Too Unexpected to Fail: Bail-Out Policy and Sudden Freezes

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Abstract

I present a mechanism that relies on the interaction of coordination and ambiguity (Knightian uncertainty) and makes precise how a loss of confidence can arise in loan markets, leading to a systemic liquidity crisis. The paper studies a simple global-game coordination model among lenders to a financial intermediary and shows how a market haircut arises in equilibrium. I show how the haircut responds to a variety of parameters. In particular, I show that coordination is non-robust to ambiguity in investor signals and becomes fragile in an environment with ambiguity. This leads to the haircut jumping up suddenly, possibly to 100% when enough lenders are ambiguity-sensitive. Further, I show that the fragility of coordination implies that in such an environment, policy itself becomes a systemic trigger. If the regulator fails to rescue an institution that the market expects to be saved, which in turn changes market expectation about policy for other institutions even slightly, an immediate systemic collapse of liquidity ensues. The results explain both the contagious run on liquidity markets at the advent of the recent crisis as well as the liquidity market freeze after the Lehman collapse. While what matters for the possibility run is whether an institution is too-unexpected-to-fail (TUTF), it is likely that institutions typically considered too-big-to-fail (TBTF) are also likely to be TUTF. The results then show that TBTF institutions limit the spread of crises, and breaking up a TBTF increases systemic vulnerability. Further, the results cast some doubt on the efficacy of the ring-fencing policy proposed by the UK banking commission.

JEL CLASSIFICATION: G2, C7, E5.
KEYWORDS: Short-term debt, systemic liquidity crises, coordination, ambiguity, bail-out policy, liquidity policy
1 Introduction

The financial markets experienced multiple rounds of systemic liquidity runs during the last crisis. As Gorton and Metrick (2012) show, with the arrival of bad news from housing markets in 2007, there was a run on the repo market which spread from subprime housing assets to non-subprime assets with no direct connection to the housing market. A similar run took place in the asset-backed commercial paper market, even for paper not exposed to subprime mortgages (see Covitz, Liang and Suarez (2009)). Further, a widespread freeze of liquidity markets followed the failure of Lehman Brothers in September 2008. A run ensued on money-market mutual funds as well and the interbank market was severely disrupted.\footnote{The Libor-OIS spread, historically around 10 basis points, spiked sharply to above 50 basis points in August 2007 and then to an all-time high of 364 basis points just after the Lehman failure, indicating severe stress on the interbank markets.}

To understand the crisis it is important to explain why runs spread to non-subprime assets, and why a systemic liquidity crisis arose immediately after the Lehman failure.

I provide a theory based on the interaction of lender coordination and ambiguity (Knightian uncertainty) to explain the origins contagion runs in liquidity markets. Further, I show how the interaction of policy with coordination and ambiguity can give rise to a sudden systemic liquidity freeze. Gorton (2012) defines systemic risk as loss of investor confidence in financial intermediary debt. The interaction of coordination, ambiguity and policy shown here clarifies a mechanism through which such a loss of confidence arises and makes the meaning of loss of confidence precise. This is the main contribution of this paper.

I study a simple global game model of coordination among lenders lending short-term funds to a financial intermediary, and calculate the equilibrium market haircut (the fraction of funds that the intermediary must put up in order to successfully attract enough funds to run an investment project). I show how haircut responds to different parameters and how coordination itself gives rise to an inefficiency. I then show that if lenders are ambiguity averse, the introduction of any (arbitrarily small) degree of ambiguity about the signal received by lenders leads to a complete breakdown of coordination and the market haircut rises to 100%. In other words, coordination is fragile with respect to ambiguity and therefore even the smallest degree of ambiguity gives
rise to a liquidity run. Thus the advent of ambiguity in the presence of coordination requirements in rolling over loans explains runs across subprime related as well as unrelated repo markets.

The systemic collapse of liquidity does not require all lenders to be affected by ambiguity. I show that if the fraction of lenders affected by ambiguity exceeds the fraction of the project that can be funded by own stable funds, coordination collapses completely if any ambiguity is introduced. If the former fraction is below the latter one, the market haircut rises sharply when ambiguity is introduced, but not to 100%.

The problem above does not arise for a financial intermediary if the market expects it to be rescued by policymakers, i.e. given lender-of-last-resort (LOLR) support (throughout the paper, the terms rescue, bail-out and LOLR support are used interchangeably). For such an intermediary, coordination ceases to be an issue as LOLR support covers any funds not rolled over. It follows that the collapse-of-coordination cannot happen. However, this immunity depends entirely on policymakers following market expectations about institutions that are too-unexpected-to-fail (TUTF). If policy does not follow market expectations so that the latter adjusts even slightly, a complete collapse of liquidity arises through the interaction of coordination and ambiguity as described above. Thus policy itself becomes a potential trigger of systemic crisis. This offers an explanation of the tightening of credit terms for other large financial intermediaries and runs on even the money market funds in the wake of the Lehman failure. Such intermediaries and funds are typically perceived as safe (i.e. the market implicitly assumes they will receive LOLR support), but as we show, even a small change in this expectation is enough to explain these runs.

The results also show that avoiding systemic risk implies that the following three things cannot co-exist: (1) short-term debt not covered by public guarantees (2) any factor (such as complex financial innovation that makes the exposure of an intermediary to loss opaque) that is likely to introduce some (even small) degree of ambiguity in investors’ signals of fundamentals, and (3) policy making that is market-independent in the sense that the regulator does not rescue an institution that the market expects to be rescued. In other words, the presence of rollover risk and ambiguity imply that the extent of bail-out policy is determined entirely by market expectations.

