Unemployment (Fears) and Deflationary Spirals

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Abstract

The interaction of incomplete markets and sticky nominal wages is shown to magnify business cycles even though these two features – in isolation – dampen them. During recessions, fears of unemployment stir up precautionary sentiments which induces agents to save more. The additional savings may be used as investments in both a productive asset (equity) and an unproductive asset (money). But even a small rise in money demand has important consequences. The desire to hold money puts deflationary pressure on the economy, which, provided that nominal wages are sticky, increases labor costs and reduces firm profits. Lower profits repress the desire to save in equity, which increases (the fear of) unemployment, and so on. This is a powerful mechanism which causes the model to behave differently from both its complete markets version, and a version with incomplete markets but without aggregate uncertainty. In our framework, deflationary pressure means a reduction the price level, but goes together with an increase in expected inflation and a decrease in the real interest rate. Thus, our mechanism is quite different from the one emphasized in the zero-lower-bound literature. In contrast to previous results in the unemployment insurance literature, agents uniformly prefer non-trivial levels of unemployment insurance.

Keywords: Keynesian unemployment, business cycles, search friction, magnification, propagation, heterogeneous agents.

JEL Classification: E12, E24, E32, E41, J64, J65.

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1 Introduction

The empirical literature documents that workers suffer substantial losses in both earnings and consumption levels during unemployment. For instance, Kolsrud, Landais, Nilsson, and Spinnewijn (2015) use Swedish data to document that consumption expenditures drop on average by 32% during the first year of an unemployment spell.\(^1\) This observed inability to insure against unemployment spells has motivated several researchers to develop business cycle models with a focus on incomplete markets. The hope (and expectation) has been that such models would not only generate more realistic behavior for individual variables, but also be able to generate volatile and prolonged business cycles without relying on large and persistent exogenous shocks. While in existing models, individual consumption is indeed much more volatile than aggregate consumption, aggregate variables are often not substantially more volatile than their counterparts in the corresponding complete markets (or representative-agent) version. Krusell, Mukoyama, and Sahin (2010), for instance, find that imperfect risk sharing does not help in generating more volatile business cycles. McKay and Reis (2013) document that a decrease in unemployment benefits – which exacerbates market incompleteness – actually decreases the volatility of aggregate consumption.\(^2\) The reason is that a decrease in unemployment benefits increases precautionary savings, investment, the capital stock, and ultimately makes the economy as a whole better equipped to smooth consumption.

We develop a model in which the inability to insure against unemployment risk generates business cycles which are much more volatile than the corresponding complete markets version. Moreover, although the only aggregate exogenous shock has a small standard deviation, the outcome of key exercises such as changes in unemployment benefits depends crucially on whether there is aggregate uncertainty. This result is obtained by combining incomplete asset markets with incomplete adjustments of the nominal wage rate to changes in the price level.\(^3\) The impact of shocks is prolonged by Diamond-Mortensen-Pissarides search frictions in the labor market.

Before explaining why the combination of incomplete markets and sticky nominal wages amplifies business cycles, we first explain why these features by themselves dampen business cycles in our model in which aggregate fluctuations are caused by productivity shocks. First, consider a model in which there are complete markets, but nominal wages do not respond one-for-one to price level changes. A negative productivity shock induces agents to reduce their demand for money, since the present is worse than the future and agents would like to smooth consumption. Moreover, less money is needed at lower activity levels. The decline in money demand puts upward pressure on the

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\(^1\) Appendix A provides a more detailed discussion of the empirical literature investigating the behavior of individual consumption during unemployment spells.

\(^2\) As discussed in section 7.1, a decrease in unemployment benefits does increase the volatility of output in the model of McKay and Reis (2013), but the effects are small relative to our results.

\(^3\) We discuss the empirical motivation for these assumptions in section 2 and appendix A.
price level. Provided that nominal wages are sticky, the resulting downward pressure on real wages mitigates the reduction in profits caused by the direct negative effect of a decline in productivity. The result is a muted aggregate downturn, since a smaller reduction in profits implies a smaller drop in employment. Next, consider a model in which nominal wages are flexible, but workers cannot fully insure themselves against unemployment risk. Forward-looking agents understand that a persistent negative productivity shock increases the risk of being unemployed in the near future. If workers are not fully insured against this risk, the desire to save increases for precautionary reasons. However, increased savings leads to an increase in demand for all assets, including productive assets such as firm ownership. This counteracting effect alleviates the initial reduction in demand for productive assets which was induced by the direct negative effect of a reduced productivity level, and therefore dampens the increase in unemployment. In either case, sticky nominal wages or incomplete markets lead – in isolation – to a muted business cycle.

Why does the combination of incomplete markets and sticky nominal wages lead to the opposite results? As before, the increased probability of being unemployed in the near future increases agents’ desire to save more in all assets. However, the increased desire to hold money puts downward pressure on the price level, which in turn increases real labor costs and reduces profits. This latter effect counters any positive effect that increased precautionary savings might have on the demand for productive investments. Once started, this channel will reinforce itself. That is, if precautionary savings lead – through downward pressure on prices – to increased unemployment, then this will in turn lead to a further increase in precautionary savings, and so on. When does this process come to an end? At some point, the expanding number of workers searching for a new job reduces the expected cost of hiring, which makes it attractive to resume job creating investments.

In addition to endogenizing unemployment, the presence of search frictions in the labor market adds further dynamics to this propagation mechanism. First, the value of a firm – i.e. the price of equity – is forward-looking. As a consequence, a prolonged increase in real labor costs leads to a sharp reduction in economic activity already in the present, with an associated higher risk of unemployment. Second, with low job-finding rates unemployment becomes a slow moving variable. Thus, the increase in unemployment is more persistent than the reduction in productivity itself.

Our mechanism is not due to unusually restrictive monetary policy as is the case in the zero-lower-bound literature. Key in the zero-lower-bound literature is deflationary pressure that manifests itself in a reduction in expected inflation and possibly even deflation combined with the inability to reduce the policy rate to values below zero. This leads to an increase in the real interest rate, which in turn leads to a deterioration of the economy and further deflationary pressure. In our framework, however, deflationary pressure means a reduction in the price level that actually goes together with an increase in expected inflation and a substantial decrease in the real interest rate. Thus, monetary...
policy does not constitute an amplifying effect on its own.\footnote{Nevertheless, as discussed in section 7.2, there may exist more aggressive monetary policies that undo all nominal frictions. At least in theory.}

We use our framework to study the advantages of alternative unemployment insurance (UI) policies. We first document that the effects of changes in unemployment benefits on the behavior of aggregate variables and on the well-being of workers differ from the effects in other models. For example, in the model of Krusell, Mukoyama, and Sahin (2010) most agents benefit from reductions in unemployment benefits even when benefits are reduced to very low levels. We consider a permanent increase in the replacement rate from the benchmark value of 50\% of the prevailing wage rate to 55\% and document that this increase in insurance improves the welfare of all agents, provided that the policy switch occurs in a recession. This is true if wage rates adjust upwards to take into account the strengthened bargaining position of workers and if they do not.\footnote{Although wages are “just” a transfer, they have first-order welfare consequences. The reason is that the equilibrium employment level is below the social optimum. Wage increases reduce job creation and increase the gap between the equilibrium and the socially desirable employment level.} \footnote{Whether all agents prefer the switch during an expansion depends crucially to which extent wages adjust.}

There are a number of factors affecting agents’ welfare that are important for this result. As a preview of the analysis, let us mention some that operate in our model, but have not been previously emphasized in the literature. Consider a permanent increase in unemployment benefits at the onset of a recession. This obviously benefits the unemployed directly. But the employed benefit too. Firstly, they benefit because they are better insured against future income drops associated with unemployment spells. Secondly, by reducing the negative downward spiral discussed above, the employed are now less likely to be unemployed in the near future. Thirdly, and perhaps more surprisingly, employed agents also benefit because the dampening of the downward spiral implies that the value of their asset holdings drops less relative to the case in which unemployment benefits are not increased.

These features contrast with those exposed in the existing literature, in which increased unemployment benefits brings forth adverse aggregate consequences that eclipse the gains of reduced income volatility (e.g. Young (2004) and Krusell, Mukoyama, and Sahin (2010)). In particular, with lower fluctuations in individual income the precautionary motive weakens, and aggregate investment falls. The result is a decline in average employment and output, with adverse effects on welfare. This channel is important in our model as well. However, in the version of our model with aggregate uncertainty, there are two quantitatively important factors that push average employment in the opposite direction, and can overturn the negative effect associated with the reduction in precautionary savings. The first is that the demand for the productive asset can increase, because an increase in the level of unemployment benefits stabilizes asset prices as well as business cycles. The second is that the nonlinearity in the matching process is such that increases in employment during
expansions are smaller than reductions during recessions. Consequently, a reduction in volatility can lead to an increase in *average* employment (cf. Jung and Kuester (2011)).

An important aspect of our model is that precautionary savings can be used for investments in both the productive asset (firm ownership) and the unproductive asset (money). This complicates the analysis, because the numerical procedure requires a simultaneous solution for a portfolio choice problem for each agent, and for equilibrium prices. Our numerical analysis ensures that the market for firm ownership (equity) is in equilibrium and all agents owning equity discount future equity returns with the correct, that is, their own individual-specific, intertemporal marginal rate of substitution (MRS). By contrast, a typical set of assumptions in the literature is that workers jointly own the productive asset at equal shares, that these shares cannot be sold, and that discounting of the returns of this asset occurs with some average MRS or an MRS based on aggregate consumption.\(^7\),\(^8\) One exception is Krusell, Mukoyama, and Sahin (2010) who – like us – allow trade in the productive asset and discount agents’ returns on this asset with the correct marginal rate of substitution.\(^9\)

In section 2, we provide empirical motivation for the key assumptions underlying our model: sticky nominal wages and workers’ inability to insure against unemployment risk. We also discuss the relationship between savings and idiosyncratic uncertainty. In section 3, we describe the model. In section 4, we discuss the calibration of our model. In sections 5 and 6, we describe the behavior of individual and aggregate variables, respectively. In section 7, we discuss how business cycle behavior is affected by alternative UI policies.

## 2 Empirical motivation

Our model is motivated by the behavior of prices, nominal wages, and unit labor costs in the Eurozone during the recent economic downturn. The first observation is that the growth in the price level slowed considerably during the crisis relative to the trend. This is documented in the top panel of figure 1, which plots the GDP deflator for the Eurozone alongside its pre-crisis trend.\(^10\) To investigate whether nominal wages followed the slowdown in inflation, the second panel of


\(^8\)An alternative simplifying assumption is that the only agents who are allowed to invest in the productive asset are agents that are not affected by idiosyncratic risk (of any kind). Examples are Rudanko (2009), Bils, Chang, and Kim (2011), Challe, Matheron, Ragot, and Rubio-Ramirez (2014), and Challe and Ragot (2014). Bayer, Luetticke, Pham-Dao, and Tjaden (2014) analyze a more challenging problem than ours, in which firms are engaged in intertemporal decision making. However, in contrast to our model, these firms are assumed to be risk neutral, consume their own profits, and discount the future at a constant geometric rate.

\(^9\)The procedure in Krusell, Mukoyama, and Sahin (2010) is only exact if the aggregate shock can take on as many realizations as there are assets and no agents are at the short-selling constraint. Our procedure does not require such restrictions, which is important, because the fraction of agents at the constraint is nontrivial in our model.

