Jointly optimal regulation of bank capital and maturity structure*

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Abstract

In my theory, banks create excessive systemic risk through leverage and maturity mismatch, as financial constraints introduce welfare-reducing pecuniary externalities. I argue that capital and maturity regulation are complementary, and characterise optimal policy. Regulators with imperfect information can achieve efficiency with linear constraints on banks’ balance sheets. This policy limits systemic risk while allowing banks to use private information efficiently. It cannot be implemented by simply strengthening capital requirements: Capital requirements cannot target maturity mismatch and lead to inefficiently low investment. Within the Basel III framework, Net Stable Funding Ratios should be the prime regulatory tool for targeting systemic risk.

Keywords: Systemic risk, maturity mismatch, macroprudential regulation, liquidity, capital requirements, mark to funding, fire sales.

JEL classifications: G18, G21, G28 and E44.

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1 Introduction

The rationale for financial policy is that banks may not consider the adverse effect of the risks they take on other economic agents. Traditionally, this concern was motivated by the fact that taxpayers pay for failing banks due to deposit insurance or government bailouts. Financial policy accordingly focused on imposing capital requirements which constrain bank leverage and thereby align private and social incentives. Following the crisis of 2008, another concern surfaced: Banks’ individual risk can contribute to systemic risk, which causes inefficient fire sales and market freezes, thus harming the financial sector as a whole.

Systemic risk is not caused exclusively by high leverage or insufficient bank capital. It is amplified if banks rely heavily on short-term debt when raising funds for long-term lending, which is known as maturity mismatch (Brunnermeier 2009). Recent policy reforms, which are based on systemic risk concerns, target maturity mismatch as well as leverage. For instance, the global Basel III framework (BIS 2011) introduces liquidity coverage and net stable funding ratios, which explicitly target maturity structure.

This paper studies how such reforms ought to be implemented from a welfare-theoretic perspective. In particular, I address two questions:

1. Are capital and maturity regulation complementary, in the sense that using both improves efficiency?

2. How should regulation of capital and maturity structure be designed to deal with systemic risk?

My three-period model extends Lorenzoni (2008) and Shleifer and Vishny (2010). There are two fundamental financial frictions. First, banks need equity downpayments, or ’skin in the game’, to raise funding. Second, banks can only sell assets to outside buyers at fire sale prices. Fire sales are socially wasteful because outside buyers cannot extract as much surplus from them as banks.

Banks make risky long-term investments and fund themselves with equity and debt. In contrast to the previous literature, banks can choose between short-term or long-term debt contracts. Banks’ creditors face potential liquidity shocks, so they are reluctant to tie commit to long-term contracts. As a result, long-term debt is relatively expensive. Banks prefer not to use it, which creates endogenous maturity mismatch.

Fire sales can occur when banks earn low returns on their risky investments. Low returns erode their equity and their borrowing capacity due to the ’skin in the game’ constraint.
Then, banks struggle to roll over short-term debt and have to sell assets in order to repay short-term creditors.\footnote{This mechanism has been emphasised in the literature on financial amplification (Kiyotaki and Moore 1997, Bernanke, Gertler and Gilchrist 1999, Shin 2010 and He and Krishnamurthy 2012). Additional sources of financial instability include ‘margin spirals’, when ‘skin in the game’ requirements rise in bad times (Geanakoplos 2009, Brunnermeier and Pedersen 2009), uncertainty about complex financial networks (Caballero and Simek 2009) and irrational expectations (Gennaioli et al 2012).} Fire sales happen when banks are highly leveraged, or when their balance sheets feature strong maturity mismatch. However, competitive banks have no unilateral incentive to reduce leverage or maturity mismatch in order to avoid wasteful fire sales, since they take fire sale prices as given. This creates a ‘systemic externality’: A pecuniary externality which affects welfare due to the incompleteness of financial markets.\footnote{Pecuniary externalities matter in incomplete markets by the seminal result of Geanakoplos and Polemarchakis (1985). The interpretation of systemic risk as a pecuniary externality is due to Lorenzoni (2008). Other applications of this idea in financial settings include Kehoe and Levine (1993), Gromb and Vayanos (2002) and Caballero and Krishnamurthy (2003).}

To answer my first question, I consider a social planner who intervenes to reduce systemic risk. He faces two trade-offs. First, he can reduce leverage, which is socially costly because it constrains aggregate investment. Second, he can reduce maturity mismatch, which is socially costly because it prevents banks from performing valuable maturity transformation. Unless the cost of reduced maturity transformation is prohibitive, the least costly way to reduce risk is a hybrid strategy, featuring reductions in both leverage and maturity mismatch. Therefore, regulating bank capital and maturity structure are complementary strategies, and both should be employed to achieve efficiency.

For the second question, I consider the optimal choices of a regulator who can impose linear constraints on banks’ balance sheets.\footnote{The focus in this paper is therefore on ex ante regulatory constraints, which have been the core tool of the Basel accords. Another important dimension is how ex post policies, e.g. bailouts, central bank lending or quantitative easing, should be designed and how they affect the case for ex ante restrictions. Holmström and Tirole (2011), Farhi and Tirole (2012), Jeanne and Korinek (2013) and Benigno et al (2013) offer formal treatments of this issue. Combining their arguments with the analysis of this paper is an interesting topic for future research.} I assume that banks have private information about their funding costs. The regulator needs to intervene just enough to alleviate the externality, while giving banks the freedom to use their private information efficiently. The unique optimal policy is to design a constraint that replicates a ‘no fire sale condition’. The policy works through two channels: First, it automatically limits systemic risk to the socially optimal level. The systemic externality is eliminated: Private and social incentives are aligned. Second, it incentivises banks to use their private information efficiently. Banks are left to choose between reducing leverage and maturity mismatch. Since the externality is removed, banks choose to replicate the social planner’s optimal hybrid strategy.

As a corollary to this result, I show that traditional capital requirements cannot be used to...
implement the optimal policy. They cannot incentivise banks to reduce maturity mismatch. Therefore, they have to be overly aggressive on leverage in order to reduce systemic risk. This leads to inefficiently low investment. Capital requirements are therefore a blunt tool for dealing with systemic externalities.\footnote{Of course, systemic externalities are not the only source of inefficiency in the financial system. When they interact with problems caused by the external costs of bank failure, it may be optimal to combine the policy advocated here with capital requirements in order to target both externalities. The issue of interacting externalities is discussed in Goodhart et al (2013).}

Finally, I consider the implementation of the optimal policy within the Basel III framework. I argue that the Net Stable Funding Ratio (NSFR) ought to be considered Basel III’s prime weapon against systemic externalities. The optimal regulatory constraint in my model can be replicated by a NSFR. Moreover, the model can be used to derive some guidelines for the design of the NSFR in practice.

This paper is closely related to the recent literature on financial policy in the presence of pecuniary ‘systemic risk’ externalities. Korinek (2012) studies a three-period model, whereas Bianchi and Mendoza (2012) and Jeanne and Korinek (2010) compute optimal policies in calibrated infinite-horizon settings. These papers focus on reducing inefficient leverage through policies such as Pigouvian taxes on debt. In contrast, my analysis allows regulators to reduce inefficient leverage and maturity mismatch, thus introducing another dimension of policy-making.

Perotti and Suarez (2011) look at optimal ways of regulating maturity choices without modelling capital regulation. They find that Pigouvian taxes on short-term funding are preferable to direct caps, as the regulator can implement them efficiently without knowing banks’ credit opportunities. The intuition is very similar to the argument in favour of 'mark-to-funding' capital requirements in this paper.

Section 2 sets up the model. Section 3 analyses the competitive equilibrium. Section 4 analyses welfare. Section 5 considers optimal regulation. Section 6 discusses the implementation of optimal policy in the Basel III framework. Section 7 concludes.

## 2 The model

I propose a three-period model describing the behaviour of banks, who invest in long-term projects. The structure of the economy is inspired by Lorenzoni (2008) and Shleifer and Vishny (2010). This structure is extended by allowing banks to fund themselves with long-term debt, thus matching the maturities on their balance sheet.
**Time and uncertainty.** The model has three dates $t \in \{0, 1, 2\}$ and two states of the world $s \in \{g, b\}$, i.e. a good state and a bad state. State $s$ occurs with probability $\pi_s$, where $\pi_g + \pi_b = 1$. The state of the world becomes public information at the start of $t = 1$. Nobody discounts the future.

**Banks.** There is a unit measure of identical banks, who are risk-neutral. The representative bank (henceforth ‘the bank’) has an equity endowment of $e_0$ units of the consumption good at $t = 0$. It cannot raise outside equity. It makes investment decisions at $t = 0$ and state-contingent investment decisions at $t = 1$, which are described below. At $t = 2$ it realises a state-contingent profit $\Pi_s$ which it consumes immediately. There is no consumption before $t = 2$. Hence, the bank’s objective is to choose investments to maximise $E[\Pi_s]$.

**Projects.** The bank can invest in risky long-term projects at $t = 0$. The initial investment required for each project is one unit of the consumption good. One can think of these projects as being supplied by entrepreneurs, with the supply of projects being perfectly elastic at a price of one. Each project yields state-contingent cash flows at the start of $t = 1$ and $t = 2$, denoted $v_{1s} > 0$ and $v_{2s} > 0$ respectively. I assume that $v_{2b} < v_{2g} \leq 1$, so that the $t = 2$ cash flow is a partial repayment of the initial investment, and the fraction repaid is higher in the good state. Let $R = E[v_{1s} + v_{2s}] - 1$, and assume that $R > 0$, so that projects have positive expected net present value.

After cash flows are received at $t = 1$, projects are traded in a competitive secondary market. The secondary market price for projects at $t = 1$ in state $s$ is $p_s$.

**Cash.** The only alternative investment to projects is to hold cash. Cash yields zero interest.

**Bank funding.** As well as using its equity endowment, the bank can raise debt from a population of creditors, using projects as collateral. For all debt, there is an exogenous collateral constraint. As in Shleifer and Vishny (2010), this constraint takes the shape

$$\text{loan} = (1 - h) \times \text{market value of collateral}$$

where $h \in (0, 1)$ is the 'haircut' on debt. Other things being equal, a larger haircut decreases the likelihood of default, since more collateral is pledged per unit borrowed. $h$ is assumed to be large enough to rule out any default in this model. Below, I impose a lower bound on $h$ which ensures this.

