Systematic Monetary Policy
and the Forward Premium Puzzle*

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Abstract

We investigate whether systematic monetary policy is behind the forward premium puzzle, i.e. the tendency of high interest-rate currencies to appreciate, thus violating Uncovered Interest Parity (UIP). We consider a battery of monetary policy rules, which target strictly or react to either domestic or CPI inflation, in an economy influenced by both domestic and foreign shocks. Each rule has specific implications for UIP violation, which we derive explicitly and compare to empirical results. We find that most of the policy rules can account for instances of weak UIP violations in the data. However, only a forward-looking CPI rule can approximate the magnitude of frequently observed extreme UIP violations.

Keywords: Monetary policy rules, uncovered interest parity, exchange rate dynamics, currency-risk.

JEL classification codes: E52, E58, F33

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1 Introduction

The currency-risk premium has long eluded the profession. In a classic paper, Eugene Fama (1984) concretized the risk-premium puzzle: higher-interest currencies tend to appreciate. Fama’s finding, which has become a stylized fact of empirical international finance, has the textbook Uncovered Interest Parity (UIP) condition in reverse. Assuming risk neutrality of traders, UIP states that the interest-rate differential between two currencies should equal the expected depreciation rate of the higher-interest currency.

The empirical violation of UIP may however be a puzzle only if one focuses on traders’ demand for different currencies and abstracts from the supply side, where central banks influence interest rates in the pursuit of monetary policy objectives. In this regard, McCallum (1994) showed that modeling an exogenous currency-risk premium and taking into account systematic monetary policy could eliminate the forward-premium puzzle. In implementing a policy rule for stabilizing inflation and attaining a target output gap, a central bank would influence the nominally riskless interest rate and the expected depreciation rate of the home currency. The resulting premium for holding the domestic currency would then determine the demand for this currency, which the central bank would need to accommodate in order to maintain its policy course. The market-microstructure analyses in Bacchetta and van Wincoop (2007, 2010) and Burnside et al. (2009a,b) provide indirect evidence that such a mechanism would work in the presence of risk-averse traders. These articles demonstrate that, on a risk-adjusted basis, there are limited, if any, arbitrage opportunities in foreign exchange markets.

We take the argument of McCallum (1994) a step further by examining explicitly how alternative monetary policy rules would affect endogenously the currency premium. Our analysis builds on a model of a small open economy, which is affected by two exogenous stochastic shocks: a domestic shock, to the natural rate of interest, and a foreign shock, which we introduce through the real exchange rate. If the interest rate is fixed, these two shocks determine fully the evolution of inflation and the output gap over time on the basis of a reduced-form dynamic IS equation and a New Keynesian Phillips curve.

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In this setting, we introduce a central bank that uses the nominal interest rate as a tool for implementing monetary policy rules. Keeping the demand and supply of the domestic currency in the background, we consider six policy rules, divided in two groups. In each group, there is a rule that maintains the inflation rate at a target level (a “strict rule”), a simple rule that responds to the current level of inflation and the output gap (a “Taylor-type rule”), and a rule responding to the expected level of these variables (a “forward-looking rule”). The three rules in one of the groups strictly target or respond to domestic inflation; those in the other group are related to CPI inflation. In conjunction with the exogenous shocks, each policy rule gives rise to an endogenous currency premium with specific statistical properties. Thus, each rule generates a specific model-implied UIP violation, which we derive in closed form.\(^2\)

We quantify the implied UIP violations by calibrating our model as follows. First, we employ consensus values for “deep” parameters, such as the intertemporal elasticity of substitution and the dependence of the inflation rate on the output gap. Second, we adopt standard assumptions about the simple policy rules. Third, we use estimates of (i) openness to trade and (ii) the statistical properties of the real exchange rates and natural interest rates in six small open economies (Australia, Canada, New Zealand, Sweden, Switzerland, the United Kingdom) and the euro area.

We also estimate the empirical violations of UIP. For this, we use monthly data on the US-dollar exchange rates of the domestic currencies in the above seven economies and on the related interest differentials. We base the estimation on the Fama (1984) regression specification, allowing for a break at the time when a particular economy explicitly adopted inflation targeting.

We find that each of the seven exchange rates violate UIP, which is in accordance with the related literature, and that the degree of UIP violation changes with the monetary policy regime, which we believe is a novel result. In particular, during the recent period of explicit inflation targeting in Australia, Sweden, Switzerland and the euro area, the slope coefficient in a regression of the nominal depreciation rate on the corresponding interest differential is statistically significant and lies between \(-2\) and \(-4\), depending on the currency in focus. By contrast,\(^2\)

\(^{2}\)Such reverse engineering has been employed by Evans (2011), albeit in a different context.
UIP implies a coefficient of 1. For the preceding period, without explicit inflation targeting, UIP violation is generally weaker and even disappears in the case of Sweden. Interestingly, the results are reversed in the UK case, where we find that UIP violation was significantly stronger in the period before explicit inflation targeting.

Our second finding is that the three policy rules based on domestic inflation can account for the weak UIP violations (or the absence thereof) prior to the inflation targeting period in Australia, Canada, Switzerland and Sweden, and during the inflation targeting period in the United Kingdom. In other words, these rules generate a (weakly) positive slope coefficient in the Fama (1984) regression. To see why, note that the sign of this coefficient equals the sign of the covariance between the nominal exchange rate depreciation (or the real exchange rate depreciation plus CPI inflation) and the policy interest rate. Not surprisingly, the policy rules we consider generate (weakly) positive relationships between CPI inflation and the policy rate. In addition, the domestic-inflation rules preclude the policy rate from reacting to the foreign shocks driving the real exchange rate.

Our third finding is that all three policy rules based on CPI inflation can account for the direction of UIP violations observed during the inflation-targeting regimes in Australia, Sweden, Switzerland and the euro area and the pre-inflation targeting period in the United Kingdom. That said, only the forward-looking rule, reacting to expected CPI inflation, can approximate the magnitude of these UIP violations. Each of the CPI-related rules generates a negative covariance between the policy rate and real exchange rate depreciation and this covariance can explain a negative slope coefficient in the Fama (1984) regression. Of these, however, the strict and Taylor-type rules also lead to an extremely volatile policy rate, which depresses the implied regression slope very close to zero. A less volatile policy rate under the forward-looking rule leads to implied regression slopes ranging between $-0.5$ and $-1.8$, depending on the underlying parameterization.\(^3\)

Our methodology is related to macro-based models of (nominal and real) exchange rate dynamics. Recent applications include Engel and West (2006) and Mark (2009), who examined the exchange rate dynamics under various interest-rate reaction functions assuming zero currency-risk premia; these authors generate
real exchange rates implied by Taylor-type policy rules and compare them with the actual data. Overall, such models have had mixed success in replicating the key stylized facts; for a survey see Evans (2011), chapter 3.4

The fundamentals-based literature includes an external balance approach to explaining UIP violation. This is based on the observation that investors in creditor (debtor) countries with positive (negative) net foreign asset positions demand higher Foreign (Home) interest rates to compensate them for the exchange-rate risk of holding Foreign (Home) currency assets; see Gourinchas and Rey (2007).

We organize the remainder of this paper as follows. In Section 2, we summarize the forward premium puzzle and provide empirical evidence. In Section 3 we present our model. We derive analytically the model-implied violations of UIP in Section 4 and quantify these results in Section 5. We conclude in Section 6. Most of the mathematical derivations are in appendices at the end of the paper.

2 Reviewing the forward premium puzzle

This section has two parts. In the first part, we revisit the stylized facts regarding the violation of uncovered interest rate parity (UIP) and derive the implied joint properties of the currency depreciation rate, the interest rate and the currency risk premium. In the second part, we concretize the stylized facts by measuring UIP violations in six small open economies and the euro area.

2.1 Fama’s test of UIP

Let there be two currencies: domestic and foreign. The nominal exchange rate on date-$t$, denoted by $e_t$, is the date-$t$ price of the foreign currency in terms of the domestic currency. In addition, let $i_t$ and $i^*$ denote respectively the short-term interest rates on domestic- and foreign-currency bonds that are risk free in nominal terms, are issued at $t$ and mature at $t + 1$. We henceforth assume that $i^* = 0$ and refer to $i_t$ as both the policy rate and the interest differential.

