Financial frictions in the Euro Area and the United States: a Bayesian assessment

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Abstract

This paper assesses the empirical relevance of financial frictions in the Euro Area (EA) and the United States (US). It provides a comprehensive set of comparisons between two models: (i) a Smets and Wouters (2007) (SW) model with financial frictions originating in non-financial firms à la Bernanke et al. (1999) (SWBGG); and (ii) a SW model with frictions originating in financial intermediaries, à la Gertler and Karadi (2011) (SWGK). Proved that the introduction of financial frictions in either way improves the models’ fit compared to a standard SW model, the empirical comparisons reveal that the SWGK model outperforms the SWBGG model both in the EA and the US. Two main factors explain this result: first, the magnitude of the financial accelerator effect; and second, the role of the investment-specific technology shock in affecting financial variables.

Keywords: Financial frictions, DSGE models, Bayesian estimation.

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1 Introduction

In the aftermath of the financial crisis the structure of the financial system has received an increasing attention in the literature (see Brunnermeier et al., 2012, for a survey). The features of external financing are particularly relevant because of its impact on business cycle fluctuations. Since the onset of the crisis developments in credit markets have changed substantially: total financing to non-financial corporations have declined both in the Euro Area (EA) and in the United States (US). At their peak following the collapse of the Lehman Brothers, the credit spreads skyrocketed.

The dynamic stochastic general equilibrium (DSGE) literature offers alternative microfoundation of financial market frictions. The influential model of Bernanke et al. (1999) (BGG) is considered as a workhorse for the analysis of credit market imperfections in DSGE modelling. The BGG model features constrained firms that are the source of frictions in the form of a costly state verification problem (Townsend, 1979). Much of the macroeconomic literature stemming from BGG emphasizes credit market constraints on non-financial borrowers and treats financial intermediaries largely as a veil. Gertler and Karadi (2011) (GK), instead, explicitly model the banking sector as a source of financial frictions due to the presence of a moral hazard problem. Another approach is offered by the seminal paper of Stiglitz and Weiss (1981), who focus on adverse selection as a source of financial frictions (see also Christiano and Ikeda, 2011).

Given such a variety of approaches, this paper investigates which type of financial frictions is favored by the data. It empirically compares for the period 1983Q1-2008Q3 using EA and US data: (i) the Smets and Wouters (2007) (SW) model; (ii) the SWBGG model, which incorporates financial frictions à la Bernanke et al. (1999) – where lenders pay a fixed monitoring cost to observe the borrowers' realized return – in a SW economy; and (iii) the SWGK model, where the financial intermediary (FI, henceforth) faces endogenously determined balance sheet constraints. In the literature there are other papers presenting a SW economy with the financial accelerator à la BGG; examples are Queijo von Heideken (2009), Gelain (2010), Carrillo and Poilly (2013), Del Negro and Schorfheide (2013), and Christiano et al. (2014). The choice of these two modelling strategies – BGG and GK – for micro-founding financial frictions can be explained by: (i) the established importance of the BGG approach in the mainstream DSGE literature on financial frictions; (ii) the important role assigned to financial intermediaries in the GK model; and (iii) their relative analytical tractability. These two models also share a common feature, i.e. financial frictions originate in the group of agents that borrow and borrowing capacity is linked to net worth. An empirical comparison between the two approaches is novel in the DSGE arena.

As a first step, this paper finds that the introduction of financial frictions either à la BGG or GK improves the models’ fit, suggesting that these frictions are empirically relevant both in the EA and the US and this is in line with what Queijo von Heideken (2009) finds by comparing the SW and BGG models. The paper then focuses on the comparison between the SWBGG and SWGK models by examining: business cycle moments, models-implied spread, impulse responses, variance decomposition, and forecasting performance. The novel result is that the SWGK model outperforms the SWBGG model because of two main reasons. First, impulse response function
analysis reveals that the magnitude of the financial accelerator effect is different across the two models. The presence of the banking sector acts as a powerful amplification channel. The financial accelerator effect embedded in the SWGK model is indeed stronger than that in the SWBGG model for those shocks which are the main drivers of business cycle fluctuations. Disruptions in financial markets are generally associated with a rise in the credit spread and a contraction in the quantity of credit. However, while a rise in the spread causes a decline in net worth of firms in the SWBGG model, financial intermediaries benefit from a rise in the spread because of the positive effects on the relevant net worth in the SWGK model. This mechanism generally leads to a stronger rise in the spread in the SWGK model with more severe effects on investment and, hence, on output compared to the SWBGG model. Therefore, the SWGK model provides a better solution to the so-called "small shocks, large cycles" puzzle (Bernanke et al., 1996). The second reason is that the investment-specific technology shock plays a different role in the two models, both in the EA and the US: it explains a larger fraction of the spread in the SWBGG model compared to that in the SWGK model. However, this shock does not replicate the comovement between output and investment and the countercyclical behavior of the spread. Hence, its larger role in the SWBGG model provides another reasons for the better empirical performance of the SWGK model. Point forecast evaluation reveals that the SWGK model is favored also along this dimension in the EA, while in the US there is no clear evidence of an outperformed model in terms of forecasting accuracy.

The structure of the paper is as follows. Section 2 briefly presents the models. Section 3 describes the data and discusses the estimation strategy. Section 4 compares the estimated models, discusses the propagation mechanisms and presents models’ forecasting performance. Finally, Section 5 concludes. An online appendix complements the paper by providing (a) the full details of the models; (b) an analysis aiming at disentangling the effects of the magnitude of the financial frictions on impulse response functions; (c) a series of robustness checks for the empirical results.

2 The Models

This section briefly sketches the three DSGE models. Compared to the standard SW economy, the different features are: (i) a utility function comparable with Smets and Wouters (2003) and Gertler and Karadi (2011); (ii) the Dixit-Stiglitz aggregator for final output and composite labor, as in Galí et al. (2011); (iii) the price mark-up, wage mark-up and government shocks are modelled as in Smets and Wouters (2003); and (iv) the presence of financial frictions in the SWBGG and SWGK models, which changes the production side of the economy. In order to simplify the optimization problems of intermediate goods firms, retailers are the source of price stickiness.

In all models the economy is populated by: households; labor unions; labor packers; retailers; final good firms; intermediate goods firms; and the policymaker. In the SWBGG and SWGK models the economy is also populated by capital producers, while the SWGK model incorporates FI.

Households consume, save, and supply labor. A labor union differentiates labor and sets wages in a monopolistically competitive market. Competitive labor packers buy labor service from the union,
package and sell it to intermediate goods firms. The good market has a similar structure: retailers buy goods from intermediate goods firms, differentiate them and sell them in a monopolistically competitive market. The aggregate final good is produced by perfectly competitive firms assembling a continuum of intermediate goods. The policymaker sets the nominal interest rate following a Taylor rule.

In the SWBGG model, intermediate goods firms maximize the flow of discounted profits by choosing the quantity of factors for production and stipulate a financial contract to obtain funds from lenders. At the end of period $t$, firms buy capital $K_{t+1}$ that will be used throughout time $t + 1$ at the real price $Q_t$. The cost of purchased capital is then $Q_t K_{t+1}$. A fraction of capital acquisition is financed by their net worth, $N_{t+1}$, and the remainder by borrowing. In order to ensure that entrepreneurial net worth will never be enough to fully finance capital acquisitions, it is assumed that each firm survives until the next period with probability $\theta$ and her expected lifetime is consequently equal to $1/(1 - \theta)$. At the same time, the new firms entering receive a transfer, $N^e_t$, from firms who die and depart from the scene.\footnote{Following Christensen and Dib (2008), consumption of exiting firms – a small fraction of total consumption – is ignored in the general equilibrium.} There is a problem of asymmetric information about the project’s ex-post return because the return to capital is sensitive to an idiosyncratic shock. While the firm can costlessly observe the realization of the shock, the lender has to pay a fixed auditing cost to observe borrower’s return. If the firm pays in full there is no need to verify the project’s return; but in the case of default the lender verifies the state and pays the cost. As a consequence, the financial contract implies an external finance premium, $EP(\cdot)$, i.e. a difference between the cost of external and internal funds, that depends on the inverse of the firm’s leverage ratio. FI are just a “veil” in the model (Gilchrist and Zakrążek, 2011). Capital producers purchase investment and depreciated capital to transform them into capital sold to intermediate goods firms and used for production.

In the SWGK model, within each household there are two types of members at any point in time: the fraction $g$ of the household members are workers and the fraction $(1 - g)$ are bankers. The FI have a finite horizon in order to avoid the possibility of full self-financing. Every banker stays banker next period with a probability $\theta$, which is independent of history. Therefore, every period $(1 - \theta)$ bankers exit and become workers. Similarly, a number of workers become bankers, keeping the relative proportion of each type of agents constant. The household provides her new banker with a start-up transfer, which is a small fraction, $\chi$, of total assets. Each banker manages a financial intermediary. The production sector is also made of intermediate goods firms and capital producers. The optimization problem of capital producers is the same as in the SWBGG model. The intermediate goods firms finance their capital acquisitions each period by obtaining funds from the FI. While there are no financial frictions in this activity, there is a problem of moral hazard between FI and households, because the former can choose to divert a fraction $\lambda$ of available funds from the project. Hence an incentive compatibility constraint should hold in order to make households willing to deposit money in the FI; as a result, the assets the FI can acquire depend positively on their net worth.
The detailed linearized models are shown in Table 1.²

3 Data and estimation strategy

In each model there are seven orthogonal structural shocks: the technology, \( \varepsilon^t_t \); the investment-specific technology, \( \varepsilon^x_t \); the monetary policy, \( \varepsilon^r_t \); the capital quality, \( \varepsilon^k_t \); the government, \( \varepsilon^g_t \); the price mark-up, \( \varepsilon^p_t \); and the wage mark-up, \( \varepsilon^w_t \), shocks. In each model, the shocks follow an AR(1) process.

The models are estimated with quarterly data for the period 1983Q1-2008Q3, using as observables real GDP, real investment, real private consumption, hours worked, GDP deflator inflation, real wage and the nominal interest rate. The starting date is the same used by Smets and Wouters (2005), while the final quarter corresponds to the pre-crisis period since the purpose of this paper is to make a comparison among the models in “normal” times. Moreover, ending in 2008Q3 allows to avoid potential distortionary effects on the estimates of the zero lower bound on the nominal interest rate (Galí et al., 2011). Data come from the Area Wide Model database (see Fagan et al., 2005, for an explanation) for the Euro Area, and from the NIPA tables of the Bureau of Economic Analysis and the Bureau of Labor Statistics for the United States. Following Smets and Wouters (2007), all variables are logged, but the nominal interest rate which is expressed in quarterly terms. GDP, consumption, investment and wages are expressed in first differences. The inflation rate is measured as a quarterly log-difference of GDP deflator. US hours of work are multiplied by civilian employment, expressed in per capita terms and demeaned. Data on EA employment are used since there are no data available for hours worked in the Euro Area – equation (12) in Table 1.