\(^10\)The pre-crisis trend is defined as the time path the deflator would have followed if inflation beyond the forth quarter of 2007 had been equal to the average inflation rate over the five preceding years.
figure 1 displays nominal hourly earnings together with the GDP deflator. The panel also shows the realizations of both variables if they would have grown at rates equal to their pre-crisis trends. We find that nominal wages continued to grow at pre-crisis rates or above, despite a substantial reduction in inflation rates. This means that real wages increased relative to trend.\textsuperscript{11}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Key Eurozone variables before and after the financial crisis.}
\end{figure}

\textit{Notes.} Panel (a) illustrates the Eurozone GDP deflator together with its pre-crisis trend. Panel (b) illustrates nominal hourly earnings, the GDP deflator, and their associated pre-crisis trends. Panel (c) illustrates nominal unit labor costs together with the GDP deflator. Data sources are given in B.

The observed increases in real wages are not necessarily due to a combination of low inflation\textsuperscript{11}Similarly, Daly, Hobijn, and Lucking (2012) and Daly and Hobijn (2013) document that real wages increased during the recent recession in the US.
and downward nominal wage rigidity. It is possible that solid real wage growth reflects an increase in labor productivity, for example, because workers that are laid off are less productive than those that are not. To shed light on this possibility, we compare the nominal unit labor cost with the price level.\footnote{The nominal unit labor cost is defined as the cost of producing one unit of output, i.e., the nominal wage rate divided by labor productivity. The price index used as comparison is the price index used in defining labor productivity.} The results are shown in the bottom panel of figure 1. The panel shows that nominal unit labor costs have grown faster than prices since the onset of the crisis, whereas the opposite was true before the crisis. This indicates that real labor costs increased during the crisis even if one corrects for productivity.\footnote{The observation that real unit labor costs are not constant over the business cycle is interesting in itself. If the real wage rate is equal to the marginal product of capital and the marginal product is proportional to average labor productivity – properties that hold in several business cycle models – then real unit labor costs would be constant.}

These observations are consistent with the hypothesis that the combination of deflationary pressure and nominal wage stickiness increased labor costs.\footnote{Throughout this paper, we will use the term deflationary pressure broadly. In particular, we also use it – as is the case here – to indicate a slowdown in inflation relative to trend.} In principle, it is still possible that nominal wages in the Eurozone did respond fully to prices. However, in that case, it must be true that the reduction in employment is mainly due to an outflow of workers that earn low wages and could produce at low real unit labor cost, since both real wages and real unit labor costs increased. That is, it must be the case that the workers who left employment were the ones who had a wage that was low relative to their productivity. This does not seem plausible.

The pattern displayed in figure 1 is not universally true in all economic downturns. In fact, nominal unit labor cost grew at a slower rate than prices in the US, even though there was a slowdown in inflation and no slowdown in nominal wages.

3 Model

The economy consists of a unit mass of households, a large mass of potential firms, and one government. The mass of active firms is denoted $q_t$, and all firms are identical. Households are ex-ante homogenous, but differ ex-post in terms of their employment status (employed or unemployed) and their asset holdings.

\textbf{Notation.} Upper (lower) case variables denote nominal (real) variables. Variables with subscript $i$ are household specific. Variables without a subscript $i$ are either aggregate variables or variables that are identical across agents, such as prices.
3.1 Households

Each household consists of one worker who is either employed, $e_{i,t} = 1$, or unemployed, $e_{i,t} = 0$. The period-$t$ budget constraint of household $i$ is given by

$$P_t c_{i,t} + J_t (q_{i,t+1} - (1 - \delta) q_{i,t}) + M_{i,t+1} = (1 - \tau_t) W_t e_{i,t} + \mu (1 - \tau_t) W_t (1 - e_{i,t}) + D_t q_{i,t} + M_{i,t},$$

where $c_{i,t}$ denotes consumption of household $i$, $P_t$ the price of the consumption good, $M_{i,t}$ the amount of the money held at the beginning of period $t$, $W_t$ the nominal wage rate, $\tau_t$ the tax rate on nominal income, and $\mu$ the replacement rate. Money can be thought of as a liquid asset, since it facilitates transactions, whereas the other asset, equity, does not. The variable $q_{i,t}$ denotes the amount of equity held at the beginning of period $t$. One unit of equity pays out nominal dividends $D_t$. In each period, a fraction $\delta$ of all firms go out of business which leads to a corresponding loss in equity. When the term $q_{i,t+1} - (1 - \delta) q_{i,t}$ is positive, the worker is buying equity, and vice versa. The nominal value of this transaction is equal to $J_t (q_{i,t+1} - (1 - \delta) q_{i,t})$, where $J_t$ denotes the nominal price of equity ex dividend.

Households are not allowed to take short positions in equity, that is

$$q_{i,t+1} \geq 0. \quad (2)$$

The household maximizes the objective function

$$\mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{c_{i,t}^{1-\gamma} - 1}{1-\gamma} + \chi \left( \frac{M_{i,t+1}}{P_t} \right)^{1-\xi} - 1 \right) \right],$$

subject to constraints (1), (2), and with $M_{i,0}$ and $q_{i,0}$ given. The utility aspect of money comes with the salient feature that the investment portfolio of the less wealthy will be skewed towards the liquid asset.

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15 We will use the terms “liquid asset” and “money” interchangeably.
16 We assume that households hold a diversified portfolio of equity, which means that the portfolio depreciates at rate $\delta$.
17 If money and consumption enter the utility function additively, then money does not enter the Euler equation of other assets directly, which is consistent with the empirical results in Ireland (2004).
18 The least wealthy will only hold money.
The first-order conditions are given as

\[
c_{i,t}^{-\gamma} = \beta E_t \left[ c_{i,t+1}^{-\gamma} \frac{P_t}{P_{t+1}} \right] + \chi \left( \frac{M_{t+1}}{P_t} \right)^{-\zeta}, \tag{3}
\]

\[
c_{i,t}^{-\gamma} \geq \beta E_t \left[ c_{i,t+1}^{-\gamma} \left( \frac{D_{t+1} + (1 - \delta) J_{t+1}}{J_t} \right) \frac{P_t}{P_{t+1}} \right], \tag{4}
\]

\[
0 = q_{i,t+1} \left( c_{i,t}^{-\gamma} - \beta E_t \left[ c_{i,t+1}^{-\gamma} \left( \frac{D_{t+1} + (1 - \delta) J_{t+1}}{J_t} \right) \frac{P_t}{P_{t+1}} \right] \right). \tag{5}
\]

Equation (3) represents the Euler equation with respect to real money balances; equation (4) the Euler equation with respect to equity; and equation (5) captures the complementary slackness condition associated with the short-selling constraint in equation (2).\(^{19}\)

**Characteristics of the liquid asset.** The utility component of money captures the idea that money holdings facilitate transactions. This aspect of money does not play a key role in the mechanism we emphasize. This feature is helpful, however, in solving for the households’ portfolio problems.\(^{20}\) The following characteristics of the liquid asset do matter. First, in addition to the transaction motive, agents also hold money to insure themselves against unemployment risk.\(^{21}\) Second, it serves as the unit of account. In particular, wages are expressed in units of the liquid assets. This means that real wages are affected if nominal wages do not respond one-for-one to changes in the price level. Third, money is not an intermediated asset that ends up in productive use. As discussed below, shifts into and out of this safer but unproductive asset play a key role in our model. It is important for our story that there is not an additional agent in the economy who is willing to always absorb risk and finance risky investments in productive assets by issuing a safe liquid asset.\(^{22}\)

### 3.2 Active firms

An active firm produces \(z_t\) units of the output good in each period, where \(z_t\) is an exogenous stochastic variable that is identical across firms. The value of \(z_t\) follows a first-order Markov process with a low (recession) and a high (expansion) value.\(^{23}\)

\(^{19}\)The utility specification implies that agents will always choose a positive value of real money balances. Short positions in the liquid asset would become possible if the argument of the utility function is equal to \((M_{t+1} + \Phi)/P_t\) with \(\Phi > 0\) instead of \(M_{t+1}/P_t\). At higher values of \(\Phi\), agents can take larger short positions in money and are, thus, better insured against unemployment risk. Increases in \(\chi\) – while keeping \(\Phi\) equal to zero – have similar implications, since higher values of \(\chi\) imply higher average levels of financial assets.

\(^{20}\)The transactions component “anchors” the portfolio and avoids large swings in the portfolio decision.

\(^{21}\)Telyukova (2013) documents that households hold more liquid assets than they need for buying goods.

\(^{22}\)More precisely, what matters for us is that there are limits to such risk absorption and that there are periods when the economy as a whole prefers to shift out of the risky and into the safer liquid asset.

\(^{23}\)Although the model is solvable for richer processes, this simple specification for \(z_t\) helps in keeping the computational burden manageable.
There is one worker attached to each active firm. Thus, the number of active firms, \( q_t \), is equal to the economy-wide employment rate. The nominal wage rate, \( W_t \), is the only cost to the firm. Consequently, nominal firm profits, \( D_t \), are given by

\[
D_t = P_t z_t - W_t. \tag{6}
\]

The nominal wage rate is set according to the rule

\[
W_t = \omega_0 \left( \frac{z_t}{\bar{z}} \right)^{\omega_z} \left( \frac{P_t}{\bar{P}} \right)^{\omega_p} \bar{P}, \tag{7}
\]

where \( \bar{z} \) is the average productivity level, \( P_t \) is the price level, and \( \bar{P} \) is the average price level.\(^{24}\) A key aspect of this paper is on the responsiveness of nominal wages, \( W_t \), to nominal prices, \( P_t \). Therefore, we need a wage-setting rule which allows us to vary this responsiveness. The parameter \( \omega_p \) controls how responsive wages are to changes in the price level. If \( \omega_p \) is equal to one, for instance, then nominal wages adjust one-for-one to changes in \( P_t \). By contrast, nominal wages are entirely unresponsive to changes in \( P_t \) if \( \omega_p \) is equal to zero. The coefficient \( \omega_z \) indicates the sensitivity of the wage rate to changes in productivity and – together with \( \omega_p \) – controls how wages vary with business cycle conditions. The coefficient \( \omega_0 \) indicates the fraction of output that goes to the worker when \( z_t \) and \( P_t \) take on their average values, and pins down the steady state value of firm profits in real terms. When we change the level of unemployment benefits, we allow \( \omega_0 \) to change to keep average implied Nash-bargaining weight the same.

**Wages of new and existing relationships.** What matters in labor market matching models is the flexibility of wages of newly hired workers. Haefke, Sonntag, and van Rens (2013) argue that wages of new hires respond almost one-to-one to changes in labor productivity. Gertler, Huckfeldt, and Trigari (2014), however, argue that this result reflects changes in the composition of new hires and that – after correcting for such composition effects – the wages of new hires are not more cyclical than wages of existing employees. More importantly, however, what matters for our paper is whether nominal wages respond to changes in the price level, and this question is not addressed in either paper. Druant, Fabiani, Kezdi, Lamo, Martins, and Sabbatini (2009) find that many firms do not adjust wages to inflation. One would think that their results apply to new as well as old matches, since their survey evidence focuses on the firms’ main occupational groups not on the wages of individual workers. Further evidence for that wages do not fully respond to changes in the price

\(^{24}\)The specified wage is always in the worker’s bargaining set, since the wage rate exceeds unemployment benefits, there is no home production nor any disutility from working, and the probability of remaining employed exceeds the probability of finding a job. The parameters are chosen such that the wage rate is never so high that the firm would prefer to fire the worker.
level is discussed in appendix A.

### 3.3 Government

The government taxes wages to finance unemployment benefits. Since the level of unemployment benefits is equal to a fixed fraction of the wage rate – and since taxes are proportional to wage income – the government’s budget constraint can be written as

$$τ_t q_t W_t = (1 - q_t) μ (1 - τ_t) W_t.$$  \hspace{1cm} (8)

From this equation, we get an expression for the tax rate, $τ_t$, which only depends on the employment rate. That is,

$$τ_t = \frac{μ (1 - q_t)}{q_t + μ (1 - q_t)}.$$  \hspace{1cm} (9)

An increase in $q_t$ means that there is an increase in the tax base and a reduction in the number of unemployed. Both lead to a reduction in the tax rate.

