Balanced budget stimulus with tax cuts in a liquidity-constrained economy

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Abstract: This paper examines the macroeconomic effects of unexpected, exogenous, simultaneous, temporary cuts to income tax rates in an economy when the government follows a balanced budget fiscal rule and keeps money supply constant, and private agents face constraints on the ability to finance investments. The main results are that the tax cuts increase output, private consumption, and investment; the increases in output and consumption are significant and long-lasting; and the liquidity constraints play a major role in the shock’s long-term persistence. Results are obtained from calibrating a modified version of the DSGE model of liquidity and business cycles by Kiyotaki and Moore (2012). The modifications are twofold: (i) distortionary taxes to labour and dividend incomes are added, and (ii) the government follows a balanced budget fiscal rule and keeps money supply constant. Results are qualitatively robust, but quantitatively sensitive, to assumptions regarding structural parameter values, and qualitatively and quantitatively sensitive to significant variations in the persistence of tax shocks.

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1 Introduction

This paper examines the macroeconomic effects of cuts to income tax rates in an economy when the government follows a balanced budget fiscal rule and keeps money supply constant, and private agents face constraints on the ability to finance investments. The tax rate cuts are unexpected, exogenous, simultaneous, and temporary. The main results are that the tax cuts increase output, private consumption, and investment; the increases in output and consumption are significant and long-lasting; and the liquidity constraints play a major role in the shock’s long-term persistence. Liquidity constraints create demands for two assets of varying liquidity; tax cuts increase the demand for both assets; and while the tax cuts also lead to an increase in supply of the less liquid asset, the liquidity constraints restrict this increase to be small; accordingly, both asset prices increase, and amplify the internal propagation of the shock. Results are obtained from calibrating a modified version of the DSGE model of liquidity and business cycles by Kiyotaki and Moore (2012) (henceforth KM). The modifications are twofold: (i) distortionary taxes to labour and dividend incomes are added, and (ii) the government follows a balanced budget fiscal rule and keeps money supply constant. Results are qualitatively robust, but quantitatively sensitive, to assumptions regarding structural parameter values, and qualitatively and quantitatively sensitive to significant variations in the persistence of tax shocks. The paper contributes to an extensive literature on the effectiveness of fiscal policy for economic stimulation. It belongs to a narrow strand of this literature which explores balanced budget expansion. Results are consistent with those achieved by Mountford and Uhlig (2009) (henceforth MU), a member of this balanced budget research.

Tax cuts are shown to be expansionary in early works by Andersen and Jordan (1968), Giavazzi and Pagano (1990), Baxter and King (1993), Braun (1994), McGrattan (1994), Alesina and Perotti (1997), and Perotti (1999), and more recently by Romer and Romer (2010), Mertens and Ravn (2011a,b, 2012), and Monacelli et al. (2012). Support for tax cuts is also expressed in blogs by Hall and Woodford (2008), Bils and Klenow (2008), Mankiw (2008), and Barro (2009). And counterfactual experiments by Blanchard and Perotti (2002), Romer and Romer (2010), MU, and Alesina and Ardagna (2010) show that tax cuts produce larger responses than increases in government spending.

Tax cuts with a balanced budget are shown to be expansionary in Eggertsson (2010) and MU. Eggertsson (2010) obtains his results by cutting consumption taxes and simultaneously raising income and wealth taxes to perfectly compensate. MU show that completely financing an unexpected, exogenous increase in government spending with an increase in taxation causes reductions in private consumption and investment on impact, as well as in output from the second period.1 The converse of this result suggests a recipe for debt-free economic expansion. This paper complements MU by showing that the converse of their result is also true. The novelty of this paper is that while MU obtain their results from an empirical study with vector autoregressions, this paper is a theoretical investigation using a mostly neoclassical DSGE model.

The KM model is chosen for its pair of financial frictions, which resemble an essential

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1There is, however, a small increase in output on impact, with a multiplier of 1.3.
feature of the 2007/8 financial crisis. This makes the paper relevant for policy discussions on the crisis. KM is an otherwise neoclassical model, but with constraints on firms’ ability to internally and externally finance investments. Commentators argue that the cause of the crisis was the sudden and unexpected deterioration in the value of partially liquid private financial assets, which thus ruined their resaleability and suitability for use as collateral (Brunnermeier (2009), Del Negro et al. (2011), Bigio (2012) and Jermann and Quadrini (2012)). Assets’ resaleability and suitability for collateral were thus adversely affected. This event bears a striking resemblance to KM’s own negative liquidity shock.

The KM model is theoretically adjusted and/or extended in a series of recent papers. These papers can be classified into two groups. The first group uses the KM model to evaluate the unconventional policies seen in the crisis; Del Negro et al. (2011) and Driffill and Miller (2013) are members of this research, and both show that recessions would have been exacerbated had it not been for government interventions. The second KM-related group returns to the original questions posed by KM on the importance of (i) liquidity shocks for explaining business cycles, and (ii) liquidity constraints for the propagation of productivity shocks. Papers in this group include Salas-Landeau (2010), Bigio (2010, 2012), Ajello (2011), Nezafat and Slavík (2012), Shi (2012), and Jermann and Quadrini (2012).

The inclusion of distortionary taxes and a balanced budget rule is not unique in KM-related literature. Ajello (2011), Shi (2012), and Driffill and Miller (2013) have a balanced budget rule for government. Ajello (2011) also includes distortionary taxes, but he modifies the KM model more extensively than in this paper. The uniqueness of this contribution is that it is the first to examine fiscal shocks in the KM model. What is shared with these papers, the second KM-related group in particular, is showing the macroeconomic significance of KM’s liquidity constraints in propagating exogenous shocks. In this case, however, the shocks are to tax rates.

In a wider literature, the significance of this paper is that it shows how a neoclassical model can be modified to produce large responses to fiscal shocks. The New Keynesian model is the workhorse for fiscal policy research. This perhaps follows from papers like Burnside et al. (2004), which shows that the magnitude of observed responses to fiscal shocks are not matched by a standard neoclassical models, but they are matched by models that include habit formation and adjustment costs. Beyond the liquidity constraints, this model is otherwise neoclassical. This paper therefore shows that a host of New Keynesian frictions are not always needed to study fiscal policy. The KM model can be a workhorse for that purpose.

The rest of the paper is organized as follows: Section 2 describes the model in full, and derives conditions which characterize the dynamic equilibrium; Section 3 presents the main results of the tax shock; Section 4 quantifies and comments on the magnitude of shock responses using tax multipliers; Section 5 briefly gives the conclusions of sensitivity analysis on structural parameters and the persistence of tax shocks; Section 6 examines the significance of the results by relating them to similar work in the literature; and Section 7 concludes the paper and outlines avenues for future research. The technical appendix contains the model’s calibration, sensitivity analysis of the shock, algebra of proofs and derivations, an algebraic solution for steady state, and the data used in the paper.
2 The model

This section defines agents’ behaviour and derives conditions that characterize the dynamic equilibrium. The model is an adaption of the Kiyotaki and Moore (2012) framework with government and without storage. Modifications are twofold: (i) distortionary taxes on wage and dividend income are added, and (ii) different policy rules to the ones in KM are assumed.\(^2\) In particular, the government holds no equity, keeps money supply constant, and adheres to a balanced fiscal budget rule. To make the paper self-contained, a full description of the model is given. Appendix C contains detailed algebra associated with derivations, simplifications, and proofs.

2.1 The environment

The economy exists over an infinite horizon of discrete time periods. It is populated by three groups of infinitely-lived agents: entrepreneurs, workers, and a government. There is no population growth or decline, and the economy is closed to the rest of the world. Agents produce and consume a perishable general output, which also serves as the numeraire. They also exchange labour and two assets, equity and money. All markets are competitive and prices are perfectly flexible. There are no financial intermediaries; instead, agents borrow directly from other agents. The economy is also subject to random, stochastic shocks.

2.1.1 Entrepreneurs

There is a unit-mass continuum of entrepreneurs. At the start of each period all entrepreneurs are identical. They are the exclusive owners of capital and a homogeneous technology that produces the general output good with guaranteed success. At the beginning of period \(t\) the representative entrepreneur owns \(k_t\) units of capital and employs \(l_t\) hours of labour, and produces \(y_t\) units of general output at the end of the period according to

\[
y_t = A_t k_t^\gamma l_t^{1-\gamma} \tag{2.1}
\]

where \(A_t\) is a common level of total factor productivity and \(\gamma \in (0, 1)\) is the capital elasticity of output, which, given that the output market is perfectly competitive and the production function exhibits constant returns to scale, is also the share of output accruing to capital. The entrepreneur pays workers \(w_t\) units of general output for each labour-hour employed. The quantity of general output left over is the return on the capital used in production; in other words, each unit of capital used in production receives \(r_t\) units of general output as gross profit, where

\[
r_t k_t = y_t - w_t l_t \tag{2.2}
\]

Capital depreciates during the production process, and a fraction, \(\delta\), of its stock survives to the end of the period when production is complete.

\(^2\)KM have two policy rules. One prevents government’s expenditure from exploding, by limiting it to the deviation of their asset holdings from steady state. The other, for open market operations, limits the ratio of current to steady state government equity holdings to a weighted sum of productivity and liquidity impulse responses.
Some time soon after the start of the period, a fraction, \( \pi \), of entrepreneurs gain access to a homogenous investment technology that converts a unit of general output into a unit of capital with guaranteed success. Entrepreneurs are therefore identical \emph{ex ante} to when investment opportunities are revealed. \( \pi \) is independently and identically distributed across time and entrepreneurs. Who gets an opportunity to invest is exogenously determined. Those without investment opportunities carry on with what they have been doing since the start of the period, i.e. producing, consuming, and saving (by purchasing assets); they are the period’s “savers”. Those with investment opportunities are the period’s “investors”, and once their behaviour changes to pursue such opportunities. The investment technology is unrestricted in capacity, but its access expires at the end of the period. Afterwards, investors and savers revert to being \emph{ex ante} identical from the start of the next period.

The production and installation of new capital takes an entire period, so a typical investor who invests \( i_t \) units of general output has an end-of-period capital stock

\[
k_{t+1} = \delta k_t + i_t
\]

To acquire general output for investment, an investor issues new units of equity at the market price. Each of these units has a claim on the future gross profits that it contributed towards creating, i.e. each unit of equity earns \( r_t \) in dividends after period \( t \)’s production is complete. But equity depreciates in tandem with its underlying capital. Any agent is free to buy equity. An entrepreneur’s holdings of equity issued by other entrepreneurs is called his “outside” equity.

The entrepreneur possesses special skills which are costly to replicate and replace. Once an investment project is underway, his human resource is needed for the entire duration to ensure the full amount of new capital is produced. The entrepreneur, however, cannot pre-commit to being involved with the project to its end. Instead, he can guarantee that he will remain with the project for no more than an exogenously determined fraction, \( \theta \), of its duration. This implies that he can guarantee a maximum of \( \theta \) of an investment’s new capital will be produced, which further implies that he can guarantee a maximum of \( \theta \) of new output in the next period when the new capital is put to work. Consequently, the investing entrepreneur can credibly raise no more than \( \theta \) of his investment cost from equity funding. This limitation is called the “borrowing” constraint, and has its origins in Hart and Moore (1994) and Kiyotaki and Moore (1997).\(^3\)

The entrepreneur can internally finance his investment by selling his stock of outside equity. However, the entrepreneur cannot sell all of his outside equity before the window of investment opportunity closes. Instead, he can liquidate up to a fraction, \( \phi_t \), of his holdings. This limitation is called the “resaleability” constraint, and is an exogenous feature of the model.

Borrowing and resaleability constraints are together called “liquidity” constraints. The fi-

\(^3\)An alternative interpretation of \( \theta \) is constructed by Lorenzoni and Walentin (2007): the entrepreneur can “run away” with a fraction, \( 1 - \theta \), of the value of his capital at any time, simply because capital is always under his complete control. In models with formal credit markets, unlike this one, \( \theta \) is featured as a credit market friction: due to a limited ability by lenders to enforce loan contracts, lenders request collateral, and lend at most a fraction, \( \theta \), of the value of collateralized assets. The credit market friction is its more common representation, owing to Kiyotaki and Moore (1997) who show its macroeconomic significance, and to Carlstrom and Fuerst (1997) and Bernanke et al. (1999) who introduce it into dynamic macroeconomic models.
nancing gap they leave is met by the entrepreneur’s stock of a perfectly liquid, government-issued fiat money. While money is intrinsically worthless, its demand is motivated by a precaution against falling short in financing investment opportunities. Moreover, the tighter the liquidity constraints, i.e. the smaller the values of $\theta$ and $\phi_t$, the greater the need to internally finance investments, and the greater the desire to hold money balances (and conversely). That part of a project that the entrepreneur funds from his own money is called his “inside equity”. It is assumed that inside and outside equity are perfect substitutes, having the same resaleability constraint and providing the same rate of return. Inside and outside equity are therefore collectively referred to as “equity”. Equity and money are traded in competitive markets at prices $q_t$ and $p_t$, respectively, both expressed in terms of general output.\footnote{In other words, $q_t$ and $p_t$ units of general output are exchanged for 1 unit of equity and money, respectively. These are “real” prices. “Nominal” prices are the prices of a unit of general output and equity expressed in terms of money, i.e. $1/p_t$ and $p_t/q_t$, respectively. Inflation in the conventional sense is an increase in the nominal price of general output, or equivalently, a fall in the real price of money (i.e. when a unit of money is exchanged for fewer units of general output).}

For an investment opportunity requiring $i_t$ units of general output, an entrepreneur will issue $i_t$ units of equity. By the borrowing constraint, only a maximum of $\theta i_t$ will be sold, and the investor must therefore retain at least $(1 - \theta)i_t$ as new inside equity. The investor will then liquidate existing equity holdings which have since depreciated. If the entrepreneur holds $n_t$ units of equity at the start of period $t$ then a maximum of $\phi_t \delta n_t$ units can be sold, and the entrepreneur is left with at least $(1 - \phi_t) \delta n_t$ units. The entrepreneur also holds $m_t$ units of money at the start of the period, which cannot be lent or used as collateral. His equity and money holdings at the end of the period are therefore given by

\begin{align}
    n_{t+1} &\geq (1 - \theta)i_t + (1 - \phi_t) \delta n_t \\
    m_{t+1} &\geq 0
\end{align}

The gross profit paid to capital at the end of the period represents a dividend payment to the holder of the equity claim on capital. Therefore, an entrepreneur who owns $n_t$ units of equity at the start of the period earns $r_t n_t$ in gross dividends over the period. He pays a tax on dividend income to the government at a rate of $\tau_t r_t n_t$. His net dividend income is then allocated to consumption and saving, and to investment if the opportunity exists.

An investor in period $t$ consumes $c_t$ units of general output, invests $i_t$ units, and saves by acquiring $(n_{t+1}^s - i_t - \delta n_t)$ and $(m_{t+1}^s - m_t)$ units of equity and money, respectively, at their market prices. The investor thus faces a budget constraint for period $t$,

\begin{equation}
    c_t + i_t + q_t (n_{t+1}^s - i_t - \delta n_t) + p_t (m_{t+1}^s - m_t) = (1 - \tau_t^s) r_t n_t
\end{equation}

A saver in period $t$ consumes $c_t^s$ units of general output, and saves the rest of his net income by purchasing $(n_{t+1}^s - \delta n_t)$ and $(m_{t+1}^s - m_t)$ units of equity and money, respectively, at their market prices. The saver’s budget constraint for the period is

\begin{equation}
    c_t^s + q_t (n_{t+1}^s - \delta n_t) + p_t (m_{t+1}^s - m_t) = (1 - \tau_t^s) r_t n_t
\end{equation}
The representative entrepreneur considers an expected lifetime discounted utility,

$$E_t \left[ \sum_{j=t}^{\infty} \beta^{j-t} U_e(c_j) \right] = U_e(c_t) + E_t[\beta U_e(c_{t+1}) + \beta^2 U_e(c_{t+2}) + \ldots] \quad (2.7)$$

where $E_t[\cdot]$ is the expected value conditional on information available in period $t$, and $\beta \in (0, 1)$ is the subjective discount factor, or the inverse of the rate of time preference. The investor maximizes Equation (2.7) subject to his budget constraint (2.5) and liquidity constraints (2.3) and (2.4), while the saver maximizes Equation (2.7) subject to his budget constraint (2.6). Each entrepreneur’s current utility is a natural logarithm of current consumption,

$$U_e(c_t) \equiv \ln c_t$$

### 2.1.2 Workers

There is a unit-mass continuum of identical workers. They are the exclusive owners of labour. They do not own capital or have investment opportunities. In period $t$ each worker supplies $l_w^t$ hours of labour to entrepreneurs in exchange for a gross hourly wage of $w_t$ units of general output. The worker then pays the government a tax on wage income at a rate of $\tau_{wl}$. The worker holds two assets: $n_w^t$ units of equity and $m_w^t$ units of money. Each unit of equity earns gross dividends of $r_t$ units of general output per period, depreciates at a constant rate of $(1-\delta)$ per period, and is subject to the resaleability constraint, $\phi_t$. Money does not have any of these features. The worker pays the government a tax on dividend income at a rate of $\tau_{rn}$. The worker’s human resource is non-transferable across time. He therefore cannot borrow or have negative net worth. His equity and money holdings are then always non-negative, i.e. for all $t$,

$$n_w^t \geq 0 \quad \text{and} \quad m_w^t \geq 0 \quad (2.8)$$

The worker consumes $c_w^t$ units of general output. The rest of his net income is saved by purchasing $(n_{t+1}^w - \delta n_t^w)$ and $(m_{t+1}^w - m_t^w)$ units of equity and money, respectively, at their prevailing market prices. His budget constraint is given by

$$c_w^t + q_t(n_{t+1}^w - \delta n_t^w) + p_t(m_{t+1}^w - m_t^w) = (1-\tau_{wl}^t)w_t l_w^t + (1-\tau_{rn}^t)r_t m_t^w \quad (2.9)$$

Subject to Equations (2.8) and (2.9), the worker maximises an expected lifetime discounted utility,

$$E_t \left[ \sum_{j=t}^{\infty} \beta^{j-t} U_w(c_j^w, l_j^w) \right] = U_w(c_t^w, l_t^w) + E_t[\beta U_w(c_{t+1}^w, l_{t+1}^w) + \beta^2 U_w(c_{t+2}^w, l_{t+2}^w) + \ldots] \quad (2.10)$$

The worker’s utility is additively separable in consumption and leisure,

$$U_w(c^w, l^w) = c^w - \frac{\omega}{1+\nu}(l^w)^{1+\nu}$$

where $\omega$ is the relative weight of labour in utility and $\nu$ is the inverse Frisch elasticity of labour.
2.1.3 Government

The government is both the monetary and fiscal authority. It costlessly issues fiat money, which it exchanges with the private sector for general output. By issuing (or withdrawing, respectively) money over the period, it earns (or pays, respectively) $p_t(M_{t+1} - M_t)$ units of general output, where $M_t$ and $M_{t+1}$ are the stocks of money in circulation at the start and end of the period, respectively. Fiat money is the only liability of the government. The only asset it owns is a stock of privately-issued equity. At the start of period $t$ it holds $N_{t}^{g}$ units of equity, which earn dividends at a rate of $r_t$ units of general output over the period, depreciate at a rate of $(1 - \delta)$ per period, and are subject to the resaleability constraint, $\phi_t$. Equity holdings at the end of the period are therefore lower bounded as

$$N_{t+1}^{g} \geq (1 - \phi_t)\delta N_{t}^{g} \quad (2.11)$$

The government holds equity for the conduct of open market operations (OMOs), in which $|N_{t+1}^{g} - \delta N_{t}^{g}|$ units are discretionately exchanged for money.

The government consumes $G_t$ units of general output, and collects $T_t$ in taxes from entrepreneurs and workers according to

$$T_t = r_t^r n_t N_t + \tau_t^{wl} w_t L_t \quad (2.12)$$

where $N_t$ is the total private sector’s equity holdings at the beginning of period $t$, and $L_t$ is the aggregate number of labour hours used in the period’s production. The government’s spending is assumed to not directly affect the utility of workers and entrepreneurs or create any production externalities.6

The government balances its overall budget in every period. Therefore, its consumption and net OMO purchases is financed from taxes, dividends, and seignorage, according to

$$G_t + q_t(N_{t+1}^{g} - \delta N_{t}^{g}) = T_t + r_t N_t^{g} + p_t(M_{t+1} - M_t) \quad (2.13)$$

2.1.4 Exogenous variables

The economy is subject to exogenous stochastic shocks to $\{A_t, \phi_t, N_t^{g}, M_t, \tau_t^{r}, \tau_t^{wl}\}$. These variables evolve according to the same stationary AR(1) process,

$$A_t = (1 - \rho_A)A + \rho_A A_{t-1} + u_t^A \quad (2.14)$$

$$\phi_t = (1 - \rho_\phi)\phi + \rho_\phi \phi_{t-1} + u_t^\phi \quad (2.15)$$

5Nezafat and Slavík (2012) point out that this utility specification is unusual in the RBC literature, but shows that quantitative results are not sensitive to the choice of functional form. Alternative specifications are the non-separable or log-separable form of King et al. (1988) and the form of Greenwood et al. (1988) whereby disutility from work directly affects utility from consumption.

6Canova and Pappa (2011) note that if a change in government spending affects private agents’ utility (as in Bouakez and Rebei (2007)) and/or creates production externalities (as in Baxter and King (1993)) then the output response is amplified.
\[
N^g_t = (1 - \rho_{N^g})N^g + \rho_{N^g} N^g_{t-1} + u^N_{t^g}
\]
\[
M_t = (1 - \rho_M)M + \rho_M M_{t-1} + u^M_{t^m}
\]
\[
\tau^r_{t^n} = (1 - \rho_{\tau^r_{t^n}})\tau^r_{t^n} + \rho_{\tau^r_{t^n}} \tau^r_{t^n-1} + u^\tau_{t^n}
\]
\[
\tau^e_{t^w} = (1 - \rho_{\tau^e_{t^w}})\tau^e_{t^w} + \rho_{\tau^e_{t^w}} \tau^e_{t^w-1} + u^\tau_{t^w}
\]

where, for \(X \in \{A, \phi, N^g, M, \tau^r_{t^n}, \tau^e_{t^w}\}\), \(X\) denotes the steady state value of variable \(X_{t^*}\), \(\rho_X\) parameterizes the degree of persistency of a shock to \(X\), \(u^X_{t}\) is an exogenously determined, independently, identically, and Normally distributed innovation with zero mean and constant variance \(\sigma^2_{uX}\), and \(E[u^Y_{t} u^X_{t}] = 0\) for \(X \neq Y \in \{A, \phi, N^g, M, \tau^r_{t^n}, \tau^e_{t^w}\}\). It is assumed that \(|\rho_X| < 1\) so that exogenous shocks are temporary. 7

2.2 Equilibrium

2.2.1 Entrepreneurs

With the investment technology having unlimited capacity, investors are assumed to liquidate and borrow the maximum amount of equity that the liquidity constraints allow. Then the budget constraint (2.3) becomes binding with equality,

\[
n^i_{t+1} = (1 - \theta)i_t + (1 - \phi_t)\delta n_t
\]

which, if re-structured, gives the entrepreneur’s investment for the period,

\[
i_t = \left(\frac{1}{1 - \theta}\right) n^i_{t+1} - \left(\frac{1 - \phi_t}{1 - \theta}\right) \delta n_t
\]

The investor’s budget constraint (2.5) is simplified by substituting Equation (2.21) to obtain (see Appendix C.1)

\[
c^i_t + q^R_{t^i} n^i_{t+1} = (1 - \tau^r_{t^n})r_t n_t + \left[\phi_t q_{t^i} + (1 - \phi_t)q^R_{t^i}\right] \delta n_t + p_t (m_t - m^i_{t+1})
\]

where

\[
q^R_{t^i} = \frac{1 - \theta q_{t^i}}{1 - \theta}
\]

The RHS of Equation (2.22) is the investor’s net worth: his net dividends from equity holdings, the value of depreciated equity (where a resalable fraction, \(\phi_t\), is valued at the market price and the non-resalable fraction is valued at \(q^R_{t^i}\)), and net sales of money. The LHS expresses what he does with his net worth.