If UIP holds, then the expected exchange-rate depreciation equals the interest differential:

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4See also Chinn (2008), Mark (2009) and Molodtsova and Papell (2009).
In his seminal contribution, Fama (1984) examined UIP’s empirical fit with the regression specification:

\[ \Delta e_{t+1} = \alpha + \beta i_t + u_{t+1} \]  

(2)

where the residual term, \( u_{t+1} \), is independent of any variable observed on or prior to date \( t \). Given rational expectations, UIP then implies that the OLS parameter estimates should not be significantly different from \( \alpha = 0 \) and \( \beta = 1 \), respectively. However, Fama (1984) and most of the subsequent empirical studies of UIP report significantly negative estimates \( \hat{\beta} \). In other words, the UIP assumption is violated, indicating that countries with higher (lower) interest rates tend to have currencies which are expected to appreciate (depreciate).

Below, we attempt to explain the empirical finding of \( \hat{\beta} < 0 \) by modelling systematic monetary policy. To this end, we relax the UIP assumption in (1):

\[ E_t \Delta e_{t+1} = i_t - \xi_t \]  

(3)

where \( \xi_t \) is a premium that compensates investors for the perceived risk of holding domestic versus foreign bonds. If monetary policy creates a systematic link between the interest differential and the currency risk premium – i.e. if \( \text{Cov} (\xi_t, i_t) \neq 0 \) – then an OLS estimate of the slope coefficient in regression specification (2) would not converge to unity. Instead:

\[ \hat{\beta} \rightarrow_p 1 - \frac{\text{Cov}(\xi_t, i_t)}{V(i_t)} = \frac{\text{Cov}(\Delta e_{t+1}, i_t)}{V(i_t)} \]  

(4)

which allows us to henceforth keep \( \xi_t \) in the background and conduct the analysis in terms of \( \text{Cov}(\Delta e_{t+1}, i_t), V(i_t) \) and their drivers.

Equation (4) pins down the conditions under which systematic monetary policy can explain the empirical violation of UIP. First, a policy rule underpinning \( \hat{\beta} < 0 \) is a rule that implies negative co-movement of realized exchange rate depreciation and the policy rate: \( \text{Cov}(\Delta e_{t+1}, i_t) < 0 \). Second, once the sign of \( \hat{\beta} \) is accounted for, the implied relative size of \( \text{Cov}(\Delta e_{t+1}, i_t) \) and \( V(i_t) \) should match the empirical magnitude of \( \hat{\beta} \).
2.2 Empirical evidence

We investigate violations of UIP in the case of seven currencies: Australian dollar (AUD), British pound (GBP), Canadian dollar (CAD), Swedish krona (SEK), Swiss franc (CHF), New Zealand dollar (NZD) and the euro. For each of these currencies, we consider the exchange rate with the US dollar (USD) and run the Fama regression as specified in (2). We use monthly series of the spot exchange rates on the 25th of each month (or, if a weekend, the last preceding work day). We combine these series with matching series of one-month forward exchange rates, which we use to calculate the interest differential on the basis of covered interest parity.

In running each regression, we attempt to identify whether the change in monetary policy regime coincided with a change in the degree of UIP violation. For the first five currencies, the available data cover a period during which the corresponding central bank did not have an explicit inflation target and a subsequent period of explicit inflation targeting.⁵ To reflect this, we introduce a dummy variable that equals 1 during the first regime and 0 thereafter. In the case of NZD and the euro, the data cover only the inflation targeting period.

The regression results, which we report in Table 1, deliver two messages. First, five out of the seven exchange rates have experienced a period of strong UIP violation. In accordance with previous empirical studies, we estimate slope coefficients that lie between $-2$ and $-4$. This range is in line with Engel’s (2011) estimates for advanced economies’ US-dollar exchange rates over a similar period. Second, in the case of four currencies for which there are long enough time series data, the degree of UIP violation changes with the monetary policy regime. The introduction of explicit inflation targeting coincides with greater UIP violation by the exchange rates of the AUD, CHF and SEK with the USD. Interestingly, the degree of UIP violation by the GBP-USD exchange rate was stronger before the introduction of inflation targeting.

⁵For each country, we adopt Ball and Sheridan’s (2003) convention for the starting date of IT as the quarter when the central bank announced it began targeting constant inflation. Our dataset extends to 2008:08 to avoid the turbulence associated with the global financial crisis. All exchange rates are in US dollars per unit of foreign currency. Data source: BIS.
3 The model

We now describe the analytic environment we work in. First, we specify the exogenous shocks affecting the modelled economy. Second, we state the two standard equations that govern the evolution of inflation and the output gap over time. Third, we specify six monetary policy rules that we consider as candidates for explaining the empirical violations of UIP.

3.1 Real uncertainty

There are two exogenous sources of uncertainty, stemming from the natural real rate of interest and the real exchange rate. We assume that the log real exchange rate, $q_t$, and the deviation of the natural real rate from its (positive) long-run mean, $r_t^n$, follow mutually independent AR(1) processes:

$$r_t^n = \rho r_{t-1}^n + \varepsilon_t$$
$$q_t = \mu q_{t-1} + \eta_t$$

where $\rho \in [0,1)$, $\mu \in [0,1)$ and, in line with long-run PPP, the steady-state real exchange rate is zero, $E(q_t) = 0$. In addition, $\varepsilon_t$ and $\eta_t$ are i.i.d. zero-mean variables with time-invariant variances $V(\varepsilon)$ and $V(\eta)$, respectively, and $Cov(\varepsilon_t, \eta_{t+j}) = 0$ for any $j$. Below, we will make use of the following ratio between the variances of the two exogenous innovations: $\nu \equiv \frac{V(\eta)}{V(\varepsilon)}$.

We think of the exogenous natural rate as corresponding to shocks of domestic origin and the real exchange rate as capturing foreign shocks. The natural rate would be related to the evolution of both real and financial frictions in the economy and has been studied as an exogenous variable by Benati and Vitale (2007) and Laubach and Williams (2003). In turn, in our framework of perfect pass-through, the real exchange rate is tightly linked with the terms of trade. Kehoe and Ruhl (2008) derive conditions under which terms-of-trade shocks are not identical to productivity shocks, and so exert independent influence on the economy.
3.2 Dynamics in the economy

We adopt a standard closed-economy joint determination of domestic inflation \((\pi_t^H)\) and the output gap \((x_t)\) specified by a New Keynesian Phillips curve and a dynamic IS relationship:

\[
\pi_t^H = \kappa x_t + b E_t \pi_{t+1}^H, \quad \kappa > 0, \; 0 < b < 1 
\]
\[
x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}^H - r_t^H), \quad \sigma > 0
\]

where \(i_t - E_t \pi_{t+1}^H\) is the ex-ante real interest rate, and \(\sigma\) measures the representative household’s intertemporal risk-aversion; equivalently, its inverse elasticity of intertemporal substitution.

Then, we generalize our model to an open-economy setting by following Galí and Monacelli (2005). Namely, we employ a utility function with a constant elasticity of substitution between home and foreign goods, and assume constant foreign-currency prices and perfect exchange-rate pass-through. The latter assumption is important as it gives rise to the following relationship between CPI inflation, \(\pi_t = p_t - p_{t-1}\), and domestic inflation:

\[
\pi_t = (1 - \alpha) \pi_t^H + \alpha \Delta e_t
\]

where \(\alpha \in [0, 1]\) is the share of domestic consumption allocated to foreign goods and is, thus, an index of the economy’s openness. Further, under perfect pass-through, the (log) real exchange rate level is approximately proportional to the country’s (log) effective terms of trade, with the coefficient of proportionality decreasing in openness; see Galí (2008). We can thus map the real exchange rate uncertainty captured by AR(1) process (6) onto fluctuations in the country’s underlying (exogenous) terms of trade.