### 3.4 Firm creation and equity market

Agents that would like to increase their equity position in firm ownership, i.e., agents for whom $q_{i,t+1} - (1 - δ) q_{i,t} > 0$, can do so by buying equity at the price $J_t$ from agents that would like to sell equity, i.e., from agents for whom $q_{i,t+1} - (1 - δ) q_{i,t} < 0$. Alternatively, agents who would like to obtain additional equity can also acquire new firms by creating them. How many new firms are created by investing $v_{i,t}$ real units depends on the number of unemployed workers, $u_t$, and the aggregate amount invested, $v_t$. In particular, the aggregate number of new firms created is equal to

$$h_t \equiv q_t - (1 - δ) q_{t-1} = ψ v_t^{θ} u_t^{1−η},$$  \hspace{1cm} (10)

and an individual investment of $v_{i,t}$ results in $(h_t / v_t) v_{i,t}$ new firms. In equilibrium, the cost of creating one new firm, $v_t / h_t$, has to be equal to the real market price, $J_t / P_t$, since new firms are identical to existing firms. Setting $v_t / h_t$ equal to $J_t / P_t$ and using equation (10) gives

$$v_t = \left( \frac{ψ J_t}{P_t} \right)^{1/(1−η)} u_t.$$  \hspace{1cm} (11)

Thus, investment in new firms is increasing in $J_t / P_t$ and increasing in the mass of workers looking for a job, $u_t$.

Equilibrium in the equity market requires that the supply of equity is equal to the demand for
equity. That is,

\[ h_t + \int_{i \in A_-} \left( (1 - \delta) q_i - q(e_i, q_i, M_i; s_t) \right) dF_t(e_i, q_i, M_i) = \int_{i \in A_+} (q(e_i, q_i, M_i; s_t) - (1 - \delta) q_i) dF_t(e_i, q_i, M_i), \]

with

\[ A_- = \{ i : q(e_i, q_i, M_i; s_t) - (1 - \delta) q_i \leq 0 \}, \]
\[ A_+ = \{ i : q(e_i, q_i, M_i; s_t) - (1 - \delta) q_i \geq 0 \}, \]

and where \( F_t(e_i, q_i, M_i) \) denotes the cross-sectional cumulative distribution function in period \( t \) of the three individual state variables: the employment state, \( e_i \), money holdings, \( M_i \), and equity holdings, \( q_i \). The variable \( s_t \) denotes the set of aggregate state variables and its elements are discussed in Section 3.6.

Combining the last three equations gives

\[ \psi^{1/(1-\eta)} \left( \frac{J_t}{P_t} \right)^{\eta/(1-\eta)} u_t = \int_{i \in A} (q(e_i, q_i, M_i; s_t) - (1 - \delta) q_i) dF_t(e_i, q_i, M_i), \]

with \( A = \{ A_+ \cup A_- \} \).

Our representation of the “matching market” looks somewhat different than usual. As documented in appendix C, however, it is equivalent to the standard search-and-matching setup. Our way of “telling the story” has two advantages. First, there is only one type of investor, namely the household. That is, we do not have entrepreneurs who fulfill a crucial arbitrage role in the standard setup, but attach no value to their existence or activities pursued. Second, all agents in our economy have access to the same two assets: firm ownership and money. By contrast, households and entrepreneurs have different investment opportunities in the standard setup.\(^{25}\)

### 3.5 Money market

Equilibrium in the market for money holdings requires that the net demand of households wanting to increase their money holdings is equal to the net supply of households wanting to decrease their

\(^{25}\)There is one other minor difference. In our formulation, there is no parameter for the cost of posting a vacancy and there is no variable representing the number of vacancies. Our version only contains the product, i.e., the total amount invested in creating new firms. In the standard setup, the vacancy posting cost parameter is not identified unless one has data on vacancies. The reason is that different combinations of this parameter and the scalings coefficient of the matching function can generate the exact same model outcomes as long as the number of vacancies are not taken into consideration.
money holdings. That is,

\[ \int_{i \in \mathcal{B}_-} (M_i - M(e_i, q_i, M_i; s_i)) \, dF_i(e_i, q_i, M_i) \]

\[ = \int_{i \in \mathcal{B}_+} (M(e_i, q_i, M_i; s_i) - M_i) \, dF_i(e_i, q_i, M_i), \quad (14) \]

with

\[ \mathcal{B}_- = \{ i : M(e_i, q_i, M_i; s_i) - M_i \leq 0 \}, \]

\[ \mathcal{B}_+ = \{ i : M(e_i, q_i, M_i; s_i) - M_i \geq 0 \}. \]

Money supply, \( \overline{M} \), is constant in the benchmark economy. In section 7.2, we discuss the role of monetary policy.

### 3.6 Equilibrium and model solution

In equilibrium, the following conditions hold: (i) asset demands are determined by the households’ optimality conditions, (ii) the cost of creating a new firm equals the market price of an existing firm, (iii) the demand for equity from households that want to buy equity equals the creation of new firms plus the supply of equity from households that want to sell equity, (iv) the demand for the liquid assets from households that want to increase their holdings is equal to the supply from households that want to reduce their holdings, and (v) the government’s budget constraint is satisfied.

The state variables for agent \( i \) are individual asset holdings, employment status, and the aggregate state variables. The latter consist of the aggregate productivity level, \( z_t \), and the cross-sectional joint distribution of employment status and asset holdings, \( F_t \). We use an algorithm similar to the one used in Krusell and Smith (1998) to solve for the laws of motion of aggregate variables. The numerical procedure is discussed in appendix G.2.

### 3.7 Discounting firm profits correctly with heterogeneous ownership

With incomplete markets and heterogeneous firm ownership, the question arises how to discount future firm profits. In our model, each and every firm owner discounts firm profits as indicated by the agent’s individual optimality condition. The reason is that agents can buy and sell equity. This means that the Euler equation for equity is satisfied with equality for all investors holding equity, which implies that all firm owners discount the proceeds of the equity investment with the correct, i.e., their own, individual-specific, MRS.\(^{26}\) Our numerical algorithm ensures that market prices

\(^{26}\)Krusell, Mukoyama, and Sahin (2010) describe a procedure to discount firm profits (almost) correctly. They assume that the number of assets is equal to the number of realizations of the aggregate shock. Firm profits can then be
and quantities are such that the equilibrium conditions as well as each agent’s Euler equations are satisfied.

In our model, all agents can choose to invest in the risky productive asset and in the less risky and unproductive asset. Previous research studying precautionary savings and idiosyncratic risk often assumes that agents can only trade in the unproductive asset. The productive asset is then subject to some form of communal ownership, with fixed ownership shares that can never be sold no matter how keen an agent would be to do so.\textsuperscript{27} Aggregate investment decisions in the productive asset are then determined by an “Euler equation” using an MRS based either on aggregate consumption; on an average of the marginal rate of substitution of all agents; or on risk neutral geometric discounting. Another approach is to assume that there exist two distinct types of agents: One type of agent faces idiosyncratic risk but cannot invest in the productive asset; the other who can invest in the productive asset, but is not affected by any type of idiosyncratic risk. Since there is no ex-post heterogeneity within the group of the latter type, their analysis lends itself to a representative agent, which then dictates the aggregate investment decisions in the economy.\textsuperscript{28} Both approaches simplify the analysis considerably, but both direct any possible consequences of precautionary savings induced by idiosyncratic risk towards the unproductive asset only, which limits our understanding of the effect of idiosyncratic risk on business cycles.

A long outstanding and unresolved debate in corporate finance deals with firm decision making when owners are heterogeneous and markets are incomplete. This is not an issue here since active firms do not take any decisions beyond that of having entered the market.\textsuperscript{29} If firms had to make such decisions, we would have to deal with this challenging issue and specify how firm decisions are made. Conditional on this specification, however, our approach can still be used to solve for equilibrium prices, and firm owners would still discount firm profits correctly.

\section{Calibration}

This section starts with a discussion of the parameter values that play a key role in generating the results, followed by a discussion of the remaining and less crucial parameter values. The model period is one quarter. Targets are constructed using Eurozone data from 1980 to 2012.\textsuperscript{30} We focus discounted with the prices of the two corresponding contingent claims and this would be exactly correct if borrowing or short-sell constraints are not binding for any investor. Our procedure allows investors to be constrained and the number of realizations of the aggregate shock can exceed the number of assets.


\textsuperscript{28}Examples are Rudanko (2009), Bils, Chang, and Kim (2011), McKay and Reis (2013), Challe, Matheron, Ragot, and Rubio-Ramirez (2014), and Challe and Ragot (2014).

\textsuperscript{29}Note that firm creation is a static decision and all agents in the economy would compare the cost of creating one firm, \(v_t/h_t\), and its market value, \(J_t/P_t\), in the same way.

\textsuperscript{30}Details about data sources are given in appendix B.
on the Eurozone for two reasons. First, as discussed in section 2, Eurozone inflation slowed down during the crisis and nominal unit labor costs did not. Second, many economists have warned of the risks of a deflationary spiral in the Eurozone.\footnote{According to the May 2014 survey of the Centre for Macroeconomics, half of the macroeconomists in the panel thought that there was a significant risk of sustained negative inflation in the coming two years. Details can be found at http://cfmsurvey.org/surveys/euro-area-deflation-and-risk-uk-economy-may-2014. For our story, inflation does not have to be negative. It is sufficient if deflationary pressure lowers inflation, which increases real labor costs when nominal wages are sticky.}

**Key parameter values.** Regarding the choice of key parameter values, our strategy is to show that our main results can be generated with conservative choices. For example, we set the coefficient of relative risk aversion equal to 2. Even though risk aversion is not that high, the differences between our heterogeneous-agent model and the representative-agent version are substantial. There are also important differences between agents’ behavior in the heterogeneous-agent model without aggregate uncertainty and the analogue in the model with aggregate uncertainty.

The incidence and duration of unemployment spells are obviously important. The probability of job destruction, $\delta$, and the parameter characterizing efficiency in the matching market, $\psi$, are chosen to ensure that the unemployment rate and the expected duration of an unemployment spell in the economy without aggregate risk match their observed counterparts, which are equal to 10.7\% and 3.57 quarters, respectively.\footnote{Finding the right parameter values requires solving the model numerous times, which is very computer intensive for the model with aggregate uncertainty. For that reason, we calibrate these parameters by matching the targets in the model without aggregate uncertainty. The corresponding statistics in the model with aggregate uncertainty are somewhat different; the average unemployment rate is equal to 11.7\% and the average duration is equal to 4.03 quarters.} These numbers correspond to a 3.36\% quarterly job separation rate and a value for $\psi$ equal to 0.574, implying a quarterly matching probability of 28\%.

Unemployment insurance regimes vary a lot across countries in Europe. Esser, Ferrarini, Nelson, Palme, and Sjöberg (2013) report that net replacement rates for insured workers vary from 20\% in Malta to just above 90\% in Portugal. Most countries have net replacement rates between 50\% and 70\% with an average duration of around one year. Coverage ratios vary from about 50\% in Italy to 100\% in Finland, Ireland, and Greece. Net replacement rates for workers that are not covered are much lower. In most countries, these are less than 40\%. In the model, unemployment benefits are set equal to 50\% and – for computational convenience – are assumed to last for the entire duration of the unemployment spell. A replacement rate of 50\% is possibly a bit less than the average observed, but this is compensated for by the longer duration of unemployment benefits in the model and the universal coverage.