\(q^R_{t^i}\) is an effective payment the entrepreneur makes to himself, or his replacement cost, for every unit of inside equity purchased. For an investment cost \(i_t\), the investor raises as much as \(\theta i_t q_{t^i}\) from issuing new equity. This is the investment’s external finance. The remainder of the investment cost, \((i_t - \theta i_t q_{t^i})\), is internally financed from liquid funds, obtained from a combination of liquidating outside equity and from money stocks. \((i_t - \theta i_t q_{t^i})\) therefore represents the total effective payment the investor makes to himself to purchase a fraction of his own new equity.

7With an estimated DSGE model, Mertens and Ravn (2011a) show that responses are different in magnitude, but not in direction, between temporary and permanent tax shocks.
issue. Put differently, for every unit of investment, the entrepreneur retains \((1 - \theta)\) units as inside equity which require an effective payment of \((1 - \theta q_t)\) to himself. Notice from Equation (2.23) that the higher the market price of equity, the more funds he can raise, and the less he is required to spend on accumulating inside equity, i.e. the smaller his effective payment, \(q_t^R\); and conversely.

Alternatively, substituting Equation (2.20) into Equation (2.5) gives the investor’s resource constraint (see Appendix C.1),

\[
c_i^t + (1 - \theta q_t) i_t = (1 - \tau_t^{rn}) r_t m_t + \phi_t q_t \delta n_t + p_t (m_t - m_i^{t+1}) \tag{2.24}
\]

The RHS of Equation (2.24) is the total liquid resources available to the investor in period \(t\): net dividends from equity, a resaleable portion of equity holdings, and net sales of money. The LHS says how he uses these resources: for consumption and financing that portion of his investment for which he cannot borrow.

At the start of period \(t\) the representative investor and saver make optimal choices on \(\{c_i^t, i_t, m_i^{t+1}, n_i^{t+1}\}\) and \(\{c_s^t, n_s^{t+1}, m_s^{t+1}\}\), respectively, conditional on all available information. But their choices are made with uncertainty about investment opportunities in the future. Entrepreneurs’ first order conditions yield an Euler equation (see Appendix C.2),

\[
\pi E_t \left[ \frac{1}{q_t} \left[ (1 - \tau_t^{rn}) r_t^{t+1} + \phi_t^{t+1} \delta q_t^{t+1} + [1 - \phi_t^{t+1}] \delta q_t^{R_t} \right] U'(c_i^{t+1}) \right] + (1 - \pi) E_t \left[ \frac{1}{q_t} \left[ (1 - \tau_t^{rn}) r_t^{t+1} + \delta q_t^{t+1} \right] U'(c_s^{t+1}) \right] = E_t \left[ \frac{p_t^{t+1}}{p_t} \left( \pi U'(c_i^{t+1}) + (1 - \pi) U'(c_s^{t+1}) \right) \right] \tag{2.25}
\]

Once entrepreneurs identify their optimal paths, the Euler equation (2.25) describes their expectation that \(1/q_t\) additional units of equity and \(1/p_t\) additional units of money provide the same marginal utility from consumption. The expression on the RHS of (2.25) is the expected marginal benefit of holding \(1/p_t\) additional units of money. The expression on the LHS of (2.25) is the expected marginal benefit of holding \(1/q_t\) additional units of equity. Each unit is expected to return a net dividend of \((1 - \tau_t^{rn}) r_{t+1}\) plus a depreciated value. If there is an investment opportunity, a resaleable fraction of depreciated equity, \(\delta \phi_{t+1}\), will be valued at the market price, \(q_{t+1}\), while the non-resaleable fraction, \(\delta (1 - \phi_{t+1})\), will be valued at its replacement cost \(q_t^R\). If there is no investment opportunity, the depreciated value will be \(\delta q_{t+1}\).

**Claim 1.** \(q_t \neq 1 \iff m_i^{t+1} = 0\)

**Proof of Claim 1.** See Appendix C.4.

The market price of equity is critical for economic activity. An investor needs at least 1 unit of general output for every unit of equity issued. If \(q_t < 1\) then the entrepreneur will not raise enough funds in the market to fulfil his ambition of investing \(i_t\). Investors will abandon their opportunities, and then all entrepreneurs will become savers. If \(q_t > 1\) then an entrepreneur will be able to materialize his opportunity to invest and sell as much equity as he can within budget.
and liquidity constraints. The following assumption is therefore made to restrict attention to the case where investment takes place in the economy:

**Assumption 1.** \( q_t > 1 \)

By Claim 1 and Assumption 1, an investor will not have any money at the end of a period of investment, i.e. \( m_{t+1} = 0 \). He exhausts all of his money in the pursuit of an investment opportunity. In the next period, up to when new investment opportunities are revealed, the current period’s investors will be able to replenish their money stocks.

The entrepreneur’s logarithmic utility function provides a standard feature that his consumption in each period is a stable fraction, \((1 - \beta)\), of his net worth in that period. From Equations \((2.6)\) and \((2.22)\), Claim 1 and Assumption 1, a typical investor and saver therefore consume, respectively,

\[
\begin{align*}
    c^i_t &= (1 - \beta)([1 - \tau^R_t] r_t n_t + [\phi_t q_t + (1 - \phi_t) q^R_t] \delta n_t + p_t m_t) \\
    c^s_t &= (1 - \beta)([1 - \tau^R_t] r_t n_t + q_t \delta n_t + p_t m_t)
\end{align*}
\]

The difference in consumption between the two types of entrepreneurs is given by

\[
    c^s_t - c^i_t = (q_t - q^R_t)(1 - \phi_t)\delta n_t
\]

Assumption 1 therefore implies \( c^s_t > c^i_t \). Indeed, as entrepreneurs utilize equity and money towards investment financing, they intertemporally substitute consumption away from an investing period and towards a saving period. During a period of saving they accumulate equity and money, and do so in an optimal fashion according to the Euler equation\((2.25)\).

Assumption 1 also implies (see Appendix C.5)

\[
\begin{align*}
    \frac{[1 - \tau^R_t] r_{t+1} + \phi_{t+1} \delta q_{t+1} + (1 - \phi_{t+1}) \delta q^R_{t+1}}{q_t} < \frac{[1 - \tau^R_t] r_{t+1} + \delta q_{t+1}}{q_t}
\end{align*}
\]

i.e. an investor’s equity portfolio generates a lower rate of return than a saver’s equity portfolio. This is because of the limited resaleability of equity for an investor, which forces him to own inside equity that is valued negatively to the market price of equity. Hence, the return on equity is correlated with consumption. This correlation, along with the resaleability constraint, is what makes equity risky. Money, on the other hand, is free from these risks. Its return does not depend on having an investment opportunity, and it is perfectly liquid; hence two reasons why entrepreneurs hold money. Saving entrepreneurs also accumulate money in preparation for when they receive investment opportunities and expect to face financing constraints.

The Euler equation \((2.25)\) simplifies to a portfolio balance equation (see Appendix C.3),

\[
\pi E_t \left[ \frac{p_{t+1}}{q_t} - \left( \frac{[1 - \tau^R_t] r_{t+1} + \phi_{t+1} \delta q_{t+1} + (1 - \phi_{t+1}) \delta q^R_{t+1}}{q_t} \right) \right]
\]

11
\[
E_t \left[ \frac{(1-\tau_{t+1}) q_{t+1} + \delta q_{t+1}}{1-\tau^n_{t+1}} r_{t+1} n_{t+1} + q_{t+1} \delta n_{t+1} + p_{t+1} m_{t+1}} - \frac{p_{t+1}}{p_t} \right]
\]  
(2.29)

Equation (2.29) reflects the portfolio balance theory of Tobin (1958, 1969) and demonstrates substitution between assets when their relative price changes. If \( q_t \) rises, for example, then equity’s expected return falls, and entrepreneurs substitute towards money. The substitution itself creates an increase in demand for money, which, \textit{ceteris paribus}, raises \( p_t \). Substitution moves back and forth until expected portfolio returns between having and not having an investment opportunity are equal. The LHS of Equation (2.29) expresses an expected excess return on money over equity if the entrepreneur becomes an investor. The RHS expresses an expected excess return on equity over money if he becomes a saver. The portfolio balance equation says that the \textit{ex ante} identical entrepreneur equates the expected marginal benefits of receiving and not receiving an investment opportunity. He does this by varying how many units of equity and money he holds.

### 2.2.2 Workers

At the start of period \( t \) the representative worker chooses \( \{c^w_t, l^w_t, n^w_{t+1}, m^w_{t+1}\} \) to maximize his expected discounted utility, subject to his budget constraint. The individual worker optimally supplies labour until the marginal disutility of work (or equivalently, the marginal utility of leisure) equals the real wage rate. First order conditions yield his supply of labour (see Appendix C.6),

\[
l^w_t = \left[ \frac{(1-\tau^n_{t+1}) w_t}{\omega} \right]^{\frac{1}{\nu}}
\]  
(2.30)

**Claim 2.** \( n^w_{t+j} = 0 \) and \( m^w_{t+j} = 0 \), for \( j = 0, 1, 2, \ldots \), i.e. the worker will always choose not to hold equity and money.

**Proof of Claim 2.** From Equations (C.9), (C.11) and (C.12) in Appendix C.6, if the worker decides to hold equity and money, i.e. if \( n^w_{t+1} \neq 0 \) and \( m^w_{t+1} \neq 0 \) then

\[
\frac{\delta q_{t+1} + (1-\tau^n_{t+1}) r_{t+1}}{q_t} = \frac{p_{t+1}}{p_t} = \frac{1}{\beta}
\]  
(2.31)

Equation (2.31) says that holding equity and money will not provide any superior (expected) gains above the discounted marginal utility from consumption, \( 1/\beta \). If the worker has one more unit of general output, he gains as much by consuming it as he expects to gain by saving it. Then there is no reason for the worker to save. The worker saves only if there is a marginal benefit from doing so.  

By Claim 2, the worker’s budget constraint (2.9) simplifies to

\[
c^w_t = (1-\tau^n_{t+1}) w_t l^w_t
\]  
(2.32)

i.e. in each period the worker’s consumes his entire net wage earnings, thus making him non-
2.2.3 The labour market

From the production function (2.1), the marginal product of labour is

\[
\frac{\partial y_t}{\partial l_t} = (1 - \gamma) A_t \left( \frac{k_t}{l_t} \right)^\gamma
\]

From Equation (2.2), the first order condition for gross profit maximization with respect to labour is

\[
\frac{\partial y_t}{\partial l_t} - w_t = 0
\]

\[
\Rightarrow (1 - \gamma) A_t \left( \frac{k_t}{l_t} \right)^\gamma - w_t = 0
\]

\[
\Rightarrow l_t = k_t \left[ \frac{(1 - \gamma) A_t}{w_t} \right]^\frac{1}{\gamma}
\]

which is the entrepreneur’s demand for labour, given his capital stock.

With an aggregate capital stock, \( K_t \), owned entirely by entrepreneurs, it follows that the aggregate demand for labour is

\[
L^D_t = K_t \left[ \frac{(1 - \gamma) A_t}{w_t} \right]^\frac{1}{\gamma}
\] (2.33)

Given the homogeneity and unit mass of the worker population, from Equation (2.30), the aggregate labour supply labour is

\[
L^S_t = \left[ \frac{(1 - \tau_{w}) w_t}{\omega} \right]^\frac{1}{\nu}
\] (2.34)

The inverse versions of these labour market functions (2.33) and (2.34) are, respectively,

\[
w^D_t = \frac{(1 - \gamma) A_t K^\gamma_t}{(L_t)^\gamma}
\] (2.35)

\[
w^S_t = \left[ \frac{\omega}{1 - \tau_{w}^{\nu}} \right] (L_t)^\nu
\] (2.36)

Notice from Equation (2.35) that the slope of the inverse aggregate labour demand function

---

8 The non-Ricardian feature is how this model departs from the standard Real Business Cycle model and starts to resemble Keynesian IS-LM. Driffill and Miller (2013) algebraically show that the KM model is fundamentally IS-LM by simplifying it down to two equations that resemble IS and LM functions. If workers are Ricardian, as in the standard RBC model, then a cut in the income tax rate increases the present value of disposable income, and thus creates a positive wealth effect that induces a rise in saving and a drop in consumption. But here, a cut in the income tax rate increases workers’ consumption; this will be seen in Section 3. The non-Ricardian feature of this model arises endogenously. Elsewhere in the literature, where such behaviour is exogenously assumed to hold (in Gali et al. (2007), for example), it is justified by such things as lack of access to financial markets, myopia, or fear of saving. Campbell and Mankiw (1989) provide empirical support for the existence of non-Ricardian behaviour, while Mankiw (2000) reviews microeconomic evidence that supports such behaviour.
depends negatively on capital elasticity of output, $\gamma$, and positively on productivity, $A_t$, and the capital stock, $K_t$. The function’s curvature inversely depends on $\gamma$. Also, notice from Equation (2.36) that supply’s slope varies positively with the relative utility weight, $\omega$, and the rate of tax on wages, $\tau^w_t$, while its curvature varies positively with the inverse Frisch elasticity, $\nu$.

Given that wages are flexible, the labour market always clears and full employment is guaranteed. The market clears when $L^S_t = L^D_t$, which gives the equilibrium real wage and employment (see Appendix C.7),

$$w_t = K^\frac{\nu}{\gamma+\nu} \omega^\frac{\gamma}{\gamma+\nu} (1 - \tau^w_t)^{\frac{-\gamma}{\gamma+\nu}} [(1 - \gamma) A_t]^{\frac{\nu}{\gamma+\nu}}$$  \hspace{1cm} (2.37)

$$L_t = K^\frac{\gamma}{\gamma+\nu} \omega^\frac{1}{\gamma+\nu} [(1 - \tau^w_t)(1 - \gamma) A_t]^{\frac{1}{\gamma+\nu}}$$  \hspace{1cm} (2.38)

### 2.2.4 The equity market

Since every unit of equity produces a unit of capital, then the total stocks of equity and capital are always equal, i.e.

$$K_t = N_t + N^g_t$$  \hspace{1cm} (2.39)

The law of motion for the aggregate capital stock is

$$K_{t+1} = I_t + \delta K_t$$  \hspace{1cm} (2.40)

Over the period, investors issue new equity amounting to $\theta$ of their investments, $I_t$, and sell $\phi_t$ of their depreciated equity holdings, $\pi \delta N_t$, to savers. And since investors do not hold money, savers are the only participants in government OMOs. The stock of equity held by savers at the end of the period is therefore

$$N^s_{t+1} = (1 - \pi) \delta N_t + \theta I_t + \phi_t \pi \delta N_t - (N^g_{t+1} - \delta N^g_t)$$  \hspace{1cm} (2.41)

Equation (2.41) can be re-expressed as an equity market clearing condition,

$$(N^s_{t+1} - \delta N^s_t) + (N^s_{t+1} - \delta N^s_t) = \phi_t \pi \delta N_t + \theta I_t$$  \hspace{1cm} (2.42)

The RHS of Equation (2.42) is the supply of equity, which originates from investors’ need for finance: they liquidate a fraction, $\phi_t$, of the stock they own, $\pi N_t$, and they issue $\theta I_t$ new units. The expression $(N^s_{t+1} - \delta N^s_t)$ on the LHS represents private demand for equity, which comes from savers. The government buys $(N^g_{t+1} - \delta N^g_t)$ units of equity (or sells, in which case this expression enters negatively in Equation (2.42)).

### 2.2.5 Aggregate demand and supply

Aggregate private consumption is

$$C_t = C^i_t + C^s_t + C^w_t$$  \hspace{1cm} (2.43)
where, from Equations (2.26), (2.27) and (2.29), consumption by investors, savers, and workers are, respectively,

\[ C_i^t = \pi(1 - \beta)([1 - \tau_r^m]r_t N_t + \phi_t q_t + (1 - \phi_t) q^R_{t+1}) \delta N_t + p_t M_t \] (2.44)
\[ C_s^t = (1 - \pi)(1 - \beta)([1 - \tau_r^m]r_t N_t + q_t \delta N_t + p_t M_t) \] (2.45)
\[ C_w^t = (1 - \tau_w^m)w_t L_t \] (2.46)

As savers accumulate equity and money over the period, their aggregate asset portfolio at the end of period \( t \) satisfies an aggregate portfolio balance equation,

\[
\pi E_t \left[ \frac{(p_{t+1} + 1)}{p_t} - \left( \frac{[1 - \tau_r^{m+1}]r_{t+1} + \phi_{t+1} q_{t+1} + [1 - \phi_{t+1}] q^R_{t+1}}{q_t} \right) \delta N_{t+1}^s + p_{t+1} M_{t+1} \right] = (1 - \pi) E_t \left[ \frac{[1 - \tau_r^{m+1}]r_{t+1} + q_{t+1} \delta N_{t+1}^s + p_{t+1} M_{t+1}}{1 - \tau_r^{m+1} r_{t+1} N_{t+1}^s} \right]
\] (2.47)

From Equation (2.24), total investment spending is

\[
(1 - \theta q_t) I_t = ([1 - \tau_r^m]r_t + \phi_t q_t) \pi N_t + \pi p_t M_t - C_i^t
\] (2.48)

From Equation (2.1), aggregate supply is

\[ Y_t = A_t K_t^\gamma L_t^{1 - \gamma} \] (2.49)

From Equation (2.2), aggregate gross profits are

\[ r_t K_t = Y_t - w_t L_t \] (2.50)

which gives gross rate of return to capital (see Appendix C.8),

\[ r_t = a_t K_t^{\alpha - 1} \] (2.51)

where

\[ a_t \equiv \gamma \left( \frac{(1 - \gamma)(1 - \tau_w^m)}{\omega} \right)^{\frac{1 - \gamma}{\nu + \gamma}} A_t^{\frac{1 + \nu}{\nu + \gamma}} \] (2.52)
\[ \alpha \equiv \frac{\gamma(1 + \nu)}{\nu + \gamma} \] (2.53)

Finally, the goods market clears when

\[ Y_t = C_t + I_t + G_t \] (2.54)

In summary, the dynamic equilibrium of the model is characterized by equations (2.12), (2.13), (2.37), (2.38), (2.39), (2.40), (2.42), (2.43), (2.44), (2.45), (2.46), (2.47), (2.48), (2.49),
2.3 Steady state

To introduce a balanced budget fiscal policy rule for government, the following assumption is made about steady state.\textsuperscript{9}

Assumption 2. In steady state, the government holds no equity, i.e. $N^g = 0$.

For any equity the government buys or sells, the exchange is made with money. If the government does not hold equity in steady state then the money supply does not change. This implies the government earns no dividends or seignorage revenue, and must therefore balance its fiscal budget to satisfy Equation (2.13). Assumption 2 together with exogeneity of money supply (by Equation (2.17)) imply that the government holds no equity in every period, and its budget constraint (2.13) becomes $G_t = T_t$ for all $t$.

3 Impulse response analysis

This section presents the results of simulating unexpected, exogenous, temporary cuts to both tax rates (a “tax shock”). The cuts to wage and dividend income tax rates are equivalent to 1% and 2.8% of national output, respectively. Tax collections fall by 3.3% \textit{ceteris paribus}, and by 3.1% with endogenous changes in tax bases. From the next period, the tax shock deteriorates at an assumed rate of 5% per quarter, and both tax rates asymptotically increase towards their steady state levels. The main results of this section are that the shock increase output, private consumption, and investment; the increases in output and consumption are significant and long-lasting; and the liquidity constraints play a major role in the shock’s long-term persistence.

The shock is simulated by the quantitative technique of calibration. Appendix A describes the calibration process in detail. Structural parameters are set to the “baseline” values in Table 4 (in Appendix A); these values are taken from similar models in the related literature. Steady state levels and autoregressive (shock) parameters for exogenous variables are set according to values listed in Table 5 (in Appendix A). All parameter values are based on quarterly data. Results are presented as impulse responses, which are graphically illustrated Figure 1 and summarized in Table 1.

3.1 Immediate responses

Labour market. In response to the cut in the rate of tax on wage income, workers increase their labour supply at each and every wage (from Equation (2.34)).\textsuperscript{10} In reply, entrepreneurs

\textsuperscript{9}With Assumption 2 the implied steady state system is simple enough to algebraically solve by hand; this solution is given in Appendix D. Without Assumption 2 the implied steady state system is too complex to algebraically solve by hand; nevertheless, a numerical simulation reveals that in this steady state the government holds an insignificantly small quantity of equity. With or without Assumption 2, the steady state system offers two positive roots for $q$. One root is greater than 1, i.e. high enough to guarantee investment, and is therefore accepted as equity’s steady state price.

\textsuperscript{10}More precisely, the baseline setting for $\nu$ implies that the inverse aggregate labour supply function (2.36) is linear through the origin, as illustrated in Figure 12. The shock therefore \textit{pivots} the supply curve outwards.
Figure 1: Impulse responses of a temporary, negative tax shock: baseline scenario

NOTES: Vertical axes measure percentage deviation from steady state. Horizontal axes measure quarters after the shock, starting from quarter 1. The blue dot indicates the quarter 1 response. Impulse responses for \{K_t, N^*_s\} occur at the end of the quarter.
Table 1: Impulse responses of a tax shock: baseline scenario

<table>
<thead>
<tr>
<th></th>
<th>Impulse responses (%)</th>
<th>Quarters to Largest 10% of Qu. 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quarter: 1 2 4 8 20 200</td>
<td>Largest</td>
</tr>
<tr>
<td>$T_t$</td>
<td>-3.1 -2.9 -2.6 -2 -0.9 0.1</td>
<td>-3.1</td>
</tr>
<tr>
<td>$Y_t$</td>
<td>1.1 1.2 1.3 1.5 1.8 0.2</td>
<td>1.8</td>
</tr>
<tr>
<td>$I_t$</td>
<td>1.9 1.8 1.7 1.4 0.9 0</td>
<td>1.9</td>
</tr>
<tr>
<td>$C_t$</td>
<td>2.3 2.3 2.2 2.1 1.7 0.2</td>
<td>2.3</td>
</tr>
<tr>
<td>$C^{w}_t$</td>
<td>1.7 1.6 1.6 1.5 1.3 0.1</td>
<td>1.7</td>
</tr>
<tr>
<td>$C^l_t$</td>
<td>0 0 0 0 0 0</td>
<td>0</td>
</tr>
<tr>
<td>$C^s_t$</td>
<td>0.6 0.6 0.6 0.5 0.4 0</td>
<td>0.6</td>
</tr>
<tr>
<td>$N^s_t$</td>
<td>0.3 2.1 5.3 10.4 19.2 3.4</td>
<td>22.2</td>
</tr>
<tr>
<td>$w_t$</td>
<td>-0.9 -0.8 -0.6 -0.2 0.5 0.2</td>
<td>0.9</td>
</tr>
<tr>
<td>$L_t$</td>
<td>0.4 0.4 0.4 0.4 0.3 0</td>
<td>0.4</td>
</tr>
<tr>
<td>$r_t$</td>
<td>0 0 0 0 0 0</td>
<td>0</td>
</tr>
<tr>
<td>$K_t$</td>
<td>1.9 3.6 6.8 12 20.8 3.6</td>
<td>23.6</td>
</tr>
<tr>
<td>$p_t$</td>
<td>6.6 6.3 5.7 4.7 2.6 0</td>
<td>6.6</td>
</tr>
<tr>
<td>$q_t$</td>
<td>3.2 3.1 2.7 2.2 1 0</td>
<td>3.2</td>
</tr>
</tbody>
</table>

NOTES: This table gives impulse responses at certain periods after the shock, and the period of time it takes for impulse responses to reach their peak and to get within 10% of their quarter 1 magnitudes. Convergence of 200 quarters means some time after 200 quarters, and not in the 200th quarter. Impulse responses for stock variables $\{K_t, N^s_t\}$ occur at the end of the quarter.

Output. Output increases because of more employment (from Equation (2.49)). The other determinants of output are unchanged by the shock. Total factor productivity is exogenously determined (by Equation (2.14)), and the period’s capital stock is determined before the shock hits.

Entrepreneurs. The cut in the dividend income tax improves the net worth of all entrepreneurs. As a result, these agents all increase their consumption (by Equation (2.44) and Equation (2.45)). Those who receive investment opportunities early in the quarter spend more on investment (from Equation (2.48)), while savers increase their demand for both assets.