The following set of measurement equations show the link between the observables in the dataset and the endogenous variables of the DSGE model:

\[
\begin{bmatrix}
\Delta Y^o_t \\
\Delta C^o_t \\
\Delta I^o_t \\
\Delta W^o_t \\
L^o_t \\
\pi^o_t \\
\bar{r}^n_t \\
\end{bmatrix} =
\begin{bmatrix}
\gamma \\
\gamma \\
\gamma \\
\gamma \\
0 \\
\bar{\pi} \\
\bar{r}^n \\
\end{bmatrix}
+ 
\begin{bmatrix}
\hat{Y}_t - \hat{Y}_{t-1} \\
\hat{C}_t - \hat{C}_{t-1} \\
\hat{I}_t - \hat{I}_{t-1} \\
\hat{W}_t - \hat{W}_{t-1} \\
\hat{L}_t \\
\hat{\pi}_t \\
\hat{\bar{r}}_t \\
\end{bmatrix}
\]  

(1)

where \( \gamma \) is the common quarterly trend growth rate of GDP, consumption, investment and wages; \( \bar{\pi} \) is the steady-state quarterly inflation rate; and \( \bar{r}^n \) is the steady-state quarterly nominal interest rate. A hat over a variable indicates the log-deviation from steady state.

² More details are available in the online appendix.
³ The Smets and Wouters (2007) model features a risk premium shock, which is meant to proxy frictions in the process of financial intermediation (not explicitly modelled). Since the SWBGG and SWGK models provide an explicit microfoundation for financial frictions, the risk premium shock has been replaced with the financial shock proposed by Gertler and Karadi. Exercises for the Euro Area (Villa, 2013) – with a different filtering technique – show that the main result of the paper still holds in the presence of the risk premium shock instead of the capital quality shock.
3.1 Calibration and priors

The parameters which cannot be identified in the dataset and/or are related to steady state values of the variables are calibrated. The time period in the model corresponds to one quarter in the data.

Table 2 shows the calibration of the parameters common to both models. The discount factor, $\beta$, is equal to 0.99, implying a quarterly steady state real interest rate of 1%; the capital income share, $\alpha$, is equal to 0.33. The depreciation rate is equal to 0.025, corresponding to an annual depreciation rate of 10%. The ratio of government spending to GDP is equal to 0.20. The elasticities of substitution in goods and labor markets are equal to 6 in order to target a gross steady state mark up of 1.20, as in Christiano et al. (2014), among many others.

The calibration of the financial parameters is shown in Table 3. The parameter $\theta$ represents the survival rate of intermediate goods firms in the SWBGG model and of FI in the SWGK model. This parameter is set equal to 0.972 implying an expected working life for bankers and firms of almost a decade; this value is close to those in BGG and GK. In the SWBGG model the parameter pinning down the steady state spread, $S$, is set to match the steady state spread of 150 basis points. Following BGG, the ratio of capital to net worth is set to 2, implying that 50% of firm’s capital expenditures are externally financed. As long as the calibration of the SWGK model is concerned, the fraction of assets given to new bankers, $\chi$, and the fraction of assets that can be diverted, $\lambda$, are equal to 0.001 and 0.515, respectively, to target the same steady state spread of 150 basis points and a steady state leverage ratio of 4.4 The online appendix investigates the robustness of the main results to the calibration of the financial parameters.

Table 4 shows the assumptions for the prior distributions of the estimated parameters for both models; the prior distributions are the same for both countries. The choice of the functional forms of parameters and the location of the prior mean correspond to a large extent to those in Smets and Wouters (2003, 2007) where applicable. The prior of some model-specific parameters are as follows. The parameter measuring the inverse of the Frisch elasticity of labor supply follows a Normal distribution with a prior mean of 0.33, the value used by Gertler and Karadi (2011), and a loose standard deviation of 0.25. The elasticity of external finance premium with respect to leverage of firms is assumed to follow a Beta distribution with prior mean of 0.05 and standard deviation of 0.05, which implies $[0.002, 0.151]$ 90% prior interval.

4 Models comparison

This section performs empirical comparisons among the models estimated both for the Euro Area and United States, with a focus on the SWBGG and SWGK models since this is the novel part of the paper. The comparison is made along the following dimensions: (i) the estimated parameters and the likelihood race; (ii) simulated business cycle moments versus those in the data and models-implied spread; (iii) impulse response functions and variance decomposition; and (iv) the forecasting

\footnote{Compared to Gertler and Karadi (2011), the higher value of the steady state spread is targeted with a higher calibration of the parameter $\lambda$.}
performance. The online appendix presents some robustness exercises.

### 4.1 Estimated parameters and likelihood race

The mean of the estimated parameters for each model is computed with two chains of the Metropolis-Hastings algorithm with a sample of 250,000 draws. Table 4 reports the posterior mean with 95% probability intervals in parentheses of the SWBGG and SWGK models for the Euro Area and the United States. Most parameters are remarkably similar across the two models. As in Smets and Wouters (2005), the fact that in almost all the cases the posterior estimate of a parameter in one model falls in the estimated confidence band for the same parameter of the other model can be considered as a rough measure of similarity.

As far as the Euro Area is concerned, the degree of price stickiness reveals that firms adjust prices about every year and half, with a higher degree of stickiness in SWGK model. The Calvo parameter for wage stickiness reveals that the average duration of wage contracts is slightly more than a year, lower than the degree of price stickiness, as in Smets and Wouters (2003). There is a moderate degree of price indexation and a higher degree of wage indexation similar to previous estimates for the EA (Adolfson et al., 2008; Gerali et al., 2010). The mean of the parameter measuring the elasticity of capital utilization is higher than its prior mean, revealing that capital utilization is more costly than assumed a-priori. The estimated value in both models favors high costs of capital utilization, suggesting a minor role for this internal propagation mechanism, in line with the literature (e.g. Adolfson et al., 2007; Christoffel et al., 2008). The estimates of the parameter measuring the Taylor rule reaction to inflation are also in line with previous estimates for the EA, with a higher value in the SWBGG model. There is also evidence of short-term reaction to the current change in the output gap. Turning to the exogenous shock processes, all shocks are quite persistent but the wage mark-up shock. The mean of the standard errors of the shocks is in line with the similar studies of the EA, but the standard deviation of the investment-specific technology shock which is higher.

As far as the US economy is concerned, the two models with financial frictions feature a higher degree of price stickiness compared to the value found by Smets and Wouters (2007), since average length of price contract is more than two years. Our results are in line with Del Negro et al. (2013). Similarly to the EA, the estimates of the parameter measuring the elasticity of capital utilization are higher than the prior mean, rather in line with the value found by Smets and Wouters for the period 1984-2004, equal to 0.69. There is evidence of a relatively low external superficial habit in consumption, similarly to the results by De Graeve (2008). The estimates of the shock processes are generally similar among the models, but in the SWBGG model the investment-specific technology shock has a lower persistence and volatility compared to the SWGK model.

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5Version 4.3.3 of the Dynare toolbox for Matlab is used for the computations. For details on the Bayesian estimation procedure see Fernández-Villaverde (2010), among others.

6In a calibrated real business cycle model Villa (2012) shows that capital utilization play a limited role in amplifying the effects of the shocks hitting the economy. It should also be noted that parameter $\zeta$ is a transformation of parameter $\psi$, estimated by Smets and Wouters (2003). In particular, $\zeta = 1/(1 + \psi)$. Following Smets and Wouters (2007), the parameter $\zeta \in [0, 1]$ is estimated.
The estimated mean of the elasticity of the external finance premium with respect to the leverage position is equal to 0.04 in the Euro Area and to 0.05 in the United States. Also in this case the posterior estimate of this parameter for one country falls in the estimated confidence band for the same parameter of the other country. An estimated elasticity different from zero is a first piece of evidence in favor of a model with financial frictions.

The Bayes factor (BF) is used to judge the relative fit of the models, as in An and Schorfheide (2007), among many others. According to Jeffreys (1998), a BF of $3 - 10$ provides “slight” evidence in favor of model $i$; a BF in the range $[10 - 100]$ provides “strong to very strong” evidence in favor of model $i$; and a BF greater than 100 provides “decisive evidence”.

The main results are shown in Table 5. As a first step, the comparison is made with the SW model featuring perfect financial markets. The introduction of financial frictions à la BGG leads to an improvement of the marginal likelihood for both EA and US data, suggesting that these frictions are empirically relevant. This result confirms the findings by Queijo von Heideken (2009), among others. Second, the empirical relevance of financial frictions in the Euro Area is stronger in the model featuring frictions at the level of financial intermediaries. The Bayes factor points to “decisive” evidence in favor of the SWGK model versus the SWBGG model, as shown in the last row of Table 5. This result is true also for the United States: the comparison between the two models with financial frictions provides “decisive” evidence in favor of the SWGK model.

4.2 Business cycle moments and models-implied spread

In order to investigate which features of the data are better captured by the SWGK model, this subsection first shows the comparison between the moments generated by the models and those in the data, as in Iacoviello and Neri (2010). It then investigates whether the estimated models generates proxies of the spread with business cycle fluctuations in line with the data.

Table 6 shows relative standard deviations, cross-correlations and autocorrelations of output, investment, consumption, inflation and the nominal interest rate within the 90% highest posterior

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7 Queijo von Heideken (2009) finds that financial frictions are larger in the Euro Area compared to United States since the estimates of monitoring costs are higher in the first country. Our estimation strategy is not necessarily in contrast with her results. Following Meier and Muller (2006), Kamber et al. (2012) and Fernández and Gulam (2014), from the value of the elasticity it is possible to trace out the values of the deep parameters of the financial contract, which are also consistent with the calibrated parameters shown in Tables 2 and 3. As far as the Euro Area is concerned, these values are: a share of monitoring costs equal to 0.22; a standard deviation of the idiosyncratic shock to firm’s return equal to 0.25; the resulting quarterly business failure rate is 0.024. For the United States they are: a share of monitoring costs equal to 0.17; a standard deviation of the idiosyncratic shock to firm’s return equal to 0.32; the resulting quarterly business failure rate is 0.025.

8 Such a comparison is based on the marginal likelihood of alternative models. Let $m_i$ be a given model, with $m_i \in M$, $\theta$ the parameter vector and $p_i(\theta|m_i)$ the prior density for model $m_i$. The marginal likelihood for a given model $m_i$ and common dataset $Y$ is

$$L(Y|m_i) = \int_\theta L(Y|\theta,m_i)p_i(\theta|m_i)d\theta,$$

(2)

where $L(Y|\theta,m_i)$ is the likelihood function for the observed data $Y$ conditional on the parameter vector and on the model; and $L(Y|m_i)$ is the marginal data density. The Bayes factor is the ratio between the marginal likelihoods. The log data density of the three models is computed with the Geweke (1999)’s modified harmonic mean estimator.
As far as the Euro Area is concerned, the comparison of the relative standard deviations of investment and consumption (with respect to output) shows that for both the SWBGG and SWGK models the confidence bands contain the empirical standard deviations. The SWGK model fits all the variables better compared to the SWBGG model in terms of the value of the posterior mean. However, both models perform poorly in replicating the relative standard deviations of interest rate and inflation since the models’ business cycle statistics are outside the empirical standard deviations.

The comparison of the cross-correlation with output reveals that the SWBGG and the SWGK models cannot reproduce the cross-correlation of investment and the SWBGG performs worse. The SWGK model fits the data better than the SWBGG model in terms of cross-correlations of consumption, inflation and interest rate, although results are rather similar since business cycle statistics of the data are within confidence bands of both models.