The collateral constraint in (1) is taken as given. Lorenzoni (2008) conducts a similar analysis with endogenous haircut constraints, derived from limited pledgeability of returns as in Hart

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5The effect of this assumption on my results is discussed in Section 4.
6For any state-contingent variable $Y_s$, its expectation is defined as $E[Y_s] = \pi_g Y_g + \pi_b Y_b$. 

and Moore (1994) and Holmström and Tirole (1997). Generalising the analysis of this paper to the case of endogenous haircuts is a subject for future research. I would not expect this extension to change the nature of my results on equilibrium, welfare and regulation.

The bank can propose short-term and long-term debt contracts to creditors. Short-term debt contracts can be issued at $t = 0$ and $t = 1$. The bank repays the principal plus an interest payment one period later. Long-term debt contracts are available only at $t = 0$. The bank repays the principal plus an interest payment at $t = 2$.

The bank will ensure that creditors’ participation constraints bind, leaving creditors indifferent between lending and not lending. Hence, the preferences of creditors will determine the cost of short-term and long-term debt.

In particular, creditors are risk-averse and have potential liquidity needs at $t = 1$ as in Diamond and Dybvig (1983). Their outside option is to hold cash. They like to hold liquid assets such as short-term bank debt, because this allows them to fulfill any liquidity needs that may arise. Assuming that the bank can make take-it-or-leave-it offers to creditors, it does not need to pay interest on its short-term debt. However, potential liquidity needs make creditors reluctant to hold illiquid assets, such as long-term debt contracts. The bank has to offer a liquidity risk premium on long-term debt, which takes the shape of a non-zero interest payment.

Appendix B derives the cost of debt contracts explicitly from creditor preferences. Here, I simply state it in reduced form:

- Short-term debt is costless. No interest payment is required.

- Long-term debt is costly. If the bank raises $L$ units of long-term debt, the interest payment required is $\bar{r} (L) \geq 0$. The function $\bar{r}$ is differentiable, strictly increasing and convex, with $\bar{r} (0) = 0$.

Note that long-term debt is costly because creditors have to be compensated for holding illiquid assets, not because creditors derive a surplus from long-term debt contracts. Hence, long-term debt is not only costly for the bank, but socially costly. This has a natural interpretation: When it uses long-term debt to fund long-term investments in projects, the bank fails to perform maturity transformation, the process of transforming short-term deposits into long-term investments. Maturity is often cited as one of the fundamental valuable functions of the banking sector. Consequently, it is a natural result that reduced maturity transformation is socially costly in this model.
Bank investments at $t = 0$. The bank invests in $n_0$ projects funded by (and pledged as collateral against) short-term debt. The collateral constraint in equation (1) requires the bank to contribute $hn_0$ of its own equity: The market value of projects at $t = 0$ is one, as this is the ratio at which consumption goods can be turned into projects. The bank repays the loan $(1 - h)n_0$ at $t = 1$ and pays no interest.

The bank invests in $\bar{n}_0$ projects funded by long-term debt. The collateral constraint requires the bank to contribute $h\bar{n}_0$ of its own equity. The bank repays the loan $(1 - h)\bar{n}_0$ at $t = 2$. Also, it makes an interest payment of $\tilde{r}((1 - h)\bar{n}_0) \equiv r(\bar{n}_0)$ at $t = 2$. The bank is obliged to hold the collateral, $\bar{n}_0$ projects, until the debt matures.\footnote{More generally, one does not need to require the bank to hold the projects themselves as collateral, but merely assets of equivalent value. However, this would not change my main results, as the bank still would not be able to liquidate the collateral to finance itself at $t = 1$.}

Any equity not spent on downpayments or unlevered projects is held as cash, denoted $c_0$. The bank does not invest in unlevered projects that are funded in full by equity. It can be shown that this assumption is without loss of generality, since the cash flow from an unlevered project can be replicated by a portfolio of projects backed by short-term debt and cash.

In summary, the bank’s choice variables at $t = 0$ are $x_0 = (c_0, n_0, \bar{n}_0) \in \mathbb{R}_+^3$, and its budget constraint is

$$e_0 = h(n_0 + \bar{n}_0) + c_0$$ (2)

Let $B_0 \subset \mathbb{R}_+^3$ be the set of bank choices $x_0$ that satisfy this budget constraint.

Bank cash flows at $t = 1$ in state $s$. After the state of the world is revealed at $t = 1$, the bank receives the cash flow $v_{1s}$ for each project it invested in at $t = 0$. It also repays its short-term creditors $(1 - h)n_0$. Hence its net cash flow is $v_{1s}(n_0 + \bar{n}_0) - (1 - h)n_0$.

The bank’s equity at $t = 1$ in state $s$, denoted $e_{1s}$, is defined as the sum of the net cash flow, retained cash and the value of its marketable assets. Marketable assets exclude the $\bar{n}_0$ projects which are pledged as collateral against long-term debt. The remaining assets are $c_0$ units of cash and $n_0$ projects with a market value of $p_s$ each. Hence, equity can be written as follows:

$$e_{1s} = c_0 + n_0[v_{1s} + p_s - (1 - h)] + \bar{n}_0 v_{1s}$$ (3)

Bank investments at $t = 1$ in state $s$. After trading in the secondary market, the bank holds $n_{1s}$ projects funded by short-term debt. The market value of these projects is $p_s n_{1s}$,
so the bank is required to contribute $hp_s n_{1s}$ of its own equity. The bank repays the loan $(1 - h) p_s n_{1s}$ at $t = 2$ and pays no interest. The bank also holds $c_{1s}$ units of cash.

In summary, the bank’s choice variables at $t = 1$ are $x_{1s} = (c_{1s}, n_{1s}) \in \mathbb{R}^2_+$, and its budget constraint is

$$e_{1s} = hp_s n_{1s} + c_{1s}$$

(4)

where $c_{1s}$ is its $t = 1$ equity, which is related to its first-period choices $x_0$ by equation (3). Let $B_{1s}(x_0) \subset \mathbb{R}^2_+$ be the set of bank choices that satisfy this budget constraint.

Bank cash flows at $t = 2$ in state $s$. The bank receives the cash flow $v_{2s}$ for each project held at $t = 1$. This includes $\bar{n}_0$ projects held as collateral against long-term debt. It repays $(1 - h) p_s n_{1s}$ to short-term creditors and $(1 - h) \bar{n}_0 + r (\bar{n}_0)$ to long-term creditors. The bank’s profit is given by the sum of the net cash flow and retained cash, which can be written as follows:

$$\Pi_s = c_{1s} + v_{2s} (n_{1s} + \bar{n}_0) - (1 - h) (p_s n_{1s} + \bar{n}_0) - r (\bar{n}_0)$$

(5)

Secondary market for projects and outside buyers. In addition to banks, there is a unit measure of outside buyers who participate in the secondary market for projects at $t = 1$. Outside buyers derive a non-stochastic utility $p$ from each project they purchase, where $p < v_{2s}$ for all $s$. Thus, outside buyers value projects at less than the cash flow bankers can extract from them. A possible motivation for this assumption is that monitoring loans to businesses and households efficiently requires expertise which outside buyers do not possess. $p$ is interpreted as a fire sale price. Shleifer and Vishny (2011) provide a theoretical and empirical survey on fire sales.

The haircut revisited. Finally, I derive a condition which ensures that the haircut $h$ is large enough to make all debt contracts risk-free. Creditors can always seize the projects pledged as collateral and liquidate them. Assume that if they liquidate in period $t$ in state $s$, they can appropriate a fraction $\theta \in (0, 1)$ of the cash flow $v_{ts}$ and sell the project in the secondary market, as in Kiyotaki and Moore (1997). It is easy to show that the following is sufficient to make all debt contracts risk free:

$$h > 1 - \min \{p, \theta v_{2s}\}$$

(6)

Competitive equilibrium. The equilibrium of this economy is defined as follows:

Definition 1. A competitive equilibrium is described by asset prices $p_s$ for $s \in \{g, b\}$ and bank choices $x_0 \in \mathbb{R}^3_+$ and $x_{1s} \in \mathbb{R}^2_+$ for $s \in \{g, b\}$ satisfying the following two criteria:
1. **Optimality.** The bank’s choices maximise $E[\Pi_s]$ subject to $x_0 \in B_0$ and $x_{1s} \in B_{1s}(x_0)$, taking asset prices as given.

2. **Market clearing.** The bank’s choices imply $n_{1s} \leq n_0$ for all $s \in \{g, b\}$. Furthermore, $p_s \geq \underline{p}$, and if $n_{1s} < n_0$, then $p_s = \underline{p}$.

The market clearing condition states that the bank’s supply of projects in the secondary market at $t = 1$ has to equal the demand of outside buyers. In particular, the price of projects has to be equal to the outside buyers’ marginal valuation ($\underline{p}$) whenever the bank is a net seller of projects at $t = 1$, or $n_{1s} < n_0$.

### 3 Competitive equilibrium

#### 3.1 Forced sales at $t = 1$

The key property of the model is that there may be fire sales in the secondary market. Fire sales in equilibrium appear unlikely at first glance. Outside buyers are only willing to pay the fire sale price $\underline{p}$ for projects, so if the bank sells projects, this will be the equilibrium price. The bank can extract a cash flow of $v_{2s} > \underline{p}$ from each project. Hence, it considers projects to be undervalued if there is a fire sale, and has no fundamental incentive to sell in the first place.

To understand why the bank can be *forced* to sell, note that it may have short-term debt maturing at $t = 1$. Having serviced this debt, it needs to raise new short-term debt to hold on to its portfolio of projects. However, due to the haircut, raising short-term debt requires an equity downpayment of $hp_s$ for each project bought. If the bank’s equity $e_{1s}$ has been eroded by a low cash flow $v_{1s}$ and low prices $p_s$, the bank will be forced to sell projects.