If the market expects an institution to fail under a liquidity run, it follows that at the advent of ambiguity in signals about returns, such an institution would experience a
liquidity run. If, on the other hand, the market expects an institution not to fail (a TUTF institution), it does not experience a liquidity run unless policy itself contributes to any lowering of expectations about rescue probability. It follows that, so long as policy conforms to market expectations, having a TUTF institution helps limit the spread of a liquidity crisis.

While our work shows the importance of considering market expectations, a different issue, much discussed in the literature, is the consequence of a financial institution being too big to fail (TBTF). What does our work imply about the notion of TBTF? In particular, does a large institution regarded as TBTF help with or exacerbate the crisis? If market expectations about which institutions are likely to be rescued is unrelated to size, TBTF would not matter. However, it is quite likely that markets expect large institutions with complex counterparty obligations to be rescued. In this case, TBTF coincides with TUTF, in which case having a TBTF institution is helpful in limiting the systemic spread of a liquidity crisis. The same reasoning shows why breaking up a TBTF institution exacerbates the systemic crisis. If broken up, none of the parts might be TUTF, in which case all parts experience liquidity runs. This increases systemic vulnerability. In the wake of the recent crisis, many policy proposals have called for breaking up TBTF institutions. The discussion above implies such a policy may be somewhat misplaced in promoting systemic stability.

The results here imply that there are two ways for regulatory policy to reduce the systemic vulnerability to ambiguity. Any policy that reduces the coordination problem helps improve the system’s defence against ambiguity. Further, any policy that creates an expectation that an intermediary would be rescued in a crisis also has the same effect. Requiring banks to fund projects fully or partially with stable funds (long term debt, equity, insured deposits) - for example the net stable funding ratio of Basel III, requiring banks to hold more liquid assets - for example the liquidity coverage ratio requirement of Basel III belong to the first category. Explicit or implicit promise of LOLR support belongs to the second.

In similar vein, the results suggest that systemic impact of the ring-fencing proposed by the UK banking commission is questionable. Consider a bank that is TUTF in the absence of ring-fencing. Indeed, ring-fencing is proposed for precisely large banks with important network positions - and such banks are would typically be considered by the market as TUTF. Placing a ring-fence now indicates to the market that the part
outside the fence is unlikely to be rescued, which makes it vulnerable to the fragility noted here, increasing systemic instability.

Let us now describe the modelling of coordination and ambiguity in the model in some detail. We adopt a standard global game approach to model coordination among several lenders lending short-term funds to a financial intermediary. We study a typical three-period model. A project is initiated period 0 using short-term credit; funds must be rolled over in period 1; if successfully rolled over, returns are realized in period 2. If funds are not rolled over, there is a fire sale, and some return is realized depending on the state of fundamentals. Hence coordination matters: if enough lenders roll-over funds in period 1, the project earns a high expected return in period 2, and a lower return otherwise. Each lender receives a signal of the state of fundamentals with some noise. With a small amount of incomplete information, the game is dominance solvable: there is a unique equilibrium that is attained by iteratively eliminating strictly dominated strategies. However, just as agents are almost, but not entirely sure of the underlying state of fundamentals, if there is even the slightest ambiguity about the signal being biased, and if the agents are ambiguity averse, coordination breaks down completely. Agents lend successfully only when the underlying state of fundamentals is so high that it is a dominant strategy to lend.

The intuition for this result is as follows. We model ambiguity using the maxmin expected utility model of Gilboa and Schmeidler (1989). Suppose the signal bias lies in the interval \([-b, b]\) (where \(b\) is arbitrarily small). Agents consider the worst case in which there is a bias of \(b\) in the signal. We show since an agent perceives a bias relative to others’ signals, when signal noise is small, the agent’s optimal action is to calculate what signal threshold \(y^*\) others, who are perceived to receive unbiased signals, arrive at. Then set own signal threshold at \(y^* + b\). In other words, the agent wants to stay away from the threshold established by others. Note that this agent is not contributing to coordination at all. The agent rolls over own funds only when others have established a coordination threshold \(y^*\), and then sets own threshold above \(y^*\). But if everyone (or, as I show, a large enough fraction) behaves this way—each trying to stay ahead of the others—coordination necessarily fails. In this case rollover happens only when it is a dominant strategy for each agent to roll over. In all other cases where coordination matters, no one rolls over, i.e. there is a run on the project and it is liquidated.

Thus starting from a equilibrium with a coordination threshold, if even the slightest
degree of ambiguity is introduced and agents are ambiguity averse, a liquidity run follows. Further, policy plays an important role in this process.

The results in this paper imply the following pattern of credit tightening for financial intermediaries. For intermediaries with some exposure to non-performing assets (such as subprime mortgage backed securities), lenders are likely to experience some signal ambiguity. For such intermediaries, coordination breaks down or is impaired and the cost of funds rise dramatically if the lenders perceive that a public bail-out is not likely. However, if other policy measures again convince lenders that any liquidity problems would be met with central bank liquidity support, coordination would be restored and cost of borrowing would fall. For (possibly smaller, less interconnected) intermediaries that are not expected to be rescued, cost of funds would rise dramatically at the advent of crisis (and therefore, ambiguity) and stay high.