Money market. Figure 2B illustrates the immediate effects of the shock on the money market. Demand comes from entrepreneurs and supply is fully controlled by the government. $D^M_0$ and $S^M_0$ are the initial (i.e. steady state) demand and supply curves, respectively. Savers increase their demand for money after net worth improvements; this is illustrated by a rightward
Figure 2: Asset markets and immediate effects of the tax shock

(A) Equity

\[q_t \quad N_0 \quad S_0^N \quad D_0^N \quad S_1^N \quad D_1^N \quad N_1 \]

(B) Money

\[p_t \quad M_0 \quad S_0^M \quad D_0^M \quad D_1^M \quad M_1 \]

NOTES: \(D\) denotes demand and \(S\) denotes supply. Superscripts \(N\) and \(M\) denote equity and money, respectively. Subscripts 0 and 1 denote steady state and quarter 1, respectively.

The shift of \(D_0^M\) to \(D_1^M\). With exogenously fixed money supply, the result is a higher price.

**Equity market.** Figure 2A illustrates immediate effects of the shock on the equity market. \(D_0^N\) and \(S_0^N\) are the initial (steady state) demand and supply curves, respectively. Demand comes from savers and the government; supply comes from investors. Savers increase their demand for equity following net worth improvements; this is illustrated by a rightward shift of \(D_0^N\) to \(D_1^N\). Equity’s supply increases as investors issue new issues to finance their investment; this is represented by a rightward shift of \(S_0^N\) to \(S_1^N\). The increase in supply is small; because of the borrowing constraint, investors issue a small amount of equity relative to the additional investment cost. As a consequence, equity’s market price increases.

**Internal amplification.** A demand for money exists because of both liquidity constraints; the borrowing constraint, in particular, restricts the shock-induced increase in equity supply to be small, and is therefore responsible for the asset’s higher price. Increases in both asset prices amplify the net worth improvements of entrepreneurs, which in turn increases asset demand and investment. This “internal amplification” mechanism originates from the liquidity constraints and is the cause of the long-term persistence of the shock.

Consider a counterfactual scenario. If the borrowing constraint is calibrated sufficiently loose then the shock encourages investors to issue a considerable amount of new equity, and (as if \(S_1^N\) is further to the right than drawn in Figure 2A) the shock then reduces the price of the asset. This result implies a higher expected return on equity, which encourages agents to substitute away from money. Depending on the relative sizes of this portfolio balance effect and the positive wealth effect, money’s price either increases by a smaller degree than in the baseline calibration or decreases altogether. If the latter result holds, i.e. both asset prices
fall, then the *ceteris paribus* effects on entrepreneurs from the cut in the dividend tax rate are (partially or completely) offset. In particular, a first quarter increase in investment is not achieved. Without an increase in investment, aggregate demand decreases. And, as explained in Section 3.2, investment is key to the internal propagation of the shock, particularly for output.

**Private consumption.** Aggregate private consumption’s largest contributor comes from workers. The fall in the real wage is larger than the rise in employment (see Table 1), and therefore the aggregate gross wage decreases. But because of the drop in the tax rate, workers enjoy a higher aggregate net wage and, being non-Ricardian, consume more general output. Savers and investors consume more because of improvements in the net worth. Accordingly, total consumption increases.

**Government.** The cuts to wage and dividend income tax rates are equivalent to 1% and 2.8% of national output, respectively. Tax collections fall by 3.3% *ceteris paribus*.\(^{11}\) With more output and a smaller aggregate gross wage bill, the aggregate gross dividends to capital are higher (from Equation (2.50)). These endogenous changes in tax bases together with the tax rate cuts cause tax collections to fall by 3.1% (from Equation (2.12)). The government holds no stocks of money and equity, and is therefore unaffected by asset price changes. The fiscal budget is balanced by matching the decrease in taxation with a reduction in government spending. However, the strong increase in investment together with the increase in private consumption outweigh the decrease in government spending.

### 3.2 Path to adjustment

The speed of convergence is indicated by the time a variable takes to get “close” to steady state. A variable is considered “close” to steady state if the percentage deviation is within 10% of the immediate shock response. The last column of Table 1 gives this indicator convergence. Table 1 also gives the largest impulse response by magnitude and the quarter in which this occurs.

The model’s calibration assumes that, from the second quarter, the shock deteriorates at an assumed rate of 5% per quarter, and both tax rates asymptotically increase towards their steady state levels. As the effects of the shock wear off, asset prices and private consumption converge steadily towards steady state.

The steady state level of investment creates a quantity of new capital that exactly replaces depreciated units. Since the shock increases investment above its steady state level, the capital stock increases by the end of the first quarter. In the second quarter, as tax rates start increasing towards pre-shock levels, workers reduce their labour supply; in the labour market, this produces an increase in the real wage and a decrease in employment. Long-term economic growth is thus achieved by increases in the capital stock. This is why investment is described as the “internal propagation” mechanism.

Furthermore, with more output and a lower aggregate wage bill, gross aggregate dividends are higher. This in part helps entrepreneurs enjoy higher-than-normal net worth. Investment is therefore sustained above steady state, and the capital stock continues to increase. The stock

\(^{11}\)This is measured by a specially-constructed variable \(T_t^*\), which is described in Section 4.
peaks after almost 9 years at 23.6% above steady state. By then, the level of depreciation starts to exceed investment, and the capital stock starts to return to its pre-shock level. This is why capital exhibits a hump-shaped trajectory.

Employment persistently declines since its initial response to the shock. Workers continue with their supply reduction in response to the increase in the rate of tax. The real wage increases, and even overshoots its steady state level in the 10th quarter. It peaks just after 11 years at 0.9% above steady state (after falling below by 0.9% in the first quarter). The eventual decline in the wage rate occurs when entrepreneurs reduce their labour demand when the capital stock starts decreasing.

Output remains heavily influenced by capital, and even traces the same hump-shaped trajectory. Over its adjustment path, output is kept elevated above its steady state level while it converges. It increases continuously for almost 7 years before returning towards its pre-shock level. But the return is slow, and even after 200 quarters it is still approximately 0.2% above steady state, after being 1.8% above at its peak.

4 Tax multipliers

The objective of this section is to describe the shock responses in Section 3 as either “large” or “small”. To do this, responses are quantified by tax multipliers. A variable’s response is then described as “large” if the absolute value of its tax multiplier is in excess of unity; otherwise the response is “small”. Multipliers are more suitable than impulse responses for describing the magnitude of the effects of the shock, for two reasons. Firstly, the shock is not normalized, because of its duality, and impulse responses are therefore difficult to interpret on their own. Multipliers, on the other hand, measure normalized responses. Secondly, these multipliers are constructed to measure the fall in tax collections due to cuts in both tax rates, ceteris paribus. They therefore disentangle the discretionary change in taxes (i.e. the change in tax rates) from the endogenous component (i.e. the changes in tax bases, wtL, and rtK).

Multipliers are computed according to the methodology outlined below, and results are given in Table 2. This section omits any further multiplier analysis of the tax shock, because it would say the same things as the impulse responses analysis in Section 3.

4.1 Methodology

Tax multipliers are obtained for real aggregate variables only. Impact and cumulative multipliers are calculated. Changes in both tax rates are captured by a single variable, T∗t, which represents government tax collections with tax bases held constant to their steady state levels, i.e. from Equation (2.12),

\[ T^*_t = \tau^{rt}rN + \tau^{wt}wL \tag{4.1} \]

where notation without time subscripts represent steady state (i.e. t = 0) values. Changes in T∗t therefore represent ceteris paribus changes in taxes. The deviation of taxes in period t from

\footnote{Perotti (2012) highlights the importance of separating discretionary from endogenous changes in taxes by showing they have different effects on output.}
steady state with tax bases held constant is

$$T^*_t - T = (\tau_t^{rn} - \tau^{rn})rN + (\tau_t^{wl} - \tau^{wl})wL$$  \hspace{1cm} (4.2)

If the immediate post-shock tax rates are $\tau_t^{rn}$ and $\tau_t^{wl}$ then the immediate change in $T^*_t$ is

$$T^*_1 - T = (\tau_1^{rn} - \tau^{rn})rN + (\tau_1^{wl} - \tau^{wl})wL$$  \hspace{1cm} (4.3)

Tax multipliers therefore measure the response of a variable, over a given period of time, to a drop in government tax receipts by 1 unit of general output due to discretionary cuts in both tax rates, ceteris paribus.

Impact multipliers measure the response of a real aggregate variable in period $t$ to the change in $T^*_t$ in period 1,

$$\frac{X_t - X}{T^*_t - T}$$  \hspace{1cm} (4.4)

Of special interest in this class of multipliers is the period 1 response of the variable, or the immediate impact multiplier,

$$\frac{X_1 - X}{T^*_1 - T}$$  \hspace{1cm} (4.5)

Cumulative multipliers measure the variable’s response over a period of time. They capture accumulated changes in the variable as well as accumulated changes in the policy variable. Cumulative tax multipliers are measured over 0.5, 1, 2, 3, and 4 year horizons according to

$$\frac{\sum_{t=1}^{n} (X_t - X)}{\sum_{t=1}^{n} (T^*_t - T)}$$  \hspace{1cm} (4.6)

4.2 Results: multiplier responses

Multipliers measure the change in a real variable, i.e., in terms of units of output, for every unit of general output that the government gives up in tax revenue in the first quarter, due to discretionary cuts in both tax rates ceteris paribus. A negative (positive, respectively) multiplier indicates that the variable increases (decreases, respectively) after the shock. If cumulative multipliers of a variable increase (decrease, respectively) with the measured time horizon, then the variable is converging slower (faster, respectively) than $T^*_t$ towards steady state.

Output and private consumption have very large, negative responses to the shock, both contemporaneously and cumulatively. On impact, output and consumption increase by 2.9 and 3.5 units of general output, respectively, for every unit the government gives up from tax cuts, ceteris paribus. Both their cumulative multipliers increase with the time horizon. This suggests that while the shock itself wares off, its effects on output and consumption continue to propagate. It also confirms the slow convergence that is suggested graphically by Figure 1. Investment has a moderate increase. A unitary multiplier is observed on impact of the shock, and as the shock wares off, investment converges at a uniformly proportional and slightly slower rate than $T^*_t$.

By far, the largest responses are by asset prices. They both increase substantially, with impact multipliers of –11.8 for money and –5.3 for equity, and with similar values cumulatively
Table 2: Multipliers of a tax shock: baseline scenario

<table>
<thead>
<tr>
<th></th>
<th>Immediate impact</th>
<th>Cumulative 6-month</th>
<th>Cumulative 1-year</th>
<th>Cumulative 2-year</th>
<th>Cumulative 3-year</th>
<th>Cumulative 4-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_t )</td>
<td>-2.9</td>
<td>-3.1</td>
<td>-3.4</td>
<td>-4.2</td>
<td>-4.9</td>
<td>-5.7</td>
</tr>
<tr>
<td>( I_t )</td>
<td>-1.0</td>
<td>-1.1</td>
<td>-1.1</td>
<td>-1.1</td>
<td>-1.1</td>
<td>-1.1</td>
</tr>
<tr>
<td>( C_t )</td>
<td>-3.5</td>
<td>-3.6</td>
<td>-3.7</td>
<td>-4.0</td>
<td>-4.2</td>
<td>-4.5</td>
</tr>
<tr>
<td>( C_t^w )</td>
<td>-2.1</td>
<td>-2.1</td>
<td>-2.2</td>
<td>-2.3</td>
<td>-2.5</td>
<td>-2.7</td>
</tr>
<tr>
<td>( C_t^i )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( C_t^s )</td>
<td>-0.2</td>
<td>-0.2</td>
<td>-0.2</td>
<td>-0.2</td>
<td>-0.2</td>
<td>-0.2</td>
</tr>
<tr>
<td>( w_t )</td>
<td>3.1</td>
<td>3.0</td>
<td>2.7</td>
<td>2.1</td>
<td>1.5</td>
<td>0.9</td>
</tr>
<tr>
<td>( r_t )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( p_t )</td>
<td>-11.8</td>
<td>-11.8</td>
<td>-11.8</td>
<td>-11.9</td>
<td>-11.9</td>
<td>-11.9</td>
</tr>
<tr>
<td>( q_t )</td>
<td>-5.3</td>
<td>-5.2</td>
<td>-5.2</td>
<td>-5.2</td>
<td>-5.1</td>
<td>-5.0</td>
</tr>
</tbody>
</table>

NOTES: This table shows the multiplier responses to exogenous cuts in tax rates, *ceteris paribus*. Responses are measured relative to the drop in overall tax collections while holding tax bases constant. For every 1 unit less of general output the government collects in tax revenue in the first quarter, multipliers measure the consequential changes in endogenous variables. A negative multiplier indicates that the variable increases after the drop in taxes.

5 Sensitivity analysis

The robustness of results is examined with respect to the calibration of structural parameters and the persistence of tax shocks. For brevity, the details and results of these exercises are provided in Appendix B. The overall conclusion is that results are qualitatively robust, but quantitatively sensitive, to assumptions regarding structural parameter values, and qualitatively and quantitatively sensitive to significant variations in the persistence of tax shocks.

5.1 Sensitivity to structural parameters.

Structural parameter sensitivity analysis is performed systematically by three local methods, all involving repeated simulations of the shock with combinations of sensitivity settings that are listed in Table 4. The first method uses one change in one parameter at a time; the second and third methods use combinations of two or more sensitivity parameter values. Results are quantitatively sensitive to one-at-a-time variation of three structural parameters: the subjective discount factor (\( \beta \)), capital’s share in output (\( \gamma \)), and the survival rate of capital after depreciation (\( \delta \)). The model is also sensitive to combinations of alternative parameter settings, more so when these settings go beyond those of \( \beta \), \( \gamma \), and \( \delta \). Nevertheless, from changing parameter values either one-at-a-time or in combinations, tax shock responses vary only in magnitude, and
not in direction or adjustment trajectories. Finally, comparing baseline responses to alternatives from all possible combinations of parameter values shows that, with the exception of investors’ consumption, $C_i^t$, baseline responses are not extreme.

5.2 Sensitivity to the persistence of tax shocks.

Sensitivity to the persistence of tax shocks, $\rho_{\tau_{wl}}$ and $\rho_{\tau_{rn}}$, is examined by repeatedly simulating the shock with simultaneous use of values above and below the calibrated (and fairly standard) setting. Results are quantitatively and qualitatively sensitive to the calibration of the persistence parameters for tax shocks, $\rho_{\tau_{rn}}$ and $\rho_{\tau_{wl}}$. Very small changes in the persistence parameters do very little to alter responses; but with larger parameter variations there are significant changes in trajectory and convergence. Lowering the level of persistence reveals that investment, savers’ equity, capital, investors’ consumption, and output are still slow to converge to steady state. This suggests that features in the model – in particular, the liquidity constraints – are responsible for their long-term responses the shock. The mechanism, called the “internal amplification” mechanism, is described in the analysis of the shock.

6 Discussion

6.1 Comparison of results

This paper is closely related to Mountford and Uhlig (2009) (henceforth MU), who show that an unexpected, exogenous increase in government spending that is completely financed by an increase in taxation causes reductions in private consumption and investment on impact, as well as in output from the second period. The converse of this result suggests a recipe for debt-free economic expansion. This paper complements MU by showing that the converse of their result is also true. The novelty of this paper is that while MU obtain their results from an empirical study with vector autoregressions, this paper is a theoretical investigation using a mostly neoclassical DSGE model.

Eggertsson (2010) also propose a balanced budget stimulus with tax rate cuts. He uses a New Keynesian model with sticky prices and monopolistic competition, and compares the effects of cutting different tax rates. His recipe, therefore, is to cut consumption taxes and raise wage income and wealth taxes. However, he also suggests that liquidity constraints may reverse the intended responses. Conclusions between this paper and Eggertsson (2010) are different because this paper does not feature consumption taxes or New Keynesian frictions.

Romer and Romer (2010), Mertens and Ravn (2011a,b, 2012), and Monacelli et al. (2012) estimate vector autoregressions using, as the basis of datasets, the narrative record of exogenous US fiscal shocks developed by Romer and Romer (2010). These papers explore and quantify the macroeconomic effects of changes in taxation. Their peak cumulative multipliers are given in Table 3. Despite the differences in methodology, these papers arrive at the same conclusion as this paper, i.e. they support fiscal expansion by tax reductions. Estimated multipliers for output, consumption, and investment are large and negative, and responses exhibit long-term

\[\text{[Footnote]}\]

13 There is, however, a small increase in output on impact, with a multiplier of 1.3.
14 In the case of Mertens and Ravn (2011a), Table 3 gives multipliers from unexpected tax shocks.
Table 3: Multipliers: a survey

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Employment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Romer and Romer (2010)</td>
<td>–2.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10 quarters)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mertens and Ravn (2011a)</td>
<td>–2.0</td>
<td>–2.0</td>
<td>–10.0</td>
<td>–1.0</td>
</tr>
<tr>
<td>(10 quarters)</td>
<td>(10 quarters)</td>
<td>(10 quarters)</td>
<td>(10 quarters)</td>
<td></td>
</tr>
<tr>
<td>Mertens and Ravn (2012)</td>
<td>–1.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3 quarters)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monacelli et al. (2012)</td>
<td>–2.7</td>
<td>–9.7</td>
<td>–0.5</td>
<td></td>
</tr>
<tr>
<td>(1 year)</td>
<td>(1 year)</td>
<td>(2 quarters)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTES: This table gives the peak cumulative multipliers from a 1% cut in taxation, and (in brackets) the time after the shock these multipliers are observed. A negative multiplier therefore represents an increase in the variable.

persistence. However, their results suggest weaker output and consumption responses and a much stronger investment response than the ones found here.

6.2 Relationship with the KM-related literature

The 2007/8 financial turmoil brought a wave of recent attention to Kiyotaki and Moore (2012), for two reasons. First, commentators argue that the cause of the crisis was the sudden and unexpected deterioration in the value of partially liquid private financial assets (Brunnermeier (2009), Del Negro et al. (2011), Bigio (2012) and Jermann and Quadrini (2012)). Assets' resaleability and collateral suitability were thus adversely affected. This event bears a striking resemblance to KM’s negative liquidity shock. Secondly, the government holding risky, privately-issued assets with limited resaleability was the central component of the unconventional policy responses to the crisis, whereby these assets were exchanged for safe, liquid, government-issued securities and cash. This is, in fact, KM’s main policy implication, that government can inject liquidity to counter-cyclically dampen business cycle fluctuations.

The KM model is theoretically adjusted and/or extended in a series of recent papers. These papers can be classified into two groups. The first group uses the KM model to evaluate the unconventional policies seen in the crisis; Del Negro et al. (2011) and Driffill and Miller (2013) are members of this research, and both show that recessions would have been exacerbated had it not been for government interventions. The second KM-related group returns to the original questions posed by KM on the importance of (i) liquidity shocks for explaining business cycles, and (ii) liquidity constraints for the propagation of productivity shocks. Papers in this group include Salas-Landeau (2010), Bigio (2010, 2012), Ajello (2011), Nezafat and Slavík (2012), Shi (2012), and Jermann and Quadrini (2012).

The inclusion of distortionary taxes and a balanced budget rule is not unique in KM-related literature. Ajello (2011), Shi (2012), and Driffill and Miller (2013) have a balanced budget

15The various facilities through which the US government implemented these exchanges are described in Arman-tier et al. (2008), Fleming et al. (2009), Adrian et al. (2009), and Adrian et al. (2011).
rule for government. Ajello (2011) also includes distortionary taxes, but he modifies the KM model more extensively than in this paper. The uniqueness of this contribution is that it is the first to examine fiscal shocks in the KM model. What is shared with these papers, the second KM-related group in particular, is showing the macroeconomic significance of KM’s liquidity constraints in propagating exogenous shocks. In this case, however, the shocks are to tax rates.

6.3 The significance to New Keynesian frictions

One significance of this paper is that it shows how a neoclassical model can be modified to produce large responses to fiscal shocks. The New Keynesian model is the workhorse for fiscal policy research. This perhaps follows from papers like Burnside et al. (2004), which shows that the magnitude of observed responses to fiscal shocks are not matched by a standard neoclassical models, but they are matched by models that include habit formation and adjustment costs. Beyond the liquidity constraints, this model is otherwise neoclassical. This paper therefore shows that a host of New Keynesian frictions are not always needed to study fiscal policy. The KM model can be a workhorse for that purpose.

7 Conclusion

This paper shows that cuts to income tax rates in a liquidity constrained economy increases output, investment, and private consumption. The model is a modification of the mostly neoclassical, DSGE model of liquidity and business cycles by Kiyotaki and Moore (2012). In particular, distortionary taxes and a balanced budget fiscal rule are added to KM. The model is calibrated to be consistent with the KM-related literature. Results are qualitatively robust, but quantitatively sensitive, to assumptions regarding structural parameter values, and qualitatively and quantitatively sensitive to significant variations in the persistence of tax shocks.

This paper is unique in three ways. Firstly, these results are consistent with those obtained by Mountford and Uhlig (2009); but while they use an estimated VAR, this paper complements and supports with a theoretical finding from a calibrated neoclassical model. Secondly, this paper distinguishes itself from the rest of the KM-related literature by being the first to apply the KM model to fiscal shocks; this related literature remains focused on showing the significance of liquidity shocks in explaining business cycles, and the importance of liquidity constraints for propagating productivity shocks. Thirdly, the paper shows how a neoclassical model can be modified to produce large responses to fiscal shocks.

Some opportunities for future research are suggested by this work. One extension is an examination of a cut in taxes without a balanced budget in this environment. Another useful experiment is to cut one tax rate at a time, and determine their relative merits in the economic expansion seen in this paper. It would be interesting to determine the effects of an increase in government spending, with and without a balanced budget. The model can be adjusted by adding New Keynesian type frictions, and then determine whether such frictions change the results. Finally, taking this study to the data will facilitate the calculation of multipliers that are reliable for quantitatively comparing results with the related literature.
Technical Appendix

Appendix A  Model calibration

This appendix describes the choice of parameter values with which the tax shock is simulated by the quantitative technique of calibration. All parameter values are based on quarterly data, mostly on the US economy; parameters concerning tax rates are obtained from UK data. Structural parameter values are taken from the KM-related literature; these values are summarized in Table 4. The “baseline setting” is used in Section 3 to obtain the main results of the tax shock, and the “sensitivity settings” are used in Appendix B to re-simulate the tax shock and assess the sensitivity of results to the model’s calibration. Steady state levels and autoregressive (shock) parameters of exogenous variables are computed; these values are summarized in Table 5; Appendix E provides the data used in those exercises.

A.1  Structural parameters

Table 4: Structural parameters: baseline and sensitivity settings

<table>
<thead>
<tr>
<th>Structural parameters</th>
<th>Symbol</th>
<th>Baseline setting</th>
<th>Sensitivity settings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Lower</td>
</tr>
<tr>
<td>Fraction of investment financed by equity</td>
<td>( \theta )</td>
<td>0.185</td>
<td>0.1665</td>
</tr>
<tr>
<td>Subjective discount factor</td>
<td>( \beta )</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>Capital’s share in output</td>
<td>( \gamma )</td>
<td>0.4</td>
<td>n.a.</td>
</tr>
<tr>
<td>Survival rate of capital after depreciation</td>
<td>( \delta )</td>
<td>0.975</td>
<td>n.a.</td>
</tr>
<tr>
<td>Probability of investment opportunity</td>
<td>( \pi )</td>
<td>0.05</td>
<td>0.037</td>
</tr>
<tr>
<td>Inverse Frisch elasticity of labour supply</td>
<td>( \nu )</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>Relative utility weight on labour</td>
<td>( \omega )</td>
<td>4.01</td>
<td>3.409</td>
</tr>
</tbody>
</table>

NOTES: Appendix A describes how these values are derived. The “baseline setting” is used in this section to obtain the main results of the tax shock. The “sensitivity settings” are used in Appendix B to re-simulate the tax shock and assess the sensitivity of results to the model’s calibration. All parameter values are based on quarterly data.