By the $t = 1$ budget constraint in equation (4), the maximal number of projects the bank can hold at $t = 1$ is achieved by using all its equity to raise new short-term debt, setting $c_{1s} = 0$ and $n_{1s} = e_{1s}/p_s h$. The bank’s minimum net sale is then

$$n_0 - n_{1s} = \frac{e_{1s}}{p_s h} = \frac{n_0 [(1 - p_s)(1 - h) - v_{1s}] - \tilde{n}_0 v_{1s} - c_0}{p_s h}$$  \hspace{1cm} (7)$$

where the second equality follows from equation (3). I now make assumptions imposing structure on the cash flows $v_{1s}$. These will clarify the analysis of fire sales in equilibrium. In particular, I rule out fire sales in the good state, and ensure that fire sales are possible in the bad state.
Assumption 1. The cash flows from a project at $t = 1$ satisfy the following:

\[
\begin{align*}
    v_{1g} & \geq (1 - p)(1 - h) \\
    v_{1b} & < (1 - v_{2b})(1 - h)
\end{align*}
\]

The intuitive interpretation of Assumption 1 follows from equation (7). The first condition ensures that in the good state, the right hand side of equation (7) is always negative. The bank is never forced to sell in the good state, even when the price is at its lowest possible level, i.e. $p_g = p$. This rules out fire sales in the good state. The second condition ensures that the bank is forced to sell in the bad state if it only invests in projects funded by short-term debt (setting $c_0 = \bar{n}_0 = 0$), even when prices are high with $p_b = v_{2b}$. Thus, it ensures that fire sales are a possibility in the bad state. The following result formalises this argument:

Lemma 1. In any competitive equilibrium, asset prices satisfy $p_g = v_{2g}$ and

\[
p \leq p_b \leq v_{2b}
\]

Proof. See Appendix A. \qed

3.2 Bank profit and choices at $t = 0$

Recall that the bank’s $t = 0$ investment choices are $x_0 = (n_0, \bar{n}_0, c_0) \in B_0$. I now characterise the bank’s expected profits and optimal choices in equilibrium. These will depend on investment opportunities at $t = 1$, and in particular on whether there is a fire sale in the bad state. Intuitively, it is helpful to split the bank’s profits up into two terms: Basic profits, which it earns from investment in projects, and trading profits from transactions in the secondary market at $t = 1$.

The bank earns a basic expected net return of $R$ on each of the $(n_0 + \bar{n}_0)$ projects it invests in. However, profits are reduced by the amount of interest it has to pay on long-term debt. Hence, its expected basic profits are

\[
R (n_0 + \bar{n}_0) - r (\bar{n}_0)
\]

Regarding the secondary market at $t = 1$, there are two scenarios (see Lemma 1): Projects can be priced at fair value in all states ($p_s = v_{2s}$) or they can be underpriced in the bad state ($p_b < v_{2b}$).
First, suppose that projects are priced at fair value. Then the bank does not make profits or losses by trading at \( t = 1 \), and profits will be equal to the basic profits in equation (8). Second, suppose that projects are underpriced in the bad state. Then, the bank will optimally hold as many projects as possible, setting \( n_{1b} = e_{1b}/p_{b}h \). Its net sale \((n_0 - n_{1b})\) of projects are given by the expression in equation (7).

Projects are worth \( v_{2b} \) and the market price is \( p_b < v_{2b} \), so that net sales yield a loss of \( v_{2b} - p_b \) per unit. Since the bad state occurs with probability \( \pi_b \), its expected trading profits at \( t = 1 \) are given by

\[
-\pi_b (v_{2b} - p_b) \times \text{net sales}
\]

We can now add the basic profit and trading profit in order to characterise the bank’s profit and choices at \( t = 0 \):

**Lemma 2.** In competitive equilibrium, the bank’s expected profits satisfy \( E[\Pi_s] = V(x_0, p_b) \), and its choices solve the problem

\[
\max_{x_0 \in B_0} V(x_0, p_b)
\]

where

\[
V(x_0, p_b) = e_0 + R(n_0 + \bar{n}_0) - r(\bar{n}_0) - \pi_b (v_{2b} - p_b) \frac{n_0 [(1 - p_s)(1 - h) - v_{1s}] - \bar{n}_0 v_{1s} - c_0}{p_b h}
\]

This equation illustrates the marginal investment incentives of the bank. With one marginal unit of its equity \( e_0 \), the bank can purchase \( \frac{1}{h} \) projects funded by short- or long-term debt. Hence, it will prefer short-term to long-term debt if and only if

\[
\frac{\partial V(x_0, p_b)}{\partial n_0} > \frac{\partial V(x_0, p_b)}{\partial \bar{n}_0} \tag{9}
\]

Furthermore, the bank can hold the marginal unit as cash. Hence, it will prefer to invest in a project funded by short-term debt to hoarding cash if and only if

\[
\frac{1}{h} \frac{\partial V(x_0, p_b)}{\partial n_0} > \frac{\partial V(x_0, p_b)}{\partial c_0} \tag{10}
\]

To clarify the welfare analysis, I focus on a case where the bank prefers investing in projects to hoarding cash, and prefers short-term to long-term debt in competitive equilibrium. Therefore, I make an assumption ensuring that inequalities (9) and (10) hold regardless of the asset price \( p_b \). First, I assume that the expected return on projects is high enough to prevent cash hoarding. Second, I assume that the marginal interest payment on long-term debt is high enough to discourage the bank from using long-term debt.
**Assumption 2.** The expected net return on projects $R$ and the marginal interest payment on long-term debt $r'(\bar{n}_0)$ satisfy the following:

\[
R > \pi_b (v_{2b} - p) \frac{1 - p (1 - h) - v_{1b}}{ph}
\]

\[
r'(0) > \pi_b (v_{2b} - p) \frac{(1 - p) (1 - h)}{ph}
\]

### 3.3 The competitive equilibrium

Given Assumption 2, the bank’s profit maximising choice in $t = 0$, regardless of prices, is to hold no cash ($c_0 = 0$), hold no projects funded by long-term debt ($\bar{n}_0 = 0$) and use its entire equity $e_0$ as downpayments against short-term debt, allowing it to invest in $n_0 = e_0/h$ projects.

By Assumption 1, the bank’s equity will be insufficient to cover the cost of rolling over its short-term debt in the bad state at $t = 1$. Hence, it will be forced to sell assets and there will be fire sale prices ($p_b = p$) in equilibrium. The following proposition summarises the competitive equilibrium:

**Proposition 1.** There is a unique competitive equilibrium with $p_g = v_{2g}$ and $p_b = p$. The bank’s $t = 0$ choices in competitive equilibrium are $\mathbf{x}_0 = (0, e_0/h, 0)$.

### 4 Welfare analysis

Recall that ‘the bank’ so far has been representative for a population of small identical banks. In the competitive equilibrium, banks make choices that lead to a fire sale in the bad state. A fire sale reduces the profitability of banks’ investments by making rollover problems costly: If a bank is forced to sell assets due to rollover problems, it now has to do so at a loss, since the market price is below the fair value of the asset.

In sum, aggregate actions in competitive equilibrium lead to a situation with reduced individual profitability. This creates a ‘systemic externality’: The risk-taking of individual banks, by contributing to aggregate risk-taking and low equilibrium prices, reduces the profitability of others. As pointed out by Lorenzoni (2008), systemic externalities are pecuniary externalities because they work through equilibrium prices.

Pecuniary externalities can affect welfare here because financial markets are incomplete. In particular, banks face binding financing constraints if there is a fire sale. Raising the price
would not only transfer utility from outside buyers to banks, but it would also benefit banks by relaxing this constraint. Therefore, price changes have a first order welfare effect. The argument of Greenwald and Stiglitz (1986) on the neutrality of pecuniary externalities breaks down.

Consequently, banks might be better off if they were forced to choose lower aggregate risk-taking by a social planner in order to avoid fire sales. The remainder of this section formalises this idea, and develops a simple graphical tool to aid the welfare analysis. In particular, I argue that if the social planner’s objective is to reduce systemic risk, then reducing leverage and maturity mismatch are complementary tools for doing so in an efficient manner.

4.1 Planned equilibrium

To analyse efficiency, I study the choices of a social planner who can dictate $t = 0$ bank choices $x_0 = (c_0, n_0, \tilde{n}_0)$. The planner is bound by the bank’s budget constraint in equation (2), or $x_0 \in B_0$. He leaves banks to trade in competitive markets at $t = 1$, and his choices induce a planned equilibrium:

**Definition 2.** A planned equilibrium induced by the social planner’s choices $x_0 \in B_0$ is described by asset prices $p_s$ for $s \in \{g, b\}$ and bank choices $x_{1s} \in \mathbb{R}^2$ satisfying the following two criteria:

1. **Optimality.** The bank’s choices maximise $E[\Pi_s]$ subject to $x_{1s} \in B_{1s}(x_0, p_s)$, taking asset prices and the social planner’s choices as given.

2. **Market clearing.** The bank’s choices imply $n_0 - n_{1s} \geq 0$ for all $s \in \{g, b\}$. Furthermore, $p_s \geq \underline{p}$, and if $n_0 - n_{1s} > 0$, then $p_s = \underline{p}$.

There are three groups of agents: Banks, creditors and outside buyers. Creditors and outside buyers are always indifferent between dealing with the bank and consuming their exogenous endowment, so that their utility is the same in any planned equilibrium. Hence, the social planner seeks to maximise expected bank profits in planned equilibrium. His choices are called constrained efficient.

Even though the planner has the same objective as competitive banks, he has different incentives. Crucially, he takes into account that different choices of $x_0$ induce different

---

8The issue of gender equality is not ignored. While the social planner here is male, the regulator in the next section will be female.
equilibrium prices $p_s$. In particular, they determine whether or not there is a fire sale in the bad state ($p_b = p$). Using a parallel argument to Lemma 2, it is easy to see that bank profits in planned equilibrium are given by $V(x_0, p_b)$. The social planner takes into account that $x_0$ has an indirect influence on profits through the second argument of the value function, i.e. prices, as well as a direct influence through the first argument. This is the pecuniary externality discussed above.