The inability to fully insure against unemployment risk plays a key role. It is, therefore, important that the model generates a realistic drop in consumption during an unemployment spell. While data for the Eurozone is unavailable for this purpose, Kolsrud, Landais, Nilsson, and Spinnewijn (2015) provide evidence for Sweden. They report that consumption drops on average by 34\% during the...
first year of an unemployment spell. A key parameter to target this number is the scale parameter, \( \chi \), which characterizes the liquidity benefits of money.\(^{33}\) This parameter affects the average level of financial assets held and, thus, the ability of agents to insure against unemployment spells. The literature also provides some evidence on pre-displacement wealth levels. Gruber (2001) provides evidence for the US. In section 5, we will show that our calibration is conservative. That is, we generate the targeted consumption drop without making agents unrealistically poor.

The main focus of this paper is on the interaction between sticky nominal wages and the deflationary pressure arising from uncertain job prospects. Consequently, a key role is played by \( \omega_p \), the parameter that indicates how responsive nominal wages are to changes in the price level. Our benchmark value for \( \omega_p \) is equal to 0.7, which means that a 1\% increase in the price level leads to an 0.7\% increase in nominal wages. Druant, Fabiani, Kezdi, Lamo, Martins, and Sabbatini (2009) report that only 6\% of European firms adjust wages (of their main occupational groups) more than once a year to inflation and only 50\% do so once a year.\(^{34}\)

Finally, the curvature parameter in the utility component for liquidity services, \( \zeta \), plays an important role, because it directly affects the impact of changes in future job security on the demand for the liquid asset. With more curvature, the demand for the liquid asset is less sensitive and increased concerns about future job prospects will generate less deflationary pressure. We follow Lucas (2000), and target a money demand elasticity with respect to the nominal interest rate equal to \(-0.5\). The resulting value of \( \zeta \) is equal to 2.\(^{35}\)

**Other parameter values.** Based on the empirical estimates in Petrongolo and Pissarides (2001), the elasticity of the job finding rate with respect to tightness, \( \eta \), is set equal to 0.5. The average share of the surplus received by workers, \( \omega_0 \), and the elasticity of the wage rate with respect to changes in aggregate productivity, \( \omega_z \), are set such that the standard deviation of employment relative to the standard deviation of output are in line with their empirical counterpart.\(^{36}\)

\(^{33}\)Its calibrated value is equal to 4.00e-5.

\(^{34}\)Moreover, even if firms adjust for inflation they typically do so using backward looking measures of inflation, which reduces the responsiveness to changes in inflationary pressure.

\(^{35}\)The (hypothetical) first-order condition for a bond with a risk-free nominal interest rate is given by

\[
1 = \beta (1 + R_t) \mathbb{E}_t \left[ \left( \frac{c_{i,t+1}}{c_{i,t}} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \right].
\]

Using \((1 + R_t)^{-1} \approx 1 - R_t\), we get

\[
\ln \left( \frac{M_{i,t+1}}{P_t} \right) \approx -\zeta^{-1} \ln R_t + \zeta^{-1} (\ln \chi + \gamma \ln c_{i,t}).
\]

The other key parameter in money demand functions is the elasticity with respect to income, which captures the volume of transactions. Our transactions variable is consumption and the elasticity of money demand with respect to consumption is equal to \( \gamma / \zeta \), which equals 1 for our choices for the coefficient of relative risk aversion, \( \gamma \), and \( \zeta \).

\(^{36}\)In the benchmark economy, \( \omega_0 = 0.97 \) and \( \omega_z = 0.3 \).
In our model, the presence of idiosyncratic risk lowers average real rates of return. This motivates us to set the discount factor, $\beta$, to 0.985, which is slightly below its usual value of 0.99.\textsuperscript{37} The two values for $z_t$ are 0.978 and 1.023 and the probability of switching is equal to 0.025.\textsuperscript{38} Finally, money supply, $M$, is chosen such that the average price level, $\bar{p}$, is equal to 1.

**Parameters values in the representative-agent model.** We will compare the results of our model with those generated by the corresponding representative-agent economy. Parameter values in the representative-agent model are identical to those in the heterogeneous-agent model, except for $\beta$. We choose the value of $\beta$ for the representative-agent model such that the MRS in the representative-agent model is equal to the MRS for the agents holding equity in the heterogeneous-agent model.\textsuperscript{39} Without this adjustment, the agent in the representative-agent economy would have a more shortsighted investment horizon and average employment would be lower.

## 5 Agents’ consumption, investment and portfolio decisions

In section 5.1, we describe key aspects of the behavior of individual consumption, and in particular its behavior during an unemployment spell. In section 5.2, we focus on the individuals’ investment portfolio decisions. Appendix D provides more detailed information on both topics.

### 5.1 Post-displacement consumption

The model’s parameters are calibrated such that the one-year drop equals its empirical equivalent, that is 34%. Although not targeted, the model predicts a proportional decrease over the first year similar to what is observed in the data.\textsuperscript{40} However, whereas the data indicate that the fall in consumption settles down after one year, the model suggests that this happens after two years. Nevertheless, the distribution of the duration of unemployment is such that most unemployment spells do not exceed one year.\textsuperscript{41}

There are several reasons why the drop in consumption is of such a nontrivial magnitude. One reason is, of course, that unemployment benefits are only half as big as labor income. But a key

\textsuperscript{37}At this relatively high 6\% annual discount rate, the average real rate of return is already quite low, namely 0.72\% on an annual basis in the economy with aggregate uncertainty.

\textsuperscript{38}These values are chosen to ensure that $\mathbb{E}[\ln z_t] = 0$, $\mathbb{E}_t[\ln z_{t+1}] = 0.95 \ln z_t$, and $\mathbb{E}_t[(\ln z_{t+1} - \mathbb{E}_t[\ln z_{t+1}])^2] = 0.007^2$.

\textsuperscript{39}Here we use the models without aggregate uncertainty. Notice that the expected intertemporal marginal rate of substitution, $\beta \mathbb{E}_t \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma}$, is constant and the same for all agents holding equity in the model without aggregate shocks.

\textsuperscript{40}See Kolsrud, Landais, Nilsson, and Spinnewijn (2015).

\textsuperscript{41}81\% of all unemployment spells that start in an expansion last at most four quarters. The corresponding number for recessions is 61\%.
factor affecting the magnitude of the drop is the average level of wealth at the beginning of an unemployment spell. Using US data, Gruber (2001) finds that the median agent holds enough gross financial assets to cover 73% of the average net-income loss during an unemployment spell. Moreover, in terms of net financial assets, the median agent does not even have enough to cover 10% of the average net-income loss. In our model, the median agent’s asset holdings are equal to 54% of the average net-income loss during unemployment spells. This is true for both gross and net asset holdings, since there is no debt in our model. Furthermore, Gruber (2001) documents that 38% of all workers do not have enough assets to cover 25% of the average net-income loss. In our model, the corresponding fraction is only 7%. One would expect that agents accumulate even less savings in Europe where unemployment benefits are higher. Thus, it is not the case that we rely on unrealistically low wealth levels to generate the sizable fall in post-displacement consumption observed in the data.

The question arises why infinitely-lived agents do not build a wealth buffer that insulates them better against this consumption volatility, as is the case in the model of Krusell and Smith (1998). The key parameter used to match the observed decline in post-displacement consumption is the scaling coefficient affecting the utility of money, \( \chi \). Choosing a low value for \( \chi \) to match the observed decline in consumption implies that money holdings – one of the two wealth components – are, on average, lower. The other wealth component is the value of equity holdings, \( J_t q_{i,t} \). Elevated uncertainty about future individual consumption increases the expected value of an agent’s MRS, which would increase the price of equity, \( J_t \). As a consequence, the number of new firms as well as the total number of shares outstanding would therefore rise. However, there are several reasons why this component of wealth is not very large in our model. First, the equity price, \( J_t \), cannot increase by too much, because the presence of a liquid asset with a positive transactions benefit puts a lower bound on the average real return on equity. Moreover, the nonlinearity of the matching function dampens the impact of an increase in equity prices on the creation of new firms. Lastly, the equity price depends positively on the average share of output going to firm owners, \( 1 - \omega_0 \). To generate sufficient volatility in employment we chose a relatively high value for \( \omega_0 \), which reduces the value of \( J_t \). For all these reasons, agents in our model do not build up large buffers of real money balances or equity to insure themselves against the large declines in consumption upon and during

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42In contrast to Gruber (2001), Kolsrud, Landais, Nilsson, and Spinnewijn (2015) do not provide pre-displacement wealth levels as a function of expected earnings losses. But some information is available. In particular, using an average unemployment spell duration of 4 months, the median Swedish agent’s level of gross financial assets is equal to roughly 13% of average net-income loss. In our model, calibrated to an average level of European unemployment benefits, net income drops by more (by half as opposed to one third in Sweden), but agents that become unemployed are wealthier.

43For the model of Krusell and Smith (1998), solved in Den Haan and Rendahl (2010) using a 15% unemployment replacement rate, the average post-displacement consumption drop is 5% after one year, whereas we find a 34% drop with a 50% unemployment replacement rate.
Another aspect affecting consumption during unemployment is the ability to borrow. In our model, agents cannot go short in any asset, and they would presumably hold less financial assets if they had the option to borrow. Kolsrud, Landais, Nilsson, and Spinnewijn (2015) report, however, that the amount of consumption that is financed out of an increase in debt actually decreases following a displacement. More importantly, we think that the key feature to capture is the level of the drop in consumption upon and during unemployment, and not whether this is accomplished by limited borrowing or by some borrowing combined with a lower level of financial assets.

**State dependence of consumption drop.** Figure 2 presents a scatter plot of the reduction in consumption (y-axis) and beginning-of-period cash on hand (x-axis), where both are measured in the period when the agent becomes unemployed.\(^{44}\) There are two distinct patterns, one for expansions and one for recessions.\(^{45}\)

![Figure 2: Consumption drop upon becoming unemployed.](image)

Figure 2 documents that the drop in consumption is, on average, *much* more severe if the unemployment spell initiates in a recession. The figure also underscores the non-trivial role played by the agents’ wealth levels. In particular, during recessions the decline in consumption varies from

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\(^{44}\)Cash on hand is equal to the sum of non-asset income (here unemployment benefits), money balances, dividends, and the value of equity holdings.

\(^{45}\)The level of employment is also important for the observed decline in consumption, which explains the scatter of observations. In particular, the fall in the level of consumption is smaller at the beginning of an expansion and larger at the beginning of a recession. The reason is that expected investment returns are higher (lower) at the beginning of the expansion (recession), which would put upward (downward) pressure on consumption when the income effect dominates the substitution effect.
18.9% for the richest agent to 35.1% for the poorest. This range increases during an expansion: The richest agent faces a modest drop of 8.8%, whereas the poorest agent can expect to see consumption fall by 33.9%. The latter is only slightly below the equivalent number in a recession.

There are several reasons why consumption falls by more during recessions. First, job finding rates are on average lower in recessions than in expansions. As a consequence, agents anticipate longer unemployment spells and will, for a given amount of cash on hand, therefore reduce consumption more sharply. A second factor that affects agents’ reduced consumption is the amount of cash on hand they hold. Because the price of equity declines in recessions, so does the agents’ cash on hand. Indeed, the average value of cash on hand held by a newly unemployed agent is equal to 1.26 in a recession compared to 1.68 in an expansion.

In reality, a typical worker may not face such a large decline in the value of their equity position when the economy enters a recession. After all, quite a few workers do not own equity at all. We think, however, that the cyclicality of post-displacement consumption behavior that is driven by the cyclicality of equity prices, captures real world phenomena. First, although not all workers hold equity, many hold assets such as housing that also have volatile and cyclical prices. Second, unemployed workers may receive handouts, and or loans, from affluent family members, friends, or financial intermediaries whose ability and willingness may be affected by the value of their assets, which is likely to be cyclical.