A.1.1  Liquidity constraint parameters, \( \theta \) and \( \phi_t \)

One challenging aspect of the calibration exercise is finding suitable values for \( \theta \) and \( \phi_t \). These parameters are not directly observable, and are instead fixed to empirical proxies. Other members of the KM-related literature handle this problem in different ways. One consistent theme in these papers has been a simplification that follows from earlier versions of KM: \( \theta \) and \( \phi_t \) are assumed to be equal in steady state, while outside of steady state \( \phi_t \) varies stochastically. The calibration task then comes down to finding an empirical estimate of either parameter. Del Negro et al. (2011) targets \( \phi_t \). They propose that \( \phi_t \) is a linear function of the steady state value of a “liquidity share” variable, a ratio of liquid assets (empirically, US government liabilities) to total assets (empirically, net claims of private assets). From US data over the period
1952:1 – 2008:4, the authors obtain an average liquidity share of 12.64%. Then according to
the hypothesized linear relationship, they found that a value of 0.185 for $\phi_t$ was related to a
liquidity share of 13%. KM and Driffill and Miller (2013) also calibrate with this value of $\phi$, and
this paper does the same.

Sensitivity analysis uses higher and lower settings of 0.2035 and 0.1665, respectively, which
represent relaxation and tightening of liquidity constraints by 10% relative to baseline. The
higher setting is the highest (common) value for $\theta$ and $\phi$ that allows the model to converge to
a stable, unique equilibrium.\footnote{Some members of the KM-related literature successfully calibrate with higher settings in their own unique models: Shi (2012) set $\phi = 0.273$ and Bigio (2012) set $\theta = 0.4$. Shi (2012) associates $\phi_t$ with the return on liquid assets; he uses the range that Del Negro et al. (2011) find for annual net returns on US government liabilities, i.e. 1.72% for 1-year maturities to 2.57% for 10-year maturities, and sets an intermediate return of 2% as the target to which $\phi_t$ is calibrated. Bigio (2012) follows Lorenzoni and Walentin (2007) and sets $\theta$ to match the aggregate moments of coefficients in a regression by Gilchrist and Himmelberg (1998) of the “great ratio” $I/K$ against the return on capital and Tobin’s $q$. Salas-Landeau (2010) warns against using high parameter values, after finding that the constraints need to be tight for shocks to have significant effects.}

### A.1.2 Subjective discount factor, $\beta$

Frederick et al. (2002) provide an extensive review of the literature on empirical and experimen-
tal studies of $\beta$ and observe that most arrive at values close to 1, or equivalently, quarterly rates
of time preference close to zero, which implies that agents have almost equal preferences for the
present and future. More recently, Theodoridis et al. (2012) estimate a VAR based on the Smets
and Wouters (2007) DSGE model, but with time-varying parameters, and find that $\beta$ is close
to, but less than 1, and does not vary over time. These results support the standard practice in
the DSGE literature to fix $\beta$ very close to 1. The most popular setting is a quarterly discount
factor of 0.99, which means a 1% quarterly rate of time preference. This value is selected here.
Amongst the KM-related literature, Nezafat and Slavík (2012) shares this setting.

Values above and below, but not far away from, the baseline setting for $\beta$ are chosen for
sensitivity analysis. A higher $\beta$ of 0.999 equates agents’ preferences for the present and future.
This setting appears in, for example, Fernández-Villaverde (2010), who also investigate fiscal
shocks in a calibrated DSGE model with financial frictions. A lower $\beta$ of 0.98 implies agents are
more impatient and prefer the present, and therefore discount future utility by a 2% quarterly
rate of time preference.

### A.1.3 Capital’s share in output, $\gamma$

Christensen et al. (1980) estimate an average value of 0.40 for $\gamma$ in the US between 1947 and
and not only confirm that this value still holds, but support the Kaldor (1961) fact that it
remains constant over time.

Sensitivity from $\gamma$ relies exclusively on a lower value of 0.36. This setting appears in Shi
(2012), Nezafat and Slavík (2012), and Jermann and Quadrini (2012). Lower values of 0.33 and
0.22 are used by Bigio (2012) and Fernández-Villaverde (2010), respectively. Values above the
baseline setting are uncommon in the literature, and are therefore omitted in the analysis.
A.1.4 Survival rate of capital after depreciation, $\delta$

A quarterly depreciation rate of 2.5%, or equivalently, an annual rate of $1 - (1 - 0.025)^4 \approx 10\%$, is standard in RBC studies on the US economy. Since King et al. (1988), who describe 10% as a “more realistic depreciation rate” (p. 218), this value has been widely used in DSGE calibrations. $\delta$ is therefore set to 0.975.

Like, $\gamma$, sensitivity analysis with $\delta$ relies on just one alternative setting, a higher value of 0.98. Rates above the baseline are not unusual in the literature. Nezafat and Slavík (2012), Shi (2012), and Bigio (2012), for example, use 0.9774, 0.981, and 0.9873, respectively, and at the extreme end, Fernández-Villaverde (2010) uses 0.99.

A.1.5 Probability of an investment opportunity, $\pi$

$\pi$ can be related empirically to the fraction of firms that significantly adjust their capital in a given period. From samples of US manufacturing firms, Doms and Dunne (1998) estimate this fraction at 20% in any given year, from which Del Negro et al. (2011) and KM set $\pi$ at the quarterly rate of $1 - (1 - 0.2)^{0.25} \approx 5\%$. This value is chosen for calibrating the parameter.

Cooper et al. (1999) and Cooper and Haltiwanger (2006) perform empirical studies similar to Doms and Dunne (1998) and estimate that 14% to 25% of firms significantly adjust their capital in any given year. The difference in estimates between the two sets of studies are down to what the authors consider to be a “significant adjustment” in capital stock. To Doms and Dunne (1998), a “significant adjustment” means an event where more than 10% of a firm’s capital is repaired or replaced, whereas Cooper et al. (1999) and Cooper and Haltiwanger (2006) define it as more than 20%. The interval estimate for the fraction of firms that invest in a year provide upper and lower alternative settings for $\pi$. If 14% of firms are assumed to significantly replace or repair their capital in a year then the implied value of $\pi$ is $1 - (1 - 0.14)^{0.25} \approx 3.7\%$. If 25% of firms invest then $\pi = 1 - (1 - 0.25)^{0.25} \approx 6.9\%$.

A.1.6 Frisch elasticity of labour supply, $1/\nu$

The value of $1/\nu$ in applied economics is the subject of unresolved debate. On the one hand, empirical microeconomic studies usually find small estimates, i.e. values below 1; a review of the literature by Contreras and Sinclair (2008) shows this. Early work by MaCurdy (1981) and Altonji (1986) find estimates within US data ranging from 0 to 0.5. Since then, most empirical studies, at least those whose samples are selected from males, fall within this range; for example, in Pencavel (1986) and Domeij and Flodén (2006). On the other hand, macroeconomics needs much larger elasticities for calibrating models to match observed business cycle fluctuations in aggregate variables, as Prescott (2006) insists. For example, Peterman (2012) explains that values between 2 and 4 are required to replicate empirical volatility in aggregate labour hours.

The wide micro-macro disparity on the value of $1/\nu$ is mainly due to sample selection: macroeconomic studies aggregate all individuals, whereas microeconomic studies rely on narrower samples (Peterman (2012) and Chetty et al. (2012)). The results of MaCurdy (1981),

\footnote{Alternatively, Gourio and Kashyap (2007) consider a “significant adjustment” as investment which amounts to 35% or more of beginning-of-period capital.}
for instance, are drawn from prime-aged males. The DSGE literature is fairly consistent in using elastic values. However, there is a subset that applies unitary elasticity in macroeconomic models. This is done by Christiano et al. (2005), following elasticity estimates in Rotemberg and Woodford (1999), and also by Christiano et al. (2013) and Cesa-Bianchi and Fernandez-Corugedo (2013) in their DSGE models with financial frictions. KM, Del Negro et al. (2011), and Nezafat and Slavík (2012) also calibrate with Frisch elasticity, and this paper does the same.

Sensitivity from $1/\nu$ is assessed from both elastic and inelastic settings. A higher value of 2 is used, following the recommendations of macroeconomists; this value is also used in calibrations by Shi (2012) and Bigio (2012). The upper bound of 0.5 from MaCurdy (1981) and Altonji (1986) is used as the lower sensitivity setting.

A.1.7 Relative utility weight on labour, $\omega$

$\omega$ is often calibrated with consideration of $\nu$, since together they form labour’s coefficient in the worker’s utility function. As Hall (1997) points out, researchers have different ways of representing this coefficient. $\omega$ is usually calibrated to match an average or steady state fraction of time spent in work. According to Villa and Yang (2011), the common assumption in the literature is that individuals spend 8 hours a day in work, or one third of their time. Villa and Yang (2011) assume a utility function similar to the one in this paper, and they set $\omega = 4.01$; the difference between this model and theirs is that they include habit persistence in consumption. Nevertheless, given the modelling similarities, their setting is followed here.

Villa and Yang (2011) is based on Gertler and Karadi (2011), who calibrate $\omega$ to 3.409 based on estimates by Primiceri et al. (2006). This value is taken as a lower sensitivity setting for $\omega$. For a higher sensitivity setting, the value of 8.15 that is set by Nezafat and Slavík (2012) is used. This paper shares modelling similarities with Nezafat and Slavík (2012), including the same utility specification for workers. Their calibration of $\omega$ is done to match moments in steady state with those found empirically.

A.2 Parameterization of exogenous variables

A.2.1 Autoregressive (shock) parameters, $\rho_X$ and $\sigma_{uX}$

Persistence parameters are calibrated to values less than one so that exogenous shocks are not permanent. $\rho_\phi$, $\rho_{Ng}$, $\rho_{\tau r n}$, and $\rho_{\tau w l}$ are all set to a standard value of 0.95; the settings of $\rho_A$ and $\rho_M$ are explained below. In Appendix B the tax shock is re-simulated with different parameter settings to determine the sensitivity of results to the calibration of these parameters.

It can easily be shown that the standard deviation $\sigma_X$ of exogenous variable $X_t$ implies the standard deviation of innovations to $X_t$,

$$\sigma_{uX} = \sqrt{(1 - \rho_X^2)\sigma_X^2} \quad (A.1)$$

where $X$ is the steady state value of $X_t$. For brevity, the derivation of this result is relegated

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18 Hall (1997), for instance, normalizes $\omega$ and applies a relative weight to consumption.
Table 5: Exogenous variables: steady state levels and autoregressive (shock) parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Steady state level</th>
<th>Persistence parameter</th>
<th>Standard deviation of innovations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Symbol</td>
<td>Value</td>
<td>Symbol</td>
</tr>
<tr>
<td>Aggregate productivity</td>
<td>$A$</td>
<td>1</td>
<td>$\rho_A$</td>
</tr>
<tr>
<td>Resaleable fraction of equity</td>
<td>$\phi$</td>
<td>$\theta$</td>
<td>$\rho_\phi$</td>
</tr>
<tr>
<td>Government equity</td>
<td>$N_g$</td>
<td>0</td>
<td>$\rho_{N_g}$</td>
</tr>
<tr>
<td>Money supply</td>
<td>$M$</td>
<td>1.95</td>
<td>$\rho_{M}$</td>
</tr>
<tr>
<td>Rate of tax on dividends</td>
<td>$\tau_{rn}$</td>
<td>0.207</td>
<td>$\rho_{\tau_{rn}}$</td>
</tr>
<tr>
<td>Rate of tax on wages</td>
<td>$\tau_{wl}$</td>
<td>0.231</td>
<td>$\rho_{\tau_{wl}}$</td>
</tr>
</tbody>
</table>

NOTES: Appendix A describes how these values are derived. Exogenous variables $\{A_t, \phi_t, N_g, \tau^{rn}_t, \tau^{wl}_t\}$ follow stochastic AR(1) processes (2.14), (2.15), (2.16), (2.17), (2.18) and (2.19) which have the general form $X_t = (1 - \rho_X)X + \rho_X X_{t-1} + u_t^X$ where $\rho_X$ is the persistence parameter and $u_t^X$ are innovations.

to Appendix C.10.

In an unconventional policy move, the US government started purchasing corporate equities in the third quarter of 2008, as part of the Troubled Asset Relief Program. The natural logarithm of this short time series (which is given in Table 12 in Appendix E) has a standard deviation of 0.5671. Then by Equation (A.1),

$$\sigma_{uNg} = \sqrt{(1 - \rho_{Ng}^2)\sigma_{Ng}^2} = \sqrt{(1 - 0.95^2)} \times 0.5671^2 = 0.1771$$

Individuals in the US pay tax on income from all sources, not on the type of income earned. Dividend and wage tax data is not available from the US. The UK computes taxes by the type of income, including dividend and wage taxes. Parameters related to taxation are therefore drawn from quarterly UK data (which is given in Table 14 in Appendix E). Tax rates are computed as ratios of aggregate taxes to aggregate incomes from wages and dividends. Tax liabilities are used instead of actual tax receipts, to avoid the latter’s problems with over/underpayments, late payments, etc. Standard deviations $\sigma_{\tau_{wl}} = 0.004$ and $\sigma_{\tau_{rn}} = 0.0112$ for tax rates are observed from the data. The standard deviations of innovations to tax rates are then computed by Equation (A.1):

$$\sigma_{u\tau_{wl}} = \sqrt{(1 - \rho_{\tau_{wl}}^2)\sigma_{\tau_{wl}}^2} = \sqrt{(1 - 0.95^2)} \times 0.004^2 = 0.00124$$
$$\sigma_{u\tau_{rn}} = \sqrt{(1 - \rho_{\tau_{rn}}^2)\sigma_{\tau_{rn}}^2} = \sqrt{(1 - 0.95^2)} \times 0.0112^2 = 0.00349$$

King and Rebelo (2000) estimate an AR(1) process for $A_t$ in natural logarithms and without an intercept, using quarterly US data, and obtain point estimates of 0.979 for the persistence parameter and 0.0072 for the standard deviation of the residuals. The value of 0.979 is assumed here for $\rho_A$, and by Equation (A.1),

$$\sigma_{uA} = \sqrt{(1 - \rho_A^2)\sigma_A^2} = \sqrt{(1 - 0.979^2)} \times 0.0072^2 = 0.00147$$
Figure 3: US liquid assets to total assets


NOTES: The liquidity share is calculated according to Del Negro et al. (2011). Tables 10 and 11 in Appendix E give the data and metadata, respectively.

An annual time series of the liquidity share variable of Del Negro et al. (2011) is replicated here by following the authors’ metadata. The data and metadata are given in Tables 10 and 11, respectively, in Appendix E. The series is illustrated graphically in Figure 3. The period 1957–2007 was one of relative stability for the liquidity share, within which the mean and standard deviation are 0.1110 and 0.0204, respectively. Del Negro et al. (2011) propose that the liquidity share is a linear function of $\phi_t$ with gradient of 15, which implies $\sigma_\phi = 0.00136$. Then by Equation (A.1),

$$\sigma_{u\phi} = \sqrt{(1 - \rho^2)\sigma_\phi^2} = \sqrt{(1 - 0.95^2) \times 0.00136^2} = 0.00042$$

Equation (2.17) is estimated via least squares from quarterly US data over 1987:1–2008:1 (i.e. 84 observations); estimation results are summarized in Table 6. $M_t$ is taken as the seasonally adjusted, detrended, natural logarithm of the real monetary base. Table 13 in Appendix E provides the data and describes how the series is compiled. $\rho_M$ is set to the estimated coefficient 0.952 of the lagged dependent variable in the AR(1) regression. The Dickey-Fuller test on

$$\Delta M_t = (\rho_M - 1)M_{t-1} + u_t^M$$

\(^{19}\)From Figure 3 on page 43 in Del Negro et al. (2011), the straight line appears to travel from 12.5 to 13.25, or 0.75 units along the vertical axis, and from 0.15 to 0.2, or 0.05 units along the horizontal axis, thus giving a slope of $0.75/0.05 = 15$. 

32
Table 6: Estimation of $M_t = (1 - \rho_M)M + \rho_M M_{t-1} + \epsilon_t^M$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>$t$-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 - \rho_M)M$</td>
<td>0.093483</td>
<td>0.019623</td>
<td>4.763995</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\rho_M$</td>
<td>0.951991</td>
<td>0.010284</td>
<td>92.57203</td>
<td>0.0000</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.990407</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.990292</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.004232</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.001486</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>344.9367</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-statistic</td>
<td>8569.580</td>
<td></td>
<td></td>
<td>0.024005</td>
</tr>
<tr>
<td>Prob(F-statistic)</td>
<td>0.000000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTES: This table gives the results of estimating equation (2.17) via least squares with a sample of 84 observations from 1987:1 to 2008:1. $M_t$ is the seasonally adjusted, detrended, natural log of the real US monetary base. The data is given in Table 13 in Appendix E.

Figure 4: Histogram of residuals in the estimation of $M_t = (1 - \rho_M)M + \rho_M M_{t-1} + \epsilon_t^M$

with standard $t$-statistic,

$$\frac{\hat{\rho}_M - 1}{SE(\hat{\rho}_M)} = \frac{0.951991 - 1}{0.010284} \approx -5$$

concludes that $|\rho_M| < 1$ and $M_t$ is a trend-stationary series. Figure 4 gives a histogram of the residuals of the regression. Moreover, the Jarque-Bera test statistic, with p-value of 0.45, does not provide enough statistical evidence to reject a null hypothesis that the regression residuals are normally distributed. $\sigma_{u_M}$ is set to the standard deviation of the regression residuals, 0.004207.
A.2.2 Steady state values

Values in this section are not interpretable. $A$ is normalized to 1. As previously mentioned in this section, $\phi$ is assumed to be equal to $\theta$ (i.e. 0.185). From Assumption 2, $N^g = 0$. And the estimated regression coefficients of Equation (2.17) (in Table 6) imply a value of 1.95 for $M$.

While tax rates are obtained from UK data, the country does not have flat rates of tax on wage and dividend income. In both cases the taxpayer first enjoys a taxable allowance, and any excess amount earned during the tax year is subject to tax. The rate of tax applied on this excess depends on the individual’s income for the fiscal year. $\tau^{wl}$ and $\tau^{rn}$ are set to average ratios, 0.231 and 0.207, of aggregate tax liabilities to aggregate incomes from wages and dividends, respectively (see Table 14 in Appendix E).\(^{20}\)

\(^{20}\)These rates are very similar to those computed by Gomme and Rupert (2007) from US data and following a methodology set out by Mendoza et al. (1994) and Carey and Tchilinguirian (2000). Gomme and Rupert (2007) compute income tax rates of 0.22 on wages and 0.2688 on capital. These values, however, are not adopted here for two reasons: (i) rates are obtained from the same dataset that was used to compute $\sigma_{wrl}$ and $\sigma_{rrm}$; (ii) Gomme and Rupert (2007) use data on actual government tax collections, which, as said in this paper, may be less accurate of the tax burden than tax liabilities data because of errors in tax payments. The rates obtained here are also fairly consistent with papers that focus exclusively on tax rate calculations; Barro and Sahasakul (1983, 1986), Seater (1985), and Stephenson (1998) obtain average wage income tax rates between 0.22 and 0.30 from US data between 1954 and 1994; and Mendoza et al. (1994) obtain average income tax rates between 0.17 and 0.30 for wages and 0.27 and 0.50 for capital.
Appendix B  Sensitivity analysis

This Appendix shows how responses to the tax shock vary with changes to the calibration of structural parameters (θ, β, γ, δ, π, ν, and ω) and the persistence of tax shocks (ρτrn and ρτwl). Structural parameter sensitivity analysis is performed systematically by three local methods, all involving repeated simulations of the shock with combinations of sensitivity settings that are listed in Table 4: the first method uses one change in one parameter at a time; the second and third methods use combinations of two or more sensitivity parameter values. Sensitivity to the persistence of tax shocks is examined by repeatedly simulating the shock with simultaneous use of values above and below the calibrated, and fairly standard, setting. The conclusions from this Appendix are stated in Section 5.

B.1  Sensitivity to structural parameters: one-at-a-time parameter variation

The first approach to structural parameter sensitivity is a one-at-a-time (OAT) method: one parameter is changed to one of its sensitivity settings, and all other parameters remain at the baseline; this is done for each and every parameter and for each and every sensitivity setting that is listed in Table 4.21 This exercise produces 12 sets of results, which are graphically illustrated by impulse responses in Figures 5 to 11. The magnitude of immediate impulse responses from all 12 sensitivity simulations plus the baseline are listed in Table 7.

Since their changes are due to non-uniform changes in parameter values, impulse responses are unsuitable for comparing different scenarios, or for establishing a common criteria to assess sensitivity. An indicator of sensitivity to a particular parameter is constructed for these purposes. The indicator is a ratio of the percentage change in a variable’s first quarter impulse response to the percentage change in a parameter’s value. The indicator is henceforth referred to as a “parameter elasticity of impulse response”.22 A positive elasticity means an increase (or decrease, respectively) in the parameter’s value amplifies (or dampens, respectively) the variable’s immediate impulse response relative to that of the baseline scenario. A negative elasticity means that an increase (or decrease, respectively) in the parameter’s value dampens (or amplifies, respectively) the variable’s immediate impulse response relative to that of the baseline scenario. A variable is considered sensitive to a parameter if the absolute value of the elasticity is greater than 1. The model is considered sensitive to a parameter if the majority of the variables are sensitive to that parameter. Elasticities from all 12 repeated simulations are given in Table 8.

B.1.1  Liquidity constraints

Figure 5 illustrates the difference in impulse responses among the baseline setting and higher and lower sensitivity settings of θ and φ. The graphs show little variation in impulse responses. Moreover, parameter elasticities are less than unity in absolute value for all variables except \( C_i \), \( N_s \), and \( p_t \). The model can therefore be considered not sensitive to the calibration of liquidity

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21 This method is similar to the “one-factor-at-a-time” method of Morris (1991), but without randomly selecting parameter values.

22 The parameter elasticity resembles the “elementary effects” ratio of Morris (1991).
Figure 5: Impulse responses of a tax shock: repeated simulations with lower and higher $\theta$

NOTES: Vertical axes measure percentage deviation from steady state. Horizontal axes measure quarters after the shock, starting from quarter 1. Impulse responses for $\{K_t, N^*_t\}$ occur at the end of the quarter. Table 7 gives first quarter impulse responses.
constraint parameters, *ceteris paribus*, once they are tight enough to allow the model to converge to a unique equilibrium.

The replacement cost of equity is inversely influenced by the borrowing constraint (see Appendix C.9). Either directly or indirectly through $q_t^R$, the liquidity constraints enter negatively into investors’ consumption (Equation (2.44)) and positively into investment and equity’s supply (Equations (2.42) and (2.48), respectively). Following a tax shock, the tighter the liquidity constraints (i.e. the lower the values of $\theta$ and $\phi$), then the higher the increase in investors’ consumption and the lower the increases in equity’s supply and investment; and conversely. This explains why $I_t$ and $C_i^t$ have positive and negative parameter elasticities, respectively. It also explains why money’s price is the most sensitive variable to this parameter: from a tax shock, the tighter the liquidity constraints, the smaller the increase in equity’s supply, and the greater the increase in equity’s price (as if $S_t^N$ is more to the left than it appears in Figure 2A); then by a portfolio balance effect, the greater are the increases in money’s demand and price. KM observe this movement from equity to money when liquidity constraints tighten, in what they called a “flight to liquidity”. *Ceteris paribus*, a tightening of liquidity constraints worsens the appeal of partially liquid assets (i.e. equity) and encourages agents to substitute towards more liquid assets (i.e. money), and thereby increases the price of liquid assets; and conversely.

### B.1.2 Subjective discount factor

Parameter elasticities indicate that the shock responses of all variables are very sensitive to changes in $\beta$; Figure 6 graphically illustrates this. In fact, looking at all the elasticities in Table 8 shows that $\beta$ produces the greatest amount of sensitivity in all of the OAT simulations. Elasticities are asymmetric, and indicate that the model is more sensitive to raising the parameter’s value above baseline.

$\beta$ enters negatively in entrepreneurs’ consumption (Equations (2.44) and (2.45)). *Ceteris paribus*, increasing $\beta$ means entrepreneurs are more willing to delay consumption and spend their net worth more evenly over time. As their patience increase, they consume less in the present. This explains the negative parameter elasticities for $C_i^t$ and $C_s^t$ with higher $\beta$. The consequences are higher levels of current saving and investment. Combined with a tax shock, increasing $\beta$ amplifies the shock-induced increases in asset demands. For money, this means a larger price increase compared to the baseline scenario, hence the positive parameter elasticity for $p_t$. For equity, there is also a larger supply response; the market adjusts to the shock with a smaller price increase than in the baseline scenario, hence the positive parameter elasticity for $p_t$. These variations in asset price impulse responses then propagate throughout the economy.

