td>
<td>-17.23</td>
<td>-23.59</td>
</tr>
<tr>
<td>Akaike info criterion</td>
<td>0.58</td>
<td>0.66</td>
<td>0.85</td>
</tr>
<tr>
<td>Schwarz criterion</td>
<td>0.78</td>
<td>0.83</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Notes: Sample Period 1987Q4–2004Q2

While neither model is acceptable for the period 1987Q4–2004Q2, it is possible to find shorter sample periods for which one or other of them is acceptable, as Table 8 shows. This table shows the p-values for the likelihood ratio test of the null hypothesis that the model is either the standard or the alternative inertial Taylor rule against the alternative hypothesis that the hybrid is the true model. Our alternative model is acceptable providing the sample starts in 1983Q4 and ends before 1999Q4. But if the sample begins in 1987Q4 the model is rejected. The standard inertial Taylor model by contrast is only accepted if the sample beings in 1987Q4 and ends by 1999Q4. All this points to considerable structural instability in these models, reflecting changing responses of interest rates to output gaps and inflation.

Table 8 Partial Adjustment and Correlated Shock Rules: p-values
Figure 4: Hybrid Model, estimated on sample 1987Q4 2004Q2

<table>
<thead>
<tr>
<th>Sample</th>
<th>Standard</th>
<th>Alternative Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>83Q4-93Q4</td>
<td>0.00</td>
<td>0.89</td>
</tr>
<tr>
<td>-96Q4</td>
<td>0.00</td>
<td>0.14</td>
</tr>
<tr>
<td>-99Q4</td>
<td>0.00</td>
<td>0.07</td>
</tr>
<tr>
<td>-04Q2</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>87Q4-93Q4</td>
<td>0.53</td>
<td>0.04</td>
</tr>
<tr>
<td>-96Q4</td>
<td>0.14</td>
<td>0.00</td>
</tr>
<tr>
<td>-99Q4</td>
<td>0.44</td>
<td>0.00</td>
</tr>
<tr>
<td>-04Q2</td>
<td>0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: The entries in the table are p-values of the Likelihood-Ratio Test. The null hypothesis is partial adjustment for the standard inertial Taylor Rule (columns headed ‘Standard’) or for the alternative inertial rule (columns headed ‘Alt’), with and without serially correlated shocks.

The actual values of the interest rate and the fitted values for the hybrid model are displayed in Figure 4.
The presence of serially correlated errors in the policy rule (for both specifications) may reflect the presence of serially correlated variables (other than inflation and the output gap) which have been omitted from the estimated policy rule. Driffill et al. (2006) have argued that likely candidates are indicators of financial stress related to a financial stability motive.

4 Evidence from yield curves

Rudebusch argues that the partial adjustment of monetary policy by a central bank implies that the short-term interest rate should be highly predictable. However, term structure evidence based on futures contracts suggests that there is little if any information usually available in financial markets for predicting the Fed funds rate 3-6 months ahead and no information for predicting it 6-9 months ahead. On the contrary within a 3-month horizon the 3-month eurodollar forecasts the future change of the Fed funds rate relatively well (with an $R^2$ of 0.57).

Söderlind, Södeström and Vredin (2003) note that the predictability of the short-term interest rate depends crucially on the predictability of inflation and output as well as the degree of monetary inertia. They show that, while it is relatively easy to predict the variables that enter the Taylor rule, it is very difficult to predict interest rates. They argue that this outcome might be related to an omitted variable problem in the Taylor rule, with the potentially omitted variable being not easily predictable.

In order to examine the issue of predictability empirically, we consider our estimated equations for output and inflation, and run recursive simulations for the Fed funds rate by using the different estimated policy rules for the 1987-2004 period. After having obtained one quarter, two quarters and three quarters ahead predictions of the Fed funds rate we estimate for the 1990 Q1 - 2004 Q2 period the following regressions:

\[ i_{t+1} - i_t = \psi_0 + \psi_1 (E_t i_{t+1} - i_t) + \xi_{t+1}, \]
\[ i_{t+2} - i_{t+1} = \psi_0 + \psi_1 (E_t i_{t+2} - E_t i_{t+1}) + \xi_{t+2}, \]
\[ i_{t+3} - i_{t+2} = \psi_0 + \psi_1 (E_t i_{t+3} - E_t i_{t+2}) + \xi_{t+3}. \]

The use of parameters estimated on the full sample is consistent if parameters are stable; and recursive estimations starting from 1990 Q1 support parameter stability for the different policy rules considered. Equations (30) are the analogue of the equations considered by Rudebusch based on the forecasts implied by futures contracts.\[^{19}\] Table 10

---

\[^{19}\] Equations (15), (16) and (17) in Rudebusch (2002), pages 1172-1173.
reports the estimated parameters and the corrected $R^2$ statistic for the two specifications. The results indicate that the simple framework employed here is capable of replicating the pattern found by Rudebusch quite closely.\footnote{Also Favero (2002) has shown that the predictive regressions based on model projections and Fed Funds rate futures give very similar results. But he examines only the standard specification of the inertial forward-looking Taylor rule.}