In their study of the fed funds market in the aftermath of the Lehman failure Afonso, Kovner and Schoar (2011) find precisely this pattern: “...in the days immediately after the Lehman Brothers bankruptcy the market becomes more sensitive to bank-specific characteristics, especially in the amounts lent to borrowers but also in the cost of overnight funds. In particular, large banks with high percentages of non-performing loans (NPLs) showed drastically reduced daily borrowing amounts and borrowed from fewer counterparties in the days after Lehman’s bankruptcy. However, beginning on Tuesday, September 16, 2008, once the AIG bailout was announced, the trend reversed, and spreads for the largest banks fell steeply ... the same is not true for small banks, which continued to face higher spreads.”

Indeed, the authors interpret the reduction spreads for large banks after the AIG bailout as the effect of the governments support for systemically important banks. Our paper provides a precise mechanism to articulate such an interpretation.

Our paper also contributes to the literature on global games, and shows that coordination is fragile to ambiguity in signals. Papers by ? and ? have studied the effect of ambiguity in a 2-player global game with signal noise away from the limit. In these models, a small degree of ambiguity has a small effect reducing coordination. We show that when signal noise is small, coordination breaks down, and clarify the intuition behind this result, namely, even a small degree of ambiguity results in each agent trying to beat the coordination threshold of others, making coordination impossible.

Given the result on fragility of coordination to ambiguity, our theory can be used in
general to understand the sudden collapse in any coordination-based market. Consider, for example, the sudden collapse in the auction-rate securities market in the recent crisis.\footnote{See Han and Li (2010) for a detailed empirical investigation.} While our model does not explicitly capture the institutional details of this market, at the heart of these shadow-banking markets is investor coordination, and our theory would suggest that the interaction of such coordination and ambiguity can explain why a sudden failure can arise.

The theory of systemic spread of crisis from breakdown of coordination under ambiguity also fits with events in the Great Depression. In the new preface to 2012 edition of Kindleberger (1973), DeLong and Eichengreen write:

> The 1931 crisis began, as Kindleberger observes, in a relatively minor European financial centre, Vienna, but when left untreated leapfrogged first to Berlin and then, with even graver consequences, to London and New York. This is the 20th century’s most dramatic reminder of quickly how financial crises can metastasise almost instantaneously. In 1931 they spread through a number of different channels. German banks held deposits in Vienna. Merchant banks in London had extended credits to German banks and firms to help finance the country’s foreign trade. In addition to financial links, there were psychological links: as soon as a big bank went down in Vienna, investors, having no way to know for sure, began to fear that similar problems might be lurking in the banking systems of other European countries and the US.

In the light of our theory, we can interpret the troubles in Vienna as creating some ambiguity for lenders in other European countries as well as the US, leading to coordination failure at these centres, ensuring that the crisis was replicated across these centres.

### 1.1 Relevant literature: A brief review

There is an extensive literature studying runs in financial markets. Here I briefly note papers connected to this work. The most direct intellectual antecedent is the work
of Rochet and Vives (2004). They study a global game coordination model of loan rollover in the standard three-period framework. They show that it is possible to have banks that are solvent but illiquid, and discuss policy fixes. This paper uses a similar framework and models the same problem and therefore the same result on illiquid but solvent banks reappear here, namely when $\theta \in [\theta, \theta^*)$. However, our modelling is a little different allowing a different focus: the derivation of a market haircut and study the impact on this of the interaction between coordination with ambiguity, and the ensuing impact of policies on systemic outcomes through the channel of market expectations. Our work also allows discussion of policies that serve as insurance against ambiguity and an evaluation of the question of an intermediary being TBTF.

The question of market freezes has been studied by Acharya, Gale and Yorulmazer (2011). In their work, a freeze arises from the evolution of debt-capacity of assets in repeated rounds of rollovers. Here, the model is of a traditional “panic.” Since the seminal work of Diamond and Dybvig (1983), panics have been modeled as selecting the “bad” equilibrium out multiple coordination equilibria. The global games approach allows linking equilibrium choice to fundamentals, and derivation of a unique equilibrium for every value of fundamentals. We adopt this approach and show that coordination is fragile to ambiguity: even a small degree of ambiguity leads to a collapse of coordination so that for all values of fundamentals where coordination matters, the bad equilibrium is chosen.

Apart from this “panics” approach, there is a literature–largely developed since the latest crisis–that focuses on a systemic amplification through spirals of asset price movements (fire sales). Such movements lead to loss spirals and margin spirals working through the balance sheets to amplify a crisis (Brunnermeier and Pedersen, 2009; Adrian and Shin, 2010). Our work complements this approach in the sense that a breakdown of coordination would lead to fire sales, and subsequent spirals of loss could further exacerbate the problem.

? study a model with money market trade between regions that differ in access to insured deposits, and show how this distorts the way money markets allocate funds. When risk of bank failure rises significantly, trade breaks down, which they interpret as a market freeze. Here the motivation for a run is also counterparty risk, but arising

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3See also Brunnermeier (2009) for an overview of the approach, and Shleifer and Vishny (2011) for a discussion of fire sales in general.
from incomplete information and ambiguity about such risk. Also, our result shows that the system is fragile: even very small levels of ambiguity in signals leads to a collapse of coordination, haircut jumping up, and sudden market freezes arising.