Conversely, *ceteris paribus*, lowering $\beta$ means entrepreneurs become more impatient and consume more of their net worth in the present; this implies less saving and investment, and lower asset demands and equity supply. Combined with a tax shock, lowering $\beta$ produces a smaller increase in $p_t$ and a larger increase in $q_t$. Asset price increases feed back into improvements in entrepreneurs’ net worth. Investors therefore consume more. This is why $C_i^t$ has a positive parameter elasticity with lower $\beta$. In other words, investors increase their consumption because

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23 Given that workers’ optimal behaviour involves them not saving for the future (from Equation (2.46)), then such changes are confined to entrepreneurs.
Figure 6: Impulse responses of a tax shock: repeated simulations with lower and higher $\beta$

NOTES: See the notes in Figure 5.
of the net worth improvements they enjoy from shock-induced asset price increases; varying $\beta$ up or down does not interfere with this, hence the difference in the sign of parameter elasticities for $C^*_t$. Net worth improvements also increase saving and investment. However, lowering $\beta$ only partially offsets the increase in savings but completely offsets the increase in investment, hence the negative and positive parameter elasticities for $C^*_s$ and $I_t$, respectively.

### B.1.3 Capital’s share in output

This is another parameter for which the model is sensitive to its calibrated value. Figure 7 illustrates this. Elasticities are greater than 1 in absolute value for all variables except $L_t$ and $q_t$. The only variables with negative elasticities are $q_t$ and $r_t$; impulse responses of other variables are thus smaller when $\gamma$ is lowered.

$\gamma$ enters the aggregate labour demand and production functions (Equations (2.35) and (2.49), respectively). *Ceteris paribus*, lowering $\gamma$ positions the inverse aggregate labour demand function leftwards from its baseline calibration; this is illustrated in the first graph of Figure 12. Lowering $\gamma$ therefore dampens the shock-induced increases in the real wage and labour, and hence output. This explains the positive parameter elasticities of $w_t$, $L_t$, and $Y_t$. The changes in the goods market then propagate throughout the economy.

### B.1.4 Survival rate of capital after depreciation

Figure 8 shows small differences in tax shock impulse responses between baseline and higher $\delta$ settings. But the change in $\delta$’s setting is very small, and parameter elasticities reveal the change in quarter 1 impulse responses to be relatively much larger. Elasticities are all above 20 in absolute value, making shock responses very sensitive to the parameter’s value. In fact, this parameter is the second most sensitive, after $\beta$.

*Ceteris paribus*, a higher $\delta$ means capital and equity stocks retain more of their value after depreciation each period. This effectively provides net worth improvements to entrepreneurs. The consequences are amplified when combined with the shock. However, increasing $\beta$ improves the appeal of equity. The shock-induced increase in demand for equity is thus amplified, and creates a larger fall in $q_t$ (as if $D^N_1$ is further to the right than it appears in Figure 2A). This explains the negative parameter elasticity for $p_t$. Moreover, investors are able to invest more, given net worth improvements, and issue more equity, given its greater appeal. They sacrifice consumption for much more investment, hence the negative parameter elasticity for $C^*_i$.

### B.1.5 Probability of investment opportunity

Changing the value of $\pi$ does not significantly alter impulse responses, except for $p_t$. This is seen in the deviations of impulse response graphs in Figure 9. Elasticities are fairly similar between lowering and raising the parameter’s value relative to its baseline setting.

$\pi$ enters positively into investors’ consumption (Equation (2.44)), investment (Equation (2.48)), and the supply of equity (Equation (2.42)), and negatively into savers’ consumption (Equation (2.45)). *Ceteris paribus*, raising the value of $\pi$ increases the population of investors relative to savers, and conversely. Changing the parameter’s value therefore shifts activity between
Figure 7: Impulse responses of a tax shock: repeated simulation with lower $\gamma$

NOTES: See the notes in Figure 5.
Figure 8: Impulse responses of a tax shock: repeated simulation with higher $\delta$

$T_t (= G_t)$ (taxes/gov. cons.)

$Y_t$ (output)

$I_t$ (investment)

$C_t$ (consumption)

$C_t^w$ (workers’ cons.)

$-10^{-2} C_t^i$ (investors’ cons.)

$C_t^s$ (savers’ cons.)

$N_t^s$ (savers’ equity)

$L_t$ (labour)

$-10^{-2} r_t$ (dividend rate)

$K_t (= N_t)$ (capital/equity)

$p_t$ (price of money)

$q_t$ (price of equity)

NOTES: See the notes in Figure 5.
Figure 9: Impulse responses of a tax shock: repeated simulations with lower and higher $\pi$

$T_t (= G_t)$ (taxes/gov. cons.)

$Y_t$ (output)

$I_t$ (investment)

$C_t$ (consumption)

$C_t^w$ (workers’ cons.)

$C_t^i$ (investors’ cons.)

$C_t^s$ (savers’ cons.)

$N_t^s$ (savers’ equity)

$w_t$ (real wage)

$L_t$ (labour)

$K_t (= N_t)$ (capital/equity)

$p_t$ (price of money)

$q_t$ (price of equity)

NOTES: See the notes in Figure 5.
investment and saving. The significant changes occur in the asset markets, but these are outweighed by the effects of the tax shock. The economy is therefore hardly affected by variation in the parameter’s value, hence the very small elasticities for most variables.

Since a higher \( \pi \) means a smaller population of savers, then the shock-induced increase in money’s demand is smaller compared to the baseline scenario (as if \( D_1^M \) in Figure 2B is further to the left than where it is drawn). The increase in \( p_t \) is smaller, hence the negative parameter elasticity. This implies a larger fall in the expected return on money. The portfolio balance effect is stronger, i.e. the substitution-led increase in equity’s demand is greater, which produces a greater price increase, and therefore a positive parameter elasticity for \( q_t \). The converse is true for a lower \( \pi \).

### B.1.6 Inverse Frisch elasticity of labour supply

Figure 10 shows some variation in impulse responses from changes in \( \nu \). Parameter elasticities indicate that a minority of variables are sensitive to the parameter, and even then, the elasticities are marginally above 1. Elasticities indicate that the model is more sensitive to lowering the parameter. Overall, the model cannot be considered sensitive to the calibration of \( \nu \).

Changing the value of \( \nu \) affects the economy through the aggregate labour supply function. The baseline setting \( \nu = 1 \) makes the inverse function (Equation (2.36)) linear in \( w_t \). If \( \nu < 1 \) then the inverse function is convex, and \( \nu > 1 \) makes it concave. The second graph of Figure 12 illustrates these variations in shape of labour market curves. These variations are largely responsible for any deviations of impulse responses from the baseline scenario.

### B.1.7 Relative utility weight on labour

Although large deviations in impulse responses are shown in Figure 11, these are brought on by large changes in the value of \( \omega \), especially from raising the value above baseline. Parameter elasticities provide a more accurate means of assessing sensitivity. They indicate that the model is not sensitive to raising the parameter’s value; no variable has an elasticity above 1 in absolute value. However, lowering the parameter’s value produces large elasticities for most variables. Changes in \( \omega \) in both directions have no effect on \( w_t, r_t, \) and \( q_t \), and produces the same parameter elasticity with the other variables. The overall conclusion is that the model is sensitive to lowering the parameter’s value, but not to raising it.

\( \omega \) positively determines the slope of the inverse aggregate labour supply function (Equation (2.36)). The last graph of Figure 12 illustrates how varying \( \omega \), ceteris paribus, affects the labour market. The remarks said above about changes in \( \nu \) can also be said about \( \omega \).

### B.2 Sensitivity to structural parameters: combinations of sensitivity settings

The second and third approaches both use combinations of sensitivity settings that are listed in Table 4; for ease of reference, they are called the “Sensitive Combinations” and “All Combinations” methods, respectively.

The Sensitive Combinations method (henceforth SC) uses combinations for only those structural parameters which the OAT method determines that the model is sensitive to, i.e. \( \beta, \gamma \),
Figure 10: Impulse responses of a tax shock: repeated simulations with lower and higher $\nu$

\[ T_t (= G_t) \] (taxes/gov. cons.)
\[ Y_t \] (output)
\[ I_t \] (investment)
\[ C_t \] (consumption)
\[ C_t^w \] (workers’ cons.)
\[ C_t^i \] (investors’ cons.)
\[ C_t^s \] (savers’ cons.)
\[ N_t^s \] (savers’ equity)
\[ w_t \] (real wage)
\[ L_t \] (labour)
\[ r_t \] (dividend rate)
\[ K_t (= N_t) \] (capital/equity)
\[ p_t \] (price of money)
\[ q_t \] (price of equity)

NOTES: See the notes in Figure 5.
Figure 11: Impulse responses of a tax shock: repeated simulations with lower and higher $\omega$

$T_t (= G_t)$ (taxes/gov. cons.)

$Y_t$ (output)

$I_t$ (investment)

$C_t$ (consumption)

$C^w_t$ (workers’ cons.)

$C^i_t$ (investors’ cons.)

$C^s_t$ (savers’ cons.)

$N^s_t$ (savers’ equity)

$w_t$ (real wage)

$L_t$ (labour)

$r_t$ (dividend rate)

$K_t (= N_t)$ (capital/equity)

$p_t$ (price of money)

$q_t$ (price of equity)

NOTES: See the notes in Figure 5.
NOTES: These graphs plot Equations (2.35) and (2.36) using steady state levels of the capital stock and total factor productivity and sensitivity settings for $\gamma$, $\nu$ and $\omega$ that are listed in Table 4.
and δ. The SC uses 10 combinations of parameter values. The SC builds upon the screening
that the OAT method performs, and attempts to capture two or more sensitivity settings from
β, γ, and δ that, when combined, produce shock responses that deviate significantly from the
baseline. Impulse responses for the SC are presented graphically in Figure 13. Relying on
graphical inspection, the SC concludes that responses to the tax shock vary only in magnitude
outside of those considered in the SC. The AC also avoids any selection bias that the SC may
have, despite identification by the OAT method of which parameters are key drivers of sensi-
tivity. Impulse response graphs of the AC closely resemble those in Figure 13 (they are just
more densely populated) and are not reported, to avoid repetition. The conclusion of the SC is

\[ \frac{\theta}{\beta} \]  

\[ w_t = \frac{0.9}{0.9} \]  

\[ L_t = \frac{0.4}{0.4} \]  

\[ r_t = \frac{0.0}{0.0} \]  

\[ K_t = \frac{1.9}{1.8} \]  

\[ p_t = \frac{6.7}{7.8} \]  

\[ q_t = \frac{3.2}{3.2} \]  

\[ \text{NOTES: This table gives the percentage deviations from steady state in quarter 1 for baseline and one-at-a-time sensitivity scenarios. “L” and “H” refer to the lower and higher sensitivity parameter values, respectively, that are listed in Table 4. Impulse responses for stock variables \{K_t, N_t\} occur at the end of the quarter.} \]

\[ \text{An 11th combination of } \{\beta = 0.999, \gamma = 0.4, \delta = 0.98\} \text{ does not allow the model to converge to a unique equilibrium.} \]

\[ \text{The literature lacks criteria by which results of such an analysis are to be interpreted; a survey by Andronis et al. (2009) concludes this. Here, the objective is to observe any change in direction or trajectory of impulse responses and to recognize significantly different impulse responses from those of the baseline scenario.} \]

\[ \text{An additional 217 combinations from the AC do not allow the model to converge to a unique equilibrium.} \]
Table 8: Parameter elasticities of impulse responses

<table>
<thead>
<tr>
<th></th>
<th>( \theta )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>( \pi )</th>
<th>( \nu )</th>
<th>( \omega )</th>
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<tr>
<td></td>
<td>L</td>
<td>H</td>
<td>L</td>
<td>H</td>
<td>L</td>
<td>H</td>
<td>L</td>
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<tr>
<td>( T_t )</td>
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<td>0.1</td>
<td>36.3</td>
<td>71.9</td>
<td>4.2</td>
<td>47.4</td>
<td>0.2</td>
</tr>
<tr>
<td>( Y_t )</td>
<td>0.1</td>
<td>0.1</td>
<td>36.3</td>
<td>71.9</td>
<td>3.3</td>
<td>47.4</td>
<td>0.2</td>
</tr>
<tr>
<td>( I_t )</td>
<td>0.2</td>
<td>0.2</td>
<td>50.5</td>
<td>126.0</td>
<td>4.4</td>
<td>34.4</td>
<td>0.4</td>
</tr>
<tr>
<td>( C_t )</td>
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<td>0.1</td>
<td>24.7</td>
<td>27.5</td>
<td>3.6</td>
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<td>( C_t^w )</td>
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<td>0.1</td>
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<td>71.9</td>
<td>3.3</td>
<td>47.4</td>
<td>0.2</td>
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<td>-81.8</td>
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<td>( w_t )</td>
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<td>0.1</td>
<td>20.2</td>
<td>31.4</td>
<td>2.5</td>
<td>22.4</td>
<td>0.1</td>
</tr>
<tr>
<td>( L_t )</td>
<td>0.1</td>
<td>0.1</td>
<td>20.2</td>
<td>31.4</td>
<td>1.7</td>
<td>22.4</td>
<td>0.1</td>
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<tr>
<td>( r_t )</td>
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<td>-0.1</td>
<td>-40.5</td>
<td>-34.6</td>
<td>-1.0</td>
<td>-29.4</td>
<td>-0.2</td>
</tr>
<tr>
<td>( K_t )</td>
<td>0.2</td>
<td>0.2</td>
<td>50.5</td>
<td>126.0</td>
<td>4.4</td>
<td>34.4</td>
<td>0.4</td>
</tr>
<tr>
<td>( p_t )</td>
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<td>-1.8</td>
<td>66.7</td>
<td>191.1</td>
<td>4.5</td>
<td>-40.1</td>
<td>-1.7</td>
</tr>
<tr>
<td>( q_t )</td>
<td>0.2</td>
<td>0.2</td>
<td>-28.0</td>
<td>-26.6</td>
<td>-0.2</td>
<td>20.2</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

NOTES: This table gives the percentage change in first quarter impulse responses as a ratio to the percentage change in a single parameter value. “L” and “H” refer to the lower and higher sensitivity parameter values, respectively, that are listed in Table 4. A positive elasticity means an increase (or decrease, respectively) in the parameter’s value amplifies (or dampens, respectively) the variable’s first quarter impulse response relative to that of the baseline scenario. A negative elasticity means that an increase (or decrease, respectively) in the parameter’s value dampens (or amplifies, respectively) the variable’s first quarter impulse response relative to that of the baseline scenario. A variable is considered sensitive to a parameter if the absolute value of the elasticity is greater than 1. The model is considered sensitive to a parameter if the majority of the variables are sensitive to that parameter.

Therefore supported by the AC.

Box plots of immediate impulse responses from both SC and AC are presented in Figure 14. Two conclusions are drawn from inspecting Figure 14. Firstly, with the exception of \( C_t^s \), baseline responses (marked by a red cross) are not extreme. Secondly, the AC produces more extreme first quarter impulse responses than the SC; since SC combinations are a subset of those used in the AC, then Figure 14 indicates that not only do the parameters identified by the OAT method cause sensitivity, but also certain combinations of any of the parameter values.

B.3 Sensitivity to the persistence of tax shocks

Sensitivity to the persistence of tax shocks, \( \rho_{rel} \) and \( \rho_{rrn} \), are examined by repeatedly simulating the tax shock with simultaneous use of two values above (0.99 and 0.96) and three values below (0.94, 0.88, and 0.10) the “baseline” setting (0.95). Baseline values of structural parameters are maintained. Results are illustrated graphically by impulse responses in two ways: Figure 15 gives the usual 200-quarter horizon graphs and shows the variation in adjustment path trajectories, and Figure 16 gives a close-up of the first 20 quarters and shows the divergence of trajectories.
Figure 13: Impulse responses of a tax shock: repeated simulations with combinations of sensitivity settings for $\beta$, $\gamma$ and $\delta$

NOTES: These graphs plot impulse responses from the “Sensitive Combinations” approach to structural parameter sensitivity. They show impulse responses to the tax shock from 10 repeated simulations with combinations of all sensitivity settings for $\beta$, $\gamma$ and $\delta$ that are listed in Table 4. An 11th combination of $\{\beta = 0.999, \gamma = 0.4, \delta = 0.98\}$ does not allow the model to converge to a unique equilibrium.
Figure 14: Range of immediate impulse responses of a tax shock: repeated simulations with combinations of sensitivity settings

Notes: These box plots show the 25th and 75th percentiles, median, largest and smallest immediate impulse responses from the Sensitive Combinations (labelled “Sens.”) and All Combinations (labelled “All”) approaches to structural parameter sensitivity. Vertical axes measure percentage deviation from steady state. Red crosses indicate the baseline immediate impulse responses.
after the shock’s initial impact. Table 9 gives an indicator of the speed of convergence to steady state: the time it takes for impulse responses to fall within 10% of their initial shock impacts.

Very small changes from the “baseline”, by ±1 basis point, does not significantly alter the responses of any variable. When $\rho_{\tau \text{wl}}$ and $\rho_{\tau \text{rn}}$ are both increased and decreased to 0.96 and 0.94, respectively, Figure 15 show that the shape and speed of adjustment paths change by very little. Figure 15 also shows that a persistence close to unity, i.e. an increase by 4 basis points, significantly amplifies adjustment paths. All variables except aggregate taxes, $T_t$, and asset prices, $p_t$ and $q_t$, now exhibit hump-shaped trajectories and very long shock persistence. Those that had hum-shapes before now have exaggerated humps. For any setting below 0.88, output loses its hump-shaped trajectory. At these levels of persistence, investment falls rapidly towards steady state, and is quickly outpaced by an increasing depreciation.

Reducing the persistence parameter down to very low levels reveals those variables whose shock propagations are driven by features of the model. The slowest variables to adjust are (in order) investment, savers’ equity, capital, investors’ consumption, and output. As shown in Table 9, even at the lowest persistence (an implausible 0.10), $I_t$ and $N_t^*$ take more than 200 quarters to converge; $Y_t$ takes more than 5 years, and this is due to the slow convergence of $K_t$. The impulse response analysis of the tax shock suggests that investment is supported by asset prices, and consequently net worth, being above steady state levels throughout the process of adjustment; elevated asset prices are, in turn, due to binding liquidity constraints; and once investment is above steady state and below depreciation, the capital stock increases, and so does output. The liquidity constraints are therefore an amplifying feature for the the internal propagation mechanism (i.e. investment-capital-output relationship), or an “internal amplification” mechanism.
Figure 15: Impulse responses of a tax shock: repeated simulations with varying persistence of shocks to $\tau^{wl}$ and $\tau^{rn}$

$T_t (= G_t)$ (taxes/gov. cons.)

$Y_t$ (output)

$I_t$ (investment)

$C_t$ (consumption)

$C_t^{cw}$ (workers’ cons.)

$-10^{-2} C_t^{ci}$ (investors’ cons.)

$C_t^s$ (savers’ cons.)

$N_t^s$ (savers’ equity)

$L_t$ (labour)

$-10^{-2} r_t$ (dividend rate)

$K_t (= N_t)$ (capital/equity)

$p_t$ (price of money)

$q_t$ (price of equity)

NOTES: These graphs plot impulse responses to the tax shock from repeated simulations with lower-than-baseline settings for persistence parameters $\rho^{wl}$ and $\rho^{rn}$. Figure 16 give a close-up of the first 20 quarters.
Figure 16: Impulse responses of a tax shock: repeated simulations with varying persistence of shocks to $\tau_{wl}$ and $\tau_{rn}$, 20 quarters

$T_t (= G_t) \ (\text{taxes/gov. cons.})$

$Y_t \ (\text{output})$

$I_t \ (\text{investment})$

$C_t \ (\text{consumption})$

$C_{tw}^w \ (\text{workers' cons.})$

$C_{ti}^e \ (\text{savers' cons.})$

$N_t^s \ (\text{savers' equity})$

$L_t \ (\text{labor})$

$r_{ti} \ (\text{dividend rate})$

$K_t (= N_t) \ (\text{capital/equity})$

$p_t \ (\text{price of money})$

$q_t \ (\text{price of equity})$

Notes: These graphs show the first 20 quarters of Figure 15.
Table 9: Quarters after the tax shock when impulse responses converge to within 10% of immediate responses: repeated simulations with varying persistence of shocks to \( \tau^w_t \) and \( \tau^r_t \)

<table>
<thead>
<tr>
<th>( \rho_{\tau w t} = \rho_{\tau r n} = )</th>
<th>0.99</th>
<th>0.96</th>
<th>0.95</th>
<th>0.94</th>
<th>0.90</th>
<th>0.89</th>
<th>0.88</th>
<th>0.80</th>
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<tr>
<td>( T_t )</td>
<td>90</td>
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<td>30</td>
<td>27</td>
<td>18</td>
<td>16</td>
<td>15</td>
<td>9</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>( Y_t )</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>197</td>
<td>188</td>
<td>181</td>
<td>138</td>
<td>66</td>
<td>21</td>
</tr>
<tr>
<td>( I_t )</td>
<td>200</td>
<td>116</td>
<td>93</td>
<td>76</td>
<td>38</td>
<td>33</td>
<td>29</td>
<td>14</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>( C_t )</td>
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<td>193</td>
<td>168</td>
<td>150</td>
<td>102</td>
<td>93</td>
<td>86</td>
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<td>2</td>
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<tr>
<td>( C_t^w )</td>
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<td>172</td>
<td>154</td>
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<td>98</td>
<td>91</td>
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<tr>
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<td>200</td>
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<td>194</td>
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<tr>
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<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>181</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w_t )</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>175</td>
<td>167</td>
<td>159</td>
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<tr>
<td>( L_t )</td>
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<td>200</td>
<td>200</td>
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<td>159</td>
<td>117</td>
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<tr>
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<td>18</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>1</td>
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</tbody>
</table>

NOTES: This table gives the period of time it takes for impulse responses to get within 10% of their quarter 1 magnitudes. Convergence of 200 quarters means some time after 200 quarters, and not in the 200th quarter.
Appendix C Additional algebra

C.1 The investor’s budget and resource constraints

Substituting Equation (2.21) into Equation (2.5) gives

\[
\begin{align*}
\frac{c_i^t}{1 - \theta} + \frac{1 - \phi_t}{1 - \theta} n_{t+1}^i - & \frac{1 - \phi_t}{1 - \theta} \delta n_t + q_t \left[ n_{t+1}^i - \frac{1}{1 - \theta} n_{t+1}^i + \frac{1 - \phi_t}{1 - \theta} \delta n_t - \delta n_t \right] + p_t (m_{t+1}^i - m_t) \\
= & (1 - \tau_t n_t) r_t n_t \\
\Rightarrow c_i^t + \left[ \frac{1}{1 - \theta} + q_t - q_t \frac{1}{1 - \theta} \right] n_{t+1}^i = & (1 - \tau_t n_t) r_t n_t + \left[ \frac{1 - \phi_t}{1 - \theta} - q_t \frac{1 - \phi_t}{1 - \theta} + q_t \right] \delta n_t \\
+ & p_t (m_t - m_{t+1}^i)
\end{align*}
\]

The coefficients of \( n_{t+1}^i \) and \( \delta n_t \) in the above is simplified as follows:

\[
\begin{align*}
\frac{1}{1 - \theta} + q_t - q_t \frac{1}{1 - \theta} = & \frac{1}{1 - \theta} + q_t \left[ \frac{1 - \frac{1}{1 - \theta}}{1 - \theta} \right] \\
= & \frac{1}{1 - \theta} + q_t \left[ \frac{1 - \theta - 1}{1 - \theta} \right] \\
= & \frac{1}{1 - \theta} - q_t \frac{\theta}{1 - \theta} \\
= & \frac{1 - \theta q_t}{1 - \theta} \equiv q_t R
\end{align*}
\]

\[
\begin{align*}
\frac{1 - \phi_t}{1 - \theta} - q_t \frac{1 - \phi_t}{1 - \theta} + q_t = & \frac{1 - \phi_t}{1 - \theta} + q_t \left[ \frac{1 - \frac{1 - \phi_t}{1 - \theta}}{1 - \theta} \right] \\
= & \frac{1 - \phi_t}{1 - \theta} + q_t \left[ \frac{1 - \theta - 1 + \phi_t}{1 - \theta} \right] \\
= & \frac{1 - \phi_t}{1 - \theta} + q_t \left[ \frac{-\theta + \phi_t}{1 - \theta} \right] \\
= & \frac{1 - \phi_t - \theta q_t + \phi_t q_t}{1 - \theta} \\
= & \frac{1 - \phi_t - \theta q_t + \phi_t q_t - \phi_t \theta q_t + \phi_t q_t}{1 - \theta} \\
= & \frac{(1 - \phi_t)(1 - \theta q_t) + \phi_t q_t(1 - \theta)}{1 - \theta} \\
= & \frac{(1 - \phi_t) \frac{1 - \theta q_t}{1 - \theta} + \phi_t q_t}{1 - \theta} \\
= & (1 - \phi_t) q_t R + \phi_t q_t
\end{align*}
\]

thus giving the modified budget constraint (2.22),

\[
\begin{align*}
c_i^t + q_t R n_{t+1}^i = & (1 - \tau_t n_t) r_t n_t + \left[ \phi_t q_t + (1 - \phi_t) q_t R \right] \delta n_t + p_t (m_t - m_{t+1}^i)
\end{align*}
\]

