Debts, Deficits and the Real Interest Rate in an Endogenous Growth Model

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Representative Household versus Overlapping Generations Models

• In the context of the neoclassical growth model there are two competing intertemporal general equilibrium theories of the determination of aggregate savings.


• In both classes of models, households engage in inter-temporal optimization and their savings behavior is individually optimal. However, whereas in representative household models Ricardian equivalence holds, in overlapping generations model it does not, as optimizing households do not generally take into account the welfare of future generations.
Savings Investment and the Real Interest Rate

• This paper proposes an endogenous growth framework for comparing the effects of the representative household assumption versus the overlapping generations assumption.

• The model in this paper belongs to a class of endogenous growth theories with investment adjustment costs.

• In the model proposed, savings and investment are independent decisions, unlike the standard neoclassical growth model, which, in the tradition of Ramsey (1928), Solow (1956), Cass (1965), Koopmans (1965) and Diamond (1965) does not have a separate investment theory.

• We utilize the $q$ theory of investment (see Lucas 1967, Gould 1968, Tobin 1969, Abel 1982, Hayashi 1982), assuming that firms face convex internal costs to adjusting their capital stock.

• Thus, in the model of this paper investment is not determined by aggregate savings, but savings and investment are co-determined in competitive capital markets, through adjustments in the real interest rate.
Main Conclusions

• It is shown that the representative household and overlapping generations versions of the model have similar predictions regarding the effects of technological and preference shocks.

• However, for the same parameter values, the overlapping generations model predicts lower savings and investment, higher interest rates and lower growth rates than the corresponding representative household model.

• In addition, the overlapping generations model is not characterized by Ricardian equivalence. Thus, an increase in government expenditure or public debt causes an increase in the equilibrium real interest rate and a reduction in the long run growth rate.
We assume an economy, consisting of a large number of competitive firms that produce a single homogeneous good.

\[ Y_{it} = A K_{it}^{\alpha} (h_t L_{it})^{1-\alpha} \]

Following Arrow (1962) we assume learning by doing. In particular we assume that the efficiency of labour (human capital per worker) is a linear function of the aggregate ratio of physical capital to labor. Thus,

\[ h_t = B \left( \frac{K}{L} \right)_t \]
Aggregate Production and Growth

Upon aggregation, total output turns out to be a linear function of aggregate physical capital.

\[ Y_t = \bar{A} K_t \]

Due to the linearity of the aggregate production function, the (endogenous) rate of economic growth \( g \) will be equal to the rate of net capital accumulation, which is in turn determined by the rate of investment.

\[ g = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = \left( \frac{I_t}{K_t} \right) - \delta = \bar{A} \left( \frac{I_t}{Y_t} \right) - \delta \]
The Investment Decisions of Firms

Each firm selects employment and investment in order to maximize the present value of its profits.

\[
V_{it} = \int_{s=t}^{\infty} e^{-rs} \left( Y_{is} - w_s L_{is} - \left[ 1 + \frac{\phi}{2} \left( \frac{I_{is}}{K_{is}} \right) \right] I_{is} \right) ds
\]

under the constraints,

\[
Y_{is} = A K_{is}^\alpha (h_s L_{is})^{1-\alpha}
\]

\[
\dot{K}_{is} = I_{is} - \delta K_{is}
\]
First Order Conditions at the Firm Level

\[ w_t = (1 - \alpha)A\left(\frac{K_{it}}{L_{it}}\right)^{\alpha}h_t^{1-\alpha} \]

\[ q_{it} = 1 + \phi\left(\frac{I_{it}}{K_{it}}\right) = 1 + \phi\left(\frac{\dot{K}_{it}}{K_{it}} + \delta\right) \]

\[ \left(r + \delta - \frac{\dot{q}_{it}}{q_{it}}\right)q_{it} = \alpha A\left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1}h_t^{1-\alpha} + \phi\left(\frac{\dot{K}_{it}}{K_{it}} + \delta\right)^2 \]
First Order Conditions at the Aggregate Level

\[ w_t = (1 - \alpha) \bar{A} \left( \frac{K_t}{L_t} \right) \]

\[ q_t = 1 + \phi \left( \frac{\dot{K}_t}{K_t} + \delta \right) = 1 + \phi (g + \delta) \]

\[ \left( r + \delta - \frac{\dot{q}_t}{q_t} \right) q_t = \alpha \bar{A} + \frac{\phi}{2} \left( \frac{\dot{K}_t}{K_t} + \delta \right)^2 = \alpha \bar{A} + \frac{\phi}{2} (g + \delta)^2 \]
Growth, $q$ and the Real Interest Rate

\[ g = \frac{q_t - 1}{\phi} - \delta \]

\[ (r + \delta)(1 + \phi(g + \delta)) = \alpha \bar{A} + \frac{\phi}{2} (g + \delta)^2 \]

\[ g = r - \sqrt{r^2 - \frac{2}{\phi} \left( \alpha \bar{A} - (r + \delta)(1 + \phi\delta) \right)} - \delta^2 \]
Equilibrium Investment

\[ g = r - \sqrt{r^2 - \frac{2}{\phi} \left( \alpha \bar{A} - (r + \delta)(1 + \phi \delta) \right)} - \delta^2 \]

\[ \frac{\partial g}{\partial r} = -\frac{g + \frac{1}{\phi} (1 + \phi \delta)}{\sqrt{r^2 - \frac{2}{\phi} \left( \alpha \bar{A} - (r + \delta)(1 + \phi \delta) \right)} - \delta^2} < 0 \]
The Equilibrium Investment Schedule