Table 6 finally reports the autocorrelation coefficients of order 1. Variables are more autocorrelated in the two models than in the data, as in Gabriel et al. (2011). When it comes to matching inflation, the models are from replicating its dynamics in the data. Overall, the presence financial frictions originating in financial intermediaries is preferable in the data compared to a model where financial frictions originate in non-financial firms, in particular as far as investment dynamics is concerned.

The comparison of simulated business cycle moments between the two models in the United States is similar to that in the EA. Also for the United States the two models perform poorly in replicating the relative standard deviations of inflation and interest rate, while the SWGK model gets closer to the data in replicating the cross-correlation of investment.

Figure 1 reports the models-implied spread, in percent deviations from their steady state values, within a 95% confidence band, along with the percent deviations from HP(1600) trends of a proxy of the credit spread in EA data. The series is computed as BBB rated bonds minus government AAA bonds. Since the series is shorter than the sample period, the charts refer to the period 1996Q1-2008Q3 only. The SWBGG model generates a series of the spread rather constant over time, while the SWGK model better replicates the business cycle fluctuations of this variable. Recalling that, in the estimation, the standard dataset of macroeconomic variables do not include financial data, the SWGK model produces cyclical fluctuations of the financial variable that are closer to those in the data to a certain extent.

Theoretical moments are computed from the state-space representation for 1,000 random draws from the posterior distributions – which produce 1,000 sets of theoretical moments. Data are linearly detrended.

Since the models fail in replicating the relative standard deviations of interest rate and – to a minor extent – of inflation, there could be some doubt on the overall ability of the models to fit the data. As a robustness check, the models are then estimated allowing for measurement errors in inflation and wages, as well as for a moving-average component in the price and wage mark up shocks. In such a case there is indeed an improvement in the ability of the models in replicating the relative standard deviation of inflation and, to a minor extent, the nominal interest rate. The log data density reveals that the ranking of the re-estimated models is not affected and the SWGK model is still the preferred one. Details are available in the online Appendix C. An alternative way to improve the relative standard deviations of inflation and interest rate could be to include innovations to trend inflation in the NK Phillips curve as suggested by Cogley and Sbordone (2008).
Figure 2 reports the models-implied spread, in percent deviations from their steady state values, within a 95% confidence band, along with the percent deviations from HP(1600) trends of proxies of spreads in US data (available in the ALFRED database of the St’ Louis Fed). Due to the greater availability of US data on spreads, two measures of spreads are reported in the figure: Moody’s seasoned Baa corporate bond yield (Baa) minus Moody’s seasoned Aaa corporate bond yield (Aaa); and Baa minus long-term Treasury constant maturity rate (TCM). While the SWBGG generates a series of the spread more correlated with the latter proxy, the SWGK model replicates a series of the spread more correlated with the first proxy. Both models generate a series of the spread: (i) with a comparable order of magnitude of the available proxies; and (ii) with an upward trend at the end of the sample, picking-up the end-of-sample crisis.

4.3 Impulse response functions and variance decomposition

This section discusses the impulse response functions (IRFs) and variance decomposition of the two models for the two economies in order explain why the presence of financial frictions on FI is empirically favored by the data. This section shows the effects on output, investment, inflation, net worth of firms in the SWBGG model and of FI in the SWGK model, and spread to all the seven structural shocks: capital quality (KQ), monetary policy (MP), government spending (Gov), investment-specific technology (IS), technology (TFP), price mark-up (PMU) and wage mark-up (WMU) shocks. All the shocks are set to produce a downturn. In all the figures, impulse responses are normalized so that the size of each shock is the same across the models. The solid lines represent the estimated median and the dotted lines represent the 95% confidence intervals.

The two models embed a different transmission mechanism: while in the SWBGG model the financial accelerator effect works through the firms’ balance sheet channel, in the SWGK model it works through the bank lending channel. And the dynamics of net worth is differently affected by a change in the spread in the two models. Generally speaking, the equations describing the accumulation of net worth of firms and banks – 16a and 20b, respectively, in Table 1 – are similar. Both equations contain a term for net worth of the previous period, the leverage, the rate of return on capital and the risk free rate. However, the effect of any exogenous shock on net worth of firms is dramatically different from the effects of the same shock on net worth of FI. As explained by Villa and Yang (2011), three factors affect the profits of FI: the lending volume, the spread and the leverage. Any contractionary shock affects bank’s profits either directly or indirectly. The FI decides about the amount of corporate bonds it holds subject to the incentive constraint, the expected capital return, and the deposit return. A fall in the asset price leads to a deterioration of FI balance sheets. The reduction in net worth makes the incentive constraint tighter, leading to a higher spread and, hence, profits. The resulting rise of the spread has also the effect of reducing the demand for loans by firms, leading to a further contraction in investment and, hence, output. This is the essence of the financial accelerator effect of the SWGK model. The increase in the spread after a negative shock helps financial intermediaries to rebuild faster their net worth. On the contrary, in the SWBGG model the rise in the spread causes a further fall in net worth. Hence optimizing...
financial intermediaries benefit from a rise in the spread. In most shocks the rise in the spread is indeed stronger in the SWGK model. But this causes a more pronounced fall in investment and, hence, output. The SWGK model provides a better solution to the so-called “small shocks, large cycles” puzzle (Bernanke et al., 1996).

For all the shocks estimated for the Euro Area, but the investment-specific technology and the price mark up shocks, the financial accelerator effect embedded in the SWGK model is indeed amplified compared to that in the SWBGG model as shown by Figures 3 and 4. The propagation mechanism of the capital quality shock—which is the main driver of GDP at all horizons, as shown in Table 7—is as follows. In the SWBGG model the capital quality shock affects only the demand side of the credit market through two main mechanisms: (1) due to the simulated recession, there is a fall in asset prices and the return on capital, causing a downward shift in the demand for capital; (2) the fall in net worth due to the reduction in the return on capital, of capital and the price of capital, causes an increase in leverage. This leads to a rise in the spread, and hence a further fall in investment. The fall in net worth is persistent, since this variable is well below steady state after 20 quarters. In the SWGK a capital quality shock directly translates into a shock to bank’s balance sheet due to the identity between capital and assets. Due to the presence of moral hazard, depositors require banks not to be overleveraged. Hence they are forced to reduce lending. The reduction in the lending volume makes the incentive constraint tighter, leading to a higher spread and, hence, to a higher profitability. Compared to the SWBGG model the increase of the spread from its steady state is much larger. On one hand, net worth tends to rise back to its steady state. On the other, non-financial firms observe a rise in borrowing costs and consequently reduce their demand of capital. The fall in investment is indeed much more pronounced in the SWGK model.

A contractionary monetary policy shock is shown in the third and fourth row of Figure 3. While the sign of the impact responses are similar among the models, the transmission mechanism is different. In both models an increase in the nominal interest rate reduces investment and, therefore, output. Demand downward pressures feed through changes in the output gap to inflation. This causes a downward shift in aggregate demand, which reduces inflation on impact. This standard transmission mechanism of the monetary policy shock is enhanced through its impact on credit markets. In the SWBGG model the decline in the price of capital due to the tightening of monetary policy causes a fall in net worth of intermediate goods firms. This implies that the potential divergence of interests between firms and lenders is greater and, therefore, agency costs increase. As a result, the rise in the spread further reinforces the simulated contraction in capital and investment. In the SWGK model, due to the retrenchment in investment, loans decrease as well. At the same time the fall in asset prices worsen FI’s balance sheet. The deterioration in intermediary balance sheets pushes up the spread. The increase in financing costs causes a further decline in loans and investment.

The contractionary government spending shock leads to a positive wealth effect, leading to a crowding-out effect on consumption and investment. As evident from Table 7 this shock plays a marginal role in affecting all the variables at any horizon.
A more detailed explanation is needed for the investment-specific technology shock, depicted in the last two rows of Figure 3. This shock causes a rise in the price of capital, $Q_t$, which has two opposite effects in the SWBGG model as also explained by Kamber et al. (2012): (i) investment falls as well as output; and (ii) net worth of firms increases due to the higher return on capital, equation (16a) in Table 1. The latter effect causes a fall in the spread. This in turn determines an increase in investment. This latter effect dominates in the estimated model and investment rises. Hence this shock does not replicate the positive comovement between output and investment, at least on impact. Moreover, it accounts for most of the forecast error variance of investment at longer horizons, as evident from Table 7. This result can explain why both the SWBGG and SWGK models fail to replicate the cross-correlation of investment with output reported in Table 6. In addition, contrary to empirical evidence (e.g. Aliaga-Díaz and Olivero, 2011; Agénor et al., 2014), this shock causes a procyclical response of the spread. This result is particularly interesting when combined with the variance decomposition analysis. The investment-specific technology shock explains 73% of the unconditional variance decomposition of the spread in the SWBGG model, and more than 80% of the variance decomposition of the spread at longer horizons. Hence, it could be argued that in this model investment and the spread, important variables in explaining the financial accelerator mechanism, are explained by a “counterintuitive” shock. In the SWGK model, an investment-specific technology shock exerts three main effects: (i) the price of capital rises, causing a fall in investment and output; (ii) the retrenchment in investment leads to a lower demand for lending, affecting in turn bank’s profits; and (iii) net worth of FI rises because of the higher return on capital, equation (20b) in Table 1. The first two effects acts in the direction of reducing investment, while the latter effect – which turns out to be quantitatively more important – leads to a rise in investment. Overall, the contractionary effect in output prevails. Similarly to the SWBGG model, the spread falls. Table 7 shows that investment-specific technology shock explains 17% of the unconditional variance decomposition of the spread and even lower fractions at different horizons. And this shock, although it is the main driver of investment fluctuations in particular at longer horizons, explains a lower fraction of its fluctuations compared to the SWBGG model. This difference is evident at all the horizons. The minor importance of the investment-specific technology shock – in particular on investment and on the spread – provides some explanation on the better fit of the SWGK model reported in the previous section.

The technology shock, shown in Figure 4, has a direct impact on output by making factors less productive, and leads to an increase in prices due to the contraction in aggregate supply. Investment and consumption decline due to the contraction in output. In the SWBGG model the lower return on capital and the decrease in the price of capital affects net worth of firms, which decreases. There is a moderate rise of the spread, which contributes to the fall in investment. In the SWGK model the decrease in asset prices also worsens the FI’s balance sheet. Such a deterioration leads to an increase in the spread, and hence in profits. This makes it possible to net worth to come back to steady state faster compared to the SWBGG model. The higher borrowing costs discourages the demand for it, leading to a further and a more pronounced retrenchment in investment.
A positive price mark-up shock exerts a contractionary effect on real activity leading to a decline in output, investment and consumption. The inflation rate increases and, since the Taylor rule is operating, this leads to a rise in the nominal interest rate. This shock tends to reduce both the return on and the price of capital. Hence, net worth of firms in the SWBGG model and of FI in the SWGK model falls. The role of the price mark-up shock in explaining variations in real and financial variables is very limited for both models, as shown by Table 7. Similar argument applies to the wage mark-up shock: it leads to an increase in the prices of factors for production, causing a fall in their equilibrium quantity. This exerts a contractionary effect on output, as shown by the last two rows of Figure 4. The rise in prices is accompanied by a rise in the nominal interest rate. Differently from the price mark-up shock the wage rises. This causes an increase in the marginal product of capital, $Z^k$, as also evident from equation (8) in Table 1. This, in turn, leads to a temporary rise in the return on capital and, hence, in net worth both in the SWBGG and SWGK models. In the former, the rise in net worth and the fall in leverage explains the pro-cyclical response of the spread, which falls as in Gelain (2010). In the SWGK model, instead, the spread exhibits a counter-cyclical behavior. Similarly to the price mark-up, the wage mark-up shock plays a marginal role in accounting for fluctuations in real and financial variables.