One vector of social planner’s choices $x_0$ may induce multiple planned equilibria. Intuitively, this is due to a self-fulfilling debt-deflation spiral, which implies that both high and low asset prices may clear the market: When asset prices are high, banks have no funding problems and do not sell assets. When asset prices are low, banks are forced to sell. To facilitate the analysis, I assume that the market selects the ‘better’ equilibrium without a fire sale.

**Assumption 3.** If the social planner’s choices induce multiple planned equilibria, the one with the highest equilibrium price $p_b$ is selected with probability 1.

### 4.2 Fire sales, leverage and maturity mismatch

To further study the pecuniary externality, it is helpful to derive a ‘no fire sale condition’, i.e. to examine what the social planner would have to do to avoid a fire sale. To do so, he needs to choose the $t = 0$ investments such that fair pricing in all states ($p_s = v_{2s}$) is an equilibrium. This can only be the case if, given fair prices, banks are not forced to make net sales in the bad state. Equation (7) in Section (3) characterises the number of net sales a bank is forced to make in the bad state. In order to avoid a fire sale, the social planner has to ensure that this is not a positive number. Using $p_b = v_{2b}$ and the budget constraint in equation (2) yields the no fire sales condition:

$$n_0 [1 - v_{2b} (1 - h) - v_{1b}] + \tilde{n}_0 (h - v_{1b}) \leq e_0$$

(11)

This argument leads to the following result.\footnote{I assume that the social planner always invests in some projects funded by short-term debt, or $n_0 > 0$. If he set $n_0 = 0$, then there would be no projects for sale at $t = 1$ and the secondary market would effectively shut down. The planned equilibrium price could then be anything. My assumption is without loss of generality because the first marginal project funded by short-term debt never causes a fire sale, and is therefore always profitable for the planner by Assumption 2.}

**Lemma 3.** In planned equilibrium prices satisfy $p_g = v_{2g}$. Furthermore, $p_b = v_{2b}$ if the planner’s choice $x_0$ satisfies (11), and $p_b = p$ otherwise.
Proof. See Appendix A.

The no fire sale condition gives valuable intuition about the relationship between banks’ risk taking and fire sales. In particular, it demonstrates that in order to avoid fire sales, the bank has to avoid high leverage and maturity mismatch.

High leverage is defined as a situation where the bank invests in a lot of projects per unit of its own equity. A measure of leverage in this model is the total number of projects, $n_0 + \tilde{n}_0$. By Assumption 1 and the lower bound on $h$ in equation (6), the left hand side of the no fire sale condition is increasing in both $n_0$ and $\tilde{n}_0$. Therefore, whenever leverage increases, a fire sale becomes more likely.

High maturity mismatch is defined as a situation where the bank funds a high proportion of its investment in long-term projects with short-term debt. Maturity mismatch increases when the bank funds one existing project with short-term debt rather than long-term debt, i.e. when a unit of $\tilde{n}_0$ is replaced with $n_0$. It is easy to show that the weight on $\tilde{n}_0$ on the left hand side of the no fire sale condition is less than the weight on $n_0$. Therefore, high maturity mismatch increases the left hand side and makes a fire sale more likely.

4.3 Inefficiency of competitive equilibrium

Recall that in the competitive equilibrium, the bank invests all its equity in projects funded by short-term debt, setting $n_0 = e_0/h$ and $\tilde{n}_0 = c_0 = 0$. This leads to a fire sale. Of course, if the social planner were to replicate this choice, there would also be a fire sale in planned equilibrium.

In this subsection, I establish a condition under which the social planner would refrain from replicating the competitive equilibrium and choose to avoid a fire sale instead. In other words, I establish when the competitive equilibrium is not constrained efficient, and when pecuniary externalities are welfare-reducing.

Consider an alternative vector of bank choices, denoted $x^A_0$, which features reduced aggregate leverage compared to the competitive equilibrium:

- The bank still refrains from using long-term debt, i.e. $\tilde{n}_0^A = 0$.
- The bank reduces $n_0$ just enough to satisfy the no fire sales condition in equation (11). Since $\tilde{n}_0^A = 0$, the condition implies that $n_0^A = (1 - \eta) n_0$, where

$$\eta = 1 - \frac{h}{1 - v_{2b} (1 - h) - v_{1b}}$$

(12)
If moving from competitive equilibrium to the alternative scenario increases bank profits, then the competitive equilibrium cannot be constrained efficient. There are cost and benefits: As for costs, the bank’s basic profits fall due to the reduction in projects. Projects are reduced by a fraction $\eta$, so the cost is $\eta R$ per project. As for benefits, a fire sale is avoided, so the bank avoids trading losses in the bad state.

Evaluating the expression for net sales in equation (7) at the competitive equilibrium choice, the fraction of projects the bank is forced to sell in the bad state is given by

\[
\left(1 - p\right) \left(1 - h\right) - v_{1b} \frac{1 - p}{ph}
\]

The lost return is $v_{2b} - p$ per unit, and the bad state occurs with probability $\pi_b$. Hence, the expected benefit per project of avoiding the fire sale is

\[
\pi_b \left(v_{2b} - p\right) \left(1 - p\right) \left(1 - h\right) - v_{1b} \frac{1 - p}{ph}
\]

The alternative choice is better than the competitive equilibrium if benefits outweigh costs:

**Proposition 2.** Suppose the expected net return $R$ satisfies

\[
\eta R < \pi_b \left(v_{2b} - p\right) \left(1 - p\right) \left(1 - h\right) - v_{1b} \frac{1 - p}{ph}
\]

Then bank choices in competitive equilibrium are not constrained efficient.

**Proof.** See Appendix A.

The first inequality in Assumption 2 places a lower bound on $R$, whereas the condition in the previous proposition is an upper bound. In Appendix C, I demonstrate that these bounds are consistent with each other.

### 4.4 Characterisation of constrained efficiency

I have established a condition under which the social planner can do better than to replicate the competitive equilibrium. Now, I characterise the social planner’s choices under this condition.
If the competitive equilibrium is not constrained efficient, then the social planner will choose to prevent a fire sale in the bad state by satisfying the no fire sales condition in Lemma 3. This is because at the competitive equilibrium choice, bank profits are already maximised given fire sale prices (by the optimality part of Definition 1). Thus, any alternative choice that induces a fire sale yields lower profits than the competitive equilibrium, and will never be chosen by the planner.

Given that the planner prevents a fire sale, bank profits in planned equilibrium will be \( V(x_0, v_{2b}) \). The planner’s maximisation problem then consists of maximising \( V(x_0, v_{2b}) \) subject to the no fire sale condition from Lemma 3.

\[
V(x_0, v_{2b}) = e_0 + R(n_0 + \bar{n}_0) - r(\bar{n}_0)
\]  

(13)

The social planner faces two basic trade-offs when trying to avoid fire sales: First, he can reduce leverage by investing in cash instead of projects. Second, he can reduce maturity mismatch by substituting long-term debt for short-term debt.

The first method is costly because it reduces investment in projects. The marginal cost of reducing leverage is \( R \), the net present value of a project. This second method is costly because long-term debt requires an interest payment of \( r(\bar{n}_0) \), which reflects the social cost of reduced maturity transformation. The marginal cost of reducing maturity mismatch is \( r' \), the marginal cost of long-term debt. The social planner chooses the second method if \( R \) is large relative to \( r' \).

The existence of two trade-offs illustrates the complementarity of reductions in leverage and reductions in maturity mismatch. Due to convexities in the cost of bank funding, the socially cheapest way of satisfying the no fire sale condition is likely to be a combination of those two strategies.

A simple variational argument illustrates the exact trade-off. Recall that the planner’s choice maximises \( V(x_0, v_{2b}) \) subject to the no fire sale condition. Suppose the planner uses no long-term debt, with \( \bar{n}_0 = 0 \), and consider the following deviation: Invest in \( \epsilon > 0 \) projects funded by long-term debt, and reduce projects funded by short-term debt \( (n_0) \) by just enough to still satisfy the no fire sale condition. Since long-term debt causes less funding problems than short-term debt, the required reduction in \( n_0 \) is less than \( \epsilon \). In particular, equation (11) implies that the total investment in projects increases by

\[
\epsilon \left( 1 - \frac{h - v_{1b}}{1 - v_{2b} (1 - h) - v_{1b}} \right)
\]
Using the expression for profits in (13) the marginal gain from this deviation for small \( \epsilon \) is given by

\[
\left(1 - \frac{h - v_{1b}}{1 - v_{2b}(1 - h) - v_{1b}}\right) R - r'(0)
\]

Increasing \( \bar{n}_0^E \) is optimal if and only if the gain from this deviation is strictly positive. The following result formalises this idea:

**Proposition 3.** Suppose the competitive equilibrium is not constrained efficient. Then there exists a unique constrained efficient choice \( x_0^E \). This choice uses long-term debt \( (\bar{n}_0^E > 0) \) if and only if

\[
\frac{R}{r'(0)} > 1 - \frac{h - v_{1b}}{1 - v_{2b}(1 - h) - v_{1b}}
\]

**Proof.** See Appendix A.

The condition in the previous proposition imposes a lower bound on the ratio of \( R \) to \( r'(0) \). Assumption 2 and the condition in Proposition 2 taken together impose an upper bound on this ratio. Appendix C verifies that the two bounds are consistent with each other.

### 4.5 A graphical illustration

Figure 1 illustrates the welfare analysis. The social planner dictates \( t = 0 \) bank investments \( x_0 = (c_0, n_0, \bar{n}_0) \). Given the project investments \( n_0 \) and \( \bar{n}_0 \), cash holdings are determined by the budget constraint in equation (2). There are two free choice variables, \( n_0 \) and \( \bar{n}_0 \). The feasible set, depicted as the area under the budget line, is implied by the non-negativity constraint on cash holdings:

\[
n_0 \leq \frac{e_0}{h} - \bar{n}_0
\]

The budget line is the set of combinations where this constraint holds with equality. Hence, its slope is \(-1\).

The dashed line illustrates the no fire sale condition from equation (11). The social planner needs to choose a point on or below the line in order to avoid a fire sale. The slope of the line is flatter than the budget line. This is because substituting one unit of short-term debt for one unit of long-term debt (increasing maturity mismatch) makes the condition less likely to hold. Furthermore, the intercept of the line is below the intercept of the budget line. If the
top left corner of the budget line is chosen, a fire sale is inevitable by virtue of Assumption 1.