Using relationship (9) to rewrite equation (7) yields our open-economy specification:

\[
\pi_t = \kappa x_t + \frac{a}{1 - a} [\Delta q_t + b(1 - \mu) q_t] + b E_t \pi_{t+1}^H 
\]
\[
x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}^H - r_t^H)
\]
In our context, it helps to think of $\kappa$ and $\sigma$ as the parameters governing the impact of the policy rate, $i_t$, on the output gap and inflation. In addition, we have introduced the real exchange rate in equation (10) via the following identity, $q_t \equiv e_t - p_t$, which implies that:

$$\Delta q_t = \Delta e_t - \pi_t$$

(12)

To gain preliminary intuition about the monetary policy rule that underpins a negative slope estimate in the Fama regression, we use equations (6) and (12) to rewrite equation (4) as follows:

$$\hat{\beta} \rightarrow \frac{p \text{Cov} (E_t \pi_{t+1}, i_t) + \text{Cov} (E_t \Delta q_{t+1}, i_t)}{V(i_t)} = \frac{\text{Cov} (E_t \pi_{t+1}, i_t) - (1 - \mu) \text{Cov} (q_t, i_t)}{V(i_t)}$$

(13)

For well-defined interest-rate rules, the policy rate comoves positively with expected inflation, $\text{Cov} (E_t \pi_{t+1}, i_t) > 0$. Hence, given $\mu < 1$, obtaining $\hat{\beta} < 0$ requires a positive relationship between the policy rate and the real exchange rate, $\text{Cov}(q_t, i_t) > 0$.

In Table 2 we report estimates of $\alpha$, $\mu$ and $\rho$ for the 7 economies under consideration. Note that values of $\mu$ are near one, confirming the finding of Engel (2000) that the real exchange rates of advanced economies exhibit extremely high persistence.

### 3.3 Monetary policy rules

We study six monetary policy rules divided in two groups of three. The first group comprises a rule that attains a fixed CPI inflation target, and two rules under which monetary policy reacts to current or expected future CPI inflation and the output gap. We refer to these as strict CPI, Taylor-type, and forward-looking rules, respectively, and flag them as IT, TR and FW. The three rules in the second group are the same as those in the first, except that they strictly target or respond to domestic inflation. We flag these rules as ITd, TRd and FWd.

More concretely, the strict inflation-targeting rules are as follows. Under the
strict CPI rule, the central bank sets the policy rate, \( i_t^{TR} \), so that CPI inflation is and is expected to be zero at each date: \( \pi_t^{IT} = E_t (\pi_{t+1}^{IT}) = 0 \), all \( t \). Likewise, under the strict domestic inflation-targeting rule, the policy rate \( i_t^{ITd} \) leads to 
\( \pi_t^{H,ITd} = E_t (\pi_{t+1}^{H,ITd}) = 0 \). In analyzing each of these rules, we follow Woodford (2003) and assume that the central bank can attain its target after observing the current natural rate, \( \pi_t^n \), as well as structural parameters \( b, \kappa \) and \( \sigma \).

Second, under Taylor-type rules the central bank sets the policy rate as follows:
\[
\begin{align*}
i_t^{TR} &= \bar{i} + \phi_\pi \pi_t^{TR} + \phi_x x_t^{TR} \\
i_t^{TRd} &= \bar{i} + \phi_\pi \pi_t^{H,TRd} + \phi_x x_t^{TRd}
\end{align*}
\]
(14)
(15)
where \( \phi_\pi > 1 \) and \( \phi_x \geq 0 \) satisfy the Taylor principle for determinacy, and \( \bar{i} \) measures the neutral policy rate for a zero target inflation rate.

Third, a central bank that has adopted the forward-looking rule sets the policy rate in response to expected (CPI or domestic) inflation and expected output gap deviations from their (zero) targets:
\[
\begin{align*}
i_t^{FW} &= \bar{i} + \varphi_\pi E_t \pi_{t+1}^{FW} + \varphi_x E_t x_{t+1}^{FW} \\
i_t^{FWd} &= \bar{i} + \varphi_\pi E_t \pi_{t+1}^{H,FWd} + \varphi_x E_t x_{t+1}^{FWd}
\end{align*}
\]
(16)
(17)
where \( \varphi_\pi > 1, \varphi_x \geq 0 \) and \( \bar{i} \) is the corresponding neutral policy rate.\(^6\)

Henceforth, we suppress the superscripts of the four endogenous, policy-dependent variables, \( i_t, \pi_t, \pi_t^H \) and \( x_t \), wherever this does not cause confusion.

### 4 Model-implied UIP violations

In this section, we derive the slope coefficient, \( \hat{\beta} \), that would be obtained from regressing the realized depreciation rate, \( \Delta \epsilon_{t+1} \), on the interest differential, \( i_t \), when a particular monetary policy rule is in place. Abstracting from estimation noise, we focus on the asymptotic value of \( \hat{\beta} \) in expression (13). We obtain six rule-implied coefficient estimates, which we then use to ascertain whether the

\(^6\)Note that we do not consider exogenous policy shocks. The key conclusions of the analysis would not change even if we allowed for such shocks, provided they are independent of all other variables in the model.
underlying policy rule can account for the negative empirical sign of \( \hat{\beta} \). This allows us to narrow down the set of rules for which it is worth studying the implied magnitude of \( \hat{\beta} \).

To lighten the notation, we henceforth, omit the “hat” when referring to model-implied coefficient estimates. Instead, when confusion could arise, we signal the underlying policy rule with a superscript, e.g. \( \beta^{FW} \), \( \beta^{TRd} \), etc.

### 4.1 Policy rules responding to domestic inflation

Neither the strict domestic inflation-targeting rule nor any of the two simple rules under which the policy rate reacts to domestic inflation can generate \( \beta < 0 \). The intuition behind this result is as follows. As we discussed after equation (13), a positive relationship between the policy rate \( \pi_t \) and foreign shocks, stemming from the real exchange rate \( q_t \), is a necessary condition for a well-behaved policy rule to generate \( \beta < 0 \). However, when the rule is related to domestic inflation, the policy rate responds only to domestic shocks, stemming from the natural rate, \( r^n_t \).

We derive the result explicitly in Appendix A but provide here the key underlying steps. First, note that equations (7) and (8) are the relevant ones under the three policy rules related to domestic inflation. Second, these equations imply that a policy rate that attains \( \pi^H_t = 0 \) or is determined as in equation (15) or (17) would be driven entirely by \( \pi_t \). Since \( \pi_t \) is independent of \( r^n_t \), we then obtain that

\[
\text{Cov} (E_t \pi_{t+1}, i_t) = \beta^{TRd} = 0.
\]

- Under strict domestic-inflation targeting, the monetary authority attains \( \pi^H_t = 0 \). Together with equations (9) and (12), this implies that \( \pi_t \) is driven entirely by \( q_t \). Since \( i_t \) is independent of \( r^n_t \), we then obtain that

\[
\text{Cov} (E_t \pi_{t+1}, i_t) = \beta^{TRd} = 0.
\]

- Under the Taylor-type and forward-looking rules in equations (15) and (16), equations (9) and (12) imply that \( \text{Cov} (E_t \pi_{t+1}, i_t) = \text{Cov} (E_t \pi^H_{t+1}, i_t) \). Since the policy rules are well-behaved, the latter variance is non-negative, implying \( \beta^{TRd} \geq 0 \) and \( \beta^{FWd} \geq 0 \).
4.2 Strict CPI-inflation targeting

Strict CPI targeting generates unambiguously $\beta^{IT} < 0$. To see this, note that attaining $\pi_t = E_t \pi_{t+1} = 0$ for all $t$ requires the following policy rate:

$$i_t^{IT} = r^n_t + \frac{a}{1-a} \left(1 - \mu + \frac{b(1-\mu)^2 + 2 - \mu}{\kappa \sigma}\right) q_t - \frac{a}{1-a} \frac{1}{\kappa \sigma} q_{t-1}$$

(18)

thus implying

$$\beta^{IT} = -\frac{(1-\mu) \text{Cov}(i_t^{IT}, q_t)}{V(i_t^{IT})} < 0$$

(19)

The inequality follows from $\text{Cov}(i_t^{IT}, q_t) > 0$, the intuition for which is straightforward. By equation (10), real exchange rate depreciation puts upward pressure on CPI inflation, $\pi_t$. In order to keep $\pi_t = 0$ in the face of this upward pressure, the central bank needs to lower the output gap, $x_t$. By equation (11), the central bank lowers $x_t$ by raising the policy rate. Thus, a positive shock to the real exchange rate translates into a positive movement of $i_t^{IT}$.