55
Alternatively, substituting Equation (2.20) into Equation (2.5) gives the resource constraint (2.24),

\[ c_i^t + \delta u + q_t(1 - \theta) i_t + (1 - \phi_t) \delta m_t = (1 - \tau^m_t) r_t m_t \]

\[ \Rightarrow c_i^t + \delta u + q_t(1 - \theta) i_t + q_t(1 - \phi_t) \delta m_t = (1 - \tau^m_t) r_t m_t + p_t(m_t - m_{t+1}) \]

\[ \Rightarrow c_i^t + \delta u + q_t(1 - \theta) i_t + [(1 - \phi_t) - 1] q_t \delta m_t = (1 - \tau^m_t) r_t m_t + p_t(m_t - m_{t+1}) \]

\[ \Rightarrow c_i^t + q_t(1 - \theta) i_t + (1 - \tau^m_t) r_t m_t + p_t(m_t - m_{t+1}) \]

\[ \Rightarrow c_i^t + (1 - \theta) q_t i_t = (1 - \tau^m_t) r_t m_t + \phi_t q_t \delta m_t + p_t(m_t - m_{t+1}) \]

C.2 Entrepreneurs’ first order conditions

From Equations (2.6), (2.7) and (2.22), the investor’s Lagrangian is

\[ \mathcal{L}_e^t = U_e(c_i^t) - \lambda^t_i \left( c_i^t + q_t^R n^i_{t+1} - (1 - \tau^m_t) r_t m_t - [\phi_t q_t + (1 - \phi_t) q_t^R] \delta m_t - p_t(m_t - m_{t+1}) \right) \]

\[ + \pi E_t \left[ \beta \left( \frac{1}{\lambda^t_i} \left( c_i^t + q_t^R n^i_{t+1} - (1 - \tau^m_t) r_t m_t - [\phi_t q_t + (1 - \phi_t) q_t^R] \delta m_t - p_t(m_t - m_{t+1}) \right) \right) \right] \]

\[ \Rightarrow \frac{\partial \mathcal{L}_e^t}{\partial c_i^t} = U'_e(c_i^t) - \lambda^t_i = 0 \]

\[ \Rightarrow \lambda^t_i = U'_e(c_i^t) \]

\( \text{(C.1)} \)

\[ \frac{\partial \mathcal{L}_e^t}{\partial c_i^t} = \pi E_t \left[ \beta \left( U'_e(c_i^t + 1) - \lambda^t_i \right) \right] + (1 - \pi) E_t \left[ \beta \left( U'_e(c_i^t + 1) - \lambda^t_i \right) \right] = 0 \]

\[ \Rightarrow \beta E_t \left[ \pi U'_e(c_i^t + 1) + (1 - \pi) U'_e(c_i^t + 1) \right] = \beta E_t \left[ \pi \lambda^t_i + (1 - \pi) \lambda^t_i \right] \]

\[ \Rightarrow \pi U'_e(c_i^t + 1) + (1 - \pi) U'_e(c_i^t + 1) = \pi \lambda^t_i + (1 - \pi) \lambda^t_i \]

\( \text{(C.2)} \)

\[ \frac{\partial \mathcal{L}_e^t}{\partial n^i_{t+1}} = -\lambda^t_i q_t^R + \pi E_t \left[ \beta \lambda^t_i \left( [1 - \tau^m_t] r_{t+1} + \phi_t \delta q_{t+1} + [1 - \phi_t] \delta q_t \right) \right] \]

\[ + (1 - \pi) E_t \left[ \beta \lambda^t_i \left( \delta q_{t+1} + [1 - \tau^m_t] r_{t+1} \right) \right] = 0 \]

\[ \Rightarrow \lambda^t_i q_t^R = \beta E_t \left[ \lambda^t_i \left( [1 - \tau^m_t] r_{t+1} + \phi_t \delta q_{t+1} + [1 - \phi_t] \delta q_t \right) \right] \]

\[ + \beta (1 - \pi) E_t \left[ \lambda^t_i \left( \delta q_{t+1} + [1 - \tau^m_t] r_{t+1} \right) \right] \]

\( \text{(C.3)} \)
\[
\frac{\partial \mathcal{L}_e^i}{\partial m_{t+1}^i} = -\lambda_t^i p_t + \pi E_t[\beta \lambda_{t+1}^i] + (1 - \pi) E_t[\beta \lambda_{t+1}^i p_{t+1}] \leq 0, \quad m_{t+1}^i \geq 0, \\
\text{and } \{-\lambda_t^i p_t + \pi E_t[\beta \lambda_{t+1}^i] + (1 - \pi) E_t[\beta \lambda_{t+1}^i p_{t+1}]\} m_{t+1}^i = 0 \\
\implies -\lambda_t^i p_t + \pi E_t[\beta \lambda_{t+1}^i] + (1 - \pi) E_t[\beta \lambda_{t+1}^i p_{t+1}] = 0 \quad \text{or} \quad m_{t+1}^i = 0 \\
\implies \lambda_t^i p_t = \pi E_t[\beta \lambda_{t+1}^i] + (1 - \pi) E_t[\beta \lambda_{t+1}^i p_{t+1}] \quad \text{or} \quad m_{t+1}^i = 0 \\
\implies \lambda_t^i = \beta E_t \left[ \frac{m_{t+1}^i}{p_t} \left( \pi \lambda_{t+1}^i + (1 - \pi) \lambda_{t+1}^i \right) \right] \quad \text{or} \quad m_{t+1}^i = 0 \quad (C.4)
\]

From Equations (2.6), (2.7) and (2.22), the saver’s Lagrangian is

\[
\mathcal{L}_e^s = U_e(c_t^s) - \lambda_t^s \left( c_t^s + q_t(n_{t+1}^s - \delta m_t) + p_t(m_{t+1}^s - m_t) - (1 - \tau_t^s) r_t m_t \right) + \pi E_t \left[ \beta \left( U_e(c_{t+1}^s) - \lambda_{t+1}^s \left( c_{t+1}^s + q_{t+1}(n_{t+1}^s - \delta m_{t+1}) + p_{t+1}(m_{t+1}^s - m_{t+1}) - (1 - \tau_{t+1}^s) r_{t+1} m_{t+1} \right) \right) \right]
\]

which gives first order conditions

\[
\frac{\partial \mathcal{L}_e^s}{\partial c_t^s} = U_e'(c_t^s) - \lambda_t^s = 0 \\
\implies \lambda_t^s = U_e'(c_t^s) \quad (C.5)
\]

\[
\frac{\partial \mathcal{L}_e^s}{\partial m_{t+1}^s} = -\lambda_t^s q_t + \pi E_t[\beta \lambda_{t+1}^s \left( [1 - \tau_{t+1}^s] r_{t+1} + \phi_{t+1} \lambda_{t+1}^s \right) + [1 - \phi_{t+1}] \delta q_{t+1}^s] + (1 - \pi) E_t[\beta \lambda_{t+1}^s \left( [1 - \tau_{t+1}^s] r_{t+1} + \phi_{t+1} \lambda_{t+1}^s \right) + [1 - \phi_{t+1}] \delta q_{t+1}^s] = 0 \\
\implies \lambda_t^s q_t = \beta \pi E_t \left[ \lambda_{t+1}^s \left( [1 - \tau_{t+1}^s] r_{t+1} + \phi_{t+1} \lambda_{t+1}^s \right) + [1 - \phi_{t+1}] \delta q_{t+1}^s \right] + (1 - \pi) E_t \left[ \lambda_{t+1}^s \left( [1 - \tau_{t+1}^s] r_{t+1} + \phi_{t+1} \lambda_{t+1}^s \right) + [1 - \phi_{t+1}] \delta q_{t+1}^s \right] \quad (C.6)
\]

\[
\frac{\partial \mathcal{L}_e^s}{\partial m_{t+1}^s} = -\lambda_t^s p_t + \pi E_t[\beta \lambda_{t+1}^s p_{t+1}] + (1 - \pi) E_t[\beta \lambda_{t+1}^s p_{t+1}] = 0 \\
\implies \lambda_t^s p_t = \beta \pi E_t \left[ \frac{1}{p_t} \lambda_{t+1}^s \left( [1 - \tau_{t+1}^s] r_{t+1} + \phi_{t+1} \lambda_{t+1}^s \right) + [1 - \phi_{t+1}] \delta q_{t+1}^s \right] + (1 - \pi) E_t \left[ \frac{1}{p_t} \lambda_{t+1}^s \left( [1 - \tau_{t+1}^s] r_{t+1} + \phi_{t+1} \lambda_{t+1}^s \right) + [1 - \phi_{t+1}] \delta q_{t+1}^s \right] \quad (C.7)
\]
\[ \lambda_t^s = \beta E_t \left[ \frac{P_{t+1}}{p_t} \left( \pi \lambda_{t+1}^s + (1 - \pi) \lambda_{t+1}^i \right) \right] \quad (C.8) \]

From Equations (C.7) and (C.8),

\[ \frac{\lambda_t^i}{\beta} = E_t \left[ \frac{1}{q_t} \lambda_{t+1}^i [(1 - \tau_{t+1}^n) r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R] \right] + (1 - \pi) E_t \left[ \frac{1}{q_t} \lambda_{t+1}^s [(1 - \tau_{t+1}^n) r_{t+1} + \delta q_{t+1}] \right] \]

\[ = E_t \left[ \frac{P_{t+1}}{p_t} \left( \pi \lambda_{t+1}^i + (1 - \pi) \lambda_{t+1}^s \right) \right] \]

From Equations (C.1) and (C.5), respectively, \( \lambda_{t+1}^i = U'_e(e_{t+1}^i) \) and \( \lambda_{t+1}^s = U'_e(e_{t+1}^s) \), and substituting Equation (C.2) gives

\[ \pi E_t \left[ \frac{1}{q_t} \left( (1 + \tau_{t+1}^n) r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R \right) U'_e(e_{t+1}^i) \right] \]

\[ + (1 - \pi) E_t \left[ \frac{1}{q_t} \left( [1 - \tau_{t+1}^n] r_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1} \right) U'_e(e_{t+1}^s) \right] \]

\[ = E_t \left[ \frac{P_{t+1}}{p_t} \left( \pi U'_e(e_{t+1}^i) + (1 - \pi) U'_e(e_{t+1}^s) \right) \right] \]

\[ \Rightarrow \pi E_t \left[ \left( \frac{P_{t+1}}{p_t} \right) U'_e(e_{t+1}^i) \right] + (1 - \pi) E_t \left[ \left( \frac{P_{t+1}}{p_t} \right) U'_e(e_{t+1}^s) \right] \]

\[ = \pi E_t \left[ \left( \frac{1 + \tau_{t+1}^n}{q_t} r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R \right) U'_e(e_{t+1}^i) \right] \]

\[ + (1 - \pi) E_t \left[ \left( \frac{1 + \tau_{t+1}^n}{q_t} r_{t+1} + \delta q_{t+1} \right) U'_e(e_{t+1}^s) \right] \]

\[ \Rightarrow \pi E_t \left[ \left( \frac{P_{t+1}}{p_t} \right) U'_e(e_{t+1}^i) \right] - \pi E_t \left[ \left( \frac{1 + \tau_{t+1}^n}{q_t} r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R \right) U'_e(e_{t+1}^i) \right] \]

\[ = (1 - \pi) E_t \left[ \left( \frac{1 + \tau_{t+1}^n}{q_t} r_{t+1} + \delta q_{t+1} \right) U'_e(e_{t+1}^s) \right] - (1 - \pi) E_t \left[ \left( \frac{P_{t+1}}{p_t} \right) U'_e(e_{t+1}^s) \right] \]

\[ \Rightarrow \pi E_t \left[ \left( \frac{P_{t+1}}{p_t} \right) - \left( \frac{1 + \tau_{t+1}^n}{q_t} \right) \right] U'_e(e_{t+1}^i) \]

\[ = (1 - \pi) E_t \left[ \left( \frac{1 + \tau_{t+1}^n}{q_t} r_{t+1} + \phi_{t+1} \delta q_{t+1} + [1 - \phi_{t+1}] \delta q_{t+1}^R \right) U'_e(e_{t+1}^i) \right] \]
\[
\Rightarrow \pi E_t \left[ \frac{c_{t+1}^{l+1}}{c_t} \frac{[1+\tau_{t+1}^n]r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1-\phi_{t+1}]\delta q_{t+1}^R}{q_t} \right] = (1-\pi) E_t \left[ \frac{[1+\tau_{t+1}^n]r_{t+1} + \delta q_{t+1}^R}{c_{t+1}^{l+1}} \frac{p_{t+1}}{p_t} \right]
\]

Then by Equations (2.26) and (2.27), the last line above becomes

\[
\pi E_t \left[ \frac{[1+\tau_{t+1}^n]r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1-\phi_{t+1}]\delta q_{t+1}^R}{q_t} \right] = (1-\pi) E_t \left[ \frac{[1+\tau_{t+1}^n]r_{t+1} + q_{t+1}\delta n_{t+1} + p_{t+1}m_{t+1}}{c_{t+1}^{l+1}} \right]
\]

C.4 Proof of Claim 1

The RHS of Equations (C.3) and (C.6) are identical, thus giving

\[
\lambda_t^i q_t^R = \lambda_t^s q_t
\]

and from Equations (C.4) and (C.8),

\[
m_{t+1}^i \neq 0 \iff \lambda_t^i = \lambda_t^s \iff q_t^R = q_t \iff 1 - \theta q_t = q_t \iff q_t = 1
\]

\[
\therefore m_{t+1}^i = 0 \iff q_t \neq 1
\]

C.5 Implication of Assumption 1

\[
q_t > 1 \Rightarrow \theta q_t > \theta
\]
\[
\Rightarrow 1 - \theta q_t < 1 - \theta
\]
\[
\Rightarrow \frac{1 - \theta q_t}{1 - \theta} < 1
\]

i.e. \( q_t^R < 1 \)
\[
\Rightarrow q_t^R < q_t
\]
\[
\Rightarrow q_{t+1}^R < q_{t+1}
\]
\[
\Rightarrow \frac{(1+\tau_{t+1}^n)r_{t+1} + [1-\phi_{t+1}]\delta q_{t+1}^R}{q_t} < \frac{(1+\tau_{t+1}^n)r_{t+1} + (1-\phi_{t+1})\delta q_{t+1}}{q_t}
\]
\[
\Rightarrow (1+\tau_{t+1}^n)r_{t+1} + \phi_{t+1}\delta q_{t+1} + (1 - \phi_{t+1})\delta q_{t+1}^R < (1+\tau_{t+1}^n)r_{t+1} + \delta q_{t+1}
\]
C.6 The worker’s first order conditions

From Equations (2.9) and (2.10) the worker’s Lagrangian is

\[
\mathcal{L}_w = E_t \left[ \sum_{j=t}^{\infty} \beta^{j-t} U_w(c_j^w, l_j^w) - \lambda_t^w \left( c_t^w + q_t(n_{t+1}^w - \delta n_{t+1}^w) + p_t(m_{t+1}^w - m_t^w) - (1 - \tau_t^{w\ell}) w_t l_t^w \right) - (1 - \tau_t^{r\ell}) r_t n_t^w \right] 
\]

\[
= U_w(c_t^w, l_t^w) - \lambda_t^w \left( c_t^w + q_t(n_{t+1}^w - \delta n_{t+1}^w) + p_t(m_{t+1}^w - m_t^w) - (1 - \tau_t^{w\ell}) w_t l_t^w - (1 - \tau_t^{r\ell}) r_t n_t^w \right) 
\]

\[
+ \beta E_t \left[ U_w(c_{t+1}^w, l_{t+1}^w) - \lambda_{t+1}^w \left( c_{t+1}^w + q_{t+1}(n_{t+2}^w - \delta n_{t+2}^w) + p_{t+1}(m_{t+2}^w - m_{t+1}^w) - (1 - \tau_{t+1}^{w\ell}) w_{t+1} l_{t+1}^w - (1 - \tau_{t+1}^{r\ell}) r_{t+1} n_{t+1}^w \right) \right] + \beta^2 E_t \left[ U_w(c_{t+2}^w, l_{t+2}^w) - \lambda_{t+2}^w \left( c_{t+2}^w + q_{t+2}(n_{t+3}^w - \delta n_{t+3}^w) + p_{t+2}(m_{t+3}^w - m_{t+2}^w) - (1 - \tau_{t+2}^{w\ell}) w_{t+2} l_{t+2}^w - (1 - \tau_{t+2}^{r\ell}) r_{t+2} n_{t+2}^w \right) \right] + \ldots 
\]

which gives first order conditions

\[
\frac{\partial \mathcal{L}_w}{\partial c_t^w} = \frac{\partial U_w}{\partial c_t^w} - \lambda_t^w = 0 
\]

\[
\implies \lambda_t^w = \frac{\partial U_w}{\partial c_t^w} = 1 
\]

(C.9)

\[
\frac{\partial \mathcal{L}_w}{\partial l_t^w} = \frac{\partial U_w}{\partial l_t^w} + \lambda_t^w (1 - \tau_t^{w\ell}) w_t = 0 
\]

\[
\implies \omega(l_t^w)^\nu = \lambda_t^w (1 - \tau_t^{w\ell}) w_t 
\]

(C.10)

\[
\frac{\partial \mathcal{L}_w}{\partial n_{t+1}^w} = -\lambda_t^w q_t + \beta E_t [\lambda_{t+1}^w (\delta q_{t+1} + [1 - \tau_{t+1}^{w\ell}] r_{t+1})] \leq 0, \quad n_{t+1}^w \geq 0, \quad \text{and} \quad \{ -\lambda_t^w q_t + \beta E_t [\lambda_{t+1}^w (\delta q_{t+1} + [1 - \tau_{t+1}^{w\ell}] r_{t+1})] \} n_{t+1}^w = 0 
\]

\[
\implies \lambda_t^w = \beta E_t \left[ \frac{\delta q_{t+1} + [1 - \tau_{t+1}^{w\ell}] r_{t+1}}{q_t} \lambda_{t+1}^w \right] \quad \text{or} \quad n_{t+1}^w = 0 
\]

(C.11)

\[
\frac{\partial \mathcal{L}_w}{\partial m_{t+1}^w} = -\lambda_t^w p_t + \beta E_t [\lambda_{t+1}^w p_{t+1}] = 0, \quad m_{t+1}^w \geq 0, \quad \text{and} \quad \{ -\lambda_t^w p_t + \beta E_t [\lambda_{t+1}^w p_{t+1}] \} m_{t+1}^w = 0 
\]

\[
\implies \lambda_t^w = \beta E_t \left[ \frac{p_{t+1}}{p_t} \lambda_{t+1}^w \right] \quad \text{or} \quad m_{t+1}^w = 0 
\]

(C.12)

Substituting Equation (C.9) into Equation (C.10) gives

\[
\omega(l_t^w)^\nu = (1 - \tau_t^{w\ell}) w_t \implies l_t^w = \left[ \frac{(1 - \tau_t^{w\ell}) w_t}{\omega} \right]^{1/p} 
\]

60
C.7 Labour market equilibrium

From Equation (2.33) and Equation (2.33), \( L_t^S = L_t^D \) implies

\[
\left[ \frac{(1 - \tau_t^{wl})w_t}{\omega} \right]^{\frac{1}{\nu}} = K_t \left[ \frac{(1 - \gamma)A_t}{w_t} \right]^{\frac{1}{\gamma}}
\]

\[\implies w_t^{\frac{1}{\nu}} w_t^{\frac{1}{\gamma}} = K_t \omega^\nu \left[ (1 - \gamma)A_t \right]^{\frac{1}{\gamma}} \frac{1}{(1 - \tau_t^{wl})^{\frac{1}{\nu}}}
\]

\[\implies w_t^{\frac{\gamma + \nu}{\nu \gamma}} = K_t \omega^\nu \left[ (1 - \gamma)A_t \right]^{\frac{1}{\gamma}} \frac{1}{(1 - \tau_t^{wl})^{\frac{1}{\nu}}}
\]

\[\implies w_t = \left[ K_t \omega^\nu \left[ (1 - \gamma)A_t \right]^{\frac{1}{\gamma}} \frac{1}{(1 - \tau_t^{wl})^{\frac{1}{\nu}}} \right]^{\nu \gamma + \nu - 1}
\]

Then the quantity of labour is

\[
L_t = \left[ \frac{(1 - \tau_t^{wl})w_t}{\omega} \right]^{\frac{1}{\nu}} = \left[ \frac{(1 - \tau_t^{wl})}{\omega} \right]^{\frac{1}{\nu}} w_t^{\frac{1}{\gamma}}
\]

\[= \left[ \frac{(1 - \tau_t^{wl})}{\omega} \right]^{\frac{1}{\nu}} \left[ K_t \omega^\nu \left[ (1 - \gamma)A_t \right]^{\frac{1}{\gamma}} \frac{1}{(1 - \tau_t^{wl})^{\frac{1}{\nu}}} \right]^{\frac{1}{\nu}}
\]

\[= \left[ \frac{(1 - \tau_t^{wl})}{\omega} \right]^{\frac{1}{\nu}} \left[ K_t \omega^\nu \left[ (1 - \gamma)A_t \right]^{\frac{1}{\gamma}} \right]^{\frac{1}{\nu}} \left[ (1 - \tau_t^{wl})^{\gamma + \nu - 1} \right]^{\frac{1}{\nu}}
\]

\[= \left[ \frac{K_t \omega^\nu \left[ (1 - \gamma)A_t \right]^{\frac{1}{\gamma}}}{(1 - \tau_t^{wl})^{\gamma + \nu - 1}} \right]^{\frac{1}{\nu}}
\]

\[= \left[ K_t \omega^\nu \left[ (1 - \gamma)A_t \right]^{\frac{1}{\gamma}} \right]^{\frac{1}{\nu}} (1 - \tau_t^{wl})^{\gamma + \nu - 1}
\]

\[= K_t \omega^\nu \left[ (1 - \gamma)A_t \right]^{\frac{1}{\gamma}} (1 - \tau_t^{wl})^{\gamma + \nu - 1}
\]
C.8 Aggregate output and gross profit

Substituting Equation (2.38) into Equation (2.49) gives aggregate output,

\[ Y_t = A_t K_t^\gamma \left[ K_t^{\gamma + \nu} \omega^{-\gamma - \gamma} \left((1 - \tau_t^v) (1 - \gamma) A_t \right) \right]^{1-\gamma} \]

\[ = A_t K_t^\gamma K_t^{\gamma + \nu} \omega^{-\gamma - \gamma} \left(1 - \tau_t^v\right)(1 - \gamma) A_t \]

\[ = \omega^{-\gamma - \gamma} \left(1 - \gamma\right) A_t K_t^{\gamma + \nu} (1 - \tau_t^v) \gamma(1 - \gamma) + \gamma \]

\[ = \left[ \frac{(1 - \gamma)(1 - \tau_t^v)}{\omega} \right] \frac{\gamma(1 - \gamma)}{A_t \gamma + \nu} K_t^{\gamma + \nu} \]

Then from Equations (2.37) and (2.38), (2.50) for aggregate gross profit becomes

\[ r_t K_t = \left[ \frac{(1 - \gamma)(1 - \tau_t^v)}{\omega} \right] \frac{\gamma(1 + \nu)}{A_t \gamma + \nu} K_t^{\gamma + \nu} - \left[ K_t^{\gamma + \nu} \omega^{-\gamma - \gamma} (1 - \tau_t^v) \gamma(1 - \gamma) A_t \right]^{1-\gamma} \times \]

\[ \left[ \frac{\gamma(1 + \nu)}{A_t \gamma + \nu} K_t^{\gamma + \nu} \right] \]

\[ = \left[ \frac{(1 - \gamma)(1 - \tau_t^v)}{\omega} \right] \frac{\gamma(1 + \nu)}{A_t \gamma + \nu} K_t^{\gamma + \nu} - (1 - \gamma) \frac{\gamma(1 + \nu)}{\gamma + \nu} \omega^{-\gamma - \gamma} (1 - \tau_t^v) \gamma(1 - \gamma) A_t \frac{\gamma(1 + \nu)}{A_t \gamma + \nu} K_t^{\gamma + \nu} \]

\[ = \left(1 - \gamma\right) \frac{\gamma(1 + \nu)}{\gamma + \nu} (1 - \gamma) \frac{\gamma(1 + \nu)}{A_t \gamma + \nu} K_t^{\gamma + \nu} \]

\[ = \gamma \left[ \frac{(1 - \gamma)(1 - \tau_t^v)}{\omega} \right] \frac{\gamma(1 + \nu)}{A_t \gamma + \nu} K_t^{\gamma + \nu} \]

\[ = \alpha_t K_t^\gamma \]

C.9 The \( q_t^R - \theta \) relationship

\[ q_t^R = (1 - \theta q_t)(1 - \theta)^{-1} \]

\[ \implies \frac{\partial q_t^R}{\partial \theta} = (1 - \theta)^{-1} \frac{\partial}{\partial \theta} (1 - \theta q_t) + (1 - \theta q_t) \frac{\partial}{\partial \theta} (1 - \theta)^{-1} \]

\[ = (1 - \theta)^{-1} (-q_t) + (1 - \theta q_t)(1 - \theta)^{-2} \]

\[ = - \frac{q_t}{1 - \theta} + \frac{1 - \theta q_t}{(1 - \theta)^2} \]

\[ = - \frac{q_t (1 - \theta)}{(1 - \theta)^2} + \frac{1 - \theta q_t}{(1 - \theta)^2} \]

62
\[
\frac{-q_t + \theta q_t + 1 - \theta q_t}{(1 - \theta)^2} = \frac{1 - q_t}{(1 - \theta)^2}
\]

By Assumption 1, 1 \(- q_t < 0\) and therefore \(\frac{\partial q_t^R}{\partial \theta} < 0\) for all values of \(\theta\).