Leverage and maturity mismatch can be visualised in the figure. Leverage corresponds to a high total number of projects funded by debt, which is \( n_0 + \bar{n}_0 \). Thus, leverage is high in the north-east of the figure, when the choice is close to the budget line. Maturity mismatch corresponds to a high ratio of short-term debt to total debt, which is \( n_0 / (n_0 + \bar{n}_0) \). Thus, maturity mismatch is high in the north-west of the figure.

By Proposition 1, the competitive equilibrium is at the top left corner of the feasible set, point \( C \). Banks choose maximal leverage and maturity mismatch in competitive equilibrium.

Suppose that the competitive equilibrium is not constrained efficient. Then the social planner maximises \( V(x_0, v_{2b}) \) subject to the no fire sales condition. The isoprofit curves are level curves of the function \( V(x_0, v_{2b}) \). By totally differentiating equation (13), it is easy to show that their slope is given by

\[
\frac{d n_0}{d \bar{n}_0} = -\frac{R - r' (\bar{n}_0)}{R} \tag{14}
\]

The constrained efficient point is at point \( E \), where the isoprofit curves are tangent to the no fire sale condition. This point features both reduced leverage and reduced maturity mismatch compared to the competitive equilibrium. It is important to note that the competitive equilibrium point \( C \) yields a lower profit than \( E \), although it lies above the isoprofit line.
through $E$. The isoprofit curves are drawn assuming that the no fire sale condition is satisfied. Profits at $C$ are lower because a fire sale occurs, making bank investments less profitable.

Of course, the location of point $E$ is due to how I have drawn the isoprofit curves. If they were very flat, then the constrained efficient point would be at point $A$, with $\bar{n}_0 = 0$. From the expression for the slope of isoprofit curves in (14), they are flat when the return on projects $R$ is high relative to the marginal cost of long-term debt $r'$. In drawing them steep enough to ensure an interior optimum, I have assumed that $R/r'$ satisfies the inequality in Proposition 3.

Point $A$ has another interpretation: It was the alternative choice, featuring only reduced leverage, that was considered when deriving the condition for inefficiency of competitive equilibrium. The inequality in Proposition 2 holds if and only if bank profits are higher at $A$ than in competitive equilibrium.

4.6 The role of fixed equity

This paper assumes that the bank cannot raise outside equity at any time. At $t = 0$, it only has its own equity endowment $e_0$, and at $t = 1$ its free equity is $e_1$. Neither can be supplemented with outside equity issuance. The assumption for $t = 1$ seems relatively innocuous. It is generally accepted that banks struggle to raise outside equity in times of crisis, i.e. in the bad state at $t = 1$, for example due to time pressure or debt overhang problems (see Hanson et al 2011, Admati et al 2012). Furthermore, there are no funding problems in the good state at $t = 1$, so that introducing outside equity would not change the equilibrium in this state.

The assumption that equity is fixed ex ante, i.e. at $t = 0$, is less obvious. Hence, it is important to discuss how relaxing this assumption would affect my results. I argue that it would change the nature of the social planner’s trade-offs, whilst preserving the key result that reductions in leverage and maturity mismatch are complementary tools.

Reducing leverage in my model means reducing the number of risky projects. If banks were able to raise outside equity, that would no longer be the case: They could reduce leverage by holding the number of risky projects constant while replacing debt funding with outside equity funding. Hence, the marginal cost of reducing leverage would no longer be $R$, the return on projects, but it would be related to the spread between the cost of outside equity and debt funding.

Due to the risk-aversion of creditors, it is reasonable to assume that the aggregate surplus from bank funding would be reduced when banks issue equity instead of debt: Banks would
have to pay a higher expected return on equity than on debt to compensate creditors for equity risk. In other words, there would still be a positive social marginal cost to reducing leverage, because it prevents banks from performing socially valuable risk transformation.

Consequently, even in a setting where banks can raise outside equity at $t = 0$, there would be two social trade-offs. The social planner, when trying to satisfy the no fire sale condition, would be able to reduce leverage, which would involve using socially costly outside equity, or reduce maturity mismatch, which would involve using socially costly long-term debt. It is likely that the socially cheapest way of satisfying the condition would be to use a combination of the two strategies. Therefore, my result on the complementarity of leverage and maturity mismatch reductions would continue to apply in this context.

5 Optimal regulation

The previous section establishes that the competitive equilibrium of this model may not be constrained efficient. Throughout this section, I assume that this is the case, i.e. that the inequality in Proposition 2 holds.

The inefficiency is caused by a pecuniary externality. It would be in the banks’ interest to commit to reduce leverage and maturity mismatch to avoid fire sales. However, in competitive equilibrium, prices are taken as given, and no bank has an incentive to take unilateral action. This creates a case for intervention by a social planner.

In reality, there is no social planner who can dictate bank choices. Instead, there are regulators who can put simple linear constraints on bank choices and have limited information about the funding costs of banks. This section characterises the optimal choices of such a regulator.\footnote{Studying a regulator with perfect information is not illuminating. With perfect information about all parameters of the model, a regulator can keep imposing binding constraints until she essentially dictates banks' choices. Hence, her problem is equivalent to the social planner’s problem in Section 4.} The regulator would like the constrained choices of banks to resemble the choices of a social planner. Intuitively, her objective is to constrain banks just enough to tackle externalities, while giving them the freedom and the incentives to use their information about funding costs in a socially efficient manner.

5.1 Information and regulated equilibrium

Information. The regulator faces uncertainty about the relative cost of long-term debt, which is captured by the function $r(\bar{n}_0)$. In particular, suppose that $r(\bar{n}_0) = \theta \hat{r}(\bar{n}_0)$, where $\theta$
is a random variable, which is observed by the banks but not by the regulator. \( \theta \) has support \( \Theta = [\underline{\theta}, \bar{\theta}] \), where \( 0 < \theta < \bar{\theta} \), probability distribution function \( F(\theta) \), and it is independent of the macroeconomic state \( s \). I assume that \( r(\bar{n}_0) \) satisfies Assumption 2 for all \( \theta \in \Theta \).

**Regulatory regimes.** The regulator can impose a linear constraint on banks’ choices at \( t = 0 \), requiring that choices satisfy \( Bx_0 \leq a \), where \( A \) is a \( k \times 3 \) matrix and \( a \) is a \( k \times 1 \) vector. \( k \) is the number of regulatory constraints. A **regulatory regime** is therefore a matrix \( Q = (B, a) \). The set of feasible bank choices satisfying the regulatory constraints in regime \( Q \) is defined as

\[
J(Q) = \{ x_0 \in B_0 | B'x_0 \leq a \}
\]

A regulatory regime \( Q \) is called **plausible** if it is possible for banks to satisfy the constraint, i.e. if \( J(Q) \neq \emptyset \). The regulator can set as many linear constraints as she wants, i.e. choose any \( k \geq 0 \), but incurs a cost of \( \delta > 0 \) per constraint imposed. \( \delta \) can be arbitrarily small - it is imposed to rule out redundant constraints as part of optimal policies. Let \( Q \) denote the set of all plausible regulatory regimes.\(^{11}\)

**Regulated equilibrium.** Every plausible regulatory regime induces a regulated equilibrium. The definition of a regulated equilibrium is identical to a competitive equilibrium (Definition 1), except for the fact that bank choices maximise expected profits subject to \( x_0 \in J(Q) \) instead of \( x_0 \in B_0 \). As in the analysis of planned equilibria, I abstract from multiple equilibria:

**Assumption 4.** If a regulatory regime induces multiple regulated equilibria, the one with the highest equilibrium price \( p_b \) is selected with probability 1.

This assumption ensures that each regulatory regime leads to a unique regulated equilibrium, contingent on the realisation of \( \theta \). Let \( x_0(Q, \theta) \) denote banks’ choices at \( t = 0 \) in the regulated equilibrium induced by \( Q \), and \( p_b(Q, \theta) \) the equilibrium price in the bad state.

**Efficiency.** Let \( x_0^E(\theta) \) denote the efficient choice contingent on \( \theta \), i.e. the choice of a social planner who knows \( \theta \). To keep things interesting, I assume that the social planner’s choice is indeed affected by \( \theta \):

**Assumption 5.** The efficient choices contingent on \( \theta \) satisfy \( x_0^E(\underline{\theta}) \neq x_0^E(\bar{\theta}) \).

**Optimal regulation.** Let \( V_\theta(x_0, p_b) \) denote the banks’ expected profit function from Lemma 2 for any given \( \theta \). A regulatory regime is **optimal** if it maximises *ex ante* expected value of the banks' profit function.
profits in regulated equilibrium net of regulation costs, i.e. if it solves the problem

$$\max_{Q \in \mathcal{Q}} \int_{\hat{\Theta}} V_\theta (x_0 (Q, \theta), p_b (Q, \theta)) dF (\theta) - k \delta$$

Suppose a regulatory regime achieves the choices of a social planner who knows $\theta$, i.e.

$$x_0 (Q, \theta) = x_0^E (\theta) \ \text{for all } \theta$$

Then the regime $Q$ is called fully efficient. Since the regulator can never do better than a perfectly informed social planner, any fully efficient policy must also be optimal.

Optimal policies are never truly unique. For instance, if all components of the regulatory constraint ($B$ and $a$) are simply scaled by the same positive factor, the regulated equilibrium is unaffected, since banks face exactly the same constraints as before. An optimal policy $q$ will be called essentially unique if all other optimal policies $\hat{Q}$ replicate the constraint imposed by $Q$ (i.e. $J (Q) = J (\hat{Q})$).

### 5.2 Characterisation of optimal regulation

It is intuitive that the regulator might impose a constraint which mimics the no fire sale condition derived above. To see this, recall that the only wedge between private and social incentives in this model is the pecuniary externality arising from potential fire sales. Therefore, if the regulator guarantees that there is no fire sale, banks face the same trade-offs as the social planner in Section 4: They choose the least costly way to satisfy the no fire sales condition by reducing leverage and maturity mismatch.