4.3 Policy rules responding to CPI inflation

The policy rules specified in equations (14) and (16) imply that the equilibrium inflation and output gap depend linearly on the exogenous variables:

$$\pi_t = \delta^\pi r^n_t + \zeta^\pi q_t + \theta^\pi q_{t-1} \Rightarrow E_t \pi_{t+1} = \delta^\pi \pi_t + \delta^\pi \rho r^n_t + (\mu \zeta^\pi + \theta^\pi) q_t$$

$$x_t = \delta^x r^n_t + \zeta^x q_t + \theta^x q_{t-1} \Rightarrow E_t x_{t+1} = \delta^x x_t + \delta^x \rho r^n_t + (\mu \zeta^x + \theta^x) q_t$$

(20)

where the value of the implied coefficient vector $\{\delta, \zeta, \theta\}^{\pi,x}$ changes with the policy rule.

The coefficients governing the impact of the natural interest rate on inflation and the output gap are positive, irrespective of the policy rule: i.e. $\delta^{\pi,x} > 0$. This is not surprising because well-defined policy rules are underpinned by a policy rate that responds more than proportionately to $r^n_t$ shocks. In the light of equations (14), (16) and (20) and the assumed restrictions on the policy response coefficients,
a well-behaved rule is consistent with \( \frac{d\pi}{dr^n_t} > 0 \) and \( \frac{dx}{dr^n_t} > 0 \).

Importantly, the last properties imply that shocks to the natural rate, \( r^n_t \), generate a positive relationship between CPI inflation, \( \pi_t \), and the policy rate, \( i_t \). Thus, if \( r^n_t \) were the only shock, equation (13) would imply a slope coefficient \( \beta > 0 \).

Our findings about the coefficients \( \zeta^\pi \) and \( \theta^\pi \), governing the role of the real exchange rate \( q_t \) in the economy, suggest that the relationship between \( i_t \) and \( q_t \) could be of either sign.\(^9\) Certain configurations of these coefficients imply that the positive relationship between the level of \( q_t \) and \( \pi_t \) is the main driver of the latter variable’s time variation. In the case of the Taylor-type rule in equation (14), a sufficiently high \( \text{Cov} (q_t, \pi_t^{TR}) > 0 \) translates directly into a \( \text{Cov} (q_t, i_t^{TR}) > 0 \) that is sufficiently high to imply \( \beta^{TR} < 0 \).

The forward-looking rule brings to the fore counteracting forces at work. Let us suppose again that the coefficients in expression (20) imply a high \( \text{Cov} (q_t, \pi_t^{FW}) > 0 \). On the one hand, equation (16) indicates that, for this to translate into \( \text{Cov} (q_t, i_t^{FW}) > 0 \), it is necessary that strong persistence in \( q_t \) (i.e. a high \( \mu \) in equation (6)) lead to a sufficiently high \( \text{Cov} (q_t, E_t \pi_{t+1}^{FW}) > 0 \). On the other hand, however, \( \text{Cov} (q_t, i_t^{FW}) > 0 \) translates into \( \beta^{FW} < 0 \) only if \( \mu \) is not too high.

The reason is that an increase of \( \mu \) towards 1 impairs the predictability of future changes in the real exchange rate, thus reducing the dependence of \( E_t \pi_{t+1}^{FW} \) and \( E_t x_{t+1}^{FW} \) on real-exchange rate shocks and increasing their dependence on natural-rate shocks. As argued earlier in this subsection, this implies a positive slope coefficient.

In order to determine the conditions under which the two CPI-related policy rules generate \( \beta < 0 \), we need to calibrate numerically the model. We do this in the next section.

### 5 Numerical calibration

In this section we focus on the three CPI-related monetary policy rules. We first report our result for \( \beta^{IT}, \beta^{TR} \) and \( \beta^{FW} \) under a benchmark calibration of

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\(^{9}\)Clarida, Gali and Gertler (1998) find that real exchange rate depreciations tend to induce central banks to tighten monetary policy.
the model parameters. Then, we conduct sensitivity analysis to determine which parameter values can change substantially the conclusions.

5.1 Benchmark results

The benchmark parameters, reported in Table 3, lead to a negative model-implied slope coefficient under each CPI-related rule: $\beta^{IT} < 0$, $\beta^{FW} < 0$, and $\beta^{TR} < 0$ (Table 4). That said, the magnitude of these coefficients varies drastically across rules. Whereas $\beta^{FW}$ comes close to the strongest empirical violations of UIP reported in Table 1, $\beta^{TR}$ is much smaller in absolute value and $\beta^{IT}$ is virtually zero.

Overall, the more strongly the policy rate reacts to real exchange rate shocks, the smaller is the absolute value of a negative implied slope. To see why, note first that, as argued above, for a policy rule to underpin $\beta < 0$, the real exchange rate need to dominate as a source of uncertainty. In addition, for given shocks to the real exchange rate, the policy rate needs to respond more strongly when the values of $\kappa$ and $\sigma$ are smaller. The strict CPI-targeting rule attains $\pi_t = 0$ via a policy rate that is extremely responsive to shocks. It follows from equation (18) that, while $\text{Cov} \left( i_t^{IT}, q_t \right)$ is of the order of $(\kappa \sigma)^{-1} \approx 1/0.008$, $V \left( i_t \right)$ is of the order of $(\kappa \sigma)^{-2}$. This results in a negligibly small absolute value of $\beta^{IT}$.

This general logic also applies to the simple policy rules. We note that both $i_t^{TR}$ and $i_t^{FW}$ respond to contemporaneous real exchange rate depreciation, $\Delta q_t$. However, while such a response is an end in itself under the TR rule, the FW rule prescribes it only to the extent that $\Delta q_t$ carries information about expected future depreciation, $E_t \Delta q_{t+1}$. And this information is limited under the high benchmark persistence of the real exchange rate, reported in Table 2: when $\mu = 0.95$, equation (6) leads to $\text{corr} \left( E_t \Delta q_{t+1}, q_t \right) = -\sqrt{(1-\mu)/2} \approx 0.16$. This constrains the responsiveness of $i_t^{FW}$ to the concurrent real depreciation relative to that of $i_t^{TR}$. In turn, this leads to $V(i_t^{FW}) < V(i_t^{TR})$ and, ultimately, $|\beta^{FW}| > |\beta^{TR}|$.

5.2 Sensitivity analysis

How do the model-implied slope coefficients depend on the model parameters? We address this question by analyzing $\beta^{IT}$, $\beta^{TR}$ and $\beta^{FW}$ as functions of one
parameter at a time, keeping all the other parameters at their benchmark values. We plot the results in Figures 1A-1B and relegate supporting mathematical derivations to Appendices B-D.

The slope coefficient implied by strict CPI inflation-targeting, $\beta^{IT}$, is not sensitive to changing any of the model parameters over a reasonable range. We thus conclude that this rule can explain the sign but not the magnitude of empirical estimates of the slope coefficient in Fama’s regression (2).

Turning to the other policy rules, we observe that a number of parameters have similar impact on $\beta^{TR}$ and $\beta^{FW}$. In the light of our discussion in section 4.3, the finding that a higher $\nu$ moves the implied slopes further into negative territory should not come as a surprise (Figure 1A, top-left panel). A higher $\nu$ means that real exchange rate shocks, the only shocks in our model that can induce systematic monetary policy to deliver $\beta < 0$, become a more important driver of uncertainty in the economy. For identical reasons, lower $\rho$ values, which lead to a lower unconditional volatility of the natural rate and, thus, increase the relative importance of real exchange rate shocks, result in lower $\beta$ (Figure 1A, top-right panel).

Since they influence similarly the volatility of the policy rate, the policy-rule parameters $\phi_x$, $\phi_\pi$, $\varphi_x$ and $\varphi_\pi$ also have similar effects on $\beta$ (Figure 1A, bottom panels). A higher value of any of these response coefficients increases the sensitivity of the policy rate to exogenous shocks. This raises the volatility of the policy rate, $i_t$, relative to its covariance with expected inflation, $E_t \pi_{t+1}$, and expected real exchange rate depreciation, $E_t \Delta q_{t+1}$. In the light of equation (13), this depresses the absolute value of $\beta$.