**C.10 Standard deviation of innovations to exogenous variables: derivation of Equation (A.1)**

Consider recursive substitutions of the following AR(1) model for \(X_t\):

\[
X_t = (1 - \rho_X)X + \rho_X X_{t-1} + u_t^X
\]

\[
= (1 - \rho_X)X + \rho_X \left[ (1 - \rho_X)X + \rho_X X_{t-2} + u_{t-1}^X \right] + u_t^X
\]

\[
= (1 - \rho_X)X + \rho_X (1 - \rho_X)X + \rho_X^2 X_{t-2} + \rho_X u_{t-1}^X + u_t^X
\]

\[
= (1 - \rho_X)X + \rho_X (1 - \rho_X)X + \rho_X^2 (1 - \rho_X)X + \rho_X^3 X_{t-3} + \rho_X^2 u_{t-2}^X + \rho_X u_{t-1}^X + u_t^X
\]

\[
= \ldots
\]

\[
= (1 - \rho_X)X + \rho_X (1 - \rho_X)X + \rho_X^2 (1 - \rho_X)X + \ldots + u_1^X + \rho_X u_{t-1}^X + \rho_X^2 u_{t-2}^X + \ldots
\]

where \(X\) is the steady state value of \(X_t\). Then the variance of \(X_t\) is given by

\[
\sigma_X^2 = \sigma_{uX}^2 + \rho_X^2 \sigma_{uX}^2 + \rho_X^4 \sigma_{uX}^2 + \ldots = \frac{\sigma_{uX}^2}{1 - \rho_X^2}
\]

which implies the standard deviation of innovations to \(X_t\),

\[
\sigma_{uX} = \sqrt{(1 - \rho_X^2)\sigma_X^2}
\]
Appendix D  Algebraic steady state solution

With Assumption 2, the dynamic equilibrium of the model implies the following steady state system.

\[ T = \tau^{rn} rN + \tau^{wl} wL \] (D.1)
\[ G = T \] (D.2)
\[ w = K^{\nu} \gamma \omega \gamma (1 - \tau^{wl}) \gamma (1 - \gamma) A^{\nu} \gamma \] (D.3)
\[ L = K^{\gamma} \omega^{-1} \gamma (1 - \tau^{wl}) (1 - \gamma) A^{\gamma} \nu \] (D.4)
\[ K = N \] (D.5)
\[ K = I + \delta K \] (D.6)
\[ N^s = (1 - \pi + \pi \phi) \delta N + \theta I \] (D.7)
\[ C^i = \pi (1 - \beta) [(1 - \tau^{rn}) rN + [\phi q + (1 - \phi) q^R] \delta N + pM] \] (D.8)
\[ C^s = (1 - \pi) (1 - \beta) [(1 - \tau^{rn}) rN + q \delta N + pM] \] (D.9)
\[ C^w = (1 - \tau^{wl}) wL \] (D.10)
\[ C = C^i + C^s + C^w \] (D.11)
\[ (1 - \theta q) I = (1 - \tau^{rn}) rN + \phi \delta q \pi N + \pi pM - C^i \] (D.13)
\[ rK = Y - wL \] (D.14)
\[ r = a K^{\alpha - 1} \] (D.15)
\[ Y = C + I + G \] (D.16)
\[ q^R = \frac{1 - \theta q}{1 - \theta} \] (D.17)
\[ a = \gamma \left[ \frac{(1 - \gamma) (1 - \tau^{wl})}{\omega} \right]^{\frac{1 - \gamma}{\nu + 1}} A^{\frac{1 + \nu}{\nu + \gamma}} \] (D.18)

To simplify the notation, let \[ \Sigma = \frac{1 - \phi}{1 - \theta} \]
which gives \[ (1 - \phi) q^R = (1 - \theta q) \Sigma \] (D.19)

Substituting Equations (D.8) to (D.10) into Equation (D.11),

\[ C = \pi (1 - \beta) [(1 - \tau^{rn}) rN + [\phi q + (1 - \phi) q^R] \delta N + pM] + (1 - \pi) (1 - \beta) [(1 - \tau^{rn}) rN + q \delta N + pM] + (1 - \tau^{wl}) wL \]
\[
\begin{align*}
&= (1 - \beta) \left[ (1 - \tau^{rn})r + \pi \phi \delta q + \pi (1 - \phi) \delta q^R + (1 - \pi) \delta q \right] N + (1 - \beta) pM + (1 - \tau^{wl}) wL \\
&= (1 - \beta) \left[ (1 - \tau^{rn})r + (1 - \pi + \pi \phi) \delta q + \pi (1 - \phi) \delta q^R \right] N + (1 - \beta) pM + (1 - \tau^{wl}) wL \\
\end{align*}
\]

(D.20)

Substituting Equation (D.1) into Equation (D.2),
\[
G = \tau^{rn} r^N + \tau^{wl} wL
\]

(D.21)

From Equations (D.5) and (D.6),
\[
I = (1 - \delta) N
\]

(D.22)

Then substituting Equations (D.19) to (D.22) into Equation (D.16),
\[
\begin{align*}
Y &= (1 - \beta) \left[ (1 - \tau^{rn})r + (1 - \pi + \pi \phi) \delta q + \pi (1 - \theta q) \delta \Sigma \right] N + (1 - \beta) pM + (1 - \tau^{wl}) wL \\
&+ (1 - \delta) N + \tau^{rn} r^N + \tau^{wl} wL \\
&= \left[ (1 - \beta) [ (1 - \tau^{rn})r + (1 - \pi + \pi \phi) \delta q + \pi (1 - \theta q) \delta \Sigma ] + 1 - \delta + \tau^{rn} r \right] N \\
&+ (1 - \beta) pM + wL
\end{align*}
\]

(D.23)

Substituting Equation (D.22) into Equation (D.7),
\[
N^* = \left[ (1 - \pi + \pi \phi) \delta + \theta (1 - \delta) \right] N
\]

which is written compactly as
\[
N^* = \chi N
\]

(D.24)

where
\[
\chi = (1 - \pi + \pi \phi) \delta + \theta (1 - \delta)
\]

Substituting Equations (D.8), (D.19) and (D.22) into Equation (D.13),
\[
\begin{align*}
(1 - \theta q)(1 - \delta) N &= \left[ (1 - \tau^{rn})r + \phi \delta q \right] \pi N + \pi pM \\
&- \pi (1 - \beta) [ (1 - \tau^{rn})r N + [\phi q + (1 - \theta q) \Sigma] \delta N + pM ] \\
&= \pi (1 - \tau^{rn}) r N + \pi \phi \delta q N + \pi pM - \pi (1 - \beta) (1 - \tau^{rn}) r N - \pi (1 - \beta) \phi \delta q N \\
&- \pi (1 - \beta) (1 - \theta q) \delta \Sigma N - \pi (1 - \beta) pM \\
&= \pi (1 - \tau^{rn}) r N - \pi (1 - \beta) (1 - \tau^{rn}) r N + \pi \phi \delta q N - \pi (1 - \beta) \phi \delta q N \\
&- \pi (1 - \beta) (1 - \theta q) \delta \Sigma N + \pi pM - \pi (1 - \beta) pM \\
&= [ \pi \beta (1 - \tau^{rn}) r + \pi \beta \phi \delta q - \pi (1 - \beta) (1 - \theta q) \delta \Sigma ] N + \pi \beta pM \\
\Rightarrow (1 - \delta) - (1 - \delta) \theta q &= \pi \beta (1 - \tau^{rn}) r + \pi \beta \phi \delta q - \pi (1 - \beta) (1 - \theta q) \delta \Sigma + \pi \beta \left[ \frac{pM}{N} \right] \\
&= \pi \beta (1 - \tau^{rn}) r + \pi \beta \phi \delta q - \pi (1 - \beta) \delta \Sigma + \pi (1 - \beta) \theta \delta \Sigma q + \pi \beta \left[ \frac{pM}{N} \right]
\end{align*}
\]

65
The expression on the LHS simplifies as follows:

\[
(1 - \delta)\theta + \pi \beta \phi \delta + \pi (1 - \beta) \theta \delta \Sigma q = 1 - \delta + \pi (1 - \beta) \delta \Sigma - \pi \beta (1 - \tau^r) r - \pi \beta \left[ \frac{pM}{N} \right]
\]

(D.25)

From Equations (D.5) and (D.14),

\[ wL = Y - rN \]

which when substituted into Equation (D.23), re-states the goods market clearing condition as

\[
rN = (1 - \beta) \left[ (1 - \tau^r) r + (1 - \pi + \pi \phi) \delta q + \pi (1 - \theta q) \delta \Sigma \right] + 1 - \delta + \tau^r r N + (1 - \beta) pM
\]

\[ \implies r = (1 - \beta) \left[ (1 - \tau^r) r + (1 - \pi + \pi \phi) \delta q + \pi (1 - \theta q) \delta \Sigma \right] + 1 - \delta + \tau^r r + (1 - \beta) \left[ \frac{pM}{N} \right]
\]

\[ \implies [1 - (1 - \beta)(1 - \tau^r) - \tau^r] r = (1 - \beta)(1 - \pi + \pi \phi) \delta q + \pi (1 - \beta)(1 - \theta q) \delta \Sigma
\]

\[ + 1 - \delta + (1 - \beta) \left[ \frac{pM}{N} \right]
\]

\[ \implies \beta (1 - \tau^r) r = 1 - \delta + \pi (1 - \beta) \delta \Sigma + (1 - \beta)(1 - \pi + \pi \phi - \pi \theta \Sigma) \delta q + (1 - \beta) \left[ \frac{pM}{N} \right]
\]

(D.26)

The numerator on the LHS of the portfolio balance equation (D.12) simplifies as

\[
\pi \left[ 1 - \left( \frac{[1 - \tau^r] r + \phi \delta q + (1 - \theta q) \delta \Sigma}{q} \right) \right] = \pi [q - (1 - \tau^r) r + \phi \delta q + (1 - \theta q) \delta \Sigma] \div q
\]

\[ = \pi [q - (1 - \tau^r) r - \phi \delta q - (1 - \theta q) \delta \Sigma] \div q
\]

\[ = \pi [(1 - \phi \delta + \theta \delta \Sigma) q - (1 - \tau^r) r - \delta \Sigma] \div q
\]

\[ = \pi (1 - \phi \delta + \theta \delta \Sigma) q - \pi (1 - \tau^r) r - \pi \delta \Sigma \div q
\]

and the RHS numerator simplifies as

\[
(1 - \pi) \left[ \left( \frac{[1 - \tau^r] r + \delta q}{q} \right) - 1 \right] = (1 - \pi)[(1 - \tau^r) r + \delta q - q] \div q
\]

\[ = (1 - \pi)[(1 - \tau^r) r - (1 - \delta) q] \div q
\]

\[ = [(1 - \pi)(1 - \tau^r)r - (1 - \pi)(1 - \delta)q] \div q
\]

Then the portfolio balance equation becomes

\[
\frac{\pi (1 - \phi \delta + \theta \delta \Sigma) q - \pi (1 - \tau^r) r - \pi \delta \Sigma}{(1 - \tau^r) r N^s + \phi \delta q N^s + (1 - \theta q) \Sigma \delta N^s + pM} = \frac{(1 - \pi)(1 - \tau^r) r - (1 - \pi)(1 - \delta) q}{(1 - \tau^r) r N^s + q \delta N^s + pM}
\]

\[ \implies \left[ \pi (1 - \phi \delta + \theta \delta \Sigma) q - \pi (1 - \tau^r) r - \pi \delta \Sigma \right] \left[ (1 - \tau^r) r N^s + q \delta N^s + pM \right] = \left[ (1 - \pi)(1 - \tau^r)r - (1 - \pi)(1 - \delta) q \right] \left[ (1 - \tau^r) r N^s + \phi \delta q N^s + (1 - \theta q) \Sigma \delta N^s + pM \right]
\]

The expression on the LHS simplifies as follows:

\[
\left[ \pi (1 - \phi \delta + \theta \delta \Sigma) q - \pi (1 - \tau^r) r - \pi \delta \Sigma \right] \left[ (1 - \tau^r) r N^s + q \delta N^s + pM \right]
\]

66
The portfolio balance equation now becomes:

\[
\pi (1 - \phi \delta + \theta \delta \Sigma) q - \pi (1 - \tau^r)n r - \pi \delta \Sigma \] (1 - \tau^r)n r N^s
\]

\[
+ \pi (1 - \phi \delta + \theta \delta \Sigma) q - \pi (1 - \tau^r)n r - \pi \delta \Sigma \] q \delta N^s
\]

\[
+ \pi (1 - \phi \delta + \theta \delta \Sigma) q - \pi (1 - \tau^r)n r - \pi \delta \Sigma \] p M
\]

\[
= \pi (1 - \phi \delta + \theta \delta \Sigma)(1 - \tau^r)n r q N^s - \pi (1 - \tau^r)n r^2 N^s - \pi \delta \Sigma(1 - \tau^r)n r N^s
\]

\[
+ \pi (1 - \phi \delta + \theta \delta \Sigma) q^2 \delta N^s - \pi (1 - \tau^r)n r q \delta N^s - \pi \delta \Sigma q \delta N^s
\]

\[
+ \pi (1 - \phi \delta + \theta \delta \Sigma) q \delta N^s p M - \pi (1 - \tau^r)n r p M - \pi \delta \Sigma p M
\]

\[
= \pi (1 - \delta - \phi \delta + \theta \delta \Sigma)(1 - \tau^r)n r q N^s - \pi (1 - \tau^r)n r^2 N^s - \pi \delta \Sigma(1 - \tau^r)n r N^s
\]

\[
+ \pi (1 - \phi \delta + \theta \delta \Sigma) q \delta N^s - \pi \delta \Sigma q \delta N^s + \pi (1 - \phi \delta + \theta \delta \Sigma) q \delta N^s p M - \pi (1 - \tau^r)n r p M - \pi \delta \Sigma p M
\]

and the expression on the RHS simplifies as:

\[
\left[(1 - \pi)(1 - \tau^r)n r - (1 - \pi)(1 - \delta)q\right] \left[(1 - \tau^r)n r N^s + \phi \delta q N^s + (1 - \theta q) \Sigma \delta N^s + p M\right]
\]

\[
= (1 - \pi)(1 - \tau^r)n r \left[(1 - \tau^r)n r N^s + \phi \delta q N^s + \delta \Sigma N^s - \theta \delta \Sigma q N^s + p M\right]
\]

\[
- (1 - \pi)(1 - \delta)q \left[(1 - \tau^r)n r N^s + \phi \delta q N^s + \delta \Sigma N^s - \theta \delta \Sigma q N^s + p M\right]
\]

\[
= (1 - \pi)(1 - \tau^r)n r^2 N^s + (1 - \pi)(1 - \tau^r)n \phi \delta q N^s + (1 - \pi)(1 - \tau^r)n \delta \Sigma r N^s
\]

\[
- (1 - \pi)(1 - \tau^r)n \theta \delta \Sigma r q N^s + \pi (1 - \pi)(1 - \tau^r)n r p M - (1 - \pi)(1 - \delta)(1 - \tau^r)n r q N^s
\]

\[
- (1 - \pi)(1 - \delta)q \phi \delta q^2 N^s - (1 - \pi)(1 - \delta) \Sigma \delta q N^s + (1 - \pi)(1 - \delta) \theta \delta \Sigma q^2 N^s - (1 - \pi)(1 - \pi)(1 - \delta)q p M
\]

\[
= (1 - \pi)(-1 + \delta + \phi \delta - \theta \delta \Sigma)(1 - \tau^r)n r q N^s + (1 - \pi)(1 - \tau^r)n^2 r^2 N^s + (1 - \pi)(1 - \tau^r)n \delta \Sigma r N^s
\]

\[
+ (1 - \pi)(1 - \delta) \delta (\theta \Sigma - \phi) q^2 N^s - (1 - \pi)(1 - \delta) \delta \Sigma q N^s - (1 - \pi)(1 - \delta) \phi \delta q^2 N^s - (1 - \pi)(1 - \tau^r)n r p M
\]

The portfolio balance equation now becomes:

\[
\pi (1 - \delta - \phi \delta + \theta \delta \Sigma)(1 - \tau^r)n r q N^s - \pi (1 - \tau^r)n r^2 N^s - \pi \delta \Sigma(1 - \tau^r)n r N^s
\]

\[
+ \pi (1 - \phi \delta + \theta \delta \Sigma) q^2 N^s - \pi \delta \Sigma q N^s + \pi (1 - \phi \delta + \theta \delta \Sigma) q \delta N^s p M - \pi (1 - \tau^r)n r p M - \pi \delta \Sigma p M
\]

\[
= (1 - \pi)(-1 + \delta + \phi \delta - \theta \delta \Sigma)(1 - \tau^r)n r q N^s + (1 - \pi)(1 - \tau^r)n^2 r^2 N^s
\]

\[
+ (1 - \pi)(1 - \tau^r)n \delta \Sigma r N^s + (1 - \pi)(1 - \delta) \delta (\theta \Sigma - \phi) q^2 N^s - (1 - \pi)(1 - \delta) \delta \Sigma q N^s
\]

\[
- (1 - \pi)(1 - \delta)q \phi \delta q^2 N^s - (1 - \pi)(1 - \tau^r)n r p M
\]

\[
\Rightarrow 0 = \pi (1 - \delta - \phi \delta + \theta \delta \Sigma)(1 - \tau^r)n r q N^s - \pi (1 - \tau^r)n r^2 N^s - \pi \delta \Sigma(1 - \tau^r)n r N^s
\]

\[
+ \pi (1 - \phi \delta + \theta \delta \Sigma) q^2 N^s - \pi \delta \Sigma q N^s + \pi (1 - \phi \delta + \theta \delta \Sigma) q \delta N^s p M - \pi (1 - \tau^r)n r p M - \pi \delta \Sigma p M
\]

\[
- \pi \delta \Sigma p M - (1 - \pi)(-1 + \delta + \phi \delta - \theta \delta \Sigma)(1 - \tau^r)n r q N^s - (1 - \pi)(1 - \tau^r)n^2 r^2 N^s
\]

\[
- (1 - \pi)(1 - \tau^r)n \delta \Sigma r N^s - (1 - \pi)(1 - \delta) \delta (\theta \Sigma - \phi) q^2 N^s + (1 - \pi)(1 - \delta) \delta \Sigma q N^s
\]

\[
+ (1 - \pi)(1 - \delta)q \phi \delta q^2 N^s - (1 - \pi)(1 - \tau^r)n r p M
\]

\[
= (1 - \delta - \phi \delta + \theta \delta \Sigma)(1 - \tau^r)n r q N^s - (1 - \tau^r)n^2 r^2 N^s - \delta (1 - \tau^r)n r N^s
\]
\[ \begin{align*}
+ (\pi + \phi - \pi\phi - \phi\delta - \theta\Sigma + \pi\theta\Sigma + \theta\delta\Sigma)\delta q^2 N^s + (1 - \pi - \delta)\delta\Sigma q N^s \\
+ (1 - \delta + \pi\delta - \pi\phi\delta + \pi\theta\delta\Sigma)qsM - (1 - \tau^r)rpM - \pi\delta\Sigma pM
\end{align*} \]

Finally, substituting Equation (D.24), and then dividing throughout by \(N\),

\[ \begin{align*}
0 &= (1 - \delta - \phi\delta + \theta\delta\Sigma)(1 - \tau^r)qr\chi N - (1 - \tau^r)^2 r^2 \chi N - \delta\Sigma(1 - \tau^r) r \chi N \\
+ (\pi + \phi - \pi\phi - \phi\delta - \theta\Sigma + \pi\theta\Sigma + \theta\delta\Sigma)\delta q^2 \chi N + (1 - \pi - \delta)\delta\Sigma q \chi N \\
+ (1 - \delta + \pi\delta - \pi\phi\delta + \pi\theta\delta\Sigma)qsM - (1 - \tau^r)rpM - \pi\delta\Sigma pM \\
= \chi(\pi + \phi - \pi\phi - \phi\delta - \theta\Sigma + \pi\theta\Sigma + \theta\delta\Sigma)\delta q^2 - \chi(1 - \tau^r)^2 r^2 + \chi(1 - \pi - \delta)\delta\Sigma q \\
- \chi(1 - \tau^r)\delta\Sigma r - \pi\delta\Sigma \left[ \frac{pM}{N} \right] + \chi(1 - \delta - \phi\delta + \theta\delta\Sigma)(1 - \tau^r) qr \\
- (1 - \tau^r) \left[ \frac{pM}{N} \right] r + (1 - \delta + \pi\delta - \pi\phi\delta + \pi\theta\delta\Sigma) \left[ \frac{pM}{N} \right] q
\end{align*} \] (D.27)

