Animal Spirits in Consumption: Does Confidence Influence Asset Pricing?

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Abstract

It is common sense in the financial markets and in the business community to believe that consumer confidence affects consumption growth. The economic literature supports this intuitive causation by presenting strong empirical evidence, though the conceptual foundations of the mechanism triggering this influence have so far been overlooked. This paper offers theoretical grounds to the effect of confidence on growth. We let each commodity marginal utility of consumption vary across the different states of nature. These preference shocks modify the relative willingness to pay among the single industries, thus affecting the demand for each commodity in the market. An index of such demand adjustments arises from optimization, capturing the overall tendency in the individual’s attitude towards consumption. Following the mainstream perspective, the index so constructed is interpreted as an indicator of consumer confidence. We apply this setting to an otherwise standard asset pricing framework. A calibration of the resulting model shows that accounting for individual’s attitude towards consumption eliminates the equity premium puzzle. We also run a GMM estimation, extending the robustness of our result to the case in which the joint log-normality of consumption, confidence and equity returns is assumed away.

JEL Classification: E21, E22, G12

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1 Introduction

Individuals decide how much to save and how much to consume by equating the utility loss of consuming a little less today with the utility gain of consuming a little more in the future. In quantitative terms, the exchange rate between future and present consumption is measured by the factor return on savings. It follows that the individual’s intertemporal choice is based on the comparison between the latter and the stochastic discount factor, defined by the ratio of the discounted value of marginal utility of future consumption to that of present consumption. The key importance of the stochastic discount factor in determining individual’s savings has induced many authors to enquire whether such a factor is properly

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We claim that the identity between the consumption index (the argument of each period’s utility) and aggregate consumption in real terms (as derived by data on consumer’s expenditures, deflated by a suitable price index) rather than the definition of the stochastic discount factor may be possibly misleading.

The return on savings is primarily associated to the profitability of the economic activity they are invested on. This is in turn largely affected by how the productive environment evolves during the time elapsing from the investment’s starting date to the ending date. As a result, savings are more or less stimulated according to how the underlying economic activities are expected to perform in the state of nature that obtains. So direct an argument has driven most of the literature to associate uncertainty solely with the production side of the economy, while keeping the demand side fundamentals strictly deterministic. In this consideration lies our claim that the consumption index may be misspecified. Our argument is based on the fact that, as the productive environment changes in response to the state of nature that obtains, so does the commodity bundle that individuals optimally purchase, because of variations either in the relative commodity prices, or in the appeal of consuming each commodity (an ice-cream is more appealing in a sunny day, an umbrella in a rainy day). In our opinion, it is reasonable to think that these state-dependent features should be appropriately accounted for by the individual’s willingness to pay for the different commodities.

In order to illustrate this point, we consider a horizontally differentiated set of commodities. With horizontal differentiation we mean that commodities are distinguished according to the need they satisfy. A natural assumption to make is that there exists only a limited substitutability among the different commodities. Together with a representation of the individual’s preferences such that marginal utility of each commodity is unbounded as consumption approaches zero, this assumption implies that all commodities are actively consumed. In equilibrium, the resulting commodity bundle depend on the relative commodity prices and on a set of preference shocks, introduced to reflect the state-dependent appeal individuals express for each commodity. We aggregate the effects of preference shocks and price changes on individuals’ demand by defining a composite variable, referred to as aggregate preference shock and interpreted as reflecting the individual’s overall attitude towards spending. By using this variable, the identity between the consumption index and aggregate consumption in real terms can be replaced by an expression relating the former to the product between the latter and the aggregate preference shock.

The new specification of the consumption index, therefore, leads the individual’s intertemporal choice to be influenced by the return on savings (referred to as the supply-side effect) and by the change in the individual’s attitude towards spending (referred to as the demand-side effect) rather than exclusively by the former (as in the standard framework).

We test our predictions by nesting the setting outlined above into an otherwise standard asset pricing framework. A possible way to proceed is to measure the change in the relative risk aversion (RRA)
coefficient implied by the resulting model, compared to that required by the standard setting. The object of this analysis immediately refers to the equity premium puzzle, an issue that arises empirically when the representative agent paradigm is used to relate asset prices to investors’ saving decisions. This problem, first described by Mehra and Prescott (1985), originates from observing that the real return on equities have been about six percent higher than that on Treasury bills, over the last one hundred years. The puzzle arises because consumption growth is stable, its correlation with the equity returns is moderate, so the resulting covariance is too low to explain the equity premium, unless the RRA coefficient is extremely high. The household preferences, specified by a standard CRRA utility function, are made consistent with such a large equity premium only if the coefficient of relative risk aversion is at least as large as twenty.² In contrast, empirical works that have undertaken systematic investigations of cross-sectional data on individual’s asset holdings to assess the nature of its utility function, pioneered by Friend and Blume (1975), find that the RRA coefficient is estimated to be just in excess of two.³ The difference between the estimated and the required value of the relative risk aversion gives a measure of the magnitude of the equity premium puzzle.

In the asset pricing theory, the supply-side effect typically entails the following proposition: in equilibrium, expected future high returns induce individuals to raise current saving in order to increase future consumption. The paradigm we propose adds another proposition, reflecting the demand-side effect: in equilibrium, expected future high values of the preference shocks induce individuals to raise current consumption in order to smooth the value of utility at different dates. The individual’s optimal choice is therefore tightly linked to the correlation between the aggregate preference shock and equity returns. If this is positive, then high expected future returns come with positive expected future preference shocks. The consumer is induced to raise current saving because current consumption is expensive in terms of future consumption, and to increase current consumption to counterbalance the current negative (compared to the future) effect of the aggregate preference shock on utility. Accounting for preference shocks may thus offset the effect of equity returns to consumption, so that a lower consumption volatility or a lower RRA coefficient than in the standard framework would be required to match the data, and the empirical performance of the asset pricing model would thus be improved.

In order to derive the quantitative results, our analysis follows two approaches. We first consider a calibration approach, based on the assumption of joint log-normality of the variables involved. The solution delivers a value of the RRA coefficient, as implied by the intertemporal Euler equation, which is then compared to that required by the standard framework. Second, we implement a GMM approach, which allows us to assume away the joint log-normality. This estimation also delivers a value of the RRA coefficient. Such an estimate is testable for significance against that obtained by abstracting from the aggregate preference shocks. A proxy for this latter variable is needed to perform empirical testing. Appealing to the interpretation of the aggregate preference shock as a measure for the individual’s overall attitude towards spending, we indicate the consumer confidence index (CC), and in particular the current situation (CS) component of that indicator, as a plausible variable for the purpose.⁴ The

²The acronym CRRA stands for Constant Relative Risk Aversion. This type of utility function is typically employed in most macroeconomic frameworks to represent the representative agent’s preferences. In more recent contributions that make use of such a paradigm, the magnitude of the RRA coefficient is even higher, in some cases up to 70.