5.2 Investment decisions

A key aspect of our heterogeneous-agent model is that money demand increases during recessions, whereas it decreases in the representative-agent version. In this section, we shed light on this difference.

Money demand and cash on hand. A key result of this paper is that the interaction between sticky nominal wages and the inability to insure against unemployment risk deepens recessions. An integral part of the mechanism underlying this result is the upward pressure on money demand that emerges during recessions when job prospects worsen in the model with heterogeneous agents. When the economy enters a recession, aggregate money demand is pushed in opposite directions by different factors. First, during recessions consumption smoothing motives reduce demand for all assets, including real money balances. Because of incomplete markets, however, there are two further reasons that explain why aggregate demand for money increases in our economy. As documented in figure 3, for given cash-on-hand levels, all agents demand more money during recessions. This is consistent with the observed shift towards more liquid assets during the great
recession. Lastly, unemployed agents demand more money than employed agents, and there are more unemployed agents in the economy during recessions. The first effect is dominated by the others, and aggregate money demand increases during recessions. This result stands in sharp contrast with the representative-agent version of our economy, in which aggregate money demand unambiguously decreases during recessions.

Figure 3: Money demand (real).

Notes. This figure displays the amount invested in the liquid asset as a function of beginning-of-period cash on hand for workers of the indicated employment status and for both outcomes of aggregate productivity.

To see that this is a remarkable result, consider the economy without aggregate uncertainty. The price level, wages, and the equity price are all constant in this economy. The first-order condition for equity, equation (4), then implies that the expected marginal rate of substitution does not depend on the employment status for all agents that are not at the short-sale constraint. The first-order condition for money, equation (3), then specifies that real money balances and consumption must move in the same direction. Since unemployed agents consume less than employed agents with the same level of cash on hand, they should hold less real money balances. Although this simple reasoning abstracts from the presence of the short-sale constraint, the implication is that absent

\[ J_t = \beta E_t \left[ \left( \frac{c_{i,t+1}}{c_{i,t}} \right)^{-\gamma} \right] (D + (1 - \delta)J_{t+1}), \]

which implies that the individual MRS, \( \beta E_t \left[ (c_{i,t+1}/c_{i,t})^{-\gamma} \right] \), is pinned down by aggregate prices only, and is therefore not affected by employment status.

\[ \text{20} \]
aggregate uncertainty *all* unemployed agents would hold less real money balances than employed agents with the same amount of cash on hand. 48 By contrast, as indicated in figure 3, the opposite is true in the economy with aggregate uncertainty.

**Financial assets during unemployment spells.** Consumers cushion the drop in consumption following displacement by selling equity. Although the total amount of financial assets, and the amount invested in equity, sharply decrease, the amount held in the liquid asset actually *increases* during the first two periods of an unemployment spell. The loss of wage income means that workers’ cash-on-hand levels drop when they become unemployed. This reduces the demand for real money balances. However, and as discussed above, the unemployed actually hold more money for a given level of cash on hand. The last effect dominates in the beginning of an unemployment spell.

6 Economic aggregates over the business cycle

In the previous section, we showed that the inability of agents to insure against unemployment risk means that workers face a sharp drop in consumption when they become unemployed. We also discussed how imperfect insurance affects money demand in ways that are not present in an economy with complete markets. In this section, we discuss what this implies for aggregate activity. In particular, we document and explain why the interactions between sticky nominal wages, gloomy outlooks regarding future employment prospects, and the inability to insure against unemployment risk can deepen recessions. We compare the business cycle properties of the benchmark economy with imperfect risk sharing and sticky nominal wages, i.e. \( \omega_P < 1 \), to those of an economy with full risk sharing. Subsequently, we discuss the same comparison when nominal wages are not sticky, i.e., \( \omega_P = 1 \). Before doing so, we discuss some summary statistics.

6.1 Summary statistics

Table 1 reports the standard deviation of some key variables as predicted by the model and compares the outcomes with the observed counterparts of three Eurozone countries: France, Germany, and Italy. We HP-filter the data to extract the business cycle components. The variables considered are output, \( z_t q_t \), employment, \( q_t \), the share price, \( J_t / P_t \), the price level, \( P_t \), and real unit labor costs, \( W_t / (z_t P_t) \). As the empirical counterpart of \( P_t \), we consider both the CPI and the PPI. From the firms’

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48A similar reasoning can be used to make clear that it is also remarkable that employed as well as unemployed agents hold more real money balances when aggregate economic conditions deteriorate. Envisage a partial equilibrium version of our model with aggregate uncertainty in which all prices are constant, and there is no short-sale constraint. Markets are still incomplete because the agents cannot insure fully against unemployment risk. As discussed in the text, consumption and money demand move in the same direction under these conditions. But the reduction in the job finding rate lowers consumption. Thus, the demand for real money balances must decrease as well.
perspective, the PPI would be more relevant, but from the workers’ perspective the CPI would be more appropriate. In the model, there is no such distinction.

Table 1: Model predictions vs. data counterparts.

<table>
<thead>
<tr>
<th>Standard deviation of</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>France</td>
<td>Germany</td>
</tr>
<tr>
<td>(z_t q_t)</td>
<td>0.0150</td>
<td>0.0113</td>
</tr>
<tr>
<td>(q_t)</td>
<td>0.0084</td>
<td>0.0069</td>
</tr>
<tr>
<td>(J_t / P_t)</td>
<td>0.11</td>
<td>0.14</td>
</tr>
<tr>
<td>(P_t)</td>
<td>0.0227</td>
<td>0.0101</td>
</tr>
<tr>
<td>(P_t^*)</td>
<td>0.0227</td>
<td>0.0192</td>
</tr>
<tr>
<td>(W_t / (z_t P_t))</td>
<td>0.0142</td>
<td>0.0073</td>
</tr>
<tr>
<td>(W_t / (z_t P_t^*))</td>
<td>0.0142</td>
<td>0.0224</td>
</tr>
</tbody>
</table>

Notes. The table shows the standard deviation of HP-filtered logarithm of output, \(z_t q_t\), employment, \(q_t\), real asset prices, \(J_t / P_t\), the price level, \(P_t\), and unit labor costs, \(W_t / (z_t P_t)\), for the model and for three Eurozone countries using postwar data. The variable \(P_t\) refers to the consumer price index (CPI), and \(P_t^*\) to the producer price index (PPI). The model outcome is the same, since there is no such distinction in the model. All time series include the great recession and we use the longest possible sample. Details are given in appendix B.

The model captures general volatility quite well, although some variables are too volatile and others are not volatile enough. For example, share prices are not volatile enough and real unit labor costs are not nearly as volatile as their empirical counterpart when the PPI is used. The latter indicates that the model may underestimate the negative impact of deflationary pressures. On the other hand, employment and the price level are too volatile. Output volatility matches German output volatility well, but exceeds French and Italian output volatility.

6.2 The role of imperfect insurance when nominal wages are sticky

Figure 4 shows the normalized impulse response functions (IRFs) of key aggregate variables to a negative productivity shock for our benchmark economy and for the corresponding representative-agent economy. In our benchmark calibration, productivity takes on only two values. The IRFs are calculated as follows. The starting point is period \(s\), when productivity takes on its “expansion” value and employment is equal to its mean value conditional on being in an expansion. We then calculate the following two time paths for each variable. The “no-shock” time path is the expected time path from this point onward. The “shock” time path is the expected time path when the productivity switches to the low value in period \(s + 1\). The IRF is the difference between these two time paths. Our IRFs are based on a drop in \(z_t\) from its high to its low value, which corresponds to two times the unconditional standard deviation of \(z_t\). Usually, IRFs are based on a change in \(z_t\) that is equal to one standard deviation of its innovation. To make our results comparable to the literature, we normalize our IRFs. Our two-state process for \(z_t\) is constructed to correspond to the standard AR(1) specification for \(z_t\) with an innovation standard deviation equal to 0.007. Thus we multiply all IRFs by 0.007 and divide by two times the unconditional standard deviation of \(z_t\).
incomplete risk sharing faces a much deeper recession than the economy with complete risk sharing. In particular, in response to an 0.7% drop in productivity, output falls by 1.2% in the heterogeneous-agent economy and by only 0.7% in the representative-agent economy.

The key aspect in understanding this large difference is the behavior of the price level. In the representative-agent economy, the reduction in real activity decreases the demand for money and increases the price level. In our benchmark calibration wages are sticky ($\omega_p = 0.7$), and a 1% increase in the price level leads to a 0.7% increase in nominal wages and therefore a 0.3% decrease in real wages. Figure 4 documents that the real wage drops by 0.6% in the representative-agent economy, which is driven by the direct effect of the reduction in $z_t$, since $\omega_z > 0$, and the indirect effect through $P_t$, since $\omega_P < 1$. Thus, the direct effect of the reduction of productivity, $z_t$, on profits is counteracted, because nominal wages do not fully respond to the increase in the price level. That is, our starting point is an economy in which the sluggish response of nominal wages to changes in prices actually dampens the economic downturn.

By contrast, the price level falls in the heterogeneous-agent economy. This fall is caused by an increase in the aggregate demand for the safer asset, i.e., money. To understand this different outcome, consider again figure 3, which illustrates the relationship between the demand for money as a function of beginning-of-period cash-on-hand levels during expansions and recessions, for both employed and unemployed agents. The reduction in real activity lowers cash-on-hand levels which reduces the demand for money by both employed and unemployed agents. The drop in cash-on-hand levels is substantial because the value of equity declines sharply. Nevertheless, aggregate money demand increases. As previously discussed, both employed and unemployed agents hold more money during recessions for the same cash-on-hand level. In addition, there are more unemployed agents during recessions, and unemployed agents have larger money holdings than employed at the same level of cash on hand.

As documented in figure 4, the heterogeneous-agent model predicts an increase in the real wage rate. That is, the direct effect of the reduction in $z_t$ is dominated by the indirect effect through deflationary pressure. The predicted increase in real wages is smaller and less persistent than the observed increase in real wages in the Eurozone.\(^{50}\) Our model may, thus, underestimate the negative impact of the deflationary channel.

\(^{50}\)See figure 1.
Notes. These graphs illustrate the difference between the expected time path when the economy is in an expansion in period 0 and the expected time path conditional on being in a recession in period 1. \( \omega_p = 0.7 \), i.e., nominal wages increases with 0.7% when prices increase with 1%.
Whereas sticky nominal wages reduce the depth of recessions in the representative-agent economy, they worsen recessions in the heterogeneous-agent economy. This is a quantitatively important effect, because a reduction in the price level (for any reason) starts a self-reinforcing process that deepens recessions. In particular, the reduction in the price level puts upward pressure on real wages, which reduces profits. The fall in profits reduces investment in new jobs, which in turn reduces employment. Since this reduction in employment is persistent, employment prospects worsen. With elevated risk there is a further increase in the demand for money, which in turn puts additional upward pressure on the price level, and so on. The impulse responses show that this mechanism is powerful enough to completely overturn the dampening effect that sticky nominal wages have in an economy with complete risk sharing.

Although this is a powerful mechanism, there is a counterforce. In particular, a higher unemployment rate increases the probability that a firm finds a worker, even at a given level of investment. For the results reported here, this counterforce is strong enough to ensure stability. For some parameter values, the fluctuations could very well become so large that no non-explosive solution exists.\(^{51}\)

### 6.3 Role of imperfect insurance when nominal wages are not sticky

In this section, we discuss business cycle properties when changes in the price level leave real wages unaffected, that is, \(\omega_P = 1\). Real wages are then always procyclical, since \(\omega_z > 0\). Figure 5 plots the IRFs for the heterogeneous-agent economy and the IRFs for the corresponding representative-agent economy. There are several similarities with our benchmark results, but also one essential difference. We start with the similarities.