Finally, while I use ambiguity in signals about counterparty risk to show a market freeze, the notion of ambiguity has been fruitfully employed in other strands of literature on crises. Caballero and Krishnamurthy (2008) show ambiguity in the environment combined with ambiguity aversion can give rise to liquidity hoarding in interbank markets. Easley and OHara (2010) show markets can become illiquid when traders face extreme Knightian uncertainty (ambiguity). Such ambiguity can be represented by incomplete preferences over portfolios, and show that no trade occurs at equilibrium quotes.
2 The model

There are three periods, 0, 1 and 2. The intermediary has a project that requires 1 unit of funding initially and in period 1. In period 2 the returns are realized. If in period 1 investment by the intermediary falls below 1, the project must be liquidated. Instead of this risky project, the intermediary can also invest in a riskless asset at the initial period. We normalize the riskfree net return to 0.

There is a unit mass of potential lenders. Lenders are risk neutral. Each lender lends 1 unit of funds initially, and must decide in period 1 whether to rollover the unit to the intermediary or to withdraw (wholly or partially) and invest in a safe asset that has a net return of 0.

The return from the project as well as the liquidation value depends on the underlying economic fundamentals $\theta$. If 1 unit is invested in periods 0 and 1, the project returns $\theta R$ where $R > 1$. Here $\theta$ can be thought of as the probability of success of the project and $R$ the return if the project succeeds (the return if the project fails is 0 by implication).

The liquidation value depends on market liquidity. We assume that the liquidation value is $\theta$ in state $\theta$. In other words, the state of fundamentals also indicates the state of market liquidity. As $\theta$ improves, two things happen: the return from the project improves (the project succeeds with higher probability) and the market liquidity improves, which implies that the proceeds from any early liquidation rises.

The balance sheet of the intermediary is as follows. On the liabilities side, apart from short term borrowing of 1, the intermediary has access to stable funds (long term borrowing, equity) of $\psi \in [0, 1]$. On the assets side, apart from the long term investment of 1 unit of funds, it holds $\psi$ in liquid form (cash, t-bills).

This implies that in period 1, so long as at least $(1 - \psi)$ of short-term borrowing is rolled over, the investment project can continue to period 2 since the intermediary can fund up to $\psi$ using the liquid funds.

If the project faces liquidation because of lack of funds, it is bailed-out by the policy-maker with probability $p$. The policymaker supplies enough funds so that the project continues till maturity and lenders get the same return of $\theta R$ that they would get if enough funds had been rolled over.
2.1 The payoff of lenders and the intermediary

Let us now derive the payoff of lenders, which is a function of the state of fundamentals $\theta$. Each lender lending a unit of funds is promised a payoff of $r > 1$ with limited liability so long as the project succeeds.

Therefore the expected net return of a lender, denoted by $\pi_L(L(\theta), \theta)$, is given by

$$\pi_L(L(\theta), \theta) = \begin{cases} \theta r - 1 & \text{if } \psi + L(\theta) \geq 1 \\ p\theta r + (1 - p)\theta - 1 & \text{otherwise} \end{cases}$$

2.2 Dividing the set of fundamentals

If the state of the fundamentals $\theta$ were common knowledge, we could divide the set of fundamentals as follows.

Let

$$\bar{\theta} = \frac{1}{p\rho + (1 - p)}$$

(1)

For any $\theta > \bar{\theta}$, the net expected return from rolling over is positive even if the project is liquidated. Therefore rolling over is the dominant strategy in this case.

Next, let $\underline{\theta}$ be such that $\theta r = 1$, which implies

$$\underline{\theta} = \frac{1}{r}$$

(2)

For any $\theta < \underline{\theta}$, the net expected return from rolling over is negative even if all others roll over. Therefore not rolling over is the dominant strategy in this case.

For $\theta[\bar{\theta}, \underline{\theta}]$, whether investment proceeds depends on the size of the total funds raised. In this interval a coordination problem arises, and would give rise to multiple equilibria when $\theta$ is common knowledge.

2.3 Incomplete Information and Signals

The state of fundamentals $\theta$ is drawn from a uniform distribution on the interval $[0, 1 + \Delta]$, $\Delta > 0$. Each lender receives a signal $x$ of $\theta$ in period 1, where $x$ is uniform on $[\theta -$
\( \varepsilon, \theta + \varepsilon \). Conditional on the true state \( \theta \), the signals are independently and identically distributed. After receiving the signal, lenders simultaneously decide whether to roll over their loan or withdraw. The decisions made result in aggregate loan rollover of \( L(\theta) \).

Finally, a technical requirement. To ensure that the signal intervals are well defined at the lower boundary of the relevant range of fundamentals, I assume that \( \bar{\theta} = 1/r \geq 2\varepsilon \) and \( 1 + \Delta - \bar{\theta} \geq 2\varepsilon \). Note that the latter is possible assuming \( \Delta > 2\varepsilon \) since \( \bar{\theta} \) is at most 1.

3 Equilibrium

I consider monotone equilibria (as I show later on, there are no other types of equilibria). In a monotone equilibrium, there is a threshold \( x^* \) such that agents roll over their loan if and only if \( x \geq x^* \). The aggregate size of the loan is the mass of agents who receive \( x \geq x^* \). Thus

\[
L(\theta) = \begin{cases} 
0 & \text{if } \theta < x^* - \varepsilon, \\
\frac{\theta + \varepsilon - x^*}{2\varepsilon} & \text{if } x^* - \varepsilon \leq \theta < x^* + \varepsilon, \\
1 & \text{if } \theta \geq x^* + \varepsilon.
\end{cases}
\]

Clearly, total investment increases in \( \theta \).