There is a U-shaped relationship between the parameter capturing the economy’s openness, $a$, and the implied slope coefficients under both the TR and FW rules (Figure 1B, top panel). If $a = 0$, the economy is closed and real exchange rate shocks are inconsequential. Since the remaining, natural rate shocks relate positively with the policy rate (see above), the implied slope coefficient is positive. As $a$ increases, real exchange rate shocks play a greater role and their positive relationship with the policy rate drives the implied slope coefficients in negative territory by equation (13). Importantly, by equation (10), a higher $a$ raises the sensitivity of inflation to real-exchange rate shocks, which, all
else equal, raises the volatility of inflation. Thus, if $a$ increases beyond a certain point, the response of the policy rate to the highly volatile (expected) inflation raises $Cov(E_t \pi_{t+1}, i_t) > 0$ sufficiently so that, by equation (13), the implied slope coefficient increases and eventually enters positive territory.

The implied slope coefficients $\beta^{TR}$ and $\beta^{FW}$ depend in interesting ways on the sensitivity of the output gap to changes in the policy rate, captured by $\sigma$ (Figure 1B, bottom-left panel). When $\sigma$ is large, it results directly in a highly volatile output gap (equation (11)) and, indirectly, in a highly volatile inflation (equation (10)). Thus, similarly to a large $a$, a large $\sigma$ delivers positive implied slope coefficients under each simple rule.

To understand the implications of a small $\sigma$, consider the extreme case of $\sigma = 0$. In this case, the output gap is not hit by any shocks and thus equals its steady-state level $x_t = 0$, which implies that we can solve equation (10) analytically for inflation:

$$\pi_t = \frac{a \Delta q_t}{1 - a} \Rightarrow E_t \pi_{t+1} = -\frac{a (1 - \mu)}{1 - a} q_t \quad (21)$$

This last expression, together with equations (13), (14) and (16), reveals why the implications of $\sigma = 0$ (and, more generally, those of sufficiently small levels of $\sigma$) for the implied slope coefficients differ across the two simple policy rules. Under the FW rule, the policy rate $i_t^{FW}$ rises as $E_t \pi_{t+1}^{FW}$ rises. Thus, by expression (21), $i_t$ responds negatively to the contemporaneous real exchange rate, $q_t$. Ultimately, $Cov (i_t^{FW}, E_t \pi_{t+1}^{FW}) > 0$ and $Cov (i_t^{FW}, q_t) < 0$ lead to $\beta^{FW} > 0$ by equation (13). By contrast, under the TR rule, $i_t^{TR}$ rises as $\pi_t^{TR}$ rises. Thus, by expression (21), $i_t^{TR}$ responds positively to $q_t$. And, by equation (13), it is $Cov (i_t^{TR}, q_t) > 0$ that underpins $\beta^{TR} < 0$ at low levels of $\sigma$.

In order to shed light on why $\beta^{FW} < 0$ for intermediate values of $\sigma$, it is necessary to also consider the implications of real-exchange rate persistence, to which we turn next.

The two simple policy rules imply similar regression slopes only at high levels of real exchange rate persistence, i.e. high levels of $\mu$ (Figure 1B, bottom-right panel). As $\mu \to 1$, expected changes in the real exchange rate converge to zero. As concretized by expression (13), this means that $Cov (i_t, E_t \Delta q_{t+1})$ converges to zero and the implied slope coefficients are driven by the responsiveness of the policy rate to expected inflation, $Cov (i_t, E_t \Delta \pi_{t+1})$. Since the latter covariance is
positive for well-behaved policy rules, the implied slope coefficients increase and move into positive territory as $\mu \to 1$.

As $\mu$ decreases from 1, the implied slope coefficient $\beta^{TR}$ not only turns negative but increases monotonically in absolute value. To see why, note first that a positive shock to $q_t$ raises the current inflation rate, $\pi_t$, by equation (10), thus inducing an upward revision of the policy rate, $i^{TR}_t$, and leading to $\text{Cov} \left( i^{TR}_t, q_t \right) > 0$. All else equal, mean reversion in $q_t$ (i.e. $\mu < 1$) translates $\text{Cov} \left( i^{TR}_t, q_t \right) > 0$ into $\text{Cov} \left( i^{TR}_t, E_t q_{t+1} \right) < 0$. And the stronger is this mean reversion, i.e. the lower is $\mu$, the larger is the absolute value of the latter variance. By equation (13), this explains why a lower $\mu$ leads to a lower $\beta^{TR} < 0$.

By contrast, decreasing $\mu$ from 1 has a non-monotonic impact on the regression slope $\beta^{FW}$. When the real exchange rate, $q_t$, is stationary but highly persistent, concretely when $\mu$ is around 0.95, the FW rule implies $\beta^{FW} < 0$. Earlier discussion in this subsection suggests that, in explaining this result, we should demonstrate how $\sigma > 0$ affects the impact of $\mu$ on $\beta^{FW}$. We do this in 4 steps:

1. Starting with $\sigma = 0$, equation (21) and policy rule (16) lead to a policy rate 
   $$ i^{FW}_t = -\varphi \frac{a(1-\mu)}{1-a} q_t. $$

2. Next, setting $\sigma > 0$ allows $i^{FW}_t$ to affect the inflation rate, $\pi_t$. On the basis of equations (10)-(11), step 1 suggests that the impact of $i^{FW}_t$ on $\pi^{FW}_t$ introduces a positive dependence of $\pi^{FW}_t$ on $q_t$. In Appendix D we show that this is indeed the case.

3. For a sufficiently persistent $q_t$—i.e., sufficiently high $\mu$—the positive dependence of $\pi^{FW}_t$ on $q_t$ translates into $\text{Cov} \left( q_t, E_t \pi^{FW}_{t+1} \right) > 0$ and, by the policy rule (16), into $\text{Cov} \left( i^{FW}_t, q_t \right) > 0$.

4. $\text{Cov} \left( i^{FW}_t, q_t \right) > 0$ translates into $\text{Cov} \left( i^{FW}_t, E_t q_{t+1} \right) = (\mu - 1) \text{Cov} \left( i^{FW}_t, q_t \right) < 0$. However, as shown earlier in this subsection, this leads to $\beta^{FW} < 0$ only if $\mu$ is not too close to 1.

6 Conclusion

In this paper we traced Fama’s forward premium puzzle to the interaction between systematic monetary policy and exchange rate dynamics. We presented a
“UIP-shock” interpretation of currency-risk driven by exogenous (fundamental) uncertainty about the natural rate of interest and the real exchange rate. We assumed full exchange rate pass-through, so the real exchange rate’s dynamics correspond to underlying fluctuations in countries’ terms of trade. We then determined the model-implied degree of UIP violation under six different monetary policy rules. Specifically, the implied Fama regression slopes are a function of the underlying structural parameters; the two fundamentals’ persistence and relative variability; and the monetary authority’s responsiveness to inflation- and output gap fluctuations. We show that negative Fama regression slopes cannot arise from policy rules responding only to domestic inflation fluctuations. Further, while each of the three CPI-based policy rules is consistent with the estimated sign of UIP violation, only forward-looking rules can generate violation magnitudes similar to those observed in the data.

Empirically, it should be noted that, notwithstanding the widely documented sign of UIP violation, the explanatory power of Fama’s regression is very low. In other words, although the expected excess returns from the associated “carry trade” strategies exploiting interest-rate differential are significantly positive, they are very small. That said, our framework pinpoints the source of these predictable returns to the speed of mean-reversion and relative variability of countries’ real exchange rate and natural real interest rate. It follows that more accurate estimation of these two fundamental processes becomes important. We leave these considerations as extensions for future research.
Appendix A

In this appendix we show that the three policy rules related to domestic inflation imply non-negative regression slopes: $\beta^{Trd} = 0$, $\beta^{Trd} \geq 0$ and $\beta^{Frd} \geq 0$. To lighten the notation, we do not flag the policy rule underlying the endogenous variables wherever this does not lead to confusion.