³These authors also show that the assumption of a constant relative risk aversion utility function is a fairly accurate description of household preferences. Regarding the magnitude of the proportional risk aversion, later contributions show that the RRA coefficient may take higher values, up to 7. See e.g. Pratt and Zeckhauser (1987).

⁴The consumer confidence index is a weighted average of two components, labelled current situation and future expec-
consumer confidence index is regarded by the Conference Board as “a monthly report detailing consumer attitudes and buying intentions”. Economists consider this indicator as a measure for individual’s planned spending. The financial markets, the media and the business community refer to it as to the degree of optimism on the state of the economy that individuals are expressing through their activity of spending and saving. It seems thus reasonable to deem consumer confidence as a possible proxy for a variable that summarizes the individual’s overall willingness to pay.

This work is organized as follows. Section 2 presents the theoretical model, illustrating how to derive the new version of the Euler equation that describes the equity premium. Section 3 briefly describes the data used to derive our quantitative results. Section 4 shows the quantitative results, and section 5 concludes. The appendices contain details on all relevant mathematical derivations.

2 The Model

There exists a unit continuum of horizontally differentiated commodities available for purchase, indexed by \( v \). Horizontal differentiation captures the fact that the commodities satisfy different needs, and belong to different industries (e.g. food and clothing). Formally, the commodity space is represented by the set \( V \subset \mathbb{R} : v \in [0, 1] \). The limited substitutability among the different commodities, together with the choice of a formal representation of the individual’s preferences such that marginal utility of each commodity is unbounded as consumption approaches zero, guarantees that all commodities are in equilibrium actively consumed.

We assume that the marginal value of additional consumption varies across the different states of nature, to reflect the higher appeal of consuming a particular commodity in some states of nature rather than in others. We formalize this feature of our setup by pre-multiplying quantitative consumption of each commodity \( v \in V \), denoted by \( x_v \geq 0 \), by a preference shock, denoted by \( \theta_v > 0 \). The resulting consumption index \( c_v = \theta_v x_v \) is then nested into the constant elasticity of substitution (CES) function of Dixit and Stiglitz (1977) type, which represents the aggregate consumption index:

\[
C = \left[ \int_{v \in V} (\theta_v x_v)^\alpha \, dv \right]^{\frac{1}{\alpha}},
\]

where \( \alpha \) is a parameter governing the (limited) consumption elasticity of substitution among the different commodities.

The introduction of preference shocks generates two effects. First, serving as a set of weights, these shocks dictate the distribution of relative appeal across the different commodities, thus affecting that of relative demand. The appeal for an ice-cream is higher in a sunny day than in a rainy one; just the opposite as for an umbrella: the relative demand of these two commodities must therefore adjust.

The consumer confidence index is, however, itself suitable as a proxy for the aggregate preference shocks since, as we show in section 3, its trend is driven by the current situation component.
accordingly. In this sense, each preference shock expresses relative willingness to pay (of one commodity compared to the others). We refer to this feature as to the relative effect of introducing preference shocks. Second, preference shocks positively influence the value of the consumption index. That is, lower amounts of quantitative consumption are required to match a given value of consumption index in the case of large shocks than in that of small ones. Roughly speaking, if utility is represented by a concave function, then higher preference shocks altogether increase utility and lower marginal utility of quantitative consumption: that is, they substitute for latter. In this sense, preference shocks in the aggregate indicate “general well-being”. We refer to this feature as to the absolute effect of introducing the preference shocks.

Individual’s intertemporal utility is represented by an additively separable constant relative risk aversion (CRRA) function, defined over the stream of present and future consumption indices:

\[ U = E_0 \left\{ \sum_{t\in T} \beta^t \left( \frac{C_t^{1-\gamma}}{1-\gamma} \right) \right\}, \tag{2} \]

where: \( t \in T, T = (0, 1, 2, ..., \infty) \) indicates time; \( E_0 \) denotes the expectation operator conditional on the information available at date 0; \( \beta > 0 \) is the individual’s subjective discount factor; \( \gamma > 1 \) measures the curvature of each period’s utility.

The representative agent chooses the quantity \( x_v \) to consume for each commodity \( v \in V \) in order to maximize utility (2), subject to the intertemporal budget constraint:

\[ \int_{v \in V} p_v x_v d\nu = Y^n_t - A^n_{t+1} + (1 + i^n_t) A^n_t - B^n_{t+1} + (1 + i^n_t) B^n_t, \tag{3} \]

where \( Y^n \) is the individual’s endowment; \( A^n \) denotes the representative agent’s holdings of equities, which provide a state-contingent nominal rate of return \( i^n \); \( B^n \) denotes bond holdings, which assure a safe nominal rate of return \( i^b \) (i.e. with no regard of the state of nature that obtains); \( p_v > 0 \) is the exogenously given price for each consumption unit of commodity \( v \). The superscript \( n \) stands as a remainder that all terms in the budget constraint (3) are denominated in a common (immutable) unit.

In the presence of time-additive preferences, for illustrative purposes the individual’s choice can be conveniently split into a static and a dynamic problem. At each date \( t \), the individual chooses the optimal composition of the commodity bundle by maximizing the value of the consumption index (1), taking the resources devoted to consumption at that date as given. Over time, the individual decides how to distribute the available resources intertemporally by maximising utility (2), taking the composition of

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7The effect of rising preference shock is easily measured by computing \( \frac{\partial C}{\partial \theta_v} = (x_v t)^{\alpha - 1} (x_v t) \alpha C^{1-\alpha} > 0 \). By keeping \( C \) constant, and total differentiating (1), after some algebra, we obtain: \( \int_{v \in V} (\theta_v t)^{\alpha - 1} (x_v t) \alpha - (\theta_v t)^{\alpha - 1} (x_v t)^{\alpha} < 0 \). It is easy to notice that, with rising preference shocks, a smaller amount of quantitative consumption, however distributed across the different commodities, is required for the value of the consumption index to hold constant.

8By assuming that the household has power utility, the relative risk aversion (RRA) coefficient is automatically tied to the consumption intertemporal elasticity of substitution. More precisely, the relative risk aversion coefficient is given by the reciprocal of the elasticity of the marginal utility of consumption with respect to consumption, i.e.

\[ RRA = (\varepsilon_{MU_t, C_t})^{-1} = -\frac{\partial MU_t}{\partial C_t} \frac{C_t}{MU_t} = \gamma. \]

The assumption \( \gamma > 1 \) follows from the fact that the value of the RRA coefficient is believed to be in excess of two (see section 1 for references).
the optimal commodity bundle at each date as given.