In terms of the graphical analysis in Figure 1, if the no fire sale condition is imposed on banks as a constraint, their privately optimal choice will be at the point of tangency between the condition and their isoprofit lines. This coincides with the constrained efficient point $E$, yielding an efficient allocation in regulated equilibrium.

It turns out that uncertain funding costs do not render this policy ineffective. To see this consider the no fire sale condition from equation 11. The condition only depends on bank equity $e_0$, the asset cash flows $v_{ts}$ and the haircut $h$. It is not affected by the cost of funds, and therefore, it is independent of $\theta$. Although $\theta$ is unknown, the regulator can still mimic the no fire sale condition, thus aligning private and social incentives. The regulatory regime which mimics the no fire sale condition, denoted $Q^* = (B^*, a^*)$, only involves one linear
constraint:
\[
\begin{align*}
B^* &= (0, 1 - v_{2b}(1 - h) - v_{1b}, h - v_{1b}) \\
a^* &= e_0
\end{align*}
\] (15)

Given regime \( Q^* \), banks will choose the socially optimal funding mix, even though the regulator does not this mix herself. If \( \theta \) is high, then long-term debt is expensive, and banks find it cheapest to reduce leverage rather than maturity mismatch in order to satisfy the regulatory constraint (towards point \( A \) in Figure 1). If \( \theta \) is low, banks choose to reduce maturity mismatch and leave leverage at a higher level. In other words, the constraint which replicates the no fire sale condition gives banks the incentives to use their private information in a socially efficient way.

No other regulatory regime has the power to align private and social incentives regardless of the realisation of the uncertain cost function. Any alternative regulatory regime would lead banks to pick the ‘wrong’ funding mix for some realisation of \( \theta \). The following proposition formalises the arguments in this subsection.

**Proposition 4.** Suppose the competitive equilibrium is not constrained efficient. Then the regulatory regime \( Q^* \) defined in equation (15) is optimal, fully efficient and essentially unique.

**Proof.** See Appendix A. \( \square \)

### 5.3 Inefficiency of capital requirements

The characterisation of optimal policy in Proposition 4 implies that that Capital Adequacy Requirements (CAR), which were the principal tools of the first and second Basel Accords, cannot be optimal in a setting with systemic externalities. CAR place a lower bound \( \kappa > 0 \) on the ratio of a bank’s equity (or close substitutes) to its risk-weighted assets. In other words, it places an upper bound on a (risk-weighted) measure of bank leverage. In the notation of my model, the regulatory constraint imposed by a CAR is

\[
\frac{e_0}{\omega(n_0 + \bar{n}_0)} \geq \kappa
\]

where \( \omega > 0 \) is the risk-weight on long-term projects. In the notation of this section, a CAR
can be written as a linear constraint $Q_C = (B_C, a_C)$, where

$$
B_C = (0, \kappa \omega, \kappa \omega)
$$

$$
a_C = e_0
$$

(16)

Note that in the CAR, all projects backed by debt ($n_0$ and $\bar{n}_0$) are treated equivalently, regardless of whether they are backed by long-term debt. In other words, A CAR constrains only leverage, not maturity mismatch. In contrast, Proposition 4 states that the optimal policy replicates the no fire sale condition. From equation (11), it is evident that the no fire sale condition attaches different weights to projects funded by short-term and long-term debt. Therefore, the optimal policy calls for differential treatment for short- and long-term debt. This can never be achieved by a CAR. Hence, the proposition implies the following:

**Corollary.** There exist no parameters $\kappa$ and $\omega$ such that $Q_C$ as defined in equation (16) is an optimal policy.

**Proof.** For all $\kappa$ and $\omega$, $J(Q_C) \neq J(Q^\star)$. Since the optimal policy $Q^\star$ is essentially unique, $Q_C$ cannot be optimal. \qed

To understand the shortcomings of a CAR intuitively, recall the two trade-offs discussed in Section 4. A regulator can reduce systemic risk by constraining either leverage, or maturity mismatch, or both. Constraining leverage is costly because it reduces aggregate investment. Constraining maturity mismatch is costly because it prevents banks from performing socially valuable maturity transformation. The cheapest way to curb systemic risk is often to exploit both trade-offs, which was shown in Proposition 3.

With a CAR, the regulator can only exploit the first of the trade-offs, since she only has the power to constrain leverage. Because she has to be aggressive on leverage to curb systemic risk, it will be excessively costly to do so. Even though she may be able to achieve a better outcome than the competitive equilibrium, there will still be inefficiencies.

Figure 2 illustrates. The setup of the figure is identical to Figure 1, which was described in the previous section. A CAR (with $\kappa \omega > h$) achieves a parallel downward shift of the bank’s budget constraint. The best the regulator can do with a CAR is to set $\kappa \omega$ just high enough to prevent a fire sale. Hence, she ensures that the intercepts of the no fire sale condition and the CAR coincide.

The bank’s choice in regulated equilibrium is point R, where it funds itself exclusively with short-term debt, but holds some cash. Its leverage is reduced compared to the competitive equilibrium, but its maturity mismatch is still at the maximum level.
The regulated equilibrium with the optimal CAR is still not constrained efficient. Strictly higher welfare is achieved at point E, where the banks’ isopriefit curves and the no fire sales condition are tangent. This point has more bank lending than the regulated equilibrium, but less maturity mismatch. The regulator is not able to incentivise banks to choose E because she cannot restrict maturity mismatch with a CAR.

6 Optimal regulation with Basel III tools

The previous section showed that in my model, the optimal way of regulating banks was to impose one linear constraint on their balance sheets which replicated the no fire sale condition derived in Section 4. This section discusses how optimal regulation can be implemented within the framework of the Basel III Accord (BIS 2011).

I argue that the Basel III Net Stable Funding Ratio (NSFR) can be used to achieve a policy that is equivalent to the optimal policy I have derived. Furthermore, I use my model to establish some guidelines for NSFR design. The NSFR works as follows:

1. *Available stable funding* is calculated as a weighted sum of bank liabilities. Liabilities which are likely to exacerbate funding problems are assigned low weights.
2. *Required stable funding* is calculated as a weighted sum of bank assets. Assets which are likely to exacerbate funding problems are assigned low weights.

3. The *NSFR* is calculated as the ratio of available to required stable funding. The regulatory constraint imposed on banks is

\[
NSFR = \frac{\text{Available stable funding}}{\text{Required stable funding}} \geq 1
\]

The no fire sale condition in equation (11), which optimal policy needs to replicate, can be interpreted as a NSFR condition. Rearranging terms slightly, the condition is

\[
\frac{e_0 + (1 - v_{2b})(1 - h) \bar{n}_0}{[1 - v_{2b}(1 - h) - v_{1b}](n_0 + \bar{n}_0)} \geq 1
\]

The numerator of this above can be interpreted as *available stable funding*. The liabilities side of banks’ balance sheets at \(t = 0\) consists of equity \(e_0\), outstanding short-term debt \((1 - h)n_0\) and outstanding long-term debt \((1 - h)\bar{n}_0\). The numerator of (17) can be re-written as a weighted sum of these liabilities:

\[
ASF = 0 \times \text{s.t. debt} + (1 - v_{2b}) \times \text{l.t. debt} + 1 \times \text{equity}
\]

It follows that in order to implement the optimal policy with a NSFR, the available stable funding weights should be designed along the following lines: *Short-term debt* receives a zero weight because has to be rolled over at \(t = 1\) and exacerbates funding problems. *Equity* is the most stable source of funding as it provides a cushion against losses, and therefore receives a weight of 100%. *Long-term debt*, which matches the maturity of assets, is an intermediate case. It does not have to be rolled over, but it does oblige the bank to pledge assets as long-term collateral, reducing its liquidity in a crisis. Hence, its weight is between zero and 100%. Moreover, if the market value of the pledged collateral is high, then so is the loss in liquidity. Hence, the weight of long-term debt is decreasing in \(v_{2b}\), which measures the value of the collateral in equilibrium (since \(p_b = v_{2b}\) in the absence of a fire sale).

The denominator of the ratio can be interpreted as *required stable funding*. The asset side of banks’ balance sheets at \(t = 0\) consists of \(c_0\) units of cash and \(n_0 + \bar{n}_0\) long-term projects. The denominator of (17) can be re-written as a weighted sum of assets:

\[
RSF = 0 \times \text{cash} + [1 - v_{2b}(1 - h) - v_{1b}] \times \text{projects}
\]
This implies the following guidelines for the optimal design of required stable funding weights: 

*Cash* receives a zero weight because it is perfectly liquid and risk-free. *Projects*, on the other hand, have a positive weight because they expose the bank to funding problems at \( t = 1 \). This weight is high when projects present high cash flow risk, i.e. when the cash flows in the bad state (\( v_{1b} \) and \( v_{2b} \)) are low. However, cash flow risk is not the only important metric: Holding cash flow risk constant, the weight is raised when funding conditions are tight (high \( h \)). Intuitively, tight funding conditions reduce a project’s value as collateral and therefore its power to mitigate funding problems.

The analysis in this section shows that a NSFR can deal with systemic externalities in an efficient way. The previous section showed that capital adequacy requirements are unable to do so. I do not mean to imply that capital requirements ought to be abolished. They serve a different purpose, which is to limit with the external costs of bank failure, for instance externalities caused by deposit insurance. However, if a regulator explicitly wishes to target systemic risk, then introducing a NSFR is a superior strategy to simply making capital requirements stricter.

### 7 Conclusion

This paper has analysed the joint use of capital requirements and maturity-based tools in financial regulation. My model focuses on systemic risk created by bank leverage and maturity mismatch. Risk-taking is excessive due to pecuniary externalities. Optimal regulatory policy satisfies two criteria. First, it should force banks to reduce systemic risk to the socially efficient level. Second, it should give banks the freedom to choose an efficient funding structure based on their private information about the cost of funds. I proved that both goals can be achieved with a relatively simple linear constraint on banks’ balance sheets. Capital Adequacy Requirements can theoretically be used to curb systemic risk, but they are a blunt tool, since they prevent banks from choosing an efficient funding mix, thus putting excessive constraints on investment.