The impulse responses estimated for the United States, Figures 5 and 6, replicate transmission mechanisms similar to the ones observed in the Euro Area. However, the financial accelerator effect on output of the monetary policy shock is slightly larger in the SWBGG model compared to that in the SWGK model. There are two explanations for this result: first, the larger persistence of the monetary policy shock, equal to 0.27 in the SWBGG model and to 0.23 in the SWGK model; and, second, the different financial structures of the two areas. In the Euro Area the main fraction of external finance of corporate sector is constituted by bank loans. Hence, a monetary policy shock transmitted through the bank lending channel has a stronger impact on real activity in the EA than in the US where the corporate sector has access to markets in addition to financial intermediaries (e.g. Ciccarelli et al., 2013). The analysis of the variance decomposition points to results similar to the Euro Area: the investment-specific technology shock accounts for more than 60% of the forecast error variance of the spread in the SWBGG model, while it accounts for 16% of the variance of the spread in the SWGK model. The same shock is the main driver of investment in the SWBGG model – both in terms of unconditional variance decomposition and at longer horizons. In the SWGK model, instead, the capital quality shock is the main driver of investment. As evident from impulse responses, the investment-specific technology shock features a pro-cyclical behavior of the spread and a counter-cyclical behavior of investment, contrary to the empirical evidence. Hence, its larger role in the SWBGG model could explain the better fit of the SWGK model also for the US economy.

Overall, two main factors could provide some explanations on the better fit of the SWGK model. First, financial intermediaries provide a more powerful endogenous mechanism of amplification due to the more pronounced effect on the spread after a contractionary shock. And second, the important role of investment-specific technology shock in explaining investment and the spread in the SWBGG model is likely to explain the lower fit of the model itself.
4.4 Forecasting accuracy

This subsection evaluates the two models from a forecasting viewpoint. Following Kolasa et al. (2012), Bekiros and Paccagnini (2014a) and Bekiros and Paccagnini (2014b), the out-of-sample forecast performance is based on four horizons – ranging from one up to four quarters ahead. The recursive forecasting estimation considers the evaluation period from 2001Q1 to 2007Q4 (with 28 forecast periods in the last recursivized sample). The root mean squared forecast error (RMSFE) is used to evaluate point forecasts. In particular, RMSFEs are computed based on recursive estimation sample starting from 1983Q1-2000Q4 and ending in 1983Q1-2007Q4 ($h = 1$), 1983Q1-2007Q3 ($h = 2$), 1983Q1-2007Q2 ($h = 3$) and 1983Q1-2007Q1 ($h = 4$), respectively.

Table 9 reports the RMSFEs for the two models and the two economies for the following variables: GDP growth, investment growth, inflation and the nominal interest rate. As far as the EA is concerned, the accuracy of output growth forecasts from the SWGK model is significantly higher than that from the SWBGG model at all horizons. A similar result also holds for forecasts of investment growth, where the forecasting accuracy of the SWGK model is even higher in percentage terms compared to that of GDP growth. For inflation forecasts results are mixed although the difference is not very pronounced: the RMSFEs from the SWBGG model are lower in the first and second horizons, while for the other horizons the RMSFEs from the SWGK model are lower. The SWGK model performs better in terms of forecasting the nominal interest rate. Overall, the SWGK model outperforms the SWBGG model in forecasting real as well as nominal variables, which the exception of shorter horizon forecasts of inflation.

In the United States there is no clear evidence of an outperformed model in terms of forecasting accuracy. The values of the RMSFEs are remarkably similar across the two models. The RMSFEs for GDP growth are lower in the SWBGG model at shorter horizons and in the SWGK model at longer horizons. Results are mixed also for the RMSFEs for investment growth, while inflation forecasts are better in the SWGK model at all horizons. For the nominal interest rate RMSFEs from the two models almost coincide.

5 Conclusion

Since the onset of the financial crisis the link between financial intermediation and real activity has received an increasing attention both in academia and in policy institutions. This paper provides an empirical comparison of DSGE models which have a Smets and Wouters (2007) economy in common but feature different types of financial frictions: the SWBGG model with financial frictions originating in intermediate goods firms due to a costly state verification problem à la Bernanke et al. (1999); and the SWGK model with financial frictions embedded in financial intermediaries due to a moral hazard problem à la Gertler and Karadi (2011). The main result is that the SWGK model is always favored both by EA and US data. The reasons for the better empirical performance of the SWGK are mainly two: first, impulse response function analysis reveals that the presence of the banking sector acts as a powerful amplification channel of the shocks hitting the economy.
This is caused by the different effect that a change in the spread causes on the relevant net worth in the models. As a result, the financial accelerator effect embedded in the SWGK model is stronger than that in the SWBGG model. Second, the investment-specific technology shock plays a different role in the two models, both in the EA and the US. In particular, it explains a larger fractions of the dynamics of the spread and investment in the SWBGG model compared to those in the SWGK model. However, this shock does not replicate the comovement between output and investment and the countercyclical behavior of the spread. Hence, its larger role in the SWBGG model provides another reasons for the better empirical performance of the SWGK model. This paper also finds that introducing frictions affecting the banking sector tends to improve the quality of point forecasts of GDP growth, investment growth and the nominal interest rate in the EA. In the United States, instead, no model clearly dominates the other in terms of forecasting accuracy and the root mean squared forecast errors are remarkably similar across the two models.

The results presented in this paper offer some avenues for future research. First, it would be interesting to analyze a model featuring both types of financial frictions, at firms level and in the banking sector, in order to examine the transmission mechanism of the shocks and the accelerator/attenuator effects. And second, such a model could also incorporate the same form of financial friction (costly state verification or moral hazard at both levels) in order to empirically verify which modelling device is preferred by the data. DSGE models with a comprehensive structure of financial markets would improve our understanding of business cycle fluctuations.

References


\textbf{SW model}

(1) Euler equation

\[ \frac{1}{1-h} C_t = \frac{1}{1-h} E_t \left[ C_{t+1} \right] + \frac{h}{1-h} C_{t-1} - R_t \]

(2) Phillips curve – wages

\[ \dot{W}_t = \frac{\delta}{(1+\delta)E_t} \left[ W_{t+1} \right] + \frac{1}{(1+\delta)} \dot{W}_{t-1} + \frac{\delta}{(1+\delta)} E_t \left[ \dot{W}_{t+1} \right] - \frac{1}{(1+\delta)} \dot{W}_{t-1} + \sigma \dot{\lambda}_t - \frac{h}{1-h} C_t - \dot{W}_t + \varepsilon_t \]

(3) Capital accumulation

\[ \dot{K}_{t+1} = \delta \left( \dot{K}_t + \varepsilon_t \right) + (1 - \delta) \left( \dot{K}_t + \varepsilon_t \right) \]

(4) Optimal capital utilization

\[ \dot{Z}_t = \frac{\varepsilon_t}{1-\delta} \dot{U}_t \]

(5) Investment Euler equation

\[ \dot{I}_t = \frac{1}{(1+\delta)} \left( \dot{Q}_t + \varepsilon_t \right) + \frac{1}{(1+\delta)} \dot{I}_{t-1} + \frac{\delta}{(1+\delta)} E_t \left[ \dot{I}_{t+1} \right] \]

(6) Resource constraint

\[ \dot{Y}_t = \Theta \left[ \varepsilon_t + \alpha \left( \dot{K}_t + \dot{U}_t \right) + (1 - \alpha) \dot{L}_t \right] \]

(7) Production function

\[ \dot{Y}_t = \dot{Z}_t^\alpha \left( \dot{K}_t + \dot{U}_t \right)^{1-\alpha} \]

(8) Firms’ FOCs

\[ \dot{W}_t = \dot{Z}_t^\beta - \dot{L}_t + \dot{K}_t + \dot{U}_t \]

(9) Phillips curve – prices

\[ \dot{H}_t = \frac{\sigma \rho}{\sigma \rho + \sigma \rho^p} \dot{H}_{t-1} + \frac{\beta}{\sigma \rho + \sigma \rho^p} \dot{E}_t \left[ \dot{H}_{t+1} \right] - \frac{1-\beta \rho \sigma}{\sigma \rho + \sigma \rho^p} \left[ \varepsilon_t - \alpha \dot{Z}_t \right] \]

(10) Taylor rule

\[ \dot{R}_t^p = \rho_\delta \dot{R}_{t-1} + (1 - \rho_\delta) \left( \rho_\delta \dot{H}_{t-1} + \rho \left( \dot{Y}_t - \dot{Y}_t^p \right) + \rho \Delta_y \left[ \dot{Y}_t - \dot{Y}_t^p - \left( \dot{Y}_{t-1} - \dot{Y}_{t-1}^p \right) \right]\]

(11) Fisher equation

\[ \dot{R}_t^p = \left( 1 + \frac{\beta \sigma}{\sigma \rho} \right) \dot{H}_{t+1} + \frac{\beta \sigma}{\sigma \rho} \dot{E}_t \left[ \dot{H}_{t+1} \right] - \frac{1-\beta \rho \sigma}{\sigma \rho + \sigma \rho^p} \left[ \varepsilon_t - \alpha \dot{Z}_t \right] \]

(12) Phillips curve – employment

\[ \dot{E}_t = \frac{1}{(1+\delta)} \dot{E}_{t+1} + \frac{1}{(1+\delta)} \dot{E}_{t-1} - \frac{1-\beta \rho \sigma}{\sigma \rho + \sigma \rho^p} \left( \dot{L}_t - \dot{E}_t \right) \]

(13) Price of capital

\[ \dot{Q}_t + \dot{R}_t = \frac{Z_{t+1}}{Z_{t-1} - (1-\delta)} \dot{E}_t \left[ \dot{Q}_{t+1} + \dot{e}_t \right] \]

\textbf{SWBG model: equations (1)-(12) +}

(13a) Price of capital

\[ \dot{R}_t = \frac{Z_{t+1}}{Z_{t-1} - (1-\delta)} \dot{E}_t \left[ \dot{Q}_{t+1} + \dot{e}_t \right] - \dot{Q}_{t-1} \]

(14a) External finance premium

\[ \dot{E}_t = \frac{1}{(1+\delta)} \dot{E}_{t+1} + \frac{1}{(1+\delta)} \dot{E}_{t-1} - \frac{1-\beta \rho \sigma}{\sigma \rho + \sigma \rho^p} \left( \dot{L}_t - \dot{E}_t \right) \]

(15a) Spread

\[ \dot{E}_t \left[ \dot{R}_{t+1} \right] = \dot{R}_t + \dot{E}_t \]