Equilibrium Investment

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Equilibrium Investment and Total Factor Productivity

\[ \frac{\partial g}{\partial \tilde{A}} = \frac{\alpha / \phi}{\sqrt{(r)^2 - \frac{2}{\phi} \left( \alpha \tilde{A} - (r + \delta)(1 + \phi \delta) \right)} - \delta^2} > 0 \]
Equilibrium Investment and Total Factor Productivity
Aggregate Savings

\[ S_t = Y_t - C_t - C_t^g \]

\[ s_t = \frac{S_t}{Y_t} = 1 - c_t - c_t^g \]
The Government Budget Constraint

\[ b_t = (r_t - g_t) b_t + c_t - \tau_t \]

We assume that primary government expenditure is a constant share of total output, and tax revenue rule that ensures that the government debt to output ratio is stabilised.

\[ \tau_t = \bar{c}^g + (r_t - g_t) \bar{b} \]

\[ \bar{b} = \frac{\tau_t - \bar{c}^g}{r_t - g_t} \]
Consumption under a Representative Household

We next turn to the determination of private consumption, in an economy consisting of a large number of infinitely lived, identical households. Household \( j \) maximizes,

\[
U_j = \int_{s=0}^{\infty} e^{-(\rho-n)s} \ln(c_{js}) \, ds
\]

subject to an asset accumulation equation and the household’s solvency condition,

\[
a_{js} = (r_s - n)a_{js} + w_s - \tau_s - c_{js}
\]

\[
\lim_{t \to \infty} e^{-\int_{s=0}^{t} (r_s-n) \, ds} a_{jt} = 0
\]
Euler Equations for Consumption

\[ c_{js} = (r_s - \rho)c_{js} \]

After aggregation, consumption as a share of total output will evolve according to,

\[ c_t = (r_t - \rho - g_t + n)c_t \]

On the balanced growth path, the share of consumption in total output will be constant. This requires that,

\[ g_t = r_t - \rho + n \]
The Equilibrium Savings Schedule in a RH Model
A Reduction in the Pure Rate of Time Preference in the RH Model
An Increase in Total Factor Productivity in the RH Model
Household Consumption in a Model of Overlapping Generations

We next assume a model in which population growth comes in the form of entry of new households into the economy. Thus households differ by their date of birth. All households are infinitely lived, but each generation is only concerned about its own welfare and not the welfare of forthcoming generations. The household born at instant $j$ chooses consumption to maximize,

$$U_j = \int_{s=j}^{\infty} e^{-\rho s} \ln(c_{js}) ds$$

subject to,

$$a_{js} = r_s a_{js} + w_s - \tau_s - c_{js}$$

$$\lim_{t \to \infty} e^{s=j} a_{jt} = 0$$
Aggregate Consumption in a Model of Overlapping Generations

From the Euler equation for household consumption, aggregating over cohorts, assuming that newly born households do not inherit any wealth, yields,

\[ \dot{C}_t = (r_t - \rho + n)C_t - n \rho A_t \]

where,

\[ A_t = qK_t + B_t \]

Dividing through by aggregate output, after appropriate substitutions,

\[ \dot{c}_t = (r_t - g_t - \rho + n)c_t - n \rho \left( \frac{q_t}{\bar{A}} + \bar{b} \right) \]
Equilibrium Savings in a Model of Overlapping Generations

The equilibrium private consumption to output ratio is determined by,

\[ c_E = \frac{n\rho}{(r_E - g_E - \rho + n)} \left( (q_E \bar{A}) + \bar{b} \right) \]

After substituting out for \( q \), the equilibrium savings rate is determined by,

\[ s_E = 1 - c_E - \bar{c}^g = 1 - \frac{n\rho}{(r_E - g_E - \rho + n)} \left( \frac{1 + \phi(g_E + \delta)}{\bar{A}} + \bar{b} \right) - \bar{c}^g \]
The slope of the equilibrium savings schedule in the OLG model is determined by,

\[
0 < \frac{dg}{dr} = \frac{n\rho(1 + \phi(g + \delta))}{n\rho(1 + \phi(g + \delta)) + n\rho\phi(r - g - \rho + n)} < 1
\]

The equilibrium savings locus has a positive slope. However, the slope is lower than one, which is the slope of the comparable representative household model. In addition it is straightforward to show that the slope is declining in the real interest rate. The equilibrium savings locus lies to the right of the equilibrium savings locus of the comparable representative household model, as for any growth rate, the real interest rate is higher.
Equilibrium Savings in the OLG Model

In the OLG model the equilibrium savings locus depends negatively on both government consumption and government debt relative to output. Both government expenditure and its mode of finance matter for aggregate savings.

Higher government expenditure requires higher current and future taxation as a share of output. However, because part of the future taxation will be shouldered by as yet unborn generations, a rise in government expenditure reduces aggregate savings. The reduction in private consumption is smaller than the rise in government consumption.

For the same reason, Ricardian equivalence does not hold. Government debt is considered as wealth by current generations, as the rise in lump sum taxes to finance the servicing of the debt will be partly shouldered by future generations.
Comparing the Equilibrium Growth Rate and the Real Interest Rate between the RH and OLG Models
The Effects of a Rise in Government Debt in the RH and OLG Models
Summary of Conclusions

The representative household (RH) and overlapping generations (OLG) versions of the model have similar predictions regarding the effects of technological and preference shocks. A reduction in the pure rate of time preference reduces the real interest rate and increases growth, while an increase in total factor productivity increases both the real interest rate and the growth rate.

For similar parameter values, the overlapping generations model predicts lower savings and investment, higher interest rates and lower growth rates that the corresponding representative household model.

Finally, the overlapping generations model is not characterized by Ricardian equivalence. An increase in government expenditure or public debt causes an increase in the equilibrium real interest rate and a reduction in the long run growth rate.