A.1: Domestic-inflation Taylor-type rules

Assume domestic inflation is a linear function of $r^n_t$ in equation (5):

$$\pi^H_t = \delta r^n_t \Rightarrow E_t \pi^H_{t+1} = \delta r + \delta \rho r^n_t$$  \hspace{1cm} (A.1)

and consider policy rules adjusting to domestic inflation and output gap:

$$i_t = \tau + \phi_x \pi^H_t + \phi_x x_t \hspace{0.5cm}, \hspace{0.5cm} \phi_x > 1, \hspace{0.5cm} \phi_x \geq 0 \hspace{1cm} (A.2)$$

In order to match coefficients we require two equations for the implied real interest rate, denoted $r_t$. For the first, involving only structural parameters, apply (A.1) to NKPC equation (7):

$$x_t = \frac{\delta}{\kappa} \left[ -b \tau + (1 - b \rho) r^n_t \right] \Rightarrow$$

$$E_t x_{t+1} = \frac{\delta}{\kappa} \left\{ [1 - b(1 + \rho)] \tau + \rho(1 - b \rho) r^n_t \right\}$$

Substituting these in DIS equation (8):

$$r_t = \frac{\delta(1 - b \rho)}{\kappa \sigma} \tau + \frac{\delta(1 - b \rho)}{\kappa \sigma} \left[ \rho - 1 + \frac{\kappa \sigma}{\delta(1 - b \rho)} \right] r^n_t$$ \hspace{1cm} (A.3)

For the second equation, apply (A.2) to the Fisher relation, $r_t = i_t - E_t \pi^H_{t+1}$, thus:

$$r_t = \tau - \delta \left( 1 + \frac{\phi_x b}{\kappa} \right) \tau + \delta \left[ \phi_x - \rho + \phi_x \frac{1 - b \rho}{\kappa} \right] r^n_t$$ \hspace{1cm} (A.4)

Matching coefficients on constant terms yields the equilibrium value of $\delta$:

$$\delta = \tau \left\{ \tau \left[ 1 + \phi_x \kappa^{-1} b + (\kappa \sigma)^{-1}(1 - b \rho) \right] \right\}^{-1}$$
Similarly, matching the coefficients of terms in $r^n_t$ yields:

$$
\delta = \frac{\kappa \sigma}{(1 - b \rho) (1 - \rho) + \kappa \sigma (\phi_\pi - \rho + \frac{1}{\kappa} \phi_x (1 - b \rho))} > 0 \quad (A.5)
$$

First, note that the sign of $\beta^{TRd}$ equals that of $Cov(\pi_{t+1}, i_t)$. Second, since $\pi_t$ is a linear function of $\pi_t^H$ and $q_t$, and $i_t$ is independent of $q_t$, $Cov(\pi_{t+1}, i_t) = Cov(\pi_{t+1}^H, i_t)$. Third, the sign of $Cov(\pi_{t+1}^H, i_t)$ is that of $\delta > 0$, implying $\beta^{TRd} > 0$.

### A.2: Domestic-inflation forward-looking rules

Now consider policy rules adjusting to expected domestic inflation and output gap:

$$
i_t = \bar{i} + \varphi_\pi E_t \pi_{t+1}^H + \varphi_x E_t \bar{x}_{t+1} \quad (A.9)
$$

where for determinacy the response coefficients must satisfy:

$$
\begin{align*}
\kappa(\varphi_\pi - 1) + (1 + b)\varphi_x &< 2\sigma^{-1}(1 + b) \quad (A.10) \\
\kappa(\varphi_\pi - 1) + (1 - b)\varphi_x &> 0
\end{align*}
$$

By equations (7)-(8), the first equation for $r_t$ coincides with (A.3) above. The second equation reflects the forward-looking aspect of monetary policy:

$$
r_t = \bar{i} + \delta [\varphi_\pi - 1 + \varphi_x \kappa^{-1}[1 - b(1 + \rho)]] \bar{\pi} + \delta \rho [(\varphi_\pi - 1) + \varphi_x \kappa^{-1}(1 - b \rho)] r^n_t \quad (A.11)
$$

Matching the constant coefficients in (A.3) and (A.11) yields:

$$
\delta = \frac{\bar{i}}{\bar{\pi}} \left(1 - \varphi_\pi + \varphi_x \kappa^{-1}[b(1 + \rho) - 1] + \frac{1 - b \rho}{\kappa \sigma}\right)^{-1} \quad (A.12)
$$

A second expression for $\delta$ emerges by matching $r^n_t$ coefficients:

$$
\delta = \frac{\kappa \sigma}{(1 - b \rho) (1 - \rho) + \kappa \sigma \rho (\varphi_\pi - 1 + \frac{1}{\kappa} \varphi_x (1 - b \rho))} \quad (A.13)
$$

Since (A.13) implies $\delta > 0$, it follows that $\beta^{FD} > 0$. 

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Appendix B: Strict CPI inflation-targeting (IT)

By definition, \( \pi_t = E_t \pi_{t+1} = 0 \), which implies \( e_t = q_t \) and \( E_t \Delta e_{t+1} = E_t \Delta q_{t+1} \). Then, making use of equation (10) we obtain:

\[
x_t = -\frac{a}{1 - a\kappa} \left( \Delta q_t + b(1 - \mu)q_t \right)
\]

(B.1)

\[
E_t x_{t+1} = \frac{a}{1 - a} \frac{(1 - \mu)(1 - b\mu)}{\kappa} q_t
\]

In turn, noting that expected domestic inflation now equals:

\[
E_t \pi^H_{t+1} = E_t \Delta e_{t+1} - \frac{E_t \Delta q_{t+1}}{1 - a} = \frac{a}{1 - a} (1 - \mu) q_t
\]

(B.2)

we use the IS relationship in (11) to derive the policy rate as a function of exogenous variables:

\[
i_t^{IT} = r_t^a + \left( \frac{a}{1 - a} (1 - \mu) + \frac{a}{\kappa \sigma (1 - a)} \left( b(1 - \mu)^2 + 2 - \mu \right) \right) q_t - \frac{a}{\kappa \sigma (1 - a)} q_{t-1}
\]

(B.3)

The implied Fama slope coefficient is then defined as:

\[
\beta^{IT} = \frac{\text{Cov} \left( i_t^{IT}, E_t \Delta e_{t+1} \right)}{V \left( i_t^{IT} \right)} = \frac{\text{Cov} \left( i_t^{IT}, E_t \Delta q_{t+1} \right)}{V \left( i_t^{IT} \right)}, \text{ where (B.4)}
\]

\[
\text{Cov} \left( i_t^{IT}, E_t \Delta e_{t+1} \right) = -\frac{V(\eta)}{1 + \mu} \frac{a}{1 - a} \left( 1 + \frac{b(1 - \mu) + 2}{\kappa \sigma} \right) < 0 \] (B.5)

\[
V \left( i_t^{IT} \right) = \frac{V(\varepsilon)}{1 - \rho^2} \] (B.6)

\[
+ \frac{a^2 \left( \kappa \sigma (1 - \mu) (2b(1 - \mu) + \kappa \sigma + 4) + b^2 (1 - \mu)^3 + 4b(1 - \mu)^2 + 5 - 3\mu \right)}{\kappa^2 \sigma^2 (1 - a)^2 (1 + \mu)} V(\eta)
\]
Finally, dividing (B.5) by (B.6) and dividing through by $V(\varepsilon)$ yields:

$$\beta^{IT} = -\left[ \frac{1 - \mu}{1 + \mu} \frac{a}{1 - a} \left( 1 + \frac{b(1 - \mu) + 2}{\kappa \sigma} \right) \right] /$$

$$\left[ \frac{1}{1 - \rho^2} + \frac{a^2 \left( \frac{\kappa \sigma (1 - \mu)(2b(1 - \mu) + \kappa \sigma + 4)}{+b^2 (1 - \mu)^3 + 4b(1 - \mu)^2 + 5 - 3\mu} \right)}{(1-a)^2 \kappa^2 \sigma^2 (1+\mu)} \right] \nu$$

which is unambiguously negative.