(16a) Firms’ net worth accumulation

\[ \frac{1}{\rho} \dot{E}_t \left[ \dot{N}_{t+1} \right] = \kappa \dot{R}_t - \left( \kappa - 1 \right) \dot{R}_{t-1} - \xi \left( \dot{K}_t + \dot{Q}_{t-1} \right) \]

\textbf{SWGK model: equations (1)-(12) + (13a) +}

(14b) Gain from expanding assets

\[ \dot{V}_t = \frac{1-\theta}{1-\theta} \left[ \dot{R}_t - \dot{R} \right] \dot{E}_t \left[ \dot{A}_{t+1} \right] + \frac{1-\theta}{1-\theta} \left[ \dot{R}_t \dot{E}_t \left[ \dot{R}_{t+1} \right] - \dot{R} \dot{E}_t \right] \]

(15b) Value of expanding net worth

\[ \dot{D}_t = \theta \dot{Z}_t \dot{E}_t \left[ \dot{A}_{t+1} + \dot{\lambda}_{t+1} \right] \]

(16b) Gross growth rate of net worth

\[ \dot{\lambda}_{t+1} = \frac{1}{2} \left( \dot{V} \dot{R}_t \dot{E}_t \left[ \dot{R}_{t+1} \right] + R(1 - \dot{V} \dot{R}_t - \dot{R} \dot{R} - R) \dot{e}_t \dot{e}_t \dot{r} \dot{r} \right) \]

(17b) Gross growth rate in assets

\[ \dot{A}_{t+1} = \dot{E}_t \left[ \dot{A}_{t+1} \right] + \dot{\lambda}_{t+1} - \dot{\lambda}_t \]

(18b) Leverage

\[ \dot{e}_t = \dot{D}_t + \frac{\dot{V}}{\dot{V} \dot{r}} \dot{V}_t \]

(19b) FI constraint

\[ \dot{K}_{t+1} + \dot{Q}_t = \dot{e}_t + \dot{N}_t \]

(20b) Net worth of existing FI

\[ \dot{N}_t = \dot{N}_{t-1} + \frac{1}{2} \left( \dot{V} \dot{R}_t \dot{E}_t \left[ \dot{R}_{t+1} \right] + R(1 - \dot{V} \dot{R}_t - \dot{R} \dot{R} - R) \dot{e}_t \dot{e}_t \dot{r} \dot{r} \right) \]

(21b) Net worth of new FI

\[ \dot{N}_t^n = \dot{Q}_t + \dot{K}_t \]

(22b) Total net worth

\[ \dot{N}_t = \frac{\dot{N}_t}{\dot{N}_t} \dot{N}_t^n + \frac{\dot{N}_t}{\dot{N}_t} \dot{N}_t^n \]

(23b) Spread

\[ \dot{E}_t = \dot{E}_t \left[ \dot{R}_{t+1} \right] - \dot{R}_t \]

Variables with a ‘hat’ denote a percentage deviation from steady state, while a variable without a time subscript denotes its steady-state value. The term $Y_t^n$ represents the level of output that would prevail under flexible prices and wages without the two mark-up shocks; $E_t$ is employment; $\dot{A}_{t+1} = \mu t_{t+1} - \mu t_t$; $\mu t_t = \frac{1}{1-h} \left( h \dot{C}_{t-1} - \dot{C}_t \right)$. 

Table 1: Linearized models equations
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\beta$, discount factor</td>
<td>0.99</td>
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<tr>
<td>$\alpha$, capital income share</td>
<td>0.33</td>
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<tr>
<td>$\delta$, depreciation rate</td>
<td>0.025</td>
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<tr>
<td>$\hat{G}$, government spending to GDP ratio</td>
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</tr>
<tr>
<td>$\varepsilon$, elasticity of substitution in good market</td>
<td>set to target $M = 1.20$</td>
</tr>
<tr>
<td>$\varepsilon_w$, elasticity of substitution in labor market</td>
<td>set to target $M^{we} = 1.20$</td>
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Table 2: Calibration of parameters common to both models

<table>
<thead>
<tr>
<th>Financial Parameters</th>
<th>SWBGG model</th>
<th>SWGK model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$, survival rate</td>
<td>0.972</td>
<td>0.972</td>
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<tr>
<td>$S$, steady state spread</td>
<td>150 basis point py</td>
<td>150 basis point py</td>
</tr>
<tr>
<td>$\frac{K}{N}$, leverage ratio</td>
<td>2</td>
<td>4</td>
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<tr>
<td>$\chi$, fraction of assets given to the new bankers</td>
<td>–</td>
<td>0.001</td>
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<tr>
<td>$\lambda$, fraction of divertable assets</td>
<td>–</td>
<td>0.515</td>
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Table 3: Calibration of model-specific parameters
<table>
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<tr>
<th>Parameters</th>
<th>Distr</th>
<th>Mean</th>
<th>Std./df</th>
<th>SWBGG</th>
<th>SWGK</th>
<th>SWBGG</th>
<th>SWGK</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_p$, Calvo prices</td>
<td>Beta</td>
<td>0.75</td>
<td>0.05</td>
<td>0.82 [0.77:0.87]</td>
<td>0.84 [0.79:0.88]</td>
<td>0.89 [0.86:0.93]</td>
<td>0.89 [0.86:0.93]</td>
</tr>
<tr>
<td>$\sigma_w$, Calvo wages</td>
<td>Beta</td>
<td>0.75</td>
<td>0.05</td>
<td>0.77 [0.71:0.84]</td>
<td>0.78 [0.72:0.85]</td>
<td>0.82 [0.78:0.87]</td>
<td>0.84 [0.80:0.88]</td>
</tr>
<tr>
<td>$\sigma_{pi}$, price indexation</td>
<td>Beta</td>
<td>0.5</td>
<td>0.15</td>
<td>0.15 [0.06:0.24]</td>
<td>0.15 [0.06:0.24]</td>
<td>0.35 [0.11:0.36]</td>
<td>0.36 [0.12:0.60]</td>
</tr>
<tr>
<td>$\sigma_{wi}$, wage indexation</td>
<td>Beta</td>
<td>0.5</td>
<td>0.15</td>
<td>0.37 [0.17:0.56]</td>
<td>0.39 [0.18:0.60]</td>
<td>0.31 [0.13:0.50]</td>
<td>0.34 [0.13:0.53]</td>
</tr>
<tr>
<td>$\sigma_E$, Calvo employment</td>
<td>Beta</td>
<td>0.5</td>
<td>0.15</td>
<td>0.81 [0.78:0.84]</td>
<td>0.80 [0.76:0.83]</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\zeta$, elasticity of capital util</td>
<td>Beta</td>
<td>0.25</td>
<td>0.15</td>
<td>0.95 [0.92:0.98]</td>
<td>0.95 [0.92:0.98]</td>
<td>0.79 [0.69:0.90]</td>
<td>0.83 [0.74:0.92]</td>
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<tr>
<td>$h$, habit parameter</td>
<td>Beta</td>
<td>0.7</td>
<td>0.1</td>
<td>0.69 [0.62:0.76]</td>
<td>0.65 [0.58:0.72]</td>
<td>0.48 [0.39:0.58]</td>
<td>0.44 [0.35:0.53]</td>
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<tr>
<td>$\phi$, inverse of Frisch elasticity</td>
<td>Gamma</td>
<td>0.33</td>
<td>0.25</td>
<td>1.34 [0.81:1.86]</td>
<td>1.49 [0.95:2.04]</td>
<td>1.69 [0.95:2.39]</td>
<td>1.81 [1.05:2.57]</td>
</tr>
<tr>
<td>$\chi$, elast. of external finance</td>
<td>Beta</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04 [0.03:0.05]</td>
<td>–</td>
<td>0.05 [0.04:0.07]</td>
<td>–</td>
</tr>
<tr>
<td>$\rho_y$, Taylor rule</td>
<td>Normal</td>
<td>1.7</td>
<td>0.15</td>
<td>1.80 [1.61:2.00]</td>
<td>1.73 [1.54:1.93]</td>
<td>1.83 [1.60:2.05]</td>
<td>1.89 [1.66:2.10]</td>
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<td>0.09 [0.05:0.13]</td>
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<td>0.09 [0.05:0.13]</td>
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<td>0.88 [0.86:0.91]</td>
<td>0.89 [0.86:0.91]</td>
<td>0.84 [0.81:0.88]</td>
<td>0.85 [0.82:0.87]</td>
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<tr>
<td>$\gamma$, constant growth rate</td>
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<td>0.1</td>
<td>0.30 [0.24:0.36]</td>
<td>0.30 [0.24:0.35]</td>
<td>0.35 [0.30:0.41]</td>
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<td>$\hat{\pi}$, constant inflation</td>
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<td>0.64 [0.56:0.71]</td>
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<td>0.94 [0.89:0.99]</td>
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<td>0.2</td>
<td>0.99 [0.98:0.99]</td>
<td>0.99 [0.98:0.99]</td>
<td>0.97 [0.96:0.99]</td>
<td>0.99 [0.98:0.99]</td>
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<tr>
<td>$\rho_{\nu}$, pers. of governm shock</td>
<td>Beta</td>
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<td>0.2</td>
<td>0.92 [0.86:0.98]</td>
<td>0.92 [0.87:0.99]</td>
<td>0.95 [0.92:0.98]</td>
<td>0.96 [0.93:0.98]</td>
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<tr>
<td>$\rho_{\nu}$, pers. of investmen shock</td>
<td>Beta</td>
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<td>0.2</td>
<td>0.97 [0.95:0.99]</td>
<td>0.97 [0.96:0.99]</td>
<td>0.96 [0.93:0.99]</td>
<td>0.99 [0.98:0.99]</td>
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<tr>
<td>$\rho_{\nu}$, pers. of monetary shock</td>
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<td>0.2</td>
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<td>0.24 [0.13:0.35]</td>
<td>0.27 [0.14:0.39]</td>
<td>0.23 [0.11:0.34]</td>
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<tr>
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<td>0.2</td>
<td>0.81 [0.64:0.98]</td>
<td>0.72 [0.47:0.96]</td>
<td>0.32 [0.06:0.56]</td>
<td>0.31 [0.07:0.56]</td>
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<tr>
<td>$\rho_{\nu}$, pers. of wage mark-up shock</td>
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<td>0.59 [0.42:0.74]</td>
<td>0.58 [0.41:0.76]</td>
<td>0.17 [0.06:0.29]</td>
<td>0.20 [0.07:0.32]</td>
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<tr>
<td>$\sigma_{\nu}$, std of tech shock</td>
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<td>0.1</td>
<td>2</td>
<td>1.09 [0.84:1.33]</td>
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<td>$\sigma_{\nu}$, std of government shock</td>
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<td>2</td>
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<td>2</td>
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<td>2.11 [1.19:3.02]</td>
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<td>2</td>
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<tr>
<td>$\sigma_{\nu}$, std of price mark-up shock</td>
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<td>2</td>
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<td>0.09 [0.05:0.13]</td>
<td>0.12 [0.09:0.15]</td>
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<td>$\sigma_{\nu}$, std of wage mark-up shock</td>
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<td>0.12 [0.08:0.16]</td>
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<td>0.28 [0.23:0.33]</td>
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Table 4: Prior and posterior distributions of structural parameters
Table 5: Models comparisons

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<td>SWBGG</td>
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<tr>
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<td>Bayes factor</td>
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<td><strong>Comparison between SWBGG and SWGK</strong></td>
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<td>Bayes factor</td>
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Table 6: Moments in the data *versus* the models

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<tr>
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<td>Output</td>
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<tr>
<td>Investment</td>
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<tr>
<td>Consumption</td>
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<td>1.63</td>
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<tr>
<td>Inflation</td>
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<td>Interest rate</td>
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<td><strong>Cross-correlations with output</strong></td>
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<td>Interest rate</td>
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<tr>
<td><strong>Autocorrelations of order 1</strong></td>
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<td>Output</td>
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<td>Consumption</td>
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### Horizon Structural shocks

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<th>Wage</th>
<th>Price</th>
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### Investment

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<th>Invest.</th>
<th>Capital</th>
<th>Wage</th>
<th>Price</th>
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### Spread

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<th>Invest.</th>
<th>Capital</th>
<th>Wage</th>
<th>Price</th>
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---

**Table 7**: Variance decomposition for the Euro Area. The first number refers to the SWBGG model while the second to the SWGK model.