First, given any signal cutoff \( x^* \), let us calculate the threshold \( \theta^* \) such that successful investment occurs if and only if \( \theta \geq \theta^* \). This is given by

\[
L(\theta^*) = 1 - \psi
\]

Solving,

\[
x^* = (2\psi - 1)\varepsilon + \theta^*
\]

Next, given that the project earns a high return if and only if \( \theta \geq \theta^* \), let us calculate the
signal cutoff \( x^* \). The expected payoff of an agent with signal \( x \) from a loan rollover is

\[
V(\theta^*, x) = \Pr(\theta \geq \theta^* | x) E\left( \theta r | \{\theta \geq \theta^*, x\} \right) \\
+ \Pr(\theta < \theta^* | x) E\left( p\theta r + (1 - p)\theta | \{\theta < \theta^*, x\} \right) - 1,
\]

\[
= \frac{1}{2\varepsilon} \int_{\theta^*}^{x + \varepsilon} \theta r d\theta + \frac{1}{2\varepsilon} \int_{x - \varepsilon}^{\theta^*} (p\theta r + (1 - p)\theta) d\theta - 1,
\]

\[
= \frac{1}{4\varepsilon} \left( (1 - p)(r - 1)((x - \varepsilon)^2 - \theta^2) \right) + xr - 1.
\]

For any given \( \theta^* \), \( x^* \) solves the indifference condition

\[
V(\theta^*, x^*) = 0. \quad (5)
\]

Using equation (4) to write \( x^* \) in terms of \( \theta^* \), and substituting in the expression for \( V(\theta^*, x^*) \) from above, then using (5), we get a single equation in \( \theta^* \):

\[
\frac{(1 - p)(r - 1)(\varepsilon^2 + (\theta^* + (2\psi - 1)\varepsilon)^2 - \theta^*^2)}{4\varepsilon} + \frac{1}{2}(\theta^* + (2\psi - 1)\varepsilon)(1 + r + p(r - 1)) - 1 = 0
\]

Solving, we get following (unique) rollover cutoff. The details of the derivation are relegated to the appendix.

\[
\theta^* = \frac{1 + (1 - \psi)^2(1 - p + pr)\varepsilon - \psi^2 r \varepsilon}{r - (1 - \psi)(1 - p)(r - 1)} \quad (6)
\]

I show next that the unique monotone equilibrium identified above is also the only equilibrium irrespective of the strategies considered. In particular, investing for and only for \( x \geq x^* \) is the only strategy that survives iterative elimination of strictly dominated strategies. This dominance-solvability is similar to several other applications of coordination games exhibiting strategic complementarities in payoffs. The proof is exactly similar to the uniqueness proof in Morris and Shin (2003, 2004) and omitted.

**Proposition 1.** The monotone strategy of investing if and only if \( x \geq x^* \) is the only strategy that survives iterative elimination of strictly dominated strategies, and therefore the equilibrium in monotone strategies is the unique equilibrium.

Next, taking the limit as \( \varepsilon \to 0 \), we get the rollover cutoff:

\[
\theta^* = \frac{1}{r - (1 - \psi)(1 - p)(r - 1)} \quad (7)
\]
At $\psi = 0$, $\theta^* = 1 \equiv \overline{\theta}$. Further, for $\psi = 1$ or $p = 1$, $\theta^* = 1/r \equiv \underline{\theta}$. Further, $\theta^*$ is clearly decreasing in $\psi$ as well as $p$. It follows that for any $0 < \psi < 1$ and/or $0 < p < 1$, $\underline{\theta} < \theta^* < \overline{\theta}$.

Note that $\theta^*$ is decreasing in $\psi$, $p$ and $r$. This is intuitive: higher provision of liquid funds eases the coordination problem and thereby reduces the coordination threshold, a greater chance of public rescue improves the payoff of lenders which in turn reduces the coordination problem, and finally, a higher promised payment if the project succeeds has the same effect as a higher rescue probability.

### 3.1 Equilibrium Market Haircut

The market required stable funds for any state of fundamentals $\theta$ is the minimum stable funding the intermediary must itself provide in order for successful coordination of lending to take place. In other words, it is the minimum haircut to ensure that $\theta \geq \theta^*$. Since $\theta^*$ is decreasing in $\psi$, the haircut is smallest if we can ensure coordination is just successful at $\theta$. If we support coordination at $\theta$ by making $\theta > \theta^*$, we can lower the intermediary’s own stable funding and still ensure that $\theta \geq \theta^*$. Therefore the market haircut necessary to ensure $\theta \geq \theta^*$ is given by the solution for $\psi$ to $\theta = \theta^*$, where $\theta^*$ is given by equation (7).

Solving, market required haircut for coordination to be successful at state $\theta$ is $\psi^M(\theta)$ is given by

$$\psi^M(\theta) = 1 - \frac{r\theta - 1}{(1 - p)(r - 1)\theta}$$

### 3.2 Minimum Market Required Haircut

The market required haircut calculated above depends on the promised loan interest factor $r$. This is presumably determined by supply and demand factors in the market for funds outside the scope of the model. Even though $r$ is exogenous to the model, we can determine an upper bound for $r$ which gives us a lower bound for the coordination threshold $\theta^*$, as well as a minimum market required haircut to sustain coordination at any given $\theta$.