We first consider the representative agent’s static problem of choosing, by appropriately setting the quantity $x_{v,t}$ of consumption for each commodity $v \in V$ at a generic date $t \in T$, the optimal composition of the commodity bundle $C_t$, taking the resources devoted to consumption at date $t$ as given. Note that, as long as the individual’s preferences are represented by a monotonic function of consumption such as utility (2), at each date $t$ consumption in nominal terms must equal the amount of resources available for purchase, hereafter denoted by $S_t$. The static problem can be formally stated as follows:

$$\max_{\{x_v\}_{v \in V}} \quad C_t = \left[ \int_{v \in V} (\theta_{v,t} x_{v,t})^\alpha \, dv \right]^\frac{1}{\alpha},$$

subject to:

$$\int_{v \in V} p_{v,t} x_{v,t} dv = S_t.$$

From the solution of problem (4), we obtain the preference-shock-adjusted price index:

$$P_{\theta,t} = \left[ \int_{v \in V} \left( \frac{p_{v,t}}{\theta_{v,t}} \right)^{\frac{1-\alpha}{\alpha}} \, dv \right]^{-\frac{1}{1-\alpha}}. \tag{5}$$

Not surprisingly, the consumption price deflator (5), derived from the static equilibrium conditions, depends on the preference shocks that obtain.

The optimal value of the consumption index (1) in terms of economic aggregates is given by:

$$C_t = \frac{S_t}{P_{\theta,t}}. \tag{6}$$

In equilibrium, the value of the aggregate consumption index is influenced by the preference shocks through the price deflator. Since rising values of the preference shocks lower the value of the price index, it follows that such shocks increase the value of consumption. This result represents the absolute effect of introducing preference shocks outlined above.

Finally, the demand for each commodity $v$ is expressed by:

$$x_{v,t} = (\theta_{v,t})^{\frac{1-\alpha}{\alpha}} \left( \frac{p_{v,t}}{P_{\theta,t}} \right)^{-\frac{1}{1-\alpha}} C_t, \quad \forall v \in V. \tag{7}$$

The fraction of the optimal commodity bundle represented by consumption units of each commodity is, for a given value of the consumption price deflator, inversely related to the price of that commodity. Additionally, the magnitude of this fraction is positively related to the commodity-specific preference shock that obtains. For a given $p_{v,t}$, the higher the preference shock $\theta_{v,t}$, the higher the relative demand for commodity $v$. This result represents the relative effect of introducing preference shocks pointed out above.

Consider now the representative agent’s intertemporal choice. We keep focusing entirely on the consumption-side of the economy. Firms are not endogenously considered, and equity returns are regarded as exogenous stochastic processes. We assume that the quantities of the productive assets are fixed. The effect of this is that capital gains turn to price changes, so implicitly to asset returns. The dynamic
problem can be formally stated as follows:

$$\max_{\{A_{\theta,t+1}, B_{\theta,t+1}\}} U = E_0 \left\{ \sum_{t \in \mathbb{T}} \beta^t \left( \frac{C_t}{1 - \gamma} \right)^{1-\gamma} \right\},$$

subject to:

$$C_t = Y_{\theta,t} - A_{\theta,t+1} + R_{\theta,t}^A A_{\theta,t} - B_{\theta,t+1} + R_{\theta,t}^B B_{\theta,t},$$

where

$$Y_{\theta,t} = Y_{\theta,t}^e / P_{\theta,t}, A_{\theta,t} = A_{\theta,t}^e / P_{\theta,t-1}, B_{\theta,t} = B_{\theta,t}^e / P_{\theta,t-1}, R_{\theta,t}^A = \left(1 + \frac{1}{\gamma} \right) / \Pi_{\theta,t}, \text{ with } j = \{a, b\} \text{ and } \Pi_{\theta,t} = P_{\theta,t} / P_{\theta,t-1}. \text{ It should be understood that now all terms in the budget constraint are denominated in real terms (preference-shock-adjusted consumption units at date } t).$$

From the solution of the individual’s dynamic problem, we obtain the optimal distribution of resources over time. The optimal intertemporal consumption path evolves according to the Euler equation:

$$E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{R_{\theta,t+1}^A}{R_{\theta,t+1}^B} \right] = E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right].$$

At a first sight, the Euler equation (9) is standard in all respects: consumption smoothing is affected by the state-dependent premium that equities pay over bonds. As we point out below, the effects due to preference shocks, together with those due to changes in the relative commodity prices, are however implicitly considered in the intertemporal equilibrium through the price index $P_{\theta,t}$.

The Consumer’s Animal Spirit

The price deflator (5) comprises two important effects of uncertainty on the economy. The first, which arises from purely productive shocks, is a standard asset pricing result, explicitly measured in (9) by the factor $R_{\theta,t+1}^A$. A positive supply-side shock occurring in the period from date $t$ to date $t + 1$ lowers the firms’ costs, thereby increasing their profitability (and the returns they pay to their shareholders at date $t + 1$) and/or reducing the prices of the relevant commodities at date $t + 1$. Both situations increase real returns, either raising the nominal rate of return or reducing the price index. In short, $P_{\theta,t}$ implicitly reflects the effects of supply-side shocks on firms’ profitability that are not accounted for by the nominal rates of return they yield. Naturally, the opposite obtains as a result of a negative supply-shock of this type. If the individuals anticipate these occurrences, then savings may be stimulated (or dampened). Notice that there is no implied aftermath on the individual’s demand. Accordingly, we henceforth refer to this mechanism as to the supply-side effect.

The second effect, which is additional to the standard asset pricing predictions, is more subtle and only implicitly measured in (9) by the price deflator $P_{\theta,t}$. Supply-side shocks are typically not uniform across the different industries, and the changes in prices in response to these shocks are hardly even. The resulting adjustments in the relative commodity prices are likely to cause variations in the optimal commodity bundle composition. Such variations add to those generated by the commodity-specific preference shocks (which we have illustrated when discussing the relative effect of introducing preference shocks). The resulting composition of the commodity bundle may be more satisfactory in some states of nature rather than in others, thereby influencing the value of the optimal consumption index (as we have shown

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9The asset holdings $A_{\theta,t}$ and $B_{\theta,t}$ refer to date $t - 1$, and are accordingly denominated in terms of consumption at time $t - 1$. Consistency of the budget constraint is guaranteed by the fact that the terms $R_{\theta,t}^A A_{\theta,t}$ and $R_{\theta,t}^B B_{\theta,t}$, given the presence of $\Pi_{\theta,t}$, are denominated in consumption units at date $t$. 

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when discussing the absolute effect of introducing preference shocks). If these occurrence are anticipated, then savings may once again be stimulated (or dampened). Since no consequence on the productive-side is here accounted for, we henceforth refer to this mechanism as to the demand-side effect.