---

**Table 8**: Variance decomposition for the United States. The first number refers to the SWBGG model while the second to the SWGK model.
### Euro Area

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<td>horizon 4</td>
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### United States

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<td>horizon 2</td>
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</table>

Table 9: Root mean squared forecast errors of unconditional forecasts.

---

**Figure 1**: Model implied spread versus EA data (percent deviations from steady state and HP(1600) trend, respectively); data are extracted from the ECB.

**Figure 2**: Model implied spread versus US data (percent deviations from steady state and HP(1600) trend, respectively); data are extracted from the ALFRED database of the St. Louis Fed.
Figure 3: EA estimated impulse responses to the following shocks: capital quality (KQ), monetary policy (MP), government spending (Gov), and investment-specific technology (IS) in the SWBGG and SWGK models. Solid lines represent mean IRF and dashed line represent the 95% confidence intervals. The size of the shocks is normalized to one standard deviation.

Figure 4: EA estimated impulse responses to the following shocks: TFP, price mark-up (PMU) and wage mark-up (WMU) in the SWBGG and SWGK models. Solid lines represent mean IRF and dashed line represent the 95% confidence intervals. The size of the shocks is normalized to one standard deviation.
Figure 5: US Demand shocks: capital quality (KQ), monetary policy (MP), government spending (Gov). Solid lines represent mean IRF and dashed line represent the 95% confidence intervals. The size of the shocks is normalized to one standard deviation.

Figure 6: US Supply shocks: TFP, price mark-up (PMU) and wage mark-up (WMU). Solid lines represent mean IRF and dashed line represent the 95% confidence intervals. The size of the shocks is normalized to one standard deviation.
Appendix

A The Models

This section presents the three DSGE models. Subsection A.1 presents the core Smets and Wouters (SW) model. Subsection A.2 presents the optimization problems in the SWBGG that are different from the ones in the SW model. Subsection A.3 shows the analytical aspects which are peculiar to the SWGK model.

A.1 The core SW model

A.1.1 Households

The economy is populated by a continuum of households indexed by \( j \in (0, 1) \). Following Gertler and Karadi (2011), each household's preferences are represented by the following intertemporal utility function,

\[
U_t(\cdot) = \ln(C_t - hC_{t-1}) - \frac{L_t^{1+\phi}}{1 + \phi}
\]

where \( h \) measures the degree of superficial external habits in consumption, \( L_t \) is labor supply in terms of hours worked and \( \phi \) is the inverse of the Frisch elasticity of labor supply. The representative household enters period \( t \) with real government bonds, that pay the gross real interest rate, \( R_t \), between \( t-1 \) and \( t \). During period \( t \), each household chooses to consume \( C_t \); supplies \( L_t \) hours of work; and allocates savings in government bonds, \( B_{t+1} \). Each household gains an hourly real wage, \( W_t^h/P_t \); and dividend payments, \( \Pi_t \), from firms. The government grants transfers \( TR_t \) and imposes real lump-sum taxes \( T_t \). In addition, each household owns the capital stock which she rents to intermediate goods firms at a real gross rental rate \( R^H_t \). As explained by Smets and Wouters (2003), the supply of rental services from capital can be risen either by investing, \( I_t \), or by changing the utilization rate of installed capital, \( U_t \). The law of motion of capital, \( K_t \), is equal to

\[
K_{t+1} = (1 - \delta) e^K_t K_t + x_t \left[ 1 - F \left( \frac{I_t}{I_{t-1}} \right) \right] I_t
\]

where \( \delta \) stands for depreciation and \( e^K_t \) is the capital quality shock, as in Gertler and Karadi (2011), which is meant to capture exogenous variation in the value of capital due to economic obsolescence. The adjustment cost function \( F \) satisfies the following properties: \( F(1) = F'(1) = 0 \), and \( F''(1) = \xi > 0 \). The shock to the marginal efficiency of investment, \( x_t \), follows an AR(1) process, \( \rho_x \) is an autoregressive coefficient and \( e^x_t \) is a serially uncorrelated, normally distributed shock with zero mean and standard deviation \( \sigma_x \). This shock varies the efficiency with which the final good can be transformed into physical capital.

\[11\] All households choose the same allocation in equilibrium; hence, for sake of notation, the \( j \) index is dropped.
The budget constraint is as follows:

\[ C_t + I_t + \frac{B_t}{R_t} \leq \frac{W^h_t}{P_t} L_t + B_{t-1} + R^H_t U_t K_{t-1} - \Psi(U_t) K_{t-1} + \Pi_t + TR_t - T_t \]  \hspace{1cm} (5)

where \( \Psi(U_t) \) represents the costs of changing capital utilization, with \( \zeta = \Psi''(U_t)/\Psi'(U_t) \). Maximization of equation (3) subject to (4) and (5) yields the following first-order conditions with respect to \( C_t, B_{t+1}, L_t, I_t, K_t \) and \( U_t \):

\[ U_{Ct} = m\mu_t \]  \hspace{1cm} (6)

\[ \beta R_tE_t[m\mu_{t+1}] = m\mu_t \]  \hspace{1cm} (7)

\[ -U_{Lt} = m\mu_t \frac{W^h_t}{P_t} \Leftrightarrow \frac{U_{Lt}}{U_{Ct}} = -MRS_t \equiv -\frac{W^h_t}{P_t} \]  \hspace{1cm} (8)

\[ m\mu_t = m\mu_t^k x_t \left[ 1 - f \left( \frac{I_t}{I_{t-1}} \right) - f_t \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_t}{I_{t-1}} \right) \right] + \beta E_t \left[ m\mu_{t+1}^k x_{t+1} f_t \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right] \]  \hspace{1cm} (9)

\[ m\mu_t^k = \beta E_t \left[ m\mu_{t+1}^k \left( R^H_{t+1} U_{t+1} - \Psi(U_t) \right) + (1 - \delta) e_{t+1}^k m\mu_{t+1}^k \right] \]  \hspace{1cm} (10)

\[ R^H_t = \Psi'(U_t) \]  \hspace{1cm} (11)

where \( \beta \in (0, 1) \) is the discount factor, \( m\mu_t \) is the Lagrange multiplier associated with the budget constraint and let \( \Lambda_{t,t+1} \equiv \frac{m\mu_{t+1}}{m\mu_t} \). And \( m\mu_t^k \) is the Lagrange multiplier associated with capital accumulation equation. The Tobin’s Q is the ratio of the two multipliers, i.e. \( Q_t = \frac{m\mu_t^k}{m\mu_t} \).

A.1.2 The labor market

Households supply homogeneous labor to monopolistic labor unions which differentiate it. Labor service used by intermediate goods firms is a composite of differentiated types of labor indexed by \( l \in (0, 1) \)

\[ L_t = \left[ \int_0^1 L_t(l) \frac{c_w^{-1}}{c_w - 1} dl \right] \frac{\epsilon_w}{c_w - 1} \]  \hspace{1cm} (12)

where \( \epsilon_w \) is the elasticity of substitution across different types of labor. Labor packers solve the problem of choosing the varieties of labor to minimize the cost of producing a given amount of the aggregate labor index, taking each nominal wage rate \( W_t(l) \) as given:

\[ \min_{L_t(l)} \int_0^1 W_t(l) L_t(l) dl \]  \hspace{1cm} (13)

s.t. \[ \left[ \int_0^1 L_t(l) \frac{c_w^{-1}}{c_w - 1} dl \right] \frac{\epsilon_w}{c_w - 1} \geq \bar{L} \]  \hspace{1cm} (14)

The demand for labor is given by

\[ L_t(l) = \left( \frac{W_t(l)}{W_t} \right)^{-\epsilon_w} L_t \]  \hspace{1cm} (15)
where $W_t$ is the aggregate wage index. Equations (15) and (12) imply

$$W_t = \left[ \int_0^1 W_t(l)^{1-\varepsilon_w} \, dl \right]^{1-\varepsilon_w} \tag{16}$$

Labor unions adjust wages infrequently following the Calvo scheme. Let $\sigma_w$ be the probability of keeping wages constant and $(1 - \sigma_w)$ the probability of changing wages. In other words, each period there is a constant probability $(1 - \sigma_w)$ that the union is able to adjust the wage, independently of past history. This implies that the fraction of unions setting wages at $t$ is $(1 - \sigma_w)$. For the other fraction that cannot adjust, the wage is automatically increased at the aggregate inflation rate. The wage for non-optimizing unions evolves according to the following trajectory $W^*_t(l)$,

$$W^*_t(l) \left( \frac{P_t}{P_{t-1}} \right)^{\sigma_{wi}}, \ W_t(l) \left( \frac{P_{t+1}}{P_{t-1}} \right)^{\sigma_{wi}}, \ldots, \text{where } \sigma_{wi} \text{ denotes the degree of wage indexation}.$$  

The union chooses $W^*_t$ to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} N_{t,t+s} (\beta \sigma_w)^s L_{t+s}(l) \left[ \frac{W^*_t(l)}{P_{t+s}} \left( \frac{P_{t+s-1}}{P_{t-1}} \right)^{\sigma_{wi}} - \frac{W^h_{t+s}}{P_{t+s}} \right]$$

subject to the labor demand (15), and the indexation scheme so that $L_{t+s}(l) = \left[ \frac{W^*_t(l)}{W_{t+s}} \left( \frac{P_{t+s-1}}{P_{t-1}} \right)^{\sigma_{wi}} \right]^{-\varepsilon_w}$ $L_{t+s}$. The first order condition is equal to

$$\mathbb{E}_t \sum_{s=0}^{\infty} N_{t,t+s} (\beta \sigma_w)^s L_{t+s}(l) \left[ \frac{W^*_t(l)}{P_{t+s}} \left( \frac{P_{t+s-1}}{P_{t-1}} \right)^{\sigma_{wi}} - \frac{W^h_{t+s}}{P_{t+s}} M_{w,t} \right] = 0 \tag{18}$$

where $M_{w,t} = \frac{\varepsilon_w}{\varepsilon_w-1} u_{t}^{\varepsilon_w}$ is the time varying gross wage mark-up and $u_t^{\varepsilon_w}$ is the wage mark-up shock which follows an AR (1) process, $\rho_w$ is an autoregressive coefficient and $\varepsilon_t^{um}$ is a serially uncorrelated, normally distributed shock with zero mean and standard deviation $\sigma_{um}$.