A negative productivity shock still has a direct negative effect on profits, which leads to a reduced demand for equity (firm ownership), which in turn means that fewer jobs are created. Also, increased concerns about employment prospects still induce agents in the heterogeneous-agents economy to increase their demand for money holdings, which again is strong enough to push the price level down, while it increases in the representative-agent economy.

\(^{51}\)In particular, changes in parameter values that substantially enhance the deflationary mechanism make it computationally challenging, or even impossible, to find an accurate solution. This does not prove that a stationary solution does not exist, but it would be consistent with this hypothesis. If such a solution would exist, however, it is likely to have complex nonlinear features. This result is in contrast to standard perturbation methods which impose that aggregate shocks of any size will not destabilize the economy as long as arbitrarily small shocks do not. For example, the technique developed in Reiter (2009) to solve models with heterogeneous agents relies on a perturbation solution for changes in the aggregate shock, which implies that the solution is imposed to be stable – for shocks of any size – as long as the Blanchard-Kahn conditions are satisfied, that is, when the solution is stable for small shocks. Our experience suggests that this may impose stability where there is none.
Figure 5: Impulse responses with flexible nominal wages

Notes. These graphs illustrate the difference between the expected time path when the economy is in an expansion in period 0 and the expected time path conditional on being in a recession in period 1. $\omega_p = 1$, that is, nominal wages respond 1-for-1 to price changes.
There is also a striking difference. In the economy with flexible nominal wages, recessions are less severe when agents cannot insure themselves against unemployment risk. The reason is the following. Increased uncertainty, alongside with an expected reduction in individual consumption, increase the expected value of the marginal rate of substitution. This affects the first-order condition of the liquid asset as well as the first-order condition of the productive investment, because each agent’s future revenues of both assets are (correctly) discounted by the agent’s own individual MRS. Since wages are flexible, the associated rise in the price level bears no consequence on the return on equity, and the chain of events underlying the deflationary spiral breaks down.

Thus, if the rise in precautionary savings is partially used as productive investments, then this would dampen the reduction in the demand for equity induced by the direct negative effect of the productivity shock on profits. The IRFs document that this is indeed the case when nominal wages respond one-for-one to changes in the price level. The magnitude of the dampening effect is nontrivial. Whereas the biggest drop in employment is 0.53ppt in the representative-agent economy, it is equal to 0.43ppt in the heterogeneous-agent economy. These results make clear that a researcher would bias the model predictions if this dampening aspect of precautionary savings is not allowed to operate, for example, because there is communal firm ownership.52

In our benchmark economy, we allow this channel to operate, but the effect is dominated by the interaction between sticky nominal wages and uninsurable unemployment risk. Increased uncertainty may increase the demand for equity, but it will also increase the demand for money. Increased money demand depresses the price level, which, provided that nominal wages are somewhat sticky, increases real wages. The rise in real wages reduces profits, which in turn lowers the demand for equity. This channel dominates any positive effect that precautionary savings may have on the demand for equity.

Robustness of the dampening effect. In all cases considered, we find that recessions are less severe in the heterogeneous-agent economy than in the representative-agent economy if nominal wages respond one-for-one to changes in the price level. That is, this dampening effect is very robust. During the nineties, several papers argued that an increase in idiosyncratic risk could lead to a reduction in the demand for a risky asset when investors can save through both a risky and a risk-free asset even though it would increase total savings. This effect is referred to as temperance.53 We find that this result is quite fragile for several reasons.

The first is that it is a partial equilibrium result. In general equilibrium prices adjust. This is important. Suppose that the economy as a whole can increase savings through the risky investment, but not through the risk-free investment. Then the relative price of the risk-free asset would increase

52See footnote 7 for a list of papers following this approach.
53See Kimball (1990), Kimball (1992), Gollier and Pratt (1996), and Elmendorf and Kimball (2000).
making the riskier asset more attractive. This plays a role in our economy, because the only way the economy as a whole can do something now to have more goods in the future is by investing more in the productive asset, that is, in the risky asset. There are several other features, typically present in macroeconomic models, that make temperance less likely. One is that the temperance result relies on idiosyncratic risk to be sufficiently independent of investment risk. In macroeconomic models, that is not the case. The amount of idiosyncratic risk depends on the level of the wage rate.\textsuperscript{54} But the level of the wage rate is often correlated with the return of the risky asset, since both are affected by the same shocks.\textsuperscript{55} In appendix F, we show that this feature alone can overturn the temperance result even in a partial equilibrium setting. Another feature that works against the temperance result is the short-sale constraint on equity, which directly prevents a reduction in the demand for equity, at least for some agents. In our model, diminishing returns on the transactions aspect of money also work against temperance. This makes increased investment in the risk-free asset less attractive relative to a framework in which the return remains fixed.

It may be the case that temperance can be generated in models with different utility functions, for example, if the utility function is such that the price of risk increases during recessions.\textsuperscript{56} We leave this for future research.

7 Government policy

In this section, we discuss the two components of government policy in this model: unemployment insurance and monetary policy.

7.1 Unemployment-insurance (UI) policies

In this section, we analyze the impact of alternative unemployment-insurance policies. In our model, changes in such policies affect the economy quite differently than in many other models. Our results do not only differ from those of the standard labor search business cycle model with a representative agent, but also from those with heterogeneous agents, such as the models of Krusell, Mukoyama, and Sahin (2010) and McKay and Reis (2013).

The experiments we consider are straightforward. In the first set of experiments, we solve the model for a range of values of the replacement rate, $\mu$, and we report the resulting effect on the average employment rate conditional on the economy being in an expansion and in a recession, as

\textsuperscript{54} The consequences of idiosyncratic risk are reduced when the wage rate falls. In the extreme case when the wage rate is zero, there is no unemployment risk.

\textsuperscript{55} In the model considered here, they are both directly affected by $z_t$.

\textsuperscript{56} We considered models with different degrees of risk aversion, but this does not seem to matter for this particular issue.
well as its unconditional, or expected, value. These experiments are conducted both when changes in the replacement rate affect the wage rule, and when they do not. In the second set of experiments, we also solve the model for a range of values of the replacement rate, $\mu$, but now consider an unexpected and permanent change in $\mu$ and we discuss the effect on agents’ welfare, taking into account transition dynamics. Section 7.1.1 discusses the first set of experiments and section 7.1.2 discusses the second set. In appendix E, we discuss some additional results such as the impact of the replacement rate on unemployment duration and on aggregate transition dynamics.

### 7.1.1 Changes in the replacement rate: Comparative statics

Figure 6 illustrates the impact of changes in the replacement rate on employment levels. The value of the replacement rate, $\mu$, is provided on the $x$-axis, and the resulting employment rate on the $y$-axis. The top row shows the results with sticky wages, both when changes in the replacement rate does not (left graph) and does (right graph) affect the wage-setting rule.

UI changes when nominal wages are sticky and wage-setting rule is not affected. First, consider the case without aggregate uncertainty. An increase in the replacement rate means that agents are better insured against idiosyncratic risk, which lowers the expected value of their MRS. The latter triggers a decrease in precautionary savings, which decreases investment and employment.\(^\text{57}\)

For the case with aggregate uncertainty, we report results for values of $\mu$ equal to and above 0.5.\(^\text{58}\) For values of $\mu$ above 0.6, the case with aggregate uncertainty is very similar to the case without aggregate uncertainty. The “expansion” and “recession” employment levels form a band around the no-aggregate-uncertainty employment level with a roughly constant width. As the replacement rate increases beyond 0.6, all three employment levels decrease.

When $\mu$ is between 0.5 and 0.6, however, our deflationary mechanism is quantitatively important and an increase in the replacement rate leads to a sharp decrease in aggregate volatility, that is, the band narrows substantially. For example, consider a rise in $\mu$ from 0.5 to 0.55. The increase in the replacement rate leads to a 50.3% decrease in the standard deviation of the employment rate. The reason for this decline is that improved insurance lowers the strength of the deflationary mechanism. Indeed, the standard deviation of individual consumption is reduced by 15.3%.

\(^{57}\)At low values of $\mu$, however, changes in the replacement rate have virtually no effect on the employment level. The reason is that the presence of money puts a lower bound on the expected return on equity, and therefore an upper bound on the expected MRS. As a consequence, equity prices are bounded from above, which – through the free-entry condition – implies that employment is as well. In the model without aggregate uncertainty, the presence of money implies that the real return on firm ownership cannot be less than minus the (constant) inflation rate.

\(^{58}\)At lower levels, the deflationary mechanism becomes very strong. This makes it harder to obtain an accurate solution and it might even be the case that at a sufficiently low value for $\mu$ there is no stable solution to the model.
Figure 6: Average employment and replacement rates.

Notes. The left column displays the results when wage setting is not affected by changes in the replacement rate, $\mu$. The right column displays the results when wages setting is such that the implied average Nash-bargaining weights are kept constant when $\mu$ changes. The top row presents the results when nominal wages do not fully respond to changes in the price level and the bottom row presents results when they do.

The figure also documents that the increase in $\mu$ not only decreases aggregate volatility, it can also increase the average employment rate. In particular, the increase of $\mu$ from 0.5 to 0.55 increases the average employment rate with 0.31ppt. By contrast, in the version of our model without aggregate uncertainty the same increase in the replacement rate leads to a decrease in average employment of 0.52ppt. Such comparative statics typically result in similar answers for economies with and without aggregate uncertainty, because aggregate uncertainty is relatively small. Volatility of the only aggregate exogenous random variable, productivity, is indeed modest.
in our model. Nevertheless, the economy with aggregate uncertainty responds to a change in the replacement rate quite differently than the economy without aggregate uncertainty.

Lastly, it is interesting to note the highly nonlinear effects on employment in recessions for moderate, and for large increases of the replacement rate. The average employment rate conditional on being in recessions is first increasing in \( \mu \), reaches a peak around 0.575, and then continues to decline. It should be noted that the average employment rate in recessions is no lower than its benchmark value, corresponding to \( \mu = 0.5 \), even for values of \( \mu \) as high as 0.8.

In the remainder of this section, we explain why an increase in unemployment insurance can lead to an increase in the average employment rate, when the value of the replacement rate is such that the deflationary mechanism is quantitatively important. The starting point is the 0.52ppt decrease in employment in the economy without aggregate uncertainty mentioned above. In the economy with aggregate uncertainty, there are two effects associated with an increase in the replacement rate that increases the demand for equity and, thus, job creation. The first effect is that more insurance reduces the risk of holding equity, because an increase in the replacement rate not only reduces the volatility of real activity, but also leads to a substantial reduction in stock price volatility. In fact, if \( \mu \) increases from 0.5 to 0.55 the standard deviation of the real equity price drops by 49.8%. This reduction in risk and increased demand for equity leads to more job creation and an increase in average employment of 0.42ppt.\(^{59}\) The second effect is related to the nonlinearity of the matching process; that is, increases in equity prices have a smaller effect on job creation than decreases.\(^{60}\) For the same change in the replacement rate as before, the decrease in the volatility of the real equity price increases average employment through this channel with 0.41ppt.\(^{61}\) We now have all the ingredients to explain why the employment rate increases with 0.31ppt. If we add the 0.41ppt

\[ \psi^{1/(1-\eta)} \left( \frac{J}{P} \right)^{\eta/(1-\eta)} (1-q) \approx \delta q. \]

Since \( \eta = \frac{1}{2} \), we get that

\[ q \approx 1 - \frac{\delta}{\delta + \psi^2 J/P}. \]

Thus, \( q \) is a concave function of \( J/P \).