Our aim is to disentangle the two effects outlined above, in order to study the economic repercussions of the endogenous consideration of the commodity bundle composition in the individual’s intertemporal choice. A simple way to achieve this goal is to define a preference-shock-independent price index, as obtained by assuming \( \theta_{v,t} = 1, \forall v \in V \) in (5):

\[
P_t = \left[ \int_{v \in V} (p_{v,t})^{-\frac{\alpha}{1-\alpha}} dv \right]^{-\frac{1-\alpha}{\alpha}}. \tag{10}
\]

The price deflator (10) reflects the net effect of the adjustments in the commodity prices in response to the supply-side shocks only. Together with the changes in the nominal return on equities, it fully accounts for the standard asset pricing result, excluding the consequences that uncertainty has on the demand-side of the economy. By multiplying and dividing (5) for (10), the preference-shock-adjusted price deflator can be rewritten as:

\[
P_{\theta,t} = \frac{P_t}{\Theta_t}, \tag{11}
\]

where:

\[
\Theta_t = \left[ \frac{\int_{v \in V} (\theta_{v,t})^{-\frac{\alpha}{1-\alpha}} (p_{v,t})^{-\frac{\alpha}{1-\alpha}} dv}{\int_{v \in V} (p_{v,t})^{-\frac{\alpha}{1-\alpha}} dv} \right]^{\frac{1-\alpha}{\alpha}}. \tag{12}
\]

captures the aggregate effect of the adjustments in the optimal consumption bundle composition due to the preference shocks and the variations in the relative commodity prices. The information contained in the price index \( P_{\theta,t} \) is therefore split in two parts. On the one hand, the price deflator \( P_t \) accounts for the supply-side effect (more precisely, for that part if this effect that is not already accounted for by the nominal return on equities). On the other, the variable \( \Theta_t \), which we hereafter refer to as the aggregate preference shock, reproduces the demand-side effect.

By considering the relative effect of introducing preference shocks, we can interpret equation (12) as a weighted average of the state-dependent relative willingness to pay that individuals express for each commodity \( v \). The commodity prices being the weights, \( \Theta_t \) accounts for the effects on the optimal composition of the commodity bundle of both the demand-side shocks (through preference shocks) and the supply-side shocks (through the state-dependent relative commodity prices). In addition, the absolute effect of introducing preference shocks tells us that preference shocks altogether positively affect the value of the consumption index, lower marginal utility of consumption in aggregate terms, thereby having a negative influence on the demand for aggregate quantitative consumption. These considerations suggest to interpret equation (12) as a measure of the resulting individual’s attitude towards spending.

\[\text{---}^{10}\text{The price deflator defined here disregards the demand-side shocks, yet it fully considers the supply-side shocks. The importance of this remark is best understood by contemplating the difference between this price deflator and that obtained by considering a deterministic state of nature in which } \theta_v = 1, \forall v \in V \text{ and the supply-side shocks are also absent. Denoting by } \{p_{v}^{\text{det}} \}, \forall v \in V \text{ the price set that obtains in this last case, the resulting price deflator is:}
\]

\[
P_{\text{det},t} = \left[ \int_{v \in V} (p_{v,t}^{\text{det}})^{-\frac{\alpha}{1-\alpha}} dv \right]^{-\frac{1-\alpha}{\alpha}} \neq P_t.
\]


The solution of the individual’s problem is obviously unaffected by the decomposition of $P_t$. The aggregate consumption index (6) can be rewritten as:

$$C_t = \frac{P_t}{P_{t,t}} S_t = \Theta_t X_t$$  \hspace{1cm} (13)$$

where $X_t = S_t/P_t$ is an alternative measure of aggregate consumption in real terms, as obtained by deflating nominal consumption using the preference-shock-independent price deflator (10). The demand for each commodity $v$, expressed by (7), can also be restated in terms of this measure:

$$x_{v,t} = \left(\frac{\theta_{v,t}}{\Theta_t}\right) \frac{p_{v,t}}{P_t} X_t$$  \hspace{1cm} (14)$$

Using (13), and rewriting the asset return factors in terms of preference-shock-independent aggregate consumption $X_t$, the intertemporal problem (8) can be restated as:

$$\max_{\{A_{t+1}, B_{t+1}\}_{t \in \mathbb{T}}} U = E_0 \left\{ \sum_{t \in \mathbb{T}} \beta^t \left( \frac{\Theta_t X_t}{1 - \gamma} \right) \right\}$$

subject to:

$$X_t = Y_t - A_{t+1} + R^a_t A_t - B_{t+1} + R^b_t B_t$$

where $Y_t = P_t Y_{t,t}/P_t$, $A_t = P_{t,t-1} A_{t-1}/P_t$, $B_t = P_{t,t-1} B_{t-1}/P_t$, $R^j_t = \Pi_{t,t-1} R^j_{t-1}/\Pi_t$, with $j = \{a, b\}$ and $\Pi_t = P_t/P_{t-1}$. From the solution of this problem, we obtain the same intertemporal Euler equation as that we would get by replacing (13) and the definitions of the return factors in terms of the price index (10) just stated into (9):

$$E_t \left[ \left( \frac{\Theta_{t+1}}{\Theta_t} \right)^{1-\gamma} \left( \frac{X_{t+1}}{X_t} \right)^{-\gamma} R^a_{t+1} \right] = E_t \left[ \left( \frac{\Theta_{t+1}}{\Theta_t} \right)^{1-\gamma} \left( \frac{X_{t+1}}{X_t} \right)^{-\gamma} \right]$$  \hspace{1cm} (15)$$

The interpretation of the solution equation does, however, change. In particular, the introduction of the preference-shock independent price deflator (10) and the aggregate preference shock (12) implies a new specification of the consumption index (13), which leads the individual’s intertemporal choice to be explicitly influenced by the return on savings (referred to as the supply-side effect) and by the change in the individual’s attitude towards spending (referred to as the demand-side effect) rather than exclusively by the former (as in the standard framework).

For given preference shocks, the supply-side effect entails the following process. Expected future high returns $R^a_{t+1}$ induce individuals to give up some units of current consumption $X_t$, in order to increase the demand for asset. As the number of equities holds fixed, the current asset price must increase, in turn lowering the return on equities. As a result, expected future high implies rising current saving by decreasing current consumption.\hspace{1cm}$^{11}$ On the other hand, for given equity returns, the demand-side effect entails the following process. Expected future high values of the preference shocks increases the expected variation in the aggregate preference shocks. For the value of the terms in square brackets to remain constant, the growth rate of consumption must decrease. As a result, individuals are induced to raise

\hspace{1cm}$^{11}$Alternatively, individuals may disinvest in bonds and invest the resulting resources on equities. In this case, the higher consumption growth would be due to a rise in future consumption. Obviously, a mix of these two arguments may rather apply.
current consumption (relative to future consumption).

We test our prediction by adopting two different approaches. Following Mehra and Prescott (1985), we calibrate the model. We log-linearise the Euler equation (15), derive the relative risk aversion coefficient implied by the U.S. data in the last three decades (described in the next section), and compare its value with that obtained by widely accepted estimation in the literature. In order to improve the robustness of our findings, following Favero (2001), we also implement a GMM estimation. The results obtained by applying this method, which allows to assume away the log-normal joint distribution of consumption, preference shocks and equity returns, are presented in section 4, along with those resulting from the calibration of the model.