The dynamics of the aggregate wage index is expressed as

$$W_{t+1} = \left[ (1 - \sigma_w) \left( \frac{W^*_t(l)}{P_{t+1}} \right)^{1-\varepsilon_w} + \sigma_w \left( W_t \left( \frac{P_t}{P_{t+1}} \right)^{\sigma_{wi}} \right)^{1-\varepsilon_w} \right]^{1-\varepsilon_w} \tag{19}$$

A.1.3 Goods market

Competitive final goods firms buy intermediate goods from the retailers and assemble them. Final output is a composite of intermediate goods indexed by $f \in (0, 1)$ differentiated by retailers,

$$Y_t = \left[ \int_0^1 Y_t(f)^{\frac{\varepsilon-1}{\varepsilon}} \, df \right]^{\frac{\varepsilon}{\varepsilon+1}} \tag{20}$$
where $\varepsilon$ is the elasticity of substitution across varieties of goods. Final goods firms solve the problem of choosing $Y_t(f)$ to minimize the cost of production:

$$
\min_{Y_t(f)} \int_0^1 P_t(f) Y_t(f) \, df
$$

subject to

$$
\left[ \int_0^1 Y_t(f) \left( \frac{\varepsilon}{1-\varepsilon} \right)^{\varepsilon} \, df \right]^{1-\varepsilon} \geq \bar{Y}
$$

The demand function for intermediate good $f$ is given by

$$
Y_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\varepsilon} Y_t
$$

where $P_t$ is the aggregate wage index. Equations (23) and (20) imply

$$
P_t = \left[ \int_0^1 P_t(f)^{1-\varepsilon} \, df \right]^{1/\varepsilon}
$$

Retailers simply purchase intermediate goods at a price equal to the marginal cost and differentiate them in a monopolistically competitive market, similarly to labor unions in the labor market. Retailers set nominal prices in a staggered fashion. Each retailer resets its price with probability $(1 - \sigma_p)$. For the fraction of retailers that cannot adjust, the price is automatically increased at the aggregate inflation rate. The price for non-optimizing retailers evolves according to the following trajectory $P_t^* (f)$, $P_t^* (f) \left( \frac{P_t}{P_{t-1}} \right)^{\sigma_{pi}}$, $P_t^* (f) \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\sigma_{pi}}$, ..., where $\sigma_{pi}$ denotes the degree of price indexation. The real price $\Phi_t$ charged by intermediate goods firms in the competitive market represents also the real marginal cost common to all final good firms, i.e. $MC_t = \Phi_t$.

A retailer resetting its price in period $t$ maximizes the following flow of discounted profits with respect to $P_t^*$

$$
E_t \sum_{s=0}^{\infty} (\sigma_p \beta)^s \Lambda_{t,s} Y_{t+s}(f) \left[ P_t^* (f) \left( \frac{P_{t+s-1}}{P_t-1} \right)^{\sigma_{pi}} - MC_{t+s} \right]
$$

subject to the demand function (23), and the indexation scheme so that $Y_{t+s}(f) = \left[ P_t^* (f) \left( \frac{P_{t+s-1}}{P_t-1} \right)^{\sigma_{pi}} \right]^{1-\varepsilon} Y_t$. Let $MC_t^n$ denote the nominal marginal cost. The gross mark-up charged by final good firm $f$ can be defined as $M_t(f) \equiv P_t(f)/MC_t^n = \frac{P_t(f)}{P_t} / \frac{MC_t}{P_t} = p_t(f)/MC_t$. In the symmetric equilibrium all final good firms charge the same price, $P_t(f) = P_t$, hence the relative price is unity. It follows that, in the symmetric equilibrium, the mark-up is simply the inverse of the marginal cost.

The first order condition for this problem is

$$
E_t \sum_{s=0}^{\infty} (\sigma_p \beta)^s \Lambda_{t,s} Y_{t+s}(f) \left[ P_t^* (f) \left( \frac{P_{t+s-1}}{P_t-1} \right)^{\sigma_{pi}} - M_{p,t} MC_{t+s} \right] = 0
$$

Similarly to the labor market, the gross time varying price mark up is $M_{p,t} = \frac{\varepsilon}{1-\varepsilon} u_{t,p}^n$ and $u_{t,p}^n$ is the price mark-up shock, which follows an AR(1) process, $\rho_p$ is an autoregressive coefficient and $\varepsilon_{t,p}$
is a serially uncorrelated, normally distributed shock with zero mean and standard deviation $\sigma_{pm}$.

The equation describing the dynamics for the aggregate price level is given by

$$P_{t+1} = \left(1 - \sigma_p\right)\left(P_{t+1}^*(f)\right)^{1-\varepsilon} + \sigma_p \left(P_t \left(\frac{P_t}{P_{t-1}}\right)^{\sigma_{pi}}\right)^{1-\varepsilon} \left[1 - \frac{1}{1-\varepsilon}\right]$$

Intermediate goods firms produce goods in a perfectly competitive market. They maximize the flow of discounted profits by choosing the quantity of factors for production

$$E_t\beta_t \Lambda_{t,t+1} \left[\Phi_{t+1}Y_{t+1} - R_{t+1}^H K_{t+1} - \frac{W_{t+1}L_{t+1}}{P_{t+1}}\right]$$

where $\Phi_t$ is the competitive real price at which intermediate good is sold and $R_t^H$ is the real rental price of capital. The production function follows a Cobb-Douglas technology:

$$Y_t = A_t(U_t \varepsilon_t^k K_t)^\alpha L_t^{1-\alpha} - \Theta$$

where $\Theta$ represents fixed costs in production (Smets and Wouters, 2007). $A_t$ is the transitory technology shock following an AR(1) process, $\rho_a$ is an autoregressive coefficient and $\varepsilon_t^a$ is a serially uncorrelated, normally distributed shock with zero mean and standard deviation $\sigma_a$. Maximization yields the following first order conditions with respect to capital and labor:

$$R_t^H = MP_t^K$$

$$\frac{W_t}{P_t} = MC_t MP_t^L$$

where $MP_t^K$ is the marginal product of capital and $MP_t^L$ is the marginal product of labor.

A.1.4 The policymaker and aggregation

The policymaker sets the nominal interest rate according to the following Taylor rule (SW, 2003)

$$\ln\left(\frac{R_t^n}{R_{t-1}^n}\right) = \rho_i \ln\left(\frac{R_{t-1}^n}{R_{t-1}^n}\right) + (1 - \rho_i) \left[\rho_{\Pi} \ln\left(\frac{\Pi_t}{\Pi}\right) + \rho_{\varepsilon_y} \ln\left(\frac{Y_t}{Y_{t-1}^*}\right)\right]$$

$$+ \rho_{\Delta_y} \ln\left(\frac{Y_t/Y_{t-1}}{Y_{t-1}^*/Y_{t-2}^*}\right) + \varepsilon_t^r$$

and

$$R_{t+1} = E_t \left[\frac{R_t^n}{\Pi_{t+1}}\right]$$

where $R_t^n$ is the nominal gross interest rate, $\Pi$ is the steady state inflation rate, $Y_t^*$ is the level of output that would prevail under flexible prices and wages without the two mark-up shocks, and $\varepsilon_t^r$ is the monetary policy shock.
The resource constraint completes the model,
\[ Y_t = C_t + I_t + G_t + \Psi(U_t)K_{t-1} \]  
(33)

A.2 The SWBGG model

The presence of financial frictions originating in the demand side of the credit market alters the set-up of intermediate goods firms compared to the SW economy. This section then presents the set-up of capital producers which determine the price of capital (this simplifies the optimization problem of households).

A.2.1 Households

In the SWBGG model capital producers purchase investment and depreciated capital to transform them into capital sold to firms and intermediate goods firms choose the optimal utilization rate of capital. Hence the household simply chooses consumption, labor supply and the amount of assets, which represent real deposits in the FI as well as real government bonds. Both intermediary deposits and government debt are one period real bonds that pay the gross real interest rate, \( R_t \), between \( t \) and \( t+1 \). Both instruments are riskless and are thus perfect substitutes. This optimization problem yields the first-order conditions (6), (7) and (8) respectively.

A.2.2 Capital producers

Capital producers purchase at time \( t \) investment and depreciated capital to transform them into capital sold to firms and used for production at time \( t+1 \). Capital producers also face adjustment costs for investment as in Christiano et al. (2005). The law of motion of capital is then equal to equation (4).

The profits are given by the difference between the revenue from selling capital at the relative price \( Q_t \) and the costs of buying capital from intermediate goods firms and the investment needed to build new capital. The optimality condition is a Tobin’s \( Q \) equation, which relates the price of capital to the marginal adjustment costs,

\[
1 = Q_t x_t \left[ 1 - F \left( \frac{I_t}{I_{t-1}} \right) - F' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_t}{I_{t-1}} \right) \right] + \beta E_t \left[ \Lambda_{t,t+1} Q_{t+1} x_{t+1} + f' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right] \]
(34)

A.2.3 Intermediate goods firms

Intermediate goods firms produce goods in a perfectly competitive market and they borrow in order to finance the acquisition of capital. They maximize the flow of discounted profits by choosing the quantity of factors for production. This problem is identical to that in the SW economy, described
by equations (28)–(30). In equilibrium the optimal capital demand is

\[ E_t \left[ R_{t+1}^k \right] = E_t \left[ \frac{R_{t+1}^H + (1 - \delta)Q_{t+1}e_{t+1}^k}{Q_t} \right] \] (35)

where \( E_t \left[ R_{t+1}^k \right] \) is the expected marginal external financing cost.

In addition firms also decide the optimal capital utilization rate solving the following maximization problem

\[ \max U_t Z_t^k U_t K_t - \Psi(U_t) K_t \] (36)

This optimization problem is summarized by the following equilibrium condition

\[ Z_t^k = \Psi'(U_t) \] (37)

Intermediate goods firms face also the problem of stipulating the financial contract. BGG assume that an agency problem makes external finance more expensive than internal funds and solve a financial contract that maximizes the payoff to the firms subject to the lender earning the required rate of return.\(^{12}\) Hence, in equilibrium, the marginal external financing cost must equate the external finance premium gross of the riskless real interest rate:

\[ R_{t+1}^k = EP \left( \frac{N_{t+1}}{Q_t K_{t+1}} \right) R_t \] (38)

with \( EP'(\cdot) < 0 \) and \( EP'(1) = 1 \). As the borrower’s equity stake in a project \( \frac{N_{t+1}}{Q_t K_{t+1}} \) falls, i.e. the leverage ratio rises, the loan becomes riskier and the cost of borrowing rises. Linearisation of equation (38) yields:\(^{13}\)

\[ \hat{R}_{t+1}^k = \hat{R}_t + \kappa [\hat{Q}_t + \hat{K}_{t+1} - \hat{N}_{t+1}] \] (39)

where \( \kappa \equiv -\frac{\partial R^k}{\partial N/K} \frac{N/K}{R^\kappa} = -\frac{EP'(\cdot)}{R^\kappa} K R \) measures the elasticity of the external finance premium with respect to the leverage position of intermediate goods firms.