Turning to the practical implementation of optimal policy, I demonstrated that it can be achieved within the Basel III framework if the Accord’s Net Stable Funding Ratio requirement is correctly calibrated. This suggests that the NSFR is a powerful tool if regulators are concerned about systemic risk externalities of the kind described in this paper.
A Proofs

Lemma 1

**Lemma.** In any competitive equilibrium, asset prices satisfy $p_g = v_{2g}$ and

$$p \leq p_b \leq v_{2b}$$

**Proof.** To start with, I show that in any competitive equilibrium, $p_s \leq v_{2s}$ for all $s \in \{g, b\}$. Suppose $p_s > v_{2s}$. Then the bank optimally sells all projects at $t = 1$ in state $s$, $n_{1s} = 0$. By market clearing, it follows that $n_0 = 0$. Now consider the other state $s' \neq s$. Then $p_{s'} \geq v_{2s'}$, because otherwise, the bank would choose $n_{1s'} > 0$ and the market would not clear. Hence $p_s \geq v_{2s}$ for all $s \in \{g, b\}$. But then it is easy to show that the bank could increase its profits by setting $n_0 > 0$, contradicting optimality.

Next, I show that $p_g = v_{2g}$. Suppose that $v_{2g} > p_g \geq p_{\bar{g}}$. Then it is optimal for the bank to buy as many projects as possible, setting $n_{1g} = e_{1g}/p_g h$. Market clearing is violated, since by the expression for $e_{1g}$ in equation (3),

$$n_0 - n_{1g} = -\frac{n_0 [v_{1g} + p_g - (1-h) + p_g h] + \bar{n}_0 v_{1g} + c_0}{p_g h}$$

where the last inequality follows from Assumption 1 and $c_0, \bar{n}_0 \geq 0$.

Lemma 3

Lemma 4. In planned equilibrium prices satisfy $p_g = v_{2g}$. Furthermore, $p_b = v_{2b}$ if the planner’s choice $x_0$ satisfies (11), and $p_b = p_{\bar{g}}$ otherwise.

**Proof.** To start with, I show that in any planned equilibrium, $p_s \leq v_{2s}$ for all $s \in \{g, b\}$. If $p_s > v_{2s}$, then the bank would set $n_{1s} = 0$, implying $n_0 - n_{1s} > 0$, which violates market clearing. The proof that $p_g = v_{2g}$ is identical to the argument in Lemma 1.

Next, suppose the planner’s choices satisfy (11). I show that $p_b = v_{2b}$. At this price, banks are indifferent between all feasible $t = 1$ investments, so it is sufficient that $n_{1b} = n_0$ is feasible. This is the case if and only if the minimum net sale in Equation (7) is non-positive for $s = b$. From the budget constraint (2), this is equivalent to (11) by construction.
Finally, suppose the planner’s choices do not satisfy (11). Then, the minimum net sale in Equation (7) is positive for $s = b$ when $p_b = v_{2b}$. Since the expression is decreasing in $p_b$, it is also positive for all $p_b < v_{2b}$. This implies $p_b = p$ by the definition of planned equilibrium. \hfill \square

**Proposition 2**

**Proposition.** Suppose the expected net return $R$ satisfies

$$\eta R < \pi_b \left( v_{2b} - p \right) \frac{(1 - p)(1 - h) - v_{1b}}{ph}$$

Then bank choices in competitive equilibrium are not constrained efficient.

**Proof.** Suppose $R$ satisfies the proposed condition. Consider the competitive equilibrium choice $x^C_0$ and the alternative allocation $x^A_0$ described in the text. If the planner chooses $x^A_0$, then by Lemma 3, the price in the bad state is $p_b = v_{2b}$ in planned equilibrium. Welfare is

$$V \left( x^A_0, v_{2b} \right) = e_0 + R \left( 1 - \eta \right) n^C_0$$

If the planner chooses $x^C_0$, then by Lemma 3, the price in the bad state is $p_b = p$ in planned equilibrium. Welfare is

$$V \left( x^C_0, p \right) = e_0 + \left[ R - \pi_b \left( v_{2b} - p \right) \frac{(1 - p)(1 - h) - v_{1b}}{ph} \right] n^C_0$$

The proposed condition is equivalent to $V \left( x^A_0, v_{2b} \right) < V \left( x^C_0, p \right)$, which completes the proof. \hfill \square

**Proposition 3**

**Proposition.** Suppose the competitive equilibrium is not constrained efficient. Then there exists a unique constrained efficient choice $x^E_0$. This choice uses long-term debt ($\bar{n}^E_0 > 0$) if and only if

$$\frac{R}{r^*(0)} > 1 - \frac{h - v_{1b}}{1 - v_{2b}(1 - h) - v_{1b}}$$
Proof. First, I show that no choice \( x'_0 \in B_0 \) violating the no fire sale condition can be constrained efficient. Such a \( x'_0 \) induces \( p_b = p \) in planned equilibrium. Bank profits in planned equilibrium are then given by

\[
V(x'_0, p) \leq \max_{x_0 \in B_0} V(x_0, p) = V(x_C^0, p)
\]

where the equality follows from the optimality part of the definition of competitive equilibrium. Since \( x_C^0 \) is not efficient by assumption, neither is \( x'_0 \).

Hence, any constrained efficient choice satisfies the no fire sale condition. Let \( N_0 = p - 1(v_{2b}) \) denote the set of all choices \( x_0 \) that satisfy the no fire sale condition. A choice is then constrained efficient if and only if it is a maximiser of the problem

\[
\max_{x_0 \in B_0 \cap N_0} V(x_0, v_{2b}) = \max_{x_0 \in B_0 \cap N_0} R(n_0 + \bar{n}_0) - r(\bar{n}_0)
\]

Since \( B_0 \cap N_0 \) is a compact subset of \( \mathbb{R}_+^3 \), the objective function achieves its maximum in the feasible set, confirming existence of a constrained efficient choice. Suppose there exist two such choices, \( x^E_0 \) and \( x^F_0 \), with \( x^E_0 \neq x^F_0 \). Since the objective function is strictly concave in \( \bar{n}_0 \), this can only be the case if \( \bar{n}_0^E = \bar{n}_0^F = 0 \). Optimality then implies that \( Rn^E_0 = Rn^F_0 \).

This means that \( n^E_0 = n^F_0 \) and, by the budget constraint, \( c^E_0 = c_0 - hn^E_0 = c^F_0 \). Hence, we have \( x^E_0 = x^F_0 \), a contradiction, which establishes uniqueness.

Second, I prove the statement about long-term debt. For sufficiency, suppose \( \bar{n}_0^E = 0 \). Consider an alternative investment \( x^\epsilon_0 \) which increases \( \bar{n}_0^E \) to \( \epsilon > 0 \) and decreases \( n_0 \) by \( \beta \epsilon \) units, i.e. \( x^\epsilon_0 = (c^E_0, n^E_0 - \beta \epsilon, \epsilon) \), where

\[
\beta = \frac{h - v_{1b}}{1 - v_{2b}(1 - h) - v_{2b}}
\]

It is easy to see that \( x^\epsilon_0 \in B_0 \cap N_0 \) for small enough \( \epsilon \). Constrained efficiency of \( x^E_0 \) implies that

\[
\left. \frac{dV(x_0, v_{2b})}{d\epsilon} \right|_{\epsilon=0} = \left. \frac{d}{d\epsilon} \left[ R(n^E_0 - \beta \epsilon + \epsilon) - r(\epsilon) \right] \right|_{\epsilon=0} = (1 - \beta) R - r'(0) \leq 0
\]

For necessity, suppose \( \bar{n}_0^E > 0 \). Consider an alternative investment denoted \( x^\epsilon_0 \) which decreases \( \bar{n}_0^E \) by \( \epsilon > 0 \) and increases \( n_0 \) by \( \beta \epsilon \) units, i.e. \( x^\epsilon_0 = (c_0^E, n_0^E + \beta \epsilon, \bar{n}_0^E - \epsilon) \). It is easy
to see that $x_0^E \in B_0 \cap N_0$. Constrained efficiency of $x_0^E$ implies that
\[
\frac{d}{d\epsilon} V(x_0^\epsilon, v_{2b}) \bigg|_{\epsilon=0} = \frac{d}{d\epsilon} \left[ R \left( n_0^E + \beta \epsilon + \bar{n}_0^E - \epsilon \right) - r \left( \bar{n}_0^E - \epsilon \right) \right] \bigg|_{\epsilon=0} = -\left(1 - \beta\right) R + r'\left( \bar{n}_0^E \right) \leq 0
\]
Since $r$ is strictly convex, $r'\left( \bar{n}_0^E \right) > r'\left( 0 \right)$, and therefore $(1 - \beta) R > r'\left( \bar{n}_0 \right)$.

\[\square\]

**Proposition 4**

**Proposition.** Suppose the competitive equilibrium is not constrained efficient. Then the regulatory regime $Q^*$ defined in equation (15) is optimal, fully efficient and essentially unique.

**Proof.** (i) Optimality and efficiency. The regulatory constraint imposed by $Q^*$ is the no fire sale condition in equation (11). The regulated equilibrium prices must therefore satisfy $p_s(\theta) = v_{2s}$ for all $\theta$ and $s$. Banks’ choices in regulated equilibrium solve the problem
\[
\max_{x_0 \in B_0} V_\theta (x_0, v_{2b}) \text{ subject to (11)}
\]
But, as discussed in Section 4, this is equivalent to the problem of a social planner who knows $\theta$. Hence, the regulatory regime is fully efficient.

(ii) Essential uniqueness. Since full efficiency can be achieved with one constraint, it can never be optimal to set $k > 1$. Take any optimal policy with $k = 1$, denoted $\hat{Q} = (\hat{B}, \hat{a}) \in Q$. I show that $J(\hat{Q}) = J(Q^*)$. Since policy $Q^*$ is fully efficient, $\hat{Q}$ must also be fully efficient in order to be optimal, so that $x_0(\hat{Q}, \tilde{\theta}) = x_0^E(\theta)$ for almost all $\theta$. This implies that it must not be feasible for banks to make the same choice as in competitive equilibrium, or $x_0^C \notin J(Q)$. Since unconstrained banks would choose $x_0^C$ (by Assumption 2), it follows that the regulatory constraint must bind for almost all $\theta$:
\[
\hat{B}x_0(Q, \theta) = \hat{B}x_0^E(\theta) = \hat{a}
\]
Since $x_0^E(\theta) \neq x_0^E(\tilde{\theta})$ we can find $\theta$ and $\theta'$ satisfying the previous equation and $x_0^E(\theta) \neq x_0^E(\theta')$. Letting $z = \left[ x_0^E(\theta) - x_0^E(\theta') \right]$, we have $\hat{B}z = 0$.