\(^{59}\)We calculate this as follows. If there is no aggregate uncertainty, then the increase in \( \mu \) leads to a decrease in employment of 0.52ppt and a decrease in real equity value of 5%. If there is aggregate uncertainty, then the same change in \( \mu \) leads to a decrease in the average real equity value by only 1%. This lower drop in equity value is due to the fact that there also is a decrease in aggregate uncertainty. Assuming that these effects are linear, the difference between the 5% and the 1% drop corresponds to an increase in average employment of 0.416ppt (\( = 4/5 \times 0.52 \) ppt).

\(^{60}\)Ignoring transitions – which occur fast in this model – Equation (13) implies that

\[^{61}\]We calculate this as follows. When \( \mu = 0.5 \), then the introduction of aggregate uncertainty leads to a reduction in employment of 1.01ppt of which 46ppt can be explained by the reduction in the average equity price. The remainder of 0.55ppt is, thus, due to the nonlinearity of the matching function. When \( \mu = 0.55 \), then this nonlinearity effect is only 0.14ppt. Thus, when \( \mu \) increases from 0.5 to 0.55 in the economy with aggregate uncertainty, then there is a reduction of the impact of the nonlinearity on average employment of 0.55ppt−0.14ppt=0.41ppt.
increase due to the nonlinearity of the matching function to the 0.42ppt increase due to a reduction in risk, we get an increase in the employment rate of 0.83ppt. If we subtract the 0.52ppt decrease due to the reduction in savings because of better individual insurance, we get an increase in average employment of 0.31ppt.\(^6\)

A direct effect of an increase in \(\mu\) is an increase in the tax rate. This direct effect is counteracted by the increase in the tax base.\(^6\) The indirect effect is not strong enough to decrease average tax rates, but it is strong enough to decrease the tax rate during recessions. The reason is that the reduction in employment volatility and the higher average employment rate imply that the tax base is reduced by less when the economy enters a recession. If tax rates were distortionary – which they are not in our model – then the reduction in tax rates during recessions induced by increases in the replacement rate could lead to a further dampening of business cycle fluctuations.

**The relationship between unemployment benefits and wages.** The discussion above considered an increase in unemployment insurance while leaving the wage-setting rule unchanged. This is not unreasonable given that several empirical papers find that UI benefits do not have a significant effect on wages.\(^6\) However, not all papers reach this conclusion. Schmieder, von Wachter, and Bender (2014) find that more generous UI benefits have a significant negative effect on wages and Nekoei and Weber (2015) find that UI benefits have a positive effect on re-employment wages.\(^6\)

Even though the empirical evidence is inconclusive, it is interesting to see how the results change if wages do adjust following an increase in the replacement rate. In our next exercise, we use the same wage-setting rule as before but let \(\omega_0\) – and thus average wages – increase when \(\mu\) increases to ensure that the average Nash bargaining weight implied by our wage rule remains

\(^6\)The fact that the numbers add up means that interaction of the different effects either cancels out or is negligible.

\(^6\)In our model, taxes are only used to finance unemployment benefits and are, thus, very low. The increase in revenues caused by an increase in the tax base would be higher when average tax rates are higher.

\(^6\)See, for example, Card, Chetty, and Weber (2007), Lalove (2007), van Ours and Vodopivec (2008), and Le Barbanchon (2012).

\(^6\)Wages could go down when a higher level of UI benefits prolongs unemployment spells and increases skill loss. Wages could increase if an increase in UI benefits increases workers’ outside options, or because it allows workers to find better matches. If it is the former, then higher UI benefits would decrease the surplus of the match and the share that accrues to firm owners, which in turn would negatively affect job creation.
unaffected.\textsuperscript{66,67} As pointed out in Hall and Milgrom (2008), Nash bargaining may overstate the importance of fluctuations in the value of unemployment, because the worker’s threat in bargaining is typically not leaving the relationship and becoming unemployed, but prolonging negotiations. Consequently, our procedure may overstate the upward pressure on wages following an increase in $\mu$. By considering the case when wages do not respond at all, as well as the case when wages possibly respond too much, we can bound likely outcomes of increases in the replacement rate. We leave $\omega_P$ unchanged, which implies that wages remain sticky with respect to the aggregate price level.

**UI changes when nominal wages are sticky and wage-setting rule is affected.** The top right graph of figure 6 shows the results of the experiments when the wage rule is affected by changes in the replacement rate. First consider the case without aggregate uncertainty. The results are similar to the case when the replacement rate does not affect wage setting. One difference is that the employment level decreases by more when the replacement rate increases.\textsuperscript{68} The reason is that an increase in $\mu$ now has a *direct* negative effect on firm profits as overall wages are higher.\textsuperscript{69} The increase in wages, induced by the higher level of $\mu$, also makes job creation less attractive in the economy with aggregate uncertainty. Moreover, the forces that push employment up are weaker when the wage-setting rule is affected. In particular, the standard deviation of individual consumption drops by 10.2\% instead of 15.3\%. This smaller reduction in risk means that equity remains less attractive as an investment choice. Similarly, the volatility of aggregate employment and asset prices drop by less. The reason these volatilities do not drop by more is because the increase in $\omega_0$ lowers average profits which makes profits more sensitive to changes in productivity.\textsuperscript{70} The negative effect through higher wages and the weakening of the forces that push employment in

\begin{align*}
&\text{ui changes when nominal wages are sticky and wage-setting rule is affected.}\quad \text{The top right graph of figure 6 shows the results of the experiments when the wage rule is affected by changes in the replacement rate. First consider the case without aggregate uncertainty. The results are similar to the case when the replacement rate does not affect wage setting. One difference is that the employment level decreases by more when the replacement rate increases.}\textsuperscript{68} \text{The reason is that an increase in } \mu \text{ now has a } \textit{direct} \text{ negative effect on firm profits as overall wages are higher.}\textsuperscript{69} \text{The increase in wages, induced by the higher level of } \mu, \text{ also makes job creation less attractive in the economy with aggregate uncertainty. Moreover, the forces that push employment up are weaker when the wage-setting rule is affected. In particular, the standard deviation of individual consumption drops by 10.2\% instead of 15.3\%. This smaller reduction in risk means that equity remains less attractive as an investment choice. Similarly, the volatility of aggregate employment and asset prices drop by less. The reason these volatilities do not drop by more is because the increase in } \omega_0 \text{ lowers average profits which makes profits more sensitive to changes in productivity.}\textsuperscript{70} \text{The negative effect through higher wages and the weakening of the forces that push employment in}

\textsuperscript{66} \text{We calculate a Nash bargaining weight for each agent and then adjust } \omega_0 \text{ such that the average Nash bargaining weight under the new UI scheme is the same as under the old UI scheme. In particular, we do the following. Let } V_e(y + (1 - \tau)w) \text{ be the expected utility of an employed worker with financial assets worth } y \text{ and a current wage rate equal to } w. \text{ Other state variables are suppressed. Also, } V_u(y + (1 - \tau)\mu w) \text{ is the expected utility of an unemployed worker. His utility depends on the market wage rate, } \bar{w}. \text{ Firm value } \textit{minus} \text{ the wage payment is equal to } (1 - \delta) J/P + z - w. \text{ The implied Nash bargaining weight is then equal to}

\begin{align*}
&\frac{V_e(y + (1 - \tau)w) - V_u(y + (1 - \tau)\mu \bar{w})}{V_e(y + (1 - \tau)w) - V_u(y + (1 - \tau)\mu \bar{w}) + \partial V_e(y + (1 - \tau)w) / \partial \omega_0 ((1 - \delta) J/P + z - w)}
\end{align*}

\textsuperscript{67} \text{Under Nash bargaining, workers’ wages also vary with their individual wealth level, which would increase the computational burden. One could question whether this is an empirically relevant feature. Moreover, the results in Krusell, Mukoyama, and Sahin (2010) indicate that this complication has a negligible effect on agents’ wages apart from the very poorest.}

\textsuperscript{68} \text{Note that the range of values for } \mu \text{ considered in the right column is smaller than the range considered in the left column.}

\textsuperscript{69} \text{For the same reason, the effect of } \mu \text{ on the employment level does not flatten out at low levels of } \mu.

\textsuperscript{70} \text{The value of } \omega_0 \text{ increases to 0.9722, which implies a 7\% reduction in firm profits.}
the opposite direction mean that average employment decreases with 0.31ppt when $\omega_0$ changes, whereas average employment increases with 0.31ppt when $\omega_0$ remains constant.

**UI changes when nominal wages are flexible, $\omega_P = 1$.** The bottom two graphs of figure 6 display the results when nominal wages respond one-for-one to changes in the price level and the deflationary mechanism is, thus, not present. The left graph displays the results when the replacement rate does not affect wage setting, and the right graph displays the results when wage setting is affected. The consequences of an increase in $\mu$ are quite different from the case when nominal wages are sticky. In particular, increases in the replacement rate never increase employment and never reduce volatility of aggregate variables. This happens even if the replacement rate does not affect wages. The reason is the following. As discussed in section 6.3, imperfect insurance dampens business cycles when wages are flexible (i.e. $\omega_P = 1$). As the replacement rate increases, this effect becomes less important and business cycles therefore become more volatile.

The change in $\mu$ from 0.5 to 0.55 makes the role of $\omega_P$ very clear. When wages are flexible, this increase in the replacement rate leads to an increase in the standard deviation of the aggregate employment rate of 9.3%. In contrast, when wages are sticky the same standard deviation drops by more than 50%. The results regarding risk sharing are also different. When wages are flexible, the increase in $\mu$ leads to a decrease in standard deviation of individual log consumption of only 8.4%. With sticky nominal wages the drop is equal to 15.3%.

**7.1.2 UI changes during recessions and expansions: Welfare consequences**

In this subsection, we document how changes in the replacement rate affect agents’ welfare. In particular, we calculate the effect on welfare of an unexpected and permanent switch from $\mu = 0.5$ to a new level of insurance at two different stages: (i) when the economy enters a recession at an employment rate equal to its peak during the expansionary phase; and (ii) when the economy enters an expansion at an employment rate equal to its trough during the recessionary phase. These experiments are, again, conducted both for sticky ($\omega_P = 0.7$) and flexible ($\omega_P = 1$) wages, as well when changes in the replacement rate affect the wage rule, and when they do not. As a comparison, we also show the results of these experiments for the economy without aggregate risk. The change in welfare is calculated as follows. Starting with our benchmark economy, we calculate the cash-equivalent for each agent of changing the replacement rate. That is, we calculate the change in cash on hand required to render an agent indifferent between the change in $\mu$ and

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71Although the results depend on the current values of aggregate productivity and employment, they are very similar for different histories of productivity leading to these values. Since productivity follows a two-state Markov process, and since transitional dynamics are quite fast, the employment rate which we consider in each experiment is close to its typical value at the first period of a transition.
leaving it unchanged at its benchmark value of 0.5. A positive value means that the agent is better off. Our calculations take into account the expected transition associated with the change in $\mu$.

Figure 7 reports the *average* of the cash-equivalents across all agents, relative to output, in period of implementation. The two top graphs show the results when $\omega_P = 0.7$ and when $\mu$ does and does not affect wage setting. The bottom two graphs illustrate the same results when $\omega_P = 1$. Below, we will show how these results depend on an agent’s individual wealth and employment status.

First, consider the results in the top-left graph, which corresponds to the case when nominal wages are sticky and $\mu$ does not affect the wage setting rule. In the version of the model without aggregate uncertainty, our benchmark value for the replacement rate of 0.5 happens to be close to optimal according to this average welfare criterion. Agents benefit from an increase in $\mu$ because it lowers the volatility of individual consumption, but this is more than offset by the decrease in average employment induced by a decrease in precautionary savings.