For any $\theta \geq \theta^*$, the gross payoff of an intermediary is given by $\theta(R - (1 - k)r)$ when-
ever the project succeeds, and 0 otherwise. Since the intermediary invests $\psi$, its gross payoff in the success state must exceed $\psi$ as otherwise the intermediary would necessarily receive a strictly negative net expected payoff for any $\theta \geq \theta^*$. Thus a necessary condition for the intermediary to run the project at all is:

$$\theta^*(R - (1 - \psi)r) \geq \psi.$$ 

Changing the inequality to an equality and solving for $r$ gives us the following upper bound on $r$:

$$r_{\text{max}} = 1 + \frac{R - 1}{1 - \psi(1 - \psi)(1 - p)}.$$ 

Using this upper bound, we can get a lower bound on the value of $\theta^*$, denoted by $\theta_{\text{min}}^*$:

$$\theta_{\text{min}}^* = \frac{1 - \psi(1 - \psi)(1 - p)}{R - (1 - p)(1 - \psi)(R - (1 - \psi))}.$$ 

We can now calculate the minimum market required haircut to sustain coordination at any given $\theta$ using the same logic as in the derivation of $\psi^M(\theta)$ above. Since $\theta_{\text{min}}^*$ is decreasing in $\psi$, the haircut is smallest if we can ensure coordination is just successful at $\theta$. If we support coordination at $\theta$ by making $\theta > \theta_{\text{min}}^*$, we can lower the intermediary’s own stable funding and still ensure that $\theta \geq \theta_{\text{min}}^*$. Therefore the minimum haircut necessary to ensure $\theta \geq \theta_{\text{min}}^*$ is given by the solution for $\psi$ to $\theta = \theta_{\text{min}}^*$. From this, we get

$$\psi^M_{\text{min}}(\theta) = \frac{1}{2(1 - \theta)} \left( A - \sqrt{\frac{(1 - p)A^2 + 4(1 - \theta)((R - (1 - p)(R - 1))\theta - 1)}{1 - p}} \right)$$

where

$$A = (1 - \theta) + \theta(R - 1).$$

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4In equilibrium we must have $\pi_I(\theta^*) \geq 0$. Suppose, on the contrary, that $\pi_I(\theta^*) < 0$ in equilibrium. Then there is a positive measure of values of $\theta \geq \theta^*$ for which $\pi_I(\theta) < 0$. Note also that both $\theta^*$ and $\pi_I(\cdot)$ are decreasing in $r$. If the intermediary sets a slightly lower $r$, its payoff would increase for all values of $\theta \geq \theta^*$, and further, $\theta^*$ would increase a little, therefore reducing the measure of the set of values of $\theta$ for which payoff is negative. Thus such a reduction in $r$ is strictly profitable for the intermediary. This contradiction proves that in equilibrium we must have $\pi_I(\theta^*) \geq 0$. Given this, an upper bound on $r$ is given by $\theta^*(R - r(1 - \psi)) - \psi = 0$. 

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3.3 Inefficiency

The upper bound for $r$ derived above also gives a lower bound for the inefficiency generated in the loan rollover problem considered here. A lower bound on the inefficiency generated by the coordination problem among lenders is given by

$$\max\{0, \theta_{\min}^* - \theta_{fb}\}.$$ 

**Proposition 2.** If $\psi < 1$ so that there is some short term borrowing, $\theta_{\min}^* > \theta_{fb}$. In other words, an inefficiency arises from coordination. As $\psi \to 1$, $\theta_{\min}^* - \theta_{fb} \to 0$.

**Proof:** It is straightforward to see that $\lim_{\psi \to 1} \theta_{\min}^* = 1/R = \theta_{fb}$. This proves the second part of the proposition. Next, note that

$$\frac{\partial \theta_{\min}^*}{\partial \psi} = -\frac{1}{D^2} ((1 - p)(R - 1)((1 - \psi)(2 - (1 - p)(1 - \psi)))) < 0,$$

where $D = R - (1 - \psi)(1 - p)(R - (1 - \psi))$. Further, $\theta_{fb}$ is independent of $\psi$. Therefore, as $\psi$ falls from 1, $\theta_{\min}^*$ rises above $\theta_{fb}$. This proves the first part of the proposition. \|

Thus efficiency is achieved only when the bank funds the entire project from own stable funds without any short term borrowing. Whenever some short term borrowing is required, so that there is a positive amount requiring to be rolled over, there is an inefficiency arising from the coordination problem.

4 Ambiguity and Sudden Jumps in Market Required Haircut

In this section I show that if $p < 1$ for a financial institution (the market is not certain that the institution would be rescued if it became illiquid in period 1), when environment is characterized by ambiguity (signal is ambiguous) and agents are ambiguity averse, coordination breaks down completely.

Suppose some investors face a slight ambiguity about the quality of their own signal. Specifically, suppose some investors think that their signal might have some (arbitrarily small) bias relative to the signals received by others. In other words, such an investor $i$ regards the signal as being

$$x_i = \theta + \beta_i + \epsilon_i$$
where $\beta_i \in [-b, b]$, and $b > 0$. In other words, each buyer believes that the signal they receive is drawn from some distribution from a set of distributions with means $\theta + \beta_i$, where the bias $\beta_i$ is distributed over $[-b, b]$. Note that $b$ can be arbitrarily small, so that the extent of the ambiguity could be vanishingly small.