Table 10 Predictability of the Federal Funds Rate

<table>
<thead>
<tr>
<th>Standard Specification</th>
<th>$\psi_0$</th>
<th>$\psi_1$</th>
<th>$R^2$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>One quarter ahead</td>
<td>-0.02</td>
<td>0.80</td>
<td>0.37</td>
<td>0.36</td>
</tr>
<tr>
<td>Two quarters ahead</td>
<td>-0.05</td>
<td>0.56</td>
<td>0.14</td>
<td>0.43</td>
</tr>
<tr>
<td>Three quarters ahead</td>
<td>-0.07</td>
<td>0.46</td>
<td>0.08</td>
<td>0.44</td>
</tr>
<tr>
<td>Alternative Specification</td>
<td>-0.02</td>
<td>0.83</td>
<td>0.30</td>
<td>0.38</td>
</tr>
<tr>
<td>One quarter ahead</td>
<td>-0.04</td>
<td>0.67</td>
<td>0.11</td>
<td>0.43</td>
</tr>
<tr>
<td>Two quarters ahead</td>
<td>-0.04</td>
<td>0.67</td>
<td>0.11</td>
<td>0.43</td>
</tr>
<tr>
<td>Three quarters ahead</td>
<td>-0.05</td>
<td>0.63</td>
<td>0.09</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Notes: OLS estimates. T-statistics shown below parameter estimates are based on Newey and West (1987) heteroskedasticity- and serial-correlation-corrected standard errors. $R^2$ and standard errors of the residuals are reported for the first difference of the Federal Funds rate. Sample period for estimation 1990Q1–2004Q2.

Thus we conclude that the issue of predictability of the short-term interest rate may be misleading. A partial adjustment component in empirical Taylor rules does not appear to imply that interest rates are more predictable than the yield curves suggest.

5 Conclusions

In this paper we have attempted to add to the many already-existing explanations for inertia in empirical Taylor rules. Our proposal is that the optimal interest rate rule for stabilising inflation and the output gap will typically inherit the inertia in the economic system itself. If the evolution of the output gap and inflation depends on their own lagged values, then the rule for the control variable, the interest rate, will typically do the same. When estimated empirically, a rule in which the interest rate depends on current and lagged values of the state variables
— the output gap, inflation, and so on — may look rather like one in which the interest rate depends on its own lagged values. The picture is likely to be further confused by omitted autocorrelated variables which engender a serially correlated error term in the estimated equation. We have derived a rule from a simple macroeconomic model. The optimal interest rate rule implied by crude estimates of this model looks something like a modified form of Taylor rule with inertia. When we estimate alternative forms of interest rules directly, our alternative formulation is not wholly inconsistent with the data. While it does not completely supplant the standard Taylor rule, neither does the standard rule explain the data satisfactorily. A hybrid model containing elements of both appears to perform rather better than either alone.

Rudebusch and others have pointed to the inconsistency between the apparent forecastability of interest rates implied by the inertia in estimated Taylor rules, and the lack of forecastability implied by yield curves. The future interest rates implicit in yield curves for Treasury Bills are not good forecasts of future interest rates. However, it turns out that, with the modified form of inertial Taylor rule, allowing for the need to forecast the output gap and inflation that enter the rule, there does not appear to be a significant inconsistency between the implications of the yield curve data and the direct estimates of the Taylor rule.

The results obtained here are suggestive rather than conclusive. This line of enquiry needs to be developed in a number of ways. The macroeconomic model we used contains no forward looking behaviour or other nods in the directions of microeconomic foundations. We need to use a conceptually more coherent model. We need to examine the implied forecastability of interest rates from alternative pieces of data more carefully.
Appendix

Are Interest Rates, the Output Gap, and Inflation Stationary?

Some readers may be curious as to whether the variables we have used a stationary or have unit roots. In some sense, if the US Federal Reserve is pursuing an effective policy to keep inflation low and output close to capacity, all three variables are highly likely to be stationary. In most of the empirical analysis in the paper it is assumed that the variables are stationary. However, in some of the estimated equations the dependent variables have been expressed in first differences, such as the change in the interest rate; and the independent variables have been expressed in changes and in linear combinations of lagged levels, which are stationary even if some of the individual component variables are not, providing the US Federal Reserve is following something like a Taylor Rule in the long run.

For the output gap, for the sample 1960Q4 — 2004Q2, we obtain an augmented Dickey-Fuller (ADF) test statistic of -3.55, with a p-value of 0.0076 for the null hypothesis of a unit root. On this test, a unit root is rejected. For the Federal Funds rate, over a sample 1961Q1 — 2004Q2, the ADF test statistic is -2.41, with a p-value of 0.14. Here a unit root cannot be ruled out. For inflation, over the sample 1961Q3 to 2004Q2, the ADF test statistic is -2.23, with a p-value of .20. Again, a unit root cannot be rejected. The non-rejection of a unit root in inflation and nominal interest rates is not unexpected. Both have been persistent, and there was a marked rise in both until the late seventies and early eighties, since when both have drifted back down to low single figures (at an annual percentage rate). The non-rejection may just reflect the meeting of stationary but persistent series with a test of known low power.

If a unit root in these were accepted, then it would be legitimate to estimate a long run Taylor rule from a regression of the interest rate on inflation and the output gap. Doing that for the sample 1987Q4—2004Q2 yields

\[
i_t = 0.58 + 0.83y_t + 2.14\pi_t
\]

\[
R^2 = 0.75, DW = 0.21,
\]

(T-statistics beneath estimated parameters.) The parameters are not massively dissimilar from the ‘Taylor’ values of 0.5 and 1.5. Instead we have 0.83 amd 2.14, implying a stronger long-run response. The
Figure 5: Residuals in estimated long run Taylor rule

errors from this equation are shown in figure 5. They do not look particularly stationary. The persistent fall from around 1995 to 2004 may reflect the under-measurement of the output gap as the economy grew more strongly than expected without inflation taking off, and the Fed’s allowing interest rates to remain low.
REFERENCES