3 Description of the data

This section briefly illustrates the data used to derive our quantitative results. We begin by specifying the candidate variable to serve as proxy for the aggregate preference shock $\Theta$. We believe that the US Consumer Confidence Index ($CC$), based on a survey of consumers’ opinions on the state of the economy and provided on a monthly basis from June 1977 onwards by the Conference Board, is a plausible measure of the targeted variable. As we have discussed in section 2, it is sensible to think of the aggregate preference shock as capturing the overall tendency of the individual’s attitude towards spending. Since the consumer confidence index is often defined as a measure for individual’s planned spending and as the degree of optimism on the state of the economy that individuals are expressing through their activity of spending and saving, it seems reasonable to deem this indicator as a possible proxy for the aggregate preference shock.

The Conference Board bases the index on five questions surveying consumer attitudes and expectations. Each question is given equal weight in computing the overall index. Two of the five questions ask consumers to assess present economic conditions and, jointly considered, form the present situation component ($CS$) of the consumer confidence index. The remaining three, asking opinions about the future state of the economy, make up for the expectations component ($CF$) of the indicator.

The economic literature typically interprets consumer confidence as an indicator of changes in income or consumption. There exists substantial empirical evidence that the confidence indicator is able to predict consumption growth, though robustness is weaker after controlling for standard macroeconomic variables. In particular, economists focus on the predictive power of the expectations index, disregarding empirical testing based on the present situation component, on the grounds that rational individuals are obviously aware of changes in their own economic situation. We reconsider the role of the latter component by arguing that the “snapshot” approach taken by the Conference Board in asking consumer’s evaluation

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12 For references, see section 1.
13 The indicator first appeared in January 1969, and was released every two months. The Conference Board has then expanded it to a monthly series in 1977.
14 The survey uses data on 5000 households. The five questions are: 1) how would you rate present general business conditions in your area; 2) what would you say about available jobs in your area right now; 3) six months from now, do you think business conditions in your area will be [better/same/worse]; 4) six months from now, do you think there will be [more/same/fewer] jobs available in your area; 5) how would you guess your total family income to be six months from now. The first two questions make up for the present situation component. The trend of the latter ($CS$), along with that of the overall index ($CC$), are plotted in figure 1.
15 For empirical analysis on this issue, see e.g. Acemoglu and Scott (1994), Bram and Ludvigson (1998) and Ludvigson (2004).
about current conditions perfectly matches the contemporaneous nature of preference shock we want to address.\textsuperscript{16}

In the light of the peculiarly rigorous construction of the consumer confidence index, it is possible to give the empirical results just mentioned an economic interpretation by direct comparison of the two components constituting the overall indicator, which is in line with the predictions of our model.\textsuperscript{17} As we have shown in the previous section when discussing the Euler equation (15), in a consumption-smoothing perspective individuals save more (less) when expected future states of nature ($\Theta_{t+1}$) are relatively worse (better) than the realised current state ($\Theta_t$), since such a choice would balance out utility at different dates. If the consumer confidence index is a good proxy for the aggregate preference shock, then the value of the overall indicator should be high when the present situation index ($CS$) overtakes its forward-looking counterpart ($CF$), and vice versa. By examining the time series data on consumer confidence, we notice that this is precisely the case.\textsuperscript{18} According to these observations, our proposition finds empirical support: individuals save more (less) when the present situation index is greater (smaller) than the expectations indicator. As a result, positive (negative) variations in the consumer confidence index should have a negative (positive) impact on consumption growth.

The remaining of the dataset is as follows. Since the consumer confidence index is a monthly time series, the Mehra-Prescott dataset is unsuitable for the purposes of our research. The dataset is thus reconstructed accordingly. Although it covers shorter time intervals than Mehra-Prescott’s, the number of observations actually increases.\textsuperscript{19} The equity returns are derived by the average monthly return on the Standard & Poor’s 500 Composite Index. As a series for the bond returns, we use monthly data of annual based nominal yield on three-month government Treasury Bills. In order to report data in monthly terms, we divide each observation by twelve. Then, it is converted to real terms by using the Chain-type Price Index, provided by the Bureau of Economic Analysis of the U.S. Department of Commerce. We employ this index because it is provided by the same institution as the consumption data. These in turn correspond to the sum of two series on Real Personal Consumption Expenditures: expenditures on services and expenditures on nondurables.

\textsuperscript{16}From figure 1, it should be clear that the present situation component ($CS$) and the consumer confidence ($CC$) comove, and that the choice of using the current conditions indicator or the overall index as a proxy for average quality is virtually indifferent for our purposes, as the dynamics of the latter are driven by the former. In the next section, we shall, however, test both hypotheses.\textsuperscript{17}By rigorous construction we mean the relatively homogenous “timing” (either present or six months time) used in formulating the consumer confidence index questions with respect to those of other indicators, chiefly the widespread-in-academic-research University of Michigan’s Consumer Sentiment Index. The latter comprises the following five questions: 1) do you think now is a good or bad time for people to buy major household items; 2) would you say that you (and your family living there) are better off or worse off financially than you were a year ago; 3) now turning to business conditions in the country as a whole—do you think that during the next twelve months, we’ll have good times financially or bad times or what; 4) looking ahead, which would you say is more likely—that in the country as a whole we’ll have continuous good times during the next five years or so or that we’ll have periods of widespread unemployment or depression, or what; 5) Now looking ahead—do you think that a year from now, you (and your family living there) will be better off financially, or worse off, or just about the same as now. The first two questions compose the present situation index, unsuitable as a proxy for the aggregate preference shock (the second question asks opinions about changes in consumers’ financial situation when compared to one year earlier), and incompatible with the expectations indicator (only questions 2 and 5 are directly comparable).\textsuperscript{18}This is easily seen in figure 1, once it is understood that the overall index is just a weighted average of the two components. Algebraically, $CC = aCS + (1 - a)CF$, where $a = 40\%$ is the weight attached to $CS$.\textsuperscript{19}The Consumer Confidence Index, providing monthly data for the period 1977-2003, assures 319 observations. The Mehra-Prescott dataset accounts for just 90. Allowing for time aggregation, the latter may still be used, but the number of observations (35 if we also consider the two-month release period) would be insufficient for obtaining robust results.
absolute value eight points higher than that of the Mehra-Prescott dataset (-0.14). This is due to switching from yearly to monthly data. As Boldrin, Christiano, and Fisher (1997) show, the absolute value of the autocorrelation of consumption growth is critical for determining the nature of the equity premium when power utility is used to describe the household preferences. They show that a lower autocorrelation implies that equity and risk-free assets perform more similarly as a hedge against risk. Monthly data on consumption growth show a relatively larger absolute value of autocorrelation compared to yearly data, which in turn exhibit lower autocorrelation because of time aggregation. We should thus expect a higher value of the relative risk aversion coefficient to match the equity premium. As we show below, our estimate for the RRA coefficient, computed in the same fashion as in the standard framework, is substantially larger than that found by Mehra and Prescott (1985).