The preferences of the buyers is represented by the maxmin expected utility model of Gilboa and Schmeidler (1989). Following their model, ambiguity averse investors maximize their minimum expected utility over the set of possible signal distributions. Here, the minimizing signal distribution for investor $i$ is the one with mean $\theta + b$. Investor $i$ therefore chooses the cutoff $x_i$ to maximize expected utility conditional on signal $x_i = \theta + b + \varepsilon_i$.

$$u_i(x_i, \theta^*) = \frac{1}{2\varepsilon} \int_{\theta^*}^{x_i-b+\varepsilon} \theta r d\theta + \frac{1}{2\varepsilon} \int_{x_i-b-\varepsilon}^{\theta^*} (p\theta r + (1-p)\theta) d\theta - 1$$

Integrating and rearranging terms, we get

$$u_i(x_i, \theta^*) = \frac{1}{4} (r-1)(1-p) \left( \frac{(x_i-b)^2 - \theta^{*2}}{\varepsilon} + \varepsilon \right) + \frac{1}{2} (1-p + r(1+p))(x_i-b) - 1 \quad (8)$$

Let $y^*$ be the rollover signal threshold for all others. We want to see the relation between $x_i$ and $y^*$. In the standard case without ambiguity there would be a common threshold $x_i = y^*$. In this case, $\theta^*$ and $y^*$ are such that $L(\theta^* | y^*) = 1 - \psi$. Using equation (4), this implies,

$$\frac{\theta^* + \varepsilon - y^*}{2\varepsilon} = 1 - \psi.$$  

Solving, we get $\theta^* = y^* + \varepsilon - 2\psi\varepsilon$. Substituting the value of $\theta^*$, we get payoff in terms of $x_i$ and $y^*$:

$$\hat{u}_i(x_i, y^*) = \frac{1}{4} (1-p)(r-1) \left( \frac{(x_i-b)^2 - y^{*2}}{\varepsilon} - 2y^*(1-2\psi) - \varepsilon(1-2\psi)^2 + \varepsilon \right) + \frac{1}{2} (x_i-b)(1-p + r(1+p)) - 1 \quad (9)$$

Suppose $p < 1$. If $x_i - b > y^*$, for small $\varepsilon$, the payoff becomes large and positive, so that $x_i$ must be lower in equilibrium. Thus $x_i - b \not> y^*$. Also, if $x_i - b < y^*$, for small
\( \varepsilon \), the payoff is large and negative, so that the cutoff \( x_i \) of investor \( i \) must be higher. Therefore \( x_i - b < y^* \). It follows that the only value of \( x_i \) compatible with equilibrium is given by \( x_i - b = y^* \), implying \( x_i = y^* + b \).

In other words, investor \( i \) invests above a signal cutoff that is above the cutoff of others. If all others invest above cutoff \( y^* \), \( i \) invests above cutoff \( y^* + b \). But since everyone behaves this way, it is impossible to have an interior solution for \( y^* \). In other words, coordination only succeeds when it is a dominant strategy to rollover, i.e. when \( \theta \geq \bar{\theta} \). Otherwise coordination fails. Thus for the entire set of fundamentals for which coordination matters, coordination collapses.

Note that this is a discontinuous collapse. As shown earlier, without ambiguity there is an interior threshold \( \theta^* \) beyond which coordination succeeds. Starting from this position if we introduce even the slightest degree of ambiguity in signals, coordination collapses completely and \( \theta^* \) jumps to \( \bar{\theta} \).

Any financial intermediary for which the rescue probability is \( p = 1 \), coordination is irrelevant. However, for any other value of \( p \), coordination collapses under any degree of ambiguity and there is a run by short term investors.

The calculations and discussion above prove the following result.

**Proposition 3.** Suppose \( b_i > 0 \) for all \( i \) so there is some (possibly arbitrarily small) ambiguity across all lenders. For any \( p < 1 \) coordination breaks down completely.

Next, suppose a fraction \( \alpha \in [0, 1] \) of lenders are sensitive to ambiguity. It follows from the result above that if there is any ambiguity in signals, this fraction of agents would not be useful for coordination - if others already establish some success threshold \( \theta^* \), each ambiguity-sensitive agent would want to rollover debt beyond a cutoff above \( \theta^* \). However, \( \theta^* \) itself must be established by relying on the fraction \( (1 - \alpha) \) of ambiguity-insensitive agents only. It follows that equation (3) becomes

\[
(1 - \alpha)L(\theta^*) = 1 - \psi.
\]

Since \( L(\theta^*) \leq 1 \), it is clear that coordination cannot succeed if \( \alpha > \psi \). This proves the following result:

**Proposition 4.** For any \( \alpha > \psi \), coordination breaks down completely.
4.1 Ambiguity and Haircut

An interior coordination cutoff \( \hat{\theta}^* \) can be obtained if \( \alpha < \psi \).