Efficient choices satisfy the no fire sale condition with equality, which implies $B^*z = 0$. Furthermore, letting $v = (1, h, h)$, the budget constraint in equation 2 implies that $vz = 0$. 

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Since \( z \) is a non-zero \( 3 \times 1 \) vector, there can only be two linear independent vectors which are orthogonal to it. \( \mathbf{B}^* \) and \( \mathbf{v} \) are orthogonal to \( z \) and linear independent. It follows that \( \hat{\mathbf{B}} \), which is also orthogonal to \( z \), can be expressed as a linear combination of \( \mathbf{B}^* \) and \( \mathbf{v} \):

\[
\hat{\mathbf{B}} = k_1 \mathbf{B}^* + k_2 \mathbf{v}
\]

for two scalars \( k_1 \) and \( k_2 \). Using the equation above and the fact that \( \mathbf{v}' \mathbf{x}^E_0 (\theta) = e_0 \) (by the budget constraint),

\[
\hat{a} = \hat{\mathbf{B}} \mathbf{x}^E_0 (\theta) = k_1 a^* + k_2 e_0
\]

The set of feasible choices under policy \( \hat{Q} \) is

\[
J (\hat{Q}) = \{ \mathbf{x}_0 \in B_0 | k_1 \mathbf{B}^* \mathbf{x} + k_2 \mathbf{v} \mathbf{x}_0 \leq k_1 b^* + k_2 e_0 \} = \{ \mathbf{x}_0 \in B_0 | k_1 \mathbf{B}^* \mathbf{x}_0 \leq k_1 a^* \}
\]

To establish that \( J (\hat{Q}) = J (Q^*) \), it is now sufficient to show that \( k_1 > 0 \). Recall that the competitive equilibrium choice violates the no fire sale condition, or \( \mathbf{B}^* \mathbf{x}_0^C > a^* \). If \( k_1 \leq 0 \), then \( \mathbf{x}_0^C \in J (\hat{Q}) \), contradicting full efficiency.

\[\square\]

**B  Microfoundations of the cost of debt**

In Section 2, I assume that a bank who borrows \( L \) units of the consumption good using long-term debt incurs an interest payment of \( \tilde{r} (L) \), where \( \tilde{r} : \mathbb{R}_+ \to \mathbb{R}_+ \) is differentiable, increasing and strictly convex, with \( \tilde{r} (0) = 0 \). In contrast, the interest payment on projects backed by short-term debt is zero. This appendix describes a model of optimal debt contracts providing some foundations for these assumptions. I continue to take the haircut requirement \( h \) as given, which ensures that all debt contracts are risk-free.

**Creditors.** Banks, who behave as described in the paper, borrow from risk-averse creditors. There is a unit measure of creditors, indexed by \( i \in [0, 1] \), each of whom has an endowment of \( y \) unit of the consumption good at \( t = 0 \). Creditors can store the consumption good costlessly across periods, but they cannot invest in projects. I abstract from the case where creditors do not have enough funds to meet banks’ needs, assuming that \( y \) is large enough.

As in Diamond and Dybvig (1983), creditors experience stochastic consumption needs. With probability \( \lambda \), creditor \( i \in [0, 1] \) is an early consumer and only derives utility from consump-
tion in period 1. With probability $1 - \lambda$, he is a late consumer and consumes only in period 2. The distribution of consumer types is identical and independent across creditors. Hence, his expected utility is given by

$$\lambda u(c_{1i}) + (1 - \lambda) u(c_{2i})$$

where $u$ is twice differentiable, strictly increasing and strictly concave. The distribution of consumer types is identical and independent across creditors.

**Short-term debt contracts at $t = 1$ in state $s$.** The bank proposes a short-term debt contract to each creditor $i$. A short-term debt contract consists of a principal $b^i$ and an interest payment $r^i$. The creditor gives the bank $b^i$ at $t = 1$ and receives $b^i + r^i$ from the bank at $t = 2$. No early consumers will be willing to accept contracts at $t = 1$. Late consumers, on the other hand, will be willing to accept the contract even if there is no interest ($r^i = 0$), since the repayment is riskless. It is therefore optimal for the bank to propose and $r^i = 0$ for all $i$. Hence, short-term debt at $t = 1$ is costless.

**Short-term debt contracts at $t = 0$.** By the same reasoning as above, it is optimal for the bank to offer zero interest payments, as all creditors will be willing to accept such contracts. Formally, it is optimal for the bank to propose $r^i = 0$ for all $i$. Hence, short-term debt at $t = 0$ is also costless.

**Long-term debt contracts.** The bank proposes a long-term debt contract to each creditor $i$. A long-term debt contract consists a principal $\bar{b}^i$ and an interest payment $\bar{r}^i$. The creditor gives the bank $\bar{b}^i$ at $t = 0$ and receives $\bar{b}^i + \bar{r}^i$ from the bank at $t = 2$.

Suppose $i$ turns down the contract. He will then store his endowment until he has a consumption need, and achieve utility $u(y)$. If he accepts the contract, he will store the remaining $y - \bar{b}^i$ of his endowment at $t = 0$. If he becomes an early consumer, the repayment at $t = 2$ will be worthless, and he will consume $y - \bar{b}^i$ at $t = 1$. If he becomes a late consumer, he will store $y - \bar{b}^i$ until $t = 2$ (or accept a short-term debt contract at $t = 1$ offering the same repayment) and consume $\left(y - \bar{b}^i\right) + (\bar{b}^i + \bar{r}^i)$ at $t = 2$. His participation constraint is therefore

$$\lambda u\left(y - \bar{b}^i\right) + (1 - \lambda) u\left(y + \bar{r}^i\right) \geq u\left(y\right)$$

For the optimal contracts, the participation constraint will bind for all $i$, which defines the interest payment as an implicit function of the amount borrowed, which is denoted...
$\bar{r}^i = \bar{r}(\bar{b}^i)$. It is easy to show that

$$\frac{d\bar{r}}{d\bar{b}^i} > 0 \text{ and } \frac{d^2\bar{r}}{d(\bar{b}^i)^2} > 0$$

Hence, the optimal interest payment $\bar{r}^i$ is increasing and convex in the amount borrowed from $i$. This convexity result implies that it is optimal to evenly spread borrowing across creditors.

If the bank borrows a total of $L$ units using long-term debt, it therefore sets $\bar{b}^i = L$ and $\bar{r}^i = \bar{r}(L)$ for all $i$. The minimised cost of borrowing $L$ units of long-term debt is therefore

$$\int_0^1 \bar{r}^i \, di = \bar{r}(L)$$

By the previous argument, this cost function is differentiable, increasing, strictly convex in $L$ and satisfies $\bar{r}(0) = 0$ as required.

C Properties of bounds

This appendix demonstrates that the inequality conditions in Propositions 2 and 3 are consistent with the assumptions of the model, in particular Assumption 2.

First, consider the condition in Proposition 2, which requires that

$$R < \eta^{-1} \pi_b \left( v_{2b} - p \right) \frac{(1-p) (1-h) - v_{1b}}{ph}$$

$$= \pi_b \left( v_{2b} - p \right) \frac{1 - v_{2b} (1-h) - v_{1b}}{(1-v_{2b}) (1-h) - v_{1b}} \times \frac{(1-p) (1-h) - v_{1b}}{ph}$$

I show that the upper bound on $R$, i.e. the right-hand side of this equation, is greater than the lower bound on $R$ in Assumption 2. This is equivalent to showing that

$$\frac{1 - v_{2b} (1-h) - v_{1b}}{(1-v_{2b}) (1-h) - v_{1b}} > \frac{1 - p (1-h) - v_{1b}}{(1-p) (1-h) - v_{1b}}$$

If $v_{2b} = p$, then the left- and right-hand sides are equal. By taking the derivative, it is easy to show that the right-hand side is increasing in $p$. Hence, for all $p < v_{2b}$, the inequality is
satisfied.

Second, consider the condition in Proposition 3, which requires that

\[
\frac{R}{r'(0)} > 1 - \frac{h - v_{1b}}{1 - v_{2b}(1 - h) - v_{1b}} = \frac{(1 - v_{2b})(1 - h)}{1 - v_{2b}(1 - h) - v_{1b}}
\]

I show that the lower bound, i.e. the right-hand side of this equation, is smaller than the upper bound implied by the condition in Proposition 2 and Assumption 2. This is equivalent to showing that

\[
\frac{(1 - v_{2b})(1 - h)}{1 - v_{2b}(1 - h) - v_{1b}} < \frac{1 - v_{2b}(1 - h) - v_{1b}}{(1 - v_{2b})(1 - h) - v_{1b}} \times \frac{(1 - p)(1 - h) - v_{1b}}{(1 - p)(1 - h)}
\]

(18)

If \(v_{2b} = p\), then the inequality reduces to

\[
\left[ \frac{(1 - v_{2b})(1 - h)}{1 - v_{2b}(1 - h) - v_{1b}} \right]^2 < 1
\]

\[
\Leftrightarrow (1 - v_{2b})(1 - h) < 1 - v_{2b}(1 - h) - v_{1b}
\]

\[
\Leftrightarrow 0 < h - v_{1b}
\]

By Assumption 1 and the lower bound on \(h\) in equation 6, we have

\[
h - v_{1b} > 1 - v_{2b} - v_{1b} > (1 - v_{2b})(1 - h) - v_{1b} > 0
\]

Hence, for \(v_{2b} = p\), the inequality in (18) is satisfied. By taking the derivative, it is easy to show that the right-hand side is decreasing in \(p\). Hence, for all \(p < v_{2b}\), the inequality is satisfied.
References


Geanakoplos, J. and H. M. Polemarchakis (1985, August). Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete.