4 Quantitative Results

This section evaluate empirically the predictions of the Euler equation (15) by calibrating the model on the observable U.S. data discussed in section 3. We simplify that equation by finding an exact log-linear expression for the terms in brackets.\(^{20}\) We obtain:

\[
E_t r^a_{t+1} - r^b_{t+1} = \sigma_r [\gamma \rho_{rx} \sigma_x - (1 - \gamma) \rho_{r\#} \sigma_\#].
\]  

(16)

where \(r^a_{t+1} \equiv \ln (R^a_{t+1})\) and \(r^b_{t+1} \equiv \ln (R^b_{t+1})\) represent the rates of return on equities and bonds, respectively; \(x_{t+1} = \ln (X_{t+1}/X_t)\) and \(\#_{t+1} = \ln (\Theta_{t+1}/\Theta_t)\) denote the growth rates of consumption and aggregate preference shock, respectively; \(\gamma\) gives a measure of the RRA coefficient; \(\sigma_i\) stands for the standard deviation of the variable \(i = \{r, x, \#\}; \rho_{ij}\) is the correlation coefficient between the variables \(i, j = \{r, x, \#\}, \text{ with } i \neq j\).

It is easy to make a comparison between this expression and the equation calibrated by Mehra and Prescott (1985), given by:

\[
E_t r^a_{t+1} - r^b_{t+1} = \gamma \rho_{rx} \sigma_r \sigma_x.
\]  

(17)

The second term in brackets on the right-hand side of equation (16) does not appear in equation (17). Note that by definition \(\sigma_\# > 0\), and we expect \(\gamma > 1\). The effect of this additional term thus depends on the correlation between the return on equity and the variation in the aggregate preference shock. The required magnitude of the RRA coefficient is expected to rise if \(\rho_{r\#} < 0\), and to decrease otherwise. Using the two proxies outlined in the previous section, we obtain \(\rho_{r\#} = 0.28\) when using data on the consumer confidence index (CC) and \(\rho_{r\#} = 0.13\) when using data on the present situation component of that indicator (CS). We can therefore conclude that the calibration of (16) will produce a lower value for the RRA coefficient than that resulting from the calibration of (17).

Table 1 reports the calibration of the RRA coefficient. Column (ST) reports the result obtained by calibrating equation (17). Columns (CC) and (CS) report those obtained by calibrating equation (16) using data on CC and CS, respectively. The dataset illustrated in section 3 yields the following parameterisation: the standard deviations of equity returns and consumption growth are \(\sigma_r = 0.035\) and \(\sigma_x = 0.003\) respectively; the correlation between these two variables is \(\rho_{rx} = 0.26\). The equity premium equals 0.006. It follows that the calibration of the standard model – summarised by (17) – on a monthly

\(^{20}\)For the complete derivation of this result, we refer to the Appendix B.
dataset implies a huge magnitude for the \textit{RRA} coefficient, that is $\gamma = 220$, compared to that obtained on annual data, i.e. $\gamma = 26$.\textsuperscript{21} Although a result of this kind may appear quantitatively impressive, it comes qualitatively as no surprise. As we have discussed at the end of the previous section, a rise in the value of this coefficient is in fact to be expected.

We can complete the above parameterisation by computing the standard deviation of the aggregate preference shock variations, given by $\sigma_\theta = 0.071$ when using data on \textit{CC} and $\sigma_\theta = 0.082$ when using data on \textit{CS}. Together with the values of the correlation coefficient given above, this set of values allows us to calibrate the preference-shock-augmented version of the model – summarised by (16). If we consider the consumer confidence index as the proxy for the aggregate preference shocks, then the calibrated value for the \textit{RRA} coefficient is $\gamma = 9$. If we consider the present situation component of that indicator, that value rises to $\gamma = 16$. The introduction of commodity-specific preference shocks therefore reduces the required magnitude of the \textit{RRA} coefficient by a factor of 15.

The striking result following from the calibration of our model is all the more so if we consider that it is obtained by using monthly data.\textsuperscript{22} Given that the required magnitude of the \textit{RRA} coefficient falls from 220 to 26 when we switch from monthly to annual data, it is easy to conjecture that something similar would occur if we calibrated the preference-shock-augmented model using yearly data. Our results would thus be even closer to a value for the \textit{RRA} coefficient in the range $\gamma \in [2, 4]$, where the economic literature estimates the true value of that coefficient actually lies.\textsuperscript{23}

Finally, it should be noted that the value of the \textit{RRA} coefficient obtained by calibrating (16) using data on the consumer confidence index is smaller than that obtained by using data on the present situation component of that indicator – and therefore closer to that considered in the literature as the benchmark value for $\gamma$. This is due to the fact that the variation in \textit{CC} correlates with the equity returns more than twice as much as that in \textit{CS}, whereas the standard deviation of the two measures is about of the same magnitude.

As two robustness exercises, we implement as many estimations based on a non-linear instrumental variables (GMM) estimator. Under the joint hypothesis of representative agent intertemporal optimisation and rational expectations (IOREH), the only significant variables in predicting consumption at date $t+1$ given the information at date $t$ are consumption and consumer confidence at date $t$. The conditional expectation for date $t+1$ taken at date $t$ of the term in brackets is in fact zero. Moreover, such an expression is orthogonal to any variable other than consumption and consumer confidence included in the agent’s information set at date $t$. Notice that the Euler equation does not have any implication for the contemporaneous relation between consumption and other economic variables. Denoting the terms in square brackets in (15) generically as $f(y_{t+1}, \theta)$, we have

$$E_t [f (y_{t+1}, \theta)] = 0, \quad E_t [f (y_{t+1}, \theta) z_t] = 0$$

where $y_{t+1}$ is the vector of observed variables of interest at date $t + 1$, $\theta$ is the vector of parameters to be estimated, and $z_t$ is a vector containing any economic variable observable at date $t$.

Euler equations from intertemporal optimization and rational expectations usually delivers a poten-

\textsuperscript{21}For calibration on a yearly dataset, see Mankiw and Zeldes (1991).
\textsuperscript{22}As discussed above, the calibration based on annual data is neglected on account of lack of result’s robustness.
\textsuperscript{23}For references, see section 1.
tially infinite number of valid instruments. In our application, any lagged variable is a valid instrument under the null that the IOREH model is a data generating process. The parameters can be therefore estimated by using orthogonality conditions based on the following set of instruments:

\[ \text{constant, } \frac{\Theta_{t+1-i}}{\Theta_{t-i}}, \frac{X_{t+1-i}}{X_{t-i}}, \frac{R_{t+1-i}}{R_{t+1-i}} \]

where we have chosen the number of lags such that \( i = \{1, 2, ..., 6\} \).