### Appendix C: Taylor-type policy rules (TR)

In this appendix we derive the coefficient vector $\{\delta^\pi, \zeta^\pi, \theta^\pi, \delta^x, \zeta^x, \theta^x\}$ determining the linear mapping from exogenous variables to the inflation rate and the output gap under the Taylor-based rule. To employ the method of undetermined coefficients, we start with the guess

$$\pi_t = \delta^\pi r^n_t + \zeta^\pi q_t + \theta^\pi q_{t-1} \Rightarrow E_t\pi_{t+1} = \delta^\pi \tau + \delta^\pi \rho r^n_t + (\mu \zeta^\pi + \theta^\pi) q_t \quad (C.1)$$

which implies, via equation (10), that

$$x_t = c_0 \tau + \delta^x r^n_t + \zeta^x q_t + \theta^x q_{t-1} \Rightarrow E_t x_{t+1} = c_0 \tau + \delta^x \tau + \delta^x \rho r^n_t + (\mu \zeta^x + \theta^x) q_t \quad (C.2)$$

where

$$c_0 \equiv -\frac{b \delta^\pi}{\kappa}, \quad \delta^x \equiv \frac{\delta^\pi (1 - b \rho)}{\kappa}, \quad \theta^x \equiv \frac{a}{\kappa(1 - a)} + \frac{\theta^\pi}{\kappa} \quad \text{and} \quad (C.3)$$

$$\zeta^x \equiv \frac{1}{\kappa(1 - a)} [(1 - a) \zeta^\pi - a - b(1 - a)(\mu \zeta^\pi + \theta^\pi) - ab(1 - \mu)]$$

By equations (14), (C.1) and (C.2) we obtain the implied policy rate:

$$i_t^{TR} = \bar{i} + \phi_x c_0 \tau + (\phi_x \delta^\pi + \phi_x \delta^x) r^n_t + (\phi_x \zeta^\pi + \phi_x \zeta^x) q_t + (\phi_x \theta^\pi + \phi_x \theta^x) q_{t-1} \quad (C.4)$$
At the same time, by equations (11), (C.1) and (C.2) we obtain an alternative expression for the implied policy rate:

\[
\tilde{i}_t^R = \frac{E_t x_{t+1} - x_t}{\sigma} + E_t \tilde{\pi}_{t+1}^H + r_t^n
\]

\[
= \frac{E_t x_{t+1} - x_t}{\sigma} + E_t \left( \tilde{\pi}_{t+1} - \frac{a}{1-a} \Delta \rho_{t+1} \right) + r_t^n
\]

\[
= \sigma^{-1} [\delta^x \pi - (1 - \rho) \delta^x r_t^n + (\theta^x - (1 - \mu) \zeta^x) \rho_t - \theta^x \rho_{t-1}] + \delta^x \pi + \delta^x \rho r_t^n + r_t^n + (\mu \zeta^x + \theta^x) \rho_t + \frac{a}{1-a} (1 - \mu) \rho_t
\]

(C.5)

Matching coefficients in (C.4) and (C.5):

- **Constant terms:**
  \[
  \frac{\tilde{\pi}}{\pi} = \delta^x \left( \frac{1 - b \rho}{\sigma \lambda} + 1 + \phi_x \frac{b}{\lambda} \right)
  \]

- **\( r_t^n \) terms:**
  \[
  \delta^x = \left( \phi_x - \rho + \phi_x \frac{1 - a}{\lambda} + \frac{1 - \rho}{\sigma \lambda} \right)^{-1}
  \]
  \[
  \delta^x > 0 \text{ because } \phi_x > 1, \rho \in (0, 1), b \in (0, 1)
  \]

- **\( \rho_{t-1} \) terms:**
  \[
  \theta^x = -\frac{a}{1-a} \left( 1 + \frac{\phi_x}{\lambda} \right) \sigma
  \]
  \[
  \theta^x < 0 \text{ because } a \in (0, 1)
  \]

- **\( \rho_t \) terms:**
  \[
  \zeta^x = \left( \phi_x + \frac{1 - \mu}{\sigma} \right) \left( \frac{b \phi_x}{\lambda} + \phi_x \frac{a}{1-a} \frac{1+b(1-\mu)}{\lambda} \right) + \theta^x + \frac{a(1-\mu)}{1-a} + \frac{a b \phi_x + \theta^x}{\lambda} = \frac{a}{1-a} \left( \phi_x + \frac{1 - \mu}{\sigma} \right) \frac{(1-b) \phi_x}{\lambda} (1 - \mu)
  \]
  \[
  \zeta^x > 0 \text{ because } \phi_x > 1, \mu \in (0, 1), b (0, 1), a \in (0, 1)
  \]
Finally, equation (4) yields the implied Fama slope:

$$\beta^{TR} = \frac{\delta^n \rho (\delta^{\pi} + \phi_x \delta^x)}{1 - \rho^2 (\phi_x \delta^{\pi} + \phi_x (\zeta^\pi + \theta^x)) + (\phi_x \delta^x + \phi_x (\zeta^x + \theta^x))^2} v$$  \(\text{C.9}\)

Since \(\delta^\pi > 0\) and \(\delta^x > 0\), the last expression reveals directly that \(\beta^{TR} > 0\) for small enough \(v\). In addition, it can be shown that there exists \(\mu^h > 0\) such that \(\tilde{\beta}^{TR} < 0\) for \(\mu \in (0, \mu^h)\).

**Appendix D: Forward-looking policy rules (FW )**

In determining the coefficient vector \(\{\delta^\pi, \zeta^\pi, \theta^\pi, \delta^x, \zeta^x, \theta^x\}\) under the FW rule, we parallel Appendix C. By equations (16), (C.1) and (C.2), we obtain the implied policy rate:

$$i_t^{FW} = \bar{\pi} + (\phi_x \rho \delta^\pi + \phi_x (\alpha_0 + \delta^x)) \bar{\pi} + (\phi_x \delta^\pi + \phi_x (\zeta^\pi + \theta^x)) \rho^n + (\phi_x (\mu \zeta^\pi + \theta^\pi) + \phi_x (\mu \zeta^x + \theta^x)) \rho^n$$  \(\text{D.1}\)

Matching coefficients in (C.4) and (D.1):

- Constant terms:
  $$\frac{\bar{\pi}}{\bar{\pi}} = \delta^\pi \left( \frac{1 - b \rho}{\kappa \sigma} + 1 + \frac{b}{\kappa} - \left( \phi_x + \phi_x \frac{1 - b \rho}{\kappa} \right) \right)$$

- \(r^n_t\) terms:
  $$\delta^\pi = (\kappa \sigma) [\rho \kappa \sigma (\phi_x - 1) + \phi_x \sigma \rho (1 - b \rho) + (1 - \rho) (1 - b \rho)]^{-1}$$  \(\text{D.2}\)

- \(\delta^\pi > 0\) because \(\phi_x > 1\), \(b \in (0, 1)\), \(\rho (0, 1)\), \(a \in (0, 1)\)

- \(q_{t-1}\) terms:
  $$\theta^\pi = -\frac{\alpha}{1 - \alpha} < 0$$  \(\text{D.3}\)
• $q_t$ terms:

$$
\zeta = \frac{a (\varphi - \mu) \kappa \sigma + (1 - b \mu) (\varphi \mu \sigma + 1 - \mu)}{1 - a (\varphi_{-1} - 1) \mu \kappa \sigma + (1 - b \mu) (\varphi \mu \sigma + 1 - \mu)} \quad \text{(D.4)}
$$

$$
\zeta > 0 \text{ because } \varphi > 1, \mu \in (0, 1), b \in (0, 1), a \in (0, 1)
$$

Finally, equation (4) implies:

$$
\beta^{FW} = \frac{\delta^2 \varphi_x \delta^2 + \varphi_x \delta^2}{1 - \rho^2} + \frac{(\mu (\zeta^2 + 1) + \theta^2) - 1) (\varphi_x (\mu \zeta^2 + \theta^2) + \varphi_x (\mu \zeta^2 + \theta^2) \nu)}{1 - \mu^2} \quad \text{(D.5)}
$$

Since $\delta^2 > 0$ and $\delta^2 > 0$, the last expression reveals directly that $\beta^{FW} > 0$ for small enough $\nu$. In addition, equations (20), (D.3) and (D.4) imply that Cov ($q_t, E_t \pi_{t+1}$) > 0 iff $\frac{\mu}{1 - \mu} - \frac{1 - \mu}{\kappa \sigma} > 0$. Since the left-hand side of this inequality increases strictly in $\mu$, there is a unique threshold $\mu^*$, such that Cov ($q_t, E_t \pi_{t+1}$) > 0 for $\mu \in (\mu^*, 1)$. Note that $\mu^*$ decreases with $\kappa \sigma$. 