Equations (D.25) to (D.27) form a system of 3 equations with 3 unknowns, \(q, r, \) and \(pM/N\), which is compactly expressed as

\[ \begin{align*}
\Gamma_q q + \Gamma_r r + \Gamma_s s &= \Gamma_{01} \\
\Gamma_q q^2 + \Gamma_r r^2 + \Gamma_s s &= \Gamma_{02} \\
\Gamma_q q^3 + \Gamma_r r^3 + \Gamma_s s + \Gamma_{qr} qr r + \Gamma_{qs} q s + \Gamma_{rs} r s + \Gamma_{qs3} q s &= 0
\end{align*} \] (D.30)

where

\[ \begin{align*}
q &= \frac{pM}{N} \\
\Gamma_q &= (1 - \delta)\theta + \pi\beta\phi\delta + \pi(1 - \beta)\theta\delta\Sigma \\
\Gamma_r &= -(1 - \beta)(1 - \pi + \pi\phi - \pi\theta\Sigma)\delta \\
\Gamma_s &= \pi\beta(1 - \tau^r) \\
\Gamma_{s1} &= \pi\beta \\
\Gamma_{s2} &= -(1 - \beta) \\
\Gamma_{01} &= 1 - \delta + \pi(1 - \beta)\delta\Sigma \\
\Gamma_{02} &= 1 - \delta + \pi(1 - \beta)\delta\Sigma \\
\Gamma_{qr3} &= \chi(\pi + \phi - \pi\phi - \phi\delta - \theta\Sigma + \pi\theta\Sigma + \theta\delta\Sigma)\delta \\
\Gamma_{rr3} &= \chi(1 - \tau^r)^2 \\
\Gamma_{qr} &= \chi(1 - \pi - \delta)\delta\Sigma \\
\Gamma_{rs} &= -\chi(1 - \tau^r)\delta\Sigma \\
\Gamma_{qs} &= \chi(1 - \tau^r)\delta\Sigma \\
\Gamma_{qs3} &= \chi(1 - \delta - \phi\delta + \theta\delta\Sigma)(1 - \tau^r) \\
\Gamma_{rs3} &= -(1 - \tau^r) \\
\Gamma_{qs3} &= 1 - \delta + \pi\delta - \pi\phi\delta + \pi\theta\delta\Sigma
\end{align*} \]
Since $\Gamma_{01} = \Gamma_{02}$ then equating Equations (D.28) and (D.29) gives 
\[
\Gamma_q q + \Gamma_r r + \Gamma_s s = \Gamma_q q + \Gamma_r r + \Gamma_s s 
\]
Notice that $\Gamma_r = \pi \Gamma_2$. Then the equation above becomes 
\[
\Gamma_q q + \pi \Gamma_2 r + \Gamma_s s = \Gamma_q q + \Gamma_r r + \Gamma_s s 
\]
\[
\implies (\Gamma_q - \Gamma_q) q - (1 - \pi) \Gamma_2 r + (\Gamma_s - \Gamma_s) s = 0 
\]
\[
\implies r = \left[ \frac{\Gamma_q - \Gamma_q}{(1 - \pi) \Gamma_2} \right] q + \left[ \frac{\Gamma_s - \Gamma_s}{(1 - \pi) \Gamma_2} \right] s 
\]  \hspace{1cm} \text{(D.31)}

Substituting Equation (D.31) into Equation (D.28) gives a linear relationship between $s$ and $q$, 
\[
\Gamma_q q + \Gamma_r r \left[ \left( \frac{\Gamma_q - \Gamma_q}{(1 - \pi) \Gamma_2} \right) q + \left[ \frac{\Gamma_s - \Gamma_s}{(1 - \pi) \Gamma_2} \right] s \right] + \Gamma_s s = \Gamma_{01} 
\]
\[
\implies \left[ \frac{\Gamma_q}{(1 - \pi) \Gamma_2} \right] q + \left[ \frac{\Gamma_s + (\Gamma_s - \Gamma_s) \Gamma_r}{(1 - \pi) \Gamma_2} \right] s = \Gamma_{01} 
\]
\[
\implies \left[ \frac{(1 - \pi) \Gamma q_1 + (\Gamma_q - \Gamma_q) \Gamma_r}{(1 - \pi) \Gamma_2} \right] q + \left[ \frac{(1 - \pi) \Gamma r_2 \Gamma s + (\Gamma_s - \Gamma_s) \Gamma_r}{(1 - \pi) \Gamma_2} \right] s = \Gamma_{01} 
\]
\[
\implies s = \Gamma_{01} \left[ \frac{(1 - \pi) \Gamma r_2}{(1 - \pi) \Gamma_2} \right] q - \left[ \frac{(1 - \pi) \Gamma q_1 \Gamma r_2 + (\Gamma q_1 - \Gamma q_2) \Gamma r}{(1 - \pi) \Gamma_2} \right] q 
\]
\[
= \left[ \frac{(1 - \pi) \Gamma r_2 \Gamma_{01}}{(1 - \pi) \Gamma_2 \Gamma s + (\Gamma s - \Gamma s) \Gamma r} \right] q - \left[ \frac{(1 - \pi) \Gamma q_1 \Gamma r_2 + (\Gamma q_1 - \Gamma q_2) \Gamma r}{(1 - \pi) \Gamma_2 \Gamma s + (\Gamma s - \Gamma s) \Gamma r} \right] q 
\]  \hspace{1cm} \text{(D.32)}

which is written compactly as 
\[
\text{\quad \quad} s = s_0 + s_1 q 
\]  \hspace{1cm} \text{(D.33)}

where 
\[
\text{s}_0 = \frac{(1 - \pi) \Gamma r_2 \Gamma_{01}}{(1 - \pi) \Gamma_2 \Gamma s + (\Gamma s - \Gamma s) \Gamma r} 
\]
\[
\text{s}_1 = -\frac{(1 - \pi) \Gamma q_1 \Gamma r_2 + (\Gamma q_1 - \Gamma q_2) \Gamma r}{(1 - \pi) \Gamma_2 \Gamma s + (\Gamma s - \Gamma s) \Gamma r} 
\]

Substituting Equation (D.32) into Equation (D.31) gives a linear relationship between $r$ and $q$, 
\[
r = \left[ \frac{\Gamma_q - \Gamma_q}{(1 - \pi) \Gamma_2} \right] q + \left[ \frac{\Gamma_s - \Gamma_s}{(1 - \pi) \Gamma_2} \right] \left( \left[ \frac{(1 - \pi) \Gamma r_2 \Gamma_{01}}{(1 - \pi) \Gamma_2 \Gamma s + (\Gamma s - \Gamma s) \Gamma r} \right] q \right) 
\]
\[
- \left[ \frac{(1 - \pi) \Gamma q_1 \Gamma r_2 + (\Gamma q_1 - \Gamma q_2) \Gamma r}{(1 - \pi) \Gamma_2 \Gamma s + (\Gamma s - \Gamma s) \Gamma r} \right] q 
\]
\[
= \left[ \frac{\Gamma_q - \Gamma_q}{(1 - \pi) \Gamma_2} \right] q + \left[ \frac{\Gamma_s - \Gamma_s}{(1 - \pi) \Gamma_2} \right] \left[ \frac{(1 - \pi) \Gamma r_2 \Gamma_{01}}{(1 - \pi) \Gamma_2 \Gamma s + (\Gamma s - \Gamma s) \Gamma r} \right] q 
\]
\[
- \left[ \frac{\Gamma_s - \Gamma_s}{(1 - \pi) \Gamma_2} \right] \left[ \frac{(1 - \pi) \Gamma q_1 \Gamma r_2 + (\Gamma q_1 - \Gamma q_2) \Gamma r}{(1 - \pi) \Gamma_2 \Gamma s + (\Gamma s - \Gamma s) \Gamma r} \right] q 
\]  \hspace{1cm} \text{(D.34)}
which has solutions

\[ q = \frac{\left[ (\Gamma_{q_1} - \Gamma_{q_2}) \left( (1 - \pi) \Gamma_{r_2} \Gamma_{s_1} + (\Gamma_{s_1} - \Gamma_{s_2}) \Gamma_{r_1} \right) - (\Gamma_{s_1} - \Gamma_{s_2}) \left( (1 - \pi) \Gamma_{q_1} \Gamma_{r_2} + (\Gamma_{q_1} - \Gamma_{q_2}) \Gamma_{r_1} \right) \right]}{(1 - \pi) \Gamma_{r_2} \left( (1 - \pi) \Gamma_{r_2} \Gamma_{s_1} + (\Gamma_{s_1} - \Gamma_{s_2}) \Gamma_{r_1} \right) + \left( (\Gamma_{s_1} - \Gamma_{s_2}) \Gamma_{r_1} \right)} 
\]

which is written compactly as

\[ r = r_0 + r_1 q \]  

\[ (D.35) \]

where

\[ r_0 = \frac{(\Gamma_{s_1} - \Gamma_{s_2}) \Gamma_{r_1}}{(1 - \pi) \Gamma_{r_2} \Gamma_{s_1} + (\Gamma_{s_1} - \Gamma_{s_2}) \Gamma_{r_1}} \]

\[ r_1 = \frac{(\Gamma_{s_1} - \Gamma_{s_2}) \Gamma_{r_1}}{(1 - \pi) \Gamma_{r_2} \Gamma_{s_1} + (\Gamma_{s_1} - \Gamma_{s_2}) \Gamma_{r_1}} \]

Finally, substituting Equations (D.33) and (D.35) into Equation (D.30) gives a quadratic in \( q \),

\[ 0 = \Gamma_{qq} q^2 + \Gamma_{rr} (r_0 + r_1 q)^2 + \Gamma_{qq} q + \Gamma_{rr} (r_0 + r_1 q) + \Gamma_{ss} (s_0 + s_1 q) + \Gamma_{qr} q (r_0 + r_1 q) \]

\[ + \Gamma_{rs} \left( r_0 + r_1 q \right) (s_0 + s_1 q) + \Gamma_{qs} q (s_0 + s_1 q) \]

\[ = \Gamma_{qq} q^2 + \Gamma_{rr} (r_0^2 + 2r_0 r_1 q + r_1^2 q^2) + \Gamma_{qq} q + \Gamma_{rr} (r_0 + r_1 q) + \Gamma_{ss} (s_0 + s_1 q) + \Gamma_{qr} q (r_0 + r_1 q^2) \]

\[ + \Gamma_{rs} (r_0 s_0 + r_0 s_1 q + r_1 s_0 q + r_1 s_1 q^2) + \Gamma_{qs} (s_0 q + s_1 q^2) \]

\[ = \left[ \Gamma_{qq} + \Gamma_{rr} r_1^2 + \Gamma_{qr} r_1 + \Gamma_{rs} r_1 s_1 + \Gamma_{qs} s_1 \right] q^2 + \left[ 2\Gamma_{rr} r_0 r_1 + \Gamma_{q3} + \Gamma_{r3} r_1 + \Gamma_{s3} s_1 + \Gamma_{qr} r_0 \right. \]

\[ + \Gamma_{rs} r_0 s_1 + \Gamma_{rr} r_1 s_0 + \Gamma_{qs} s_0 \] \[ q + \left[ \Gamma_{rr} r_0^2 + \Gamma_{r3} r_0 + \Gamma_{s3} s_0 + \Gamma_{rs} r_0 s_0 \right] = \mu_2 q^2 + \mu_1 q + \mu_0 \]

which has solutions

\[ q' = \frac{-\mu_1 + \sqrt{\mu_1^2 - 4\mu_0 \mu_2}}{2\mu_2} \]

\[ q'' = \frac{-\mu_1 - \sqrt{\mu_1^2 - 4\mu_0 \mu_2}}{2\mu_2} \]

70
where

\[
\begin{align*}
\mu_2 &= \Gamma_{q3} + \Gamma_{rr}r_1^2 + \Gamma_{qr}r_1 + \Gamma_{qs}s_1 + \Gamma_{qs}s_1 \\
\mu_1 &= 2\Gamma_{rr}r_0r_1 + \Gamma_{q3} + \Gamma_{r3}r_1 + \Gamma_{s3}s_1 + \Gamma_{qr}r_0 + \Gamma_{rs}r_1s_1 + \Gamma_{qs}s_0 + \Gamma_{qs}s_0 \\
\mu_0 &= \Gamma_{rr}r_0^2 + \Gamma_{r3}r_0 + \Gamma_{s3}s_0 + \Gamma_{rs}r_0s_0
\end{align*}
\]

By Assumption 1, if only one root is greater than 1, then this root is taken as the steady state value of \(q\); if both roots are greater than 1 then the smaller value is used. The steady state values of \(r\) and \(s\) are obtained from Equation (D.35) and Equation (D.33), respectively. Steady state values \(\{A, \phi, N^g, M, \tau^{re}, \tau^{w}\}\) for the model’s exogenous parameters are assumed given. With these, together with values of \(q, r,\) and \(s,\) the steady state values of the model’s other endogenous variables are easily obtained. Firstly, Equations (D.17) and (D.18) provide \(q^R\) and \(a.\) Then from Equation (D.15),

\[
K = \left(\frac{r}{q}\right)^{\frac{1}{1-\alpha}}
\]

which is also the value of \(N,\) by Equation (D.5). \(N^s\) comes from Equation (D.24) and \(p\) follows from the definition of \(s,\) i.e.

\[
p = s\frac{N}{M}
\]

Next, \(w\) and \(L\) are delivered by Equations (D.3) and (D.4), respectively. Finally, the other variables \(\{T, G, I, C^i, C^s, C^w, C, Y\}\) are calculated from equations (D.1), (D.2), (D.22), (D.8), (D.9), (D.10), (D.11), and (D.16), respectively.
Table 10: US liquid assets, capital and liquidity

<table>
<thead>
<tr>
<th>Year</th>
<th>Federal government liabilities ($b)</th>
<th>Capital ($b)</th>
<th>Liquidity share</th>
</tr>
</thead>
<tbody>
<tr>
<td>1945</td>
<td>262.4</td>
<td>471.0</td>
<td>0.3578</td>
</tr>
<tr>
<td>1946</td>
<td>238.4</td>
<td>544.9</td>
<td>0.3044</td>
</tr>
<tr>
<td>1947</td>
<td>230.1</td>
<td>635.3</td>
<td>0.2659</td>
</tr>
<tr>
<td>1948</td>
<td>222.6</td>
<td>701.2</td>
<td>0.2410</td>
</tr>
<tr>
<td>1949</td>
<td>222.3</td>
<td>736.0</td>
<td>0.2320</td>
</tr>
<tr>
<td>1950</td>
<td>221.2</td>
<td>830.6</td>
<td>0.2103</td>
</tr>
<tr>
<td>1951</td>
<td>222.3</td>
<td>928.2</td>
<td>0.1932</td>
</tr>
<tr>
<td>1952</td>
<td>229.1</td>
<td>962.1</td>
<td>0.1923</td>
</tr>
<tr>
<td>1953</td>
<td>231.1</td>
<td>992.8</td>
<td>0.1888</td>
</tr>
<tr>
<td>1954</td>
<td>232.1</td>
<td>1076.3</td>
<td>0.1774</td>
</tr>
<tr>
<td>1955</td>
<td>233.6</td>
<td>1188.9</td>
<td>0.1642</td>
</tr>
<tr>
<td>1956</td>
<td>228.8</td>
<td>1281.4</td>
<td>0.1515</td>
</tr>
<tr>
<td>1957</td>
<td>228.6</td>
<td>1314.0</td>
<td>0.1482</td>
</tr>
<tr>
<td>1958</td>
<td>235.2</td>
<td>1434.0</td>
<td>0.1409</td>
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<tr>
<td>1959</td>
<td>245.3</td>
<td>1516.4</td>
<td>0.1392</td>
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<td>1574.6</td>
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<td>1961</td>
<td>248.2</td>
<td>1700.8</td>
<td>0.1273</td>
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<tr>
<td>1962</td>
<td>253.7</td>
<td>1748.3</td>
<td>0.1267</td>
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<td>1963</td>
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<td>0.1230</td>
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<td>0.0990</td>
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<tr>
<td>1969</td>
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<td>0.0913</td>
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<tr>
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<td>4722.8</td>
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<tr>
<td>1976</td>
<td>553.2</td>
<td>5327.2</td>
<td>0.0941</td>
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<tr>
<td>1977</td>
<td>611.9</td>
<td>5856.7</td>
<td>0.0946</td>
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<tr>
<td>1978</td>
<td>673.0</td>
<td>6672.2</td>
<td>0.0916</td>
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<tr>
<td>1979</td>
<td>723.4</td>
<td>7787.0</td>
<td>0.0850</td>
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</table>

Continued on next page
<table>
<thead>
<tr>
<th>Year</th>
<th>Federal government liabilities ($b)</th>
<th>Capital ($b)</th>
<th>Liquidity share</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>823.6</td>
<td>8982.0</td>
<td>0.0840</td>
</tr>
<tr>
<td>1981</td>
<td>923.7</td>
<td>9604.3</td>
<td>0.0877</td>
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<tr>
<td>1982</td>
<td>1101.3</td>
<td>10189.0</td>
<td>0.0975</td>
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<tr>
<td>1983</td>
<td>1290.9</td>
<td>10797.8</td>
<td>0.1068</td>
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<tr>
<td>1984</td>
<td>1501.1</td>
<td>11513.4</td>
<td>0.1153</td>
</tr>
<tr>
<td>1985</td>
<td>1744.1</td>
<td>12817.3</td>
<td>0.1198</td>
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<tr>
<td>1986</td>
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<td>0.1235</td>
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<td>1987</td>
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<td>0.1244</td>
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<td>1988</td>
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<td>16078.1</td>
<td>0.1224</td>
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<td>1989</td>
<td>2404.1</td>
<td>17644.1</td>
<td>0.1199</td>
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<tr>
<td>1990</td>
<td>2671.6</td>
<td>17957.5</td>
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<td>1991</td>
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<td>19871.3</td>
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<td>0.1458</td>
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<td>1994</td>
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<td>0.1485</td>
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<td>1995</td>
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<td>0.1416</td>
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<td>0.1427</td>
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<td>1997</td>
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<td>4118.6</td>
<td>35308.8</td>
<td>0.1045</td>
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<td>4549.1</td>
<td>40199.5</td>
<td>0.1017</td>
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<td>2004</td>
<td>4930.0</td>
<td>45337.8</td>
<td>0.0981</td>
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<tr>
<td>2005</td>
<td>5273.5</td>
<td>50847.9</td>
<td>0.0940</td>
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<tr>
<td>2006</td>
<td>5475.2</td>
<td>54131.2</td>
<td>0.0919</td>
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<td>2007</td>
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<td>0.0968</td>
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<tr>
<td>2008</td>
<td>8980.9</td>
<td>45001.9</td>
<td>0.1664</td>
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<tr>
<td>2009</td>
<td>9990.4</td>
<td>44224.8</td>
<td>0.1843</td>
</tr>
<tr>
<td>2010</td>
<td>11707.4</td>
<td>46010.2</td>
<td>0.2028</td>
</tr>
<tr>
<td>2011</td>
<td>12444.2</td>
<td>46217.6</td>
<td>0.2121</td>
</tr>
</tbody>
</table>

**Average: 1957 - 2007**

0.1110

**Standard deviation: 1957 - 2007**

0.0204


*NOTES:* The liquidity share is calculated according to Del Negro et al. (2011). Table 11 gives the metadata. The liquidity share is illustrated graphically in Figure 3.
Table 11: Liquidity share measure: metadata

<table>
<thead>
<tr>
<th>Item</th>
<th>Flow of Funds Statistics reference</th>
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</thead>
<tbody>
<tr>
<td><strong>Liabilities of the federal government</strong></td>
<td></td>
</tr>
<tr>
<td>T-bills</td>
<td>Table L.105, line 21</td>
</tr>
<tr>
<td>Treasury securities</td>
<td>Table L.105, line 22</td>
</tr>
<tr>
<td>Less: Holdings by the monetary authority</td>
<td>Table L.108, line 12</td>
</tr>
<tr>
<td>Less: Holdings by the budgetary agency</td>
<td>Table L.105, line 23</td>
</tr>
<tr>
<td>Reserves</td>
<td>Table L.108, line 32</td>
</tr>
<tr>
<td>Vault cash</td>
<td>Table L.108, line 33</td>
</tr>
<tr>
<td>Currency</td>
<td>Table L.108, line 34</td>
</tr>
<tr>
<td>Currency outside banks</td>
<td>Table L.108, line 41</td>
</tr>
<tr>
<td>Less: Remittances to the federal government</td>
<td>Table L.108, line 35</td>
</tr>
<tr>
<td><strong>Capital (at market value)</strong></td>
<td></td>
</tr>
<tr>
<td>Capital owned by households:</td>
<td></td>
</tr>
<tr>
<td>Real estate</td>
<td>Table B.100, line 3</td>
</tr>
<tr>
<td>Equipment and software of non-profit organisations</td>
<td>Table B.100, line 6</td>
</tr>
<tr>
<td>Consumer durables</td>
<td>Table B.100, line 7</td>
</tr>
<tr>
<td>Capital owned by the non-corporate sector:</td>
<td></td>
</tr>
<tr>
<td>Real estate</td>
<td>Table B.103, line 3</td>
</tr>
<tr>
<td>Equipment and software</td>
<td>Table B.103, line 6</td>
</tr>
<tr>
<td>Inventories</td>
<td>Table B.103, line 9</td>
</tr>
<tr>
<td>Capital owned by the corporate sector:</td>
<td></td>
</tr>
<tr>
<td>Equity outstanding, market value</td>
<td>Table B.102, line 35</td>
</tr>
<tr>
<td>Liabilities</td>
<td>Table B.102, line 21</td>
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<tr>
<td>Less: Financial assets</td>
<td>Table B.102, line 6</td>
</tr>
<tr>
<td>Less: Government credit market instruments</td>
<td>Table F.105c, line 33</td>
</tr>
<tr>
<td>Less: Trade receivables</td>
<td>Table F.105c, line 43</td>
</tr>
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</table>

NOTES: This table follows from the appendix of Del Negro et al. (2011).
Table 12: US federal government’s corporate equity holdings

<table>
<thead>
<tr>
<th>Period</th>
<th>Equities ($m)</th>
<th>ln(Equities)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008:4</td>
<td>188,676</td>
<td>12.15</td>
</tr>
<tr>
<td>2009:1</td>
<td>223,856</td>
<td>12.32</td>
</tr>
<tr>
<td>2009:2</td>
<td>157,566</td>
<td>11.97</td>
</tr>
<tr>
<td>2009:3</td>
<td>158,847</td>
<td>11.98</td>
</tr>
<tr>
<td>2010:1</td>
<td>50,234</td>
<td>10.82</td>
</tr>
<tr>
<td>2010:2</td>
<td>49,613</td>
<td>10.81</td>
</tr>
<tr>
<td>2010:3</td>
<td>50,814</td>
<td>10.84</td>
</tr>
<tr>
<td>2010:4</td>
<td>49,928</td>
<td>10.82</td>
</tr>
<tr>
<td>2011:1</td>
<td>62,137</td>
<td>11.04</td>
</tr>
<tr>
<td>2011:2</td>
<td>65,961</td>
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<tr>
<td>2011:3</td>
<td>59,282</td>
<td>10.99</td>
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<td>57,813</td>
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<td>2012:1</td>
<td>48,156</td>
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<tr>
<td>2012:2</td>
<td>43,618</td>
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<tr>
<td>2012:3</td>
<td>41,134</td>
<td>10.62</td>
</tr>
</tbody>
</table>

**Standard deviation**: 0.5671

*Source*: Board of Governors of the Federal Reserve, US Flow of Funds Accounts, Table L.105, line 11.

*NOTES*: Equities represent those purchased by the US government from financial corporations under the Troubled Asset Relief Program. They are valued at market prices.
<table>
<thead>
<tr>
<th>Period</th>
<th>M1 ($b)</th>
<th>CPI</th>
<th>CPI, s.a.</th>
<th>Real M1</th>
<th>ln (real M1)</th>
<th>ln (real M1), detrended</th>
</tr>
</thead>
<tbody>
<tr>
<td>1987Q1</td>
<td>730.2</td>
<td>111.200</td>
<td>111.4902</td>
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<td>743.9</td>
<td>112.700</td>
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<td>1987Q3</td>
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<td>1.835790</td>
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Sources: Board of Governors of the Federal Reserve System and Bureau of Labour Statistics.
NOTES: Nominal, seasonally adjusted base money, or M1, is obtained from the Board of Governors of the Federal Reserve System; the data represents the stock at the end of the quarter. The all-items, all urban consumers, US city average CPI (1982-84 = 100) is obtained from the Bureau of Labour Statistics. The CPI is seasonally adjusted by the multiplicative moving average method. M1 is deflated by the seasonally adjusted CPI to obtain real M1. The natural logarithm of real M1 is detrended by the Hodrick-Prescott filter.
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Average: 0.231 0.207
Standard deviation: 0.0040 0.0112

Source: HM Revenue and Customs. UK taxpayers’ earnings from employment and UK dividends were obtained from HM Revenue and Customs (2012b), Tables 3.6 and 3.7, respectively. Data for the tax year 2008-09 is not available. Income tax liabilities on earnings and dividends are obtained from HM Revenue and Customs (2012a), Table 2.6. Older issues of these publications are obtained online at http://webarchive.nationalarchives.gov.uk/20120609144700/http://hmrc.gov.uk/stats/income_tax/table2-6a.pdf and http://webarchive.nationalarchives.gov.uk/*/http://hmrc.gov.uk/stats/income_distribution/menu-by-year.htm.
References