As noted above, equation (3) now becomes

\[
(1 - \alpha)L(\hat{\theta}^*) = 1 - \psi.
\]

Equation (5) is unchanged. Following the same steps as above, we get the rollover threshold:

\[
\hat{\theta}^* = \frac{1 - \alpha}{r(1 - \alpha) - (r - 1)(1 - \psi)(1 - p)}.
\]

The maximum gross payment to debt holders in period 2 if the project succeeds is

\[
\hat{r}_{\text{max}} = \frac{\psi(1 - \psi)(1 - p) - R(1 - \alpha)}{\psi(1 - \psi)(1 - p) - (1 - \alpha)}.
\]

Finally, the lowest value of cutoff that can be sustained given any specific \( \alpha \) is

\[
\hat{\theta}_{\text{min}}^* = \frac{\psi(1 - \psi)(1 - p) - (1 - \alpha)}{(R - 1 + \psi)(1 - \psi)(1 - p) - R(1 - \alpha)}.
\]

Figure 1 below plots \( \hat{\theta}_{\text{min}}^* \) as a function of \( \psi \) for different values of \( \alpha \). For \( \alpha = 0 \), \( \hat{\theta}_{\text{min}}^* \) coincides with \( \theta_{\text{min}}^* \) calculated in the case without ambiguity. The figure also shows the case of \( \alpha = 0.4 \). Now, for \( \psi < 0.4 \) coordination breaks down completely, and then for higher values of \( \psi \), we get lower values of \( \hat{\theta}_{\text{min}}^* \).

We can also use this to find the minimum haircut needed to sustain coordination at any given \( \theta \). The least-demanding way of sustaining any given \( \theta \) is to achieve it as the cutoff \( \hat{\theta}^* \).

Given any \( \alpha \), the Y-axis shows the lowest value of \( \hat{\theta}^* \) that can be sustained for any \( \psi \). We can invert this to find the lowest value of haircut needed to sustain any given \( \theta \) as \( \hat{\theta}^* \). This can be done simply by setting \( \theta = \hat{\theta}_{\text{min}}^* \) and solving for \( \psi \). In the picture, if we take any \( \theta \) on the Y-axis, then the corresponding value of \( \psi \) on the X-axis is the lowest value of haircut that can support \( \theta \) as \( \hat{\theta}^* \).

The dashed line shows, given a particular \( \theta \) (in this case \( \theta = 0.7 \)), the minimum haircut required to sustain coordination at 0.7 for different values of \( \alpha \). If \( \alpha \) is 0, this is around 0.33. However if \( \alpha = 0.4 \) (so that 40% of lenders are ambiguity-sensitive), at the advent
of ambiguity the minimum haircut to sustain $\theta = 0.7$ jumps up to around 0.55. Thus given that some fraction of lenders are ambiguity averse, there is a sudden increase in haircut at the advent of ambiguity. Of course, if $\alpha$ exceeds the available stable funds $\psi$, the haircut jumps up all the way to 100%.

5 Conclusion

I show coordination is fragile in an environment with ambiguity. In such an environment, policy itself is a systemic trigger. If the regulator fails to rescue an institution that the market expects the regulator to rescue, this can trigger a systemic collapse of liquidity.

I determine the equilibrium market haircut and show how the haircut responds to various parameters. The haircut depends on the rate of return promised by the borrower to the lenders. To eliminate the influence of this variable, which is potentially an endogenous variable, I calculate the minimum market haircut subject to the bor-
rower’s participation constraint. The minimum haircut depends on purely exogenous variables.

Next, I show how the market haircut (as well as the minimum market haircut) responds to the presence of ambiguity. I show that if there is even a very slight degree of ambiguity and lenders are ambiguity averse, coordination breaks down completely, implying that haircut rises to 100%. This also shows that coordination in general is fragile in the presence of ambiguity. The result does not require all lenders to be ambiguity averse - any fraction above a critical threshold is enough for coordination to break down completely. When the fraction of ambiguity sensitive agents are below this critical threshold, coordination weakens but does not break down completely, so that the equilibrium market haircut rises sharply, but stops short of 100%. I calculate the market haircut and the minimum market haircut in this case and show how the haircut jumps up when an arbitrarily small degree of ambiguity is introduced.

The breakdown in coordination would not happen if the lenders expect the borrowing financial intermediary to be rescued by the central bank (or some other liquidity provider of last resort). Such intermediaries would be immune to the problem of ambiguity and coordination as no coordination is needed for such institutions to be able to roll over borrowing successfully. However, if some policy signal arrives that changes the market expectation about rescue probability even slightly, the problem reappears and there is again a breakdown of coordination implying a sharp rise in market haircut. Such a policy signal can arrive if, for example, the regulators do not rescue a financial institution that the market expected to be rescued. Thus policy itself can trigger a systemic liquidity crisis.

The results imply that in the presence of short term borrowing, and the possibility of ambiguity arising in a downturn through factors such as complex security design and/or unknown exposures of financial intermediaries to problematic assets, rescue policy cannot be determined independently of market expectations if a systemic liquidity crisis is to be avoided.

Further, in the context of systemic crises, what matters is whether an institution is too-unexpected-to-fail. If markets expect large intermediaries interconnected to many other institutions (TBTF) to be TUTF, then TBTF coincides with TUTF. In this case, to the extent that an intermediary is TBTF, it helps limit the spread of a liquidity crisis. Indeed, if a TBTF institution is broken up into smaller pieces so that none of the pieces
are TUTF, this increases systemic vulnerability. Thus policies advocating breaking up TBTF institutions may be somewhat misplaced.

A similar reasoning casts doubt on the efficacy of the ring-fencing proposed by the UK banking commission. Such a policy indicates to markets that the part outside the ring fence is unlikely to be rescued in a crisis. Thus while a ring fence protects the core of a bank, our results suggest that the market expectation that a rescue is unlikely for other parts of the bank might increase systemic vulnerability.

Our results can generally be applied to understand situations where coordination markets break down suddenly. We hope to make further progress in analyzing richer models using this core idea.
References