The quantitative results of the GMM estimations are based on the monthly dataset described in section 3. The two estimates obviously differ in the choice of the proxy used for the aggregate preference shock, that is the consumer confidence index and the present situation component of that indicator. Estimation of the Euler equation (15) is implemented by using the appropriate routine in the E-Views software, using the Bartlett weights and the Newey-West criterion to choose the lag truncation parameter.

The results are reported in Table 2. Column (ST) reports the result obtained by estimating equation (15) when considering \( \Theta_{t+1}/\Theta_{t} = 1, \forall t \in T \). Columns (CC) and (CS) report those obtained by calibrating the same equation using data on CC and CS, respectively. Standard errors are shown in brackets.\(^2\) Although our estimates are qualitatively analogous to the results obtained by calibrating the model, two points are worth noting. First, while the magnitude of \( \gamma \) obtained by using the standard model is virtually unchanged, those obtained by estimating the preference-shock-augmented model are smaller than the ones resulting from calibration. In fact, they lie in the range of values indicated by the literature as the true value for the RRA coefficient. Second, the value obtained by estimating the model using data on the consumer confidence index is curiously higher than that obtained by using data on the present situation component of that indicator (4.1 vs 2.8), exactly the opposite of what our calibration predicts.

In conclusion, we present the results of the Wald tests on coefficient restrictions, conducted in order to assess whether our estimates differ significantly from the “true” value of the RRA coefficient. Those results are reported in Table 3 for the value \( \gamma = 2 \). Column (ST) reports the test applied on the estimation of the standard model. Columns (CC) and (CS) report those obtained by estimating the preference-shock-adjusted model using data on CC and CS, respectively. The probabilities associated to the statistics are shown in brackets. As expected, the test conducted on the coefficient obtained by estimating the standard model leads to reject the null hypothesis (that is, that the estimation equals the true value of the RRA coefficient). The tests conducted on the estimates based on the preference-shocks-adjusted model do not allow for such rejection, suggesting that those estimates do approach the true value of \( \gamma \).

5 Concluding remarks

This paper proposes a preference-shock-augmented specification of individual’s preferences, in order to allow for uncertainty on the demand-side of the economy. The price deflator obtained by solving the individual’s static problem is decomposed in a preference-shock-independent price index and an aggregate measure for the preference shocks. By applying this setting to an otherwise standard asset pricing model,

\(^2\)The number of asterisks indicate the significance of the t-test: one asterisk means 10% significance; two asterisks mean 5%; three asterisks mean 1%.
we derive an intertemporal Euler equation that crucially depends on the variations in the aggregate preference shock. In particular, the model predicts that consumption growth is inversely related to such variations. This prediction has been tested empirically by calibrating the relative risk aversion (RRA) coefficient to assess whether our setting reduces the empirical issue known as the equity premium puzzle. Using the consumer confidence index (and the present situation component of that indicator) as a proxy for the aggregate preference shock, we find that the model virtually eliminates the puzzle. These results hold even when the RRA coefficient is estimated by using a GMM estimator.

We think that the most attractive feature of this paper is that the peculiar representation of commodity-specific preference shocks allows for an intuitive approximation of the theoretical aggregate preference shock to observed indicators of confidence. There exists substantial evidence that consumption growth and individual’s confidence are positively correlated. Observing the data on the consumer confidence index suggests that there is evidence of lower (higher) rates of variation in this indicator when its value is relatively larger (smaller), and vice versa. Therefore, consumption growth rates are higher (lower) when the confidence variations are smaller (larger). This fact is in line with the predictions of our intertemporal Euler equation, once the value of the aggregate preference shock is approximated by that of the consumer confidence index. In short, our model provides a sensible theoretical explanation to the empirical evidence relating individual’s confidence to consumption growth.

The preference-shock-augmented setting can be easily exploited to address other asset pricing issues, such as the evaluation of options and other derivatives, or investment. Some of these issues are already the object of ongoing research. Another field which our framework straightforwardly relates to, notably for the fact that the aggregate preference shock is drawn from the decomposition of the price deflator, is monetary economics. It is arguably sensible to conjecture that individual’s confidence, once again used as the observed approximation of the aggregate preference shocks, may alter the transmission mechanism of monetary policy, as predicted by using a standard sticky-price model.

The flexibility of the framework presented here allows it to be used in virtually every study involving the derivation of a Euler equation, although only short-term models should be considered. In the short run, in fact, it is reasonable to consider that a single state of nature characterizes each date. Longer time periods, as aggregations of short-terms, usually comprise several realised state. The successive states of nature that obtain in the latter reference period, by generating different sets of preference shocks that typically end up offsetting one another, dampen the effects of these shocks on the individual’s demand, and therefore on the equilibrium conditions. If the number of consecutive states that obtain is large enough, then such effects eventually die away, making long-term preference-shock-augmented studies economically insignificant.
Appendices

A Solution of the Representative Agent’s Problem

We write the Lagrangian function:

\[ L_t = \max_{\{x_v,t\}_{v \in V}} \left[ \int_{v \in V} (\theta_{v,t} x_v,t)^\alpha dv \right]^\frac{1}{\alpha} + \lambda_t \left( S_t - \int_{v \in V} p_{v,t} x_v,t dv \right) \]

where \( \lambda_t \) is the Lagrange multiplier associated to \( L_t \). The first-order condition for the solution consists of the set of simultaneous equations:

\[
\frac{\partial L_t}{\partial \lambda_t} = S_t - \int_{v \in V} p_{v,t} x_v,t dv = 0, \quad (18)
\]

\[
\frac{\partial L_t}{\partial x_v,t} = \left[ \int_{z \in V} (\theta_{z,t} x_{z,t})^\alpha dz \right] \frac{1}{\alpha - 1} (\theta_{v,t})^{\alpha - 1} - \lambda_t p_v,t = 0, \quad \forall v \in V \quad (19)
\]

Consider the first-order condition (19) for a generic commodity \( v \). By raising both sides to the power \(-\alpha (1 - \alpha)^{-1}\), integrating across varieties, and rearranging, we obtain:

\[
\left[ \int_{z \in V} (\theta_{z,t} x_{z,t})^\alpha dz \right]^{-1} \int_{v \in V} (\theta_{v,t} x_{v,t})^\alpha dv = \lambda_t^{1 - \frac{1}{\alpha}} \int_{v \in V} (\theta_{v,t})^{-\frac{\alpha}{\alpha - 1}} \frac{p_{v,t}}{\theta_{v,t}} dv \]

After some basic algebra, the Lagrange multiplier \( \lambda_t \) can be defined as the reciprocal of the preference-shock-adjusted price deflator (5).