Aggregate entrepreneurial net worth evolves according to the following law of motion

\[ N_{t+1} = \theta [R_t^k Q_{t-1} K_t - E_{t-1}] \left[ R_{t+1}^k (Q_{t-1} K_t - N_t) \right] + (1 - \theta) N_t^e \] (40)

where the first component of the right-hand-side represents the net worth of the \( \theta \) fraction of surviving entrepreneurs net of borrowing costs carried over from the previous period, and \( N_t^e \) is the transfer that newly entering entrepreneurs receive.

Following BGG and Gabriel et al. (2011), monitoring costs are ignored in the resource constraint since, under reasonable parameterizations, they have negligible impact on model’s dynamics.

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\(^{12}\)See BGG, Appendix, for the derivation of the financial contract and for the aggregation.

\(^{13}\)A variable with a ‘hat’ denotes a percentage deviation from steady state.
A.3 The SWGK model

The presence of financial frictions à la Gertler and Karadi does not affect the optimization problem of households, which is the same as in SWBGG, although their structure is slightly different as explained in the text. This subsection first presents the features of financial intermediaries (FI) and then of intermediate goods firms.

A.3.1 Financial intermediaries

The financial intermediaries’ balance sheet simply states that net worth and deposits should be equal to the quantity of financial claims on intermediate goods firms times the price of each claim, $Q_t S_t$. Net worth (or bank capital) evolves as follows:

$$N_{t+1} = R_{t+1}^k Q_t S_t - R_t B_{t+1}$$

where $R_{t+1}^k$ represents the non-contingent real gross return on assets.

The problem of moral hazard consists in the fact that the banker can choose to divert the fraction $\lambda$ of available funds from the project and transfer them back to her household. The depositors require to be willing to supply funds to the banker that the gains from diverting assets should be less or equal than the costs of doing so:

$$\Upsilon_t \geq \lambda Q_t S_t$$

where $\Upsilon_t$ is the expected terminal wealth, defined as

$$\Upsilon_t = E_t \sum_{s=0}^{\infty} (1 - \theta)^s \beta^{s+1} \Lambda_{t,t+1+s} \left[ \left( R_{t+1+s}^k - R_{t+s} \right) Q_{t+s} S_{t+s} + R_{t+1+s} N_{t+s} \right]$$

Equation (42) translates in the following constraint for the FI,

$$Q_t S_t = lev_t N_t$$

where $lev_t$ stands for the FI leverage ratio. The agency problem introduces an endogenous balance sheet constraint for the FI.

Total net worth is the sum of net worth of existing bankers, $N^e_t$, and net worth of new bankers, $N^n_t$, which are defined as:

$$N^e_{t+1} = \theta [(R_{t+1}^k - R_t) lev_t + R_t] N_t$$

$$N^n_t = \chi Q_t S_t$$

A.3.2 Intermediate goods firms

Intermediate goods firms maximize profits in a perfectly competitive market and borrow from FI. In order to make a meaningful comparison, the three models are as closer as possible, and the optimization problems of intermediate goods firms follow SWBGG, i.e. equations (29), (30), (37)
and (35). Each intermediate goods firm finances the acquisition of capital, $K_{t+1}$, by obtaining funds from the FI. The firm issues $S_t$ state-contingent claims equal to the number of units of capital acquired and prices each claim at the price of a unit of capital $Q_t$,

$$Q_tK_{t+1} = Q_tS_t$$ (47)

B Counterfactual exercises for impulse responses

Two considerations are worth mentioning for all the shocks: (1) a change in the spread exerts a different effect on the dynamics of the relevant net worth in the SWBGG and the SWGK models; and (2) the structural parameters differ between the two models as evident from Table 4. It is important to check whether these factors could account for the difference in the transmission mechanism of the shocks between the models.

As far as the first issue is concerned, a contractionary shock generally causes a rise in the spread, a fall in net worth and a decrease in investment. However, a rise in the spread affects the dynamics of the relevant net worth in a different way, as explained in the main text: in the SWGK model the profits of financial intermediaries increase and this helps financial intermediaries to rebuild quickly their net worth. In the SWBGG model, instead, a rise in the borrowing costs causes a protracted decline in net worth of non-financial firms. This subsection shows a counterfactual exercise in order to mitigate the effects of the spread on the dynamics of net worth of financial intermediaries in the SWGK model. In order to do so, the steady state leverage is calibrated at 2, while in the baseline calibration it is equal to 4. A reduction in the steady state leverage substantially affects the dynamics of net worth, as evident from equation (20b) in Table 1. Figures 7 and 8 show that a reduction in the steady state leverage of financial intermediaries affects impulse responses. Each chart reports the mean responses in the baseline SWGK model featuring a steady state leverage of 4 – black solid line – and the counterfactual SWGK model where steady state leverage is equal to 2 – black dotted line. The blue dashed lines report the mean responses in the baseline SWBGG model. Figure 7 shows EA impulse responses to the shocks which are quantitatively more important in terms of the variance decomposition analysis. The dynamics of net worth is substantially affected for all the three shocks. The impact response of net worth in the counterfactual SWGK model is closer to that of the SWBGG model. At the same time, the rise in the spread is more pronounced in the counterfactual SWGK model compared to the SWBGG model. This is explained by the fact that the tighter incentive constraint leads to a higher spread and, hence, to a higher profitability for financial intermediaries. In the case of the capital quality and TFP shocks, the stronger amplification effect on output and investment is still present in the counterfactual SWGK model, albeit to a minor extent. In the case of the investment-specific technology shock, the moderate increase in net worth leads to a positive response of the spread in the counterfactual experiment, causing a less pronounced increase in investment and hence a more severe decline in output. With the exception of the investment-specific technology shock, overall the magnitude of the financial accelerator effect on output in the SWGK model (lev=4) is double compared to the effect obtained in a SWGK model with leverage
equal to two.

Figure 8 shows the effect of the counterfactual experiment on the models estimated for the United States. Results are similar to those for the Euro Area: the dynamics of net worth in the SWGK model is indeed affected by the value of the steady state leverage ratio. The rise in the spread is generally more pronounced in the counterfactual SWGK model compared to the SWBGG model. However, in the case of the capital quality shock the impact response of the spread in the SWBGG model is higher than that of the counterfactual SWGK model. This can also be explained by the higher estimate of elasticity of the external finance premium with respect to the leverage position of firms in the US compared to that in the EA. And the amplification effect of the SWGK model is minor in the counterfactual model compared to the baseline specification.

The role of the different structural parameters is relevant only for the price mark-up shock in the models estimated for the Euro Area. The more severe fall in output in the SWBGG model compared to that in the SWGK model, shown in Figure 4, is only explained by the higher estimated persistence of the price mark-up shock in the former model.

C Robustness analysis

This subsection illustrates whether the better fit of the SWGK model is robust: (i) to a larger estimation sample; (ii) to the calibration of the steady state leverage ratio of the SWBGG and SWGK models; and (iii) to the models’ specification. It finally presents the likelihood race when
allowing for measurement errors for inflation and wages. All these robustness exercises are based on 100,000 draws from the random walk Metropolis-Hastings algorithm.

The first row of the Table 10 shows that, even including data for the recent financial crisis, the SWGK model still performs better. The Bayes factor and the KR statistics are of the order of magnitude similar to those in Table 5. However, these results should be interpreted cautiously due to the non-linearities induced by the zero lower bound on the nominal interest rate in the latest observations.

The importance of the value of the leverage is stressed by several studies, such as Jordà et al. (2011) among many others. In the SWBGG model a change in the steady state leverage ratio, $\frac{K}{N}$, has a direct impact on equation (16a) in Table 1. In the SWGK model a change in the steady state leverage ratio, $\text{lev}$, affects the evolution of net worth of FI, equation (20b) in Table 1. In both models any change in the leverage ratio clearly influences the financial accelerator effect as also evident from the counterfactual exercises presented in Figures 7 and 8.

The leverage ratio is equal to 2 in the SWBGG model and 4 in the SWGK model as shown in the baseline calibration, Table 3. Table 10 shows how the Bayes factor is affected by changes in the leverage ratio of the two models one at a time.\footnote{For each specification the log data density of the SWGK model is computed with the modified harmonic mean estimator.} In the SWBGG model the leverage ratio of firms changes from 1.5 to 4.5, implying that from 33% to 78% of firms’ capital expenditure are externally financed. The second column of Table 10 reports the BF between the log data density of the SWGK
model and the log data density of the SWBGG model for the EA. Similarly, the last column reports this statistics between the log data density of the SWGK model and that of the SWBGG model for the US. The comparison between the SWBGG and SWGK models shows that for both economies the SWGK model is always favored by the data independently of the value of the leverage ratio in the SWBGG model. The second part of the table shows how the Bayes factor varies when the leverage ratio of financial intermediaries changes from 3 to 5.5 in the SWGK model. There is clear evidence in favor of the SWGK model also in comparison to the SWBGG model. It is also worth noting that when firms and financial intermediaries have the same leverage ratio – 3, 3.5, 4, and 4.5 – the SWGK model is always the preferred one.

Both models embed the same types of nominal and real frictions. As a further robustness check, each of the main common frictions is turned off one at a time in the spirit of Smets and Wouters (2007). This experiment makes also it possible to assess which frictions are important. The first row of Table 11 reports the log data density of the baseline estimates. No matter which friction is turned off, there is always evidence in favor of the SWGK model compared to the SWBGG model both in the EA and the US. The removal of each friction at a time has a similar effect in the models. On the side of nominal frictions, removing price stickiness implies a considerable deterioration in terms of the log data density. On the side of real frictions, the most important in terms of the log data density is investment adjustment costs. A larger value of the capital utilization elasticity implies higher marginal depreciation cost, and therefore less variation in capital utilization. Removing this friction does not imply a deterioration of the log data density; its value is even higher in all models.

As a robustness exercise, the parameter κ, measuring the elasticity of the external finance premium to the leverage position of firms in the SWBGG model, is calibrated to match steady state values of the financial variables. In this case the log data density is equal to $-357.50398$ for the SWBGG model estimated for the EA and to $-543.82403$ for the SWBGG model estimated for the US. Hence, the SWGK model is still the one favored both by EA and US data.

Table 12 shows the log data density of the models estimated allowing for measurement errors for inflation and wages, as well as for a moving-average component in the price and wage mark up shocks. The ranking of the models is not affected, being the SWGK model the preferred one.
Changes in the steady state leverage in the SWBGG model

\[ BF = \frac{L(Y|m_{SWBGG})}{L(Y|m_{SWGK})} \]

<table>
<thead>
<tr>
<th>Steady state leverage</th>
<th>SWBGG</th>
<th>SWGK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.9 x 10^2</td>
<td>5.3 x 10^2</td>
</tr>
<tr>
<td>2</td>
<td>2.8 x 10^3</td>
<td>5.6 x 10^3</td>
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Changes in the steady state leverage in the SWGK model

\[ BF = \frac{L(Y|m_{SWGK})}{L(Y|m_{SWBGG})} \]

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</tr>
</tbody>
</table>

Table 10: Models comparisons for alternative robustness exercises

Table 11: Log data density for different models’ specifications

Table 12: Relative standard deviations to output and log data density when allowing for measurement errors