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References


Table 1: Fama regressions\textsuperscript{10}

<table>
<thead>
<tr>
<th>Country</th>
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<th>$\hat{a}$</th>
<th>$\beta \times pre_{IT}$</th>
<th>$\beta$</th>
<th>$R^2$</th>
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<td>$-0.000$</td>
<td>$0.003$</td>
<td>$3.02$</td>
<td>$-3.43^*$</td>
<td>1%</td>
<td>1.92</td>
</tr>
<tr>
<td>[1977:01-2008:08]</td>
<td></td>
<td>(0.005)</td>
<td>(2.08)</td>
<td>(1.69)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>$0.004^{**}$</td>
<td>$-0.002$</td>
<td>$0.49$</td>
<td>$-1.84$</td>
<td>1%</td>
<td>1.87</td>
</tr>
<tr>
<td>[1977:01-2008:08]</td>
<td></td>
<td>(0.002)</td>
<td>(1.34)</td>
<td>(1.26)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Switzerland</td>
<td>$0.004$</td>
<td>$-0.01^{**}$</td>
<td>$3.03$</td>
<td>$-3.99^{**}$</td>
<td>0%</td>
<td>1.90</td>
</tr>
<tr>
<td>[1977:01-2008:08]</td>
<td></td>
<td>(0.006)</td>
<td>(1.98)</td>
<td>(1.85)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Great Britain</td>
<td>$0.008^{**}$</td>
<td>$-0.000$</td>
<td>$-3.24^*$</td>
<td>$0.45$</td>
<td>2%</td>
<td>2.01</td>
</tr>
<tr>
<td>[1977:01-2008:08]</td>
<td></td>
<td>(0.004)</td>
<td>(1.88)</td>
<td>(1.54)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>$0.003$</td>
<td>$-0.002$</td>
<td>$3.36^*$</td>
<td>$-2.93^{**}$</td>
<td>1%</td>
<td>1.82</td>
</tr>
<tr>
<td>[1977:01-2008:08]</td>
<td></td>
<td>(0.004)</td>
<td>(1.72)</td>
<td>(1.19)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>New Zealand</td>
<td>$0.003$</td>
<td>$-0.003$</td>
<td>$-1.97$</td>
<td></td>
<td>0%</td>
<td>1.99</td>
</tr>
<tr>
<td>[1992:08-2008:08]</td>
<td></td>
<td>(0.04)</td>
<td>(1.76)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Euro area</td>
<td>$-0.003$</td>
<td></td>
<td>$-3.66^{**}$</td>
<td></td>
<td>2%</td>
<td>1.89</td>
</tr>
<tr>
<td>[1999:01-2008:08]</td>
<td></td>
<td>(0.003)</td>
<td>(1.79)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{10}Estimates of equation (2), on the basis of monthly spot and forward exchange rates with the US dollar, allowing for a structural break. The dummy variable $pre_{IT}$ is equal to 1 prior to the break and 0 thereafter. This break occurs in 1994:10 for Australia, 1994:01 for Canada, 1999:01 for Switzerland, 1993:01 for Great Britain, 1995:01 for Sweden. Because of shorter data series, no break is allowed for in the case of New Zealand and the Euro area. Sample periods are in brackets and standard errors in parentheses. Superscripts *, ** and *** denote significance at the 10\%, 5\% and 1\% level. Strong violations of UIP in bold.
Table 2: Country characteristics\textsuperscript{11}

<table>
<thead>
<tr>
<th></th>
<th>$\hat{a}$</th>
<th>$\hat{\mu}$</th>
<th>$\hat{\sigma}_\eta$</th>
<th>$\hat{\rho}$</th>
<th>$\hat{\sigma}_\varepsilon$</th>
<th>$\hat{\nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.22</td>
<td>0.91</td>
<td>0.049</td>
<td>0.65</td>
<td>0.03</td>
<td>2.67</td>
</tr>
<tr>
<td>Canada</td>
<td>0.36</td>
<td>0.97</td>
<td>0.029</td>
<td>0.72</td>
<td>0.032</td>
<td>0.82</td>
</tr>
<tr>
<td>Euro area</td>
<td>0.33</td>
<td>0.93</td>
<td>0.030</td>
<td>0.52</td>
<td>0.012</td>
<td>6.25</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.21</td>
<td>0.90</td>
<td>0.046</td>
<td>0.60</td>
<td>0.048</td>
<td>0.92</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.39</td>
<td>0.97</td>
<td>0.031</td>
<td>0.92</td>
<td>0.032</td>
<td>0.94</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.41</td>
<td>0.91</td>
<td>0.023</td>
<td>0.90</td>
<td>0.012</td>
<td>3.67</td>
</tr>
<tr>
<td>Great Britain</td>
<td>0.29</td>
<td>0.91</td>
<td>0.033</td>
<td>0.44</td>
<td>0.03</td>
<td>1.21</td>
</tr>
</tbody>
</table>

\textsuperscript{11}The estimate of openness to trade ($\hat{a}$) is equal to the average quarterly imports-to-GDP ratio over time, in the particular country or area. The estimates of the parameters underpinning the time series properties of the real effective exchange rate ($\hat{\mu}$ and $\hat{\sigma}_\eta$) and the natural real interest rate ($\hat{\rho}$ and $\hat{\sigma}_\varepsilon$) are obtained on the basis of AR(1) specifications in line with equations (5) and (6). For the natural rate parameters, we use data on ex-post real interest rates. The relative variance is defined as $\hat{\nu} \equiv \hat{\sigma}_\eta^2/\hat{\sigma}_\varepsilon^2$. Data sources: BIS and IMF/IFS.
Table 3: Benchmark calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Based on</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$ persistence</td>
<td>$b$</td>
<td>0.968</td>
</tr>
<tr>
<td>Sensitivity of $\pi$ to $x$</td>
<td>$\kappa$</td>
<td>0.0157</td>
</tr>
<tr>
<td>Sensitivity of $x$ to $i$</td>
<td>$\sigma$</td>
<td>0.5</td>
</tr>
<tr>
<td>Openness</td>
<td>$a$</td>
<td>0.3</td>
</tr>
<tr>
<td>$q$ persistence</td>
<td>$\mu$</td>
<td>0.95</td>
</tr>
<tr>
<td>$r^n$ persistence</td>
<td>$\rho$</td>
<td>0.6</td>
</tr>
<tr>
<td>Relative volatility</td>
<td>$\nu$</td>
<td>1.4</td>
</tr>
<tr>
<td>Policy parameters</td>
<td>$\phi_\pi = \varphi_\pi$</td>
<td>1.5</td>
</tr>
<tr>
<td>(TR and FW only)</td>
<td>$\phi_x = \varphi_x$</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Table 4: Model-implied slope coefficients

<table>
<thead>
<tr>
<th></th>
<th>Strict CPI</th>
<th>TR</th>
<th>FW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Cov(\pi_{t+1}, i_t)$</td>
<td>0</td>
<td>0.05bp</td>
<td>0.01bp</td>
</tr>
<tr>
<td>$+Cov(\Delta q_{t+1}, i_t)$</td>
<td>$-34bp$</td>
<td>$-0.33bp$</td>
<td>$-0.05bp$</td>
</tr>
<tr>
<td>$= Cov(\Delta e_{t+1}, i_t)$</td>
<td>$-34bp$</td>
<td>$-0.28 bp$</td>
<td>$-0.04 bp$</td>
</tr>
<tr>
<td>$V(i_t)$</td>
<td>38700 bp</td>
<td>3.87 bp</td>
<td>0.03 bp</td>
</tr>
<tr>
<td>$\beta = \frac{Cov(\Delta r_{t+1}, i_t)}{V(i_t)}$</td>
<td>$-0.00$</td>
<td>$-0.07$</td>
<td>$-1.32$</td>
</tr>
</tbody>
</table>
Figure 1A: Sensitivity Analysis

- Model-implied β vs. ρ
- Model-implied β vs. $\phi_\pi$ or $\psi_\pi$
- Model-implied β vs. $\phi_x$ or $\psi_x$
Figure 1B: Sensitivity analysis