By multiplying both sides of the first-order condition (19) for \( x_v \), integrating across varieties, and rearranging, we also have

\[
\left[ \int_{v \in V} (\theta_{v,t} x_{v,t})^\alpha dv \right]^{\frac{1}{\alpha}} = \lambda_t \int_{v \in V} p_{v,t} x_v,t dv
\]

Using (1), (7), and (18), we obtain (6).

By raising both sides of the first order condition (19) to the power \((1 - \alpha)^{-1}\) and rearranging, we finally get:

\[ x_{v,t} = (\theta_{v,t})^{\frac{1}{1 - \alpha}} (\lambda_t p_{v,t})^{-\frac{1}{1 - \alpha}} \left[ \int_{z \in V} (\theta_{z,t} x_{z,t})^\alpha dz \right]^{\frac{1}{\alpha}} \]

Using (1), and (5), after some basic algebra we obtain (7).

The intertemporal optimization problem (8) is solved by substituting the intertemporal budget constraint for consumption into utility. The first order conditions for optimality can be stated as

\[
\frac{\partial U}{\partial A_{\theta,t+1}} = E_0 \left[ -\beta^t (C_t)^{-\gamma} + \beta^{t+1} (C_{t+1})^{-\gamma} (1 + R_{\theta,t+1}^0) \right] = 0, \quad t \in \mathbb{T} \quad (20)
\]

\[
\frac{\partial U}{\partial B_{\theta,t+1}} = E_0 \left[ -\beta^t (C_t)^{-\gamma} + \beta^{t+1} (C_{t+1})^{-\gamma} (1 + R_{\theta,t+1}^0) \right] = 0, \quad t \in \mathbb{T} \quad (21)
\]

Using the law of iterated expectations, we can rearrange the first order conditions (20) and (21) for date
\( t \) as follows:

\[
\beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 + R_{0,t+1}^a) \right] = 1
\]

\[
\beta (1 + R_{0,t+1}^b) E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] = 1
\]

Equating the left hand sides of the above equations, we obtain the Euler equation (9).

**B Exact Log-linear Euler Equation**

Define \( r_{t+1}^j = \ln \left( R_{t+1}^j \right), \quad j \in \{a, b\}, \) \( x_{t+1} = \ln (X_{t+1}/X_t) \) and \( \theta_{t+1} = \ln (\Theta_{t+1}/\Theta_t) \). Assume that the vector of the log of the stochastic variables in (15) has a joint multinormal distribution:

\[
z = \begin{bmatrix} r \\ x \\ \theta \end{bmatrix} \sim N \left( \mu = \begin{bmatrix} \mu_r \\ \mu_x \\ \mu_\theta \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_r^2 & \sigma_{rx} & \sigma_{r\theta} \\ \sigma_{rx} & \sigma_x^2 & \sigma_{x\theta} \\ \sigma_{r\theta} & \sigma_{x\theta} & \sigma_\theta^2 \end{bmatrix} \right)
\]

where \( \mu_j \) and \( \sigma_j^2 \) are respectively the mean and the variance of variable \( j = \{r, x, \theta\} \), and \( \sigma_{jj'} \) measures the covariance between the variables \( j \) and \( j' \neq j \). Defining the vectors of the exponents in (15), suitably ordered, as \( \tau_a' = \begin{bmatrix} 1 & -\gamma & 1 - \gamma \end{bmatrix} \) and \( \tau_b' = \begin{bmatrix} 0 & -\gamma & 1 - \gamma \end{bmatrix} \), the Euler equation (15) becomes:

\[
E_t [\exp(\tau_a' z_{t+1})] = \exp(r_b^t) E_t [\exp(\tau_b' z_{t+1})]
\]

Recalling that the moment generating function for the Gaussian distribution is given by \( M(\tau) = E_t [\exp(\tau' z)] = \exp(\mu' \tau + \frac{1}{2} \tau' \Sigma \tau) \), and that the relevant moments are given by \( \mu' = \sum_j \tau_j \mu_j \) and \( \tau' \Sigma \tau = \sum_j \tau_j^2 \sigma_j^2 + 2 \sum_{j' \neq j} \tau_j \tau_j' \sigma_{jj'} \), after some algebra we obtain:

\[
\exp(r_b^t) = \exp(\mu_r + \sigma_r^2/2) \exp[-\gamma \sigma_{rx} + (1 - \gamma) \sigma_{r\theta}]
\]

Considering that \( E_t [\exp(\tau_a' z_{t+1})] = \exp(\mu_r + \sigma_r^2/2) \), taking logarithms of both sides, using the definition of correlation coefficient, i.e. \( \rho_{jj'} = \sigma_{jj'}/(\sigma_j \sigma_{j'})^{1/2}, \quad j = \{r, x, \theta\} \) and \( j' \neq j \); and rearranging, we get (16).
References


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**Figures**

![Figure 1 - Consumer Confidence Index and Present Situation Component](image-url)
### Table 1 – Calibration of the Euler Equation

<table>
<thead>
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<th></th>
<th>(ST)</th>
<th>(CC)</th>
<th>(CS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ</td>
<td>219.78</td>
<td>9.2599</td>
<td>15.917</td>
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</tbody>
</table>

*Note:* Column (ST) reports the result obtained by calibrating equation (17). Columns (CC) and (CS) report those obtained by calibrating equation (16) using data on the consumer confidence index and on the current situation component of that indicator, respectively.

*Source:* The monthly dataset described in section 3, and our computations.

### Table 2 – Estimation of the Euler Equation

<table>
<thead>
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<th></th>
<th>(ST)</th>
<th>(CC)</th>
<th>(CS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ</td>
<td>220.6*** (50.22565)</td>
<td>4.0966** (1.763831)</td>
<td>2.78122* (1.568047)</td>
</tr>
</tbody>
</table>

*Note:* Column (ST) reports the result obtained by estimating equation (15) when considering no variation in the aggregate preference shock. Columns (CC) and (CS) report those obtained by estimating the same equation using data on CC and CS, respectively. Standard errors are shown in brackets. Estimates are obtained by using a non-linear instrumental variables (GMM) estimator. Instruments used include six lags of equity returns, consumption growth and, where applicable, variations in the aggregate preference shock.

*Source:* The monthly dataset described in section 3, and our computations.

### Table 3 – Wald Test on the Coefficient Restriction: θ = 2

<table>
<thead>
<tr>
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<th>(ST)</th>
<th>(CC)</th>
<th>(CS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-stat.</td>
<td>18.94415 (0.0000)</td>
<td>1.412637 (0.2355)</td>
<td>0.248211 (0.6187)</td>
</tr>
</tbody>
</table>

*Note:* Column (ST) reports the result of the test applied on the estimation of the standard model. Columns (CC) and (CS) report those obtained by estimating the preference-shock-adjusted model using data on CC and CS, respectively. Probabilities associated to the statistics are shown in brackets.

*Source:* The monthly dataset described in section 3, and our computations.