The results are quite different when aggregate uncertainty is present. An increase of $\mu$ from 0.5 to 0.55 during a recession corresponds to an average utility gain that is equivalent to 115% of quarterly per capita output. The increase in unemployment insurance is welfare improving because it leads to a decrease in the volatility of individual consumption, a decrease in aggregate volatility, and an increase in the average employment rate. Why are the welfare gains so large? A key reason is that agents are not well insured in this economy. During the first year of an unemployment spell, consumption drops on average by 34%, which equals its empirical counterpart. At some point, unemployed workers become hand-to-mouth consumers. In such a world, the level of unemployment benefits matters; when $\mu$ increases from 0.5 to 0.55, consumption volatility declines by 15.3%, when it increases to 0.7, consumption volatility declines by 44.5%. The other reason the numbers are large is that they are expressed as one lump-sum payment.72

The top-right graph of figure 7 documents that the results are qualitatively similar if the change in the replacement rate affects wage setting. Increasing the replacement rate still increases our welfare measure, but by less, especially if the increase occurs during an expansion. Moreover, increasing $\mu$ to levels above 0.64 during a recession renders negative average cash-equivalents, whereas the average cash-equivalents never turn negative when changes in the replacement rate do not affect wage setting. The reason behind the attenuated welfare gains of higher unemployment insurance is that the associated increase in wages lowers the level of employment. For the case without aggregate uncertainty – in which changes in the replacement rate cannot alter the business cycle properties – small as well as large increases in $\mu$ correspond to a decrease in our average welfare measure.

72If we use 0.18% as the discount rate – i.e., the model’s average quarterly return – then a lump sum payment equal to 115% of quarterly per capita output corresponds to a permanent increase of 0.20% of quarterly per capita output.
Figure 7: Average welfare and replacement rates.

Notes. This figure displays the average change in our welfare measure when the replacement rate, $\mu$, changes from our benchmark value of 0.5 to the indicated value on the $x$-axis. The left column displays the results when wage setting is not affected by changes in the replacement rate, $\mu$. The right column displays the results when wages setting is such that the implied average Nash-bargaining weights are kept constant when $\mu$ changes. The top row presents the results when nominal wages do not fully respond to changes in the price level and the bottom row presents results when they do.

The bottom two graphs display the results if there is no nominal wage stickiness; that is, if the deflationary mechanism is not present. If there is no aggregate uncertainty, then the price level is constant, which implies that the value of $\omega_P$ does not matter. Moreover, without nominal wage stickiness the results with and without aggregate uncertainty are very similar. It should be noted that average welfare always declines for any increase in $\mu$ beyond its benchmark value if nominal wages are flexible.
WHO BENEFITS FROM AN INCREASE IN UNEMPLOYMENT INSURANCE DURING A RECESSION? In this subsection, we discuss how an increase in the replacement rate affects different agents. We focus on an unexpected and permanent increase in $\mu$ from 0.5 to 0.55. We first discuss the case when the change occurs at the beginning of a recession. When evaluating the change, the agents take into account the transitional dynamics.

Changes in the replacement rate affect different agents for different reasons. Unemployed workers benefit immediately from the increase in unemployment benefits, since it is a direct transfer of resources from the employed to the unemployed. But employed agents benefit too. They benefit because: (i) the dampening of the downturn increases the value of their asset holdings; (ii) a higher replacement rate provides better insurance against a shortfall in income should they become unemployed; and (iii) average employment increases when $\mu$ does not affect wages, which means that all workers can expect to be less affected by unemployment. Although equity returns are positively affected by the switch to higher unemployment benefits, the return on money balances is negatively affected, since the increase in the replacement rate reduces deflationary pressure. The increase in the replacement rate increases average tax rates. During recessions, however, the reduction in the number of unemployed leads to lower tax rates, provided that $\mu$ does not affect wages.

Figure 8 displays the cash equivalent (y-axis) of the proposed change in the replacement rate as a function of the agent’s beginning-of-period cash-on-hand level (x-axis) and employment status. The figure documents that all unemployed and all employed agents prefer the switch to the higher level of unemployment insurance, irrespective of whether wages, $\omega_0$, adjust.

First, consider the dark lines which represent the results for agents that were employed last period. Among this group, the currently unemployed benefit more than the currently employed. This is not surprising given that an unemployed worker benefits directly from higher unemployment insurance. More surprisingly, rich agents benefit more than poor agents. The reason is the following. Employed agents’ invest a substantial fraction of their wealth in equity. A switch to higher unemployment benefits during recessions is beneficial to these agents, since it dampens the fall of equity prices. More over, the richer the agent, the bigger the benefit.

Second, consider the grey lines which illustrate the outcomes for agents that were unemployed last period. Richer agents in this group benefit less from the increase in $\mu$. The reason is that unemployed agents invest a larger fraction in the liquid asset and returns on this asset are negatively affected by the reduction in the deflationary pressure induced by the increase in the replacement rate. Within this group, unemployed agents also benefit more than employed agents from the increase in $\mu$ (at the same cash-on-hand level), but the difference is now quite small. It is still the case that the

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73 Cash on hand is measured at the point when it is known that the economy has entered a recession, but before it is known that $\mu$ has changed.
direct benefit of an increase in unemployment benefits and the associated increase in the job finding rate are more beneficial for the unemployed. However, when an unemployed agent has the same amount of cash on hand level as an employed agent, then this agent must have more financial assets, since unemployment benefits are less than wage income. Consequently, this worker is affected more by the negative effect of the increase in $\mu$ on the return on liquid assets.

![Figure 8: Increasing $\mu$ in the first period of a recession.](image)

**Notes.** This figure plots the welfare gains (measured as cash-on-hand equivalents relative to output) for the four possible labor market transitions. Since the change in $\mu$ affects asset prices individual portfolio shares matters, which implies that the results also depend on last period’s employment status. The label “EE”, for instance, indicates an agent who was employed in the preceding period, and remained employed in the current period, etc. The average welfare gain with $\omega_0$ unchanged equals 1.146. The average welfare gain with $\omega_0 = 0.9722$ equals 0.562.

All agents benefit less from an increase in the replacement rate when the increase is associated with higher wages. Why are the general equilibrium effects such that even employed workers who hold no equity benefit less? The associated increase in $\omega_0$ implies that the rise in the replacement rate does not dampen the downturn in real activity, nor the drop in stock prices, by as much. The benefit of an increase in the replacement rate for a poor employed worker is not affected by these different responses, since the agent is employed and does not hold equity. However, the same agent is affected by worsened future employment prospects, which are important enough to offset the increase in the agent’s wage rate.

**Who benefits from an increase in unemployment insurance during an expansion?** If the rise in the replacement rate occurs at the beginning of an expansion, the calculated cash-equivalents are
lower than those previously reported. The reason is that an increase in the replacement rate not only dampens recessions, it also dampens expansions, since the upward pressure on prices induced by a reduction in precautionary savings is smaller. It is still the case, however that all workers prefer the increase in unemployment insurance when wages are not affected. When wages do increase, however, then both the richer employed and the richer unemployed workers do not prefer to increase the replacement rate from the status quo.

### 7.1.3 Relation to the literature

In the standard search-and-matching model with a representative agent, an increase in the replacement rate that is accompanied by an increase in wages would, (i) lower average employment because firm profits fall; and (ii) increase aggregate volatility because higher wages increases the sensitivity of firm profits to shocks. If wages do not adjust, then there would be no changes at all.

In our benchmark model, in which the deflationary mechanism operates, we find that an increase in the replacement rate decreases aggregate volatility, both when the wage-setting rule does and when it does not adjust, and we find that it increases average employment when the wage-setting rule does not adjust. If the deflationary mechanism does not operate, then our framework’s predictions correspond to those of the representative-agent version. This is most clear in the bottom right graph of figure 6 which shows the employment rate as a function of $\mu$ when wages are not sticky ($\omega_P = 1$) and wages are affected. The graph shows that increases in the replacement rate always lower average employment and always increase aggregate volatility.

Our results also differ substantially from those in Krusell, Mukoyama, and Sahin (2010), who – like us – look at changes in the replacement rate in a model with incomplete risk sharing and labor market frictions. They show that 92.1% of all agents would prefer a reduction in the replacement rate from 0.4 to 0.04. As previously shown, in our benchmark economy with a replacement rate equal to 0.5, all agents would prefer a 10% increase in the replacement rate. The key difference is that we look at an economy with aggregate uncertainty. Moreover, our parameter values are such that there is a strong interaction between sticky nominal wages and imperfect insurance, which results in a deflationary mechanism that increases the volatility of business cycles and asset prices. An increase in the replacement rate weakens the deflationary mechanism and has the capacity to make all agents better off. In the version of our model without aggregate uncertainty, agents also prefer a reduction in the replacement rate as long as wages are affected through Nash bargaining, which also is the case in Krusell, Mukoyama, and Sahin (2010). Similarly, if our deflationary mechanism is not present – for example when $\omega_P = 1$, or if there is no aggregate uncertainty – then agents also prefer values of the replacement rate that are below our benchmark value, provided that wages are affected.\footnote{By contrast to Krusell, Mukoyama, and Sahin (2010), it never is the case in any of the models considered that...}
McKay and Reis (2013) consider the effects of changes in unemployment benefits on aggregate volatility in an economy with imperfect risk sharing. They find that a reduction in transfers has a close-to-zero effect on the average level of output, and actually lowers the volatility of aggregate consumption. Their approach differs from ours in that it does not include a frictional labor market and – more importantly – also no sticky nominal wages. Consequently, imperfect risk sharing does not interact with sticky nominal wages, which are the key ingredients that generate the powerful deflationary mechanism studied in this paper. Indeed, it is that precise mechanism that underlies our finding that an increase in unemployment insurance leads to a sharp decrease in aggregate volatility.

7.2 Monetary policy
A question which naturally arises in our setting is to which extent the severity of a recession is an artefact of an unrealistically tight monetary policy. That is, to which extent the assumptions of a fixed money stock together with Walrasian goods prices induce a further contractionary force that adds additional thrust to – and perhaps overshadows – the main propagation mechanism itself. To explore this possibility, we proceed in this section by calculating a shadow real and nominal interest rate that provide transparent measures of the accommodative nature of monetary policy.

As we will see, during a recession both the nominal and (expected) shadow real interest rate fall below their unconditional means, with the real rate experiencing a substantial drop. As a consequence, our choice of monetary policy is indeed accommodating, and therefore alleviates some of the adverse effects of the main propagation mechanism, and does not constitute an amplifying effect on its own. This stands in sharp contrast to models in which deep recessions are due to the combination of unusually restrictive monetary policy (for example, because the policy rate is near the zero lower bound) and high real interest rates.

The shadow interest rate. In the complete markets version of our model, calculating a shadow nominal interest rate, \( r_{t+1} \), is relatively straightforward. The Euler equation for an interest bearing lots of agents prefer extremely low replacement rates. In fact, even if \( \mu \) affects wages and in the absence of nominal wage stickiness, we find that only 40% prefer a reduction in \( \mu \) from 0.5 to 0.35 when implemented in a recession. Moreover, this fraction drops sharply for further reductions; no agent prefers a reduction of \( \mu \) from 0.5 to 0.32. These numbers are for the case without aggregate uncertainty. With aggregate uncertainty, the fraction of agents that prefer low replacement rates will be even smaller, since low replacement rates strengthen the deflationary mechanism.

75 In their model, a reduction in transfers leads to an increase the volatility of output and hours. Their effects are small relative to ours. They report that an 80% reduction in transfers leads to an 8% increase in the variance of hours, whereas our increase in the replacement rate from 0.5 to 0.55 leads to a 47.9% decrease in the standard deviation of the employment rate.
bond paying nominal return $i_{t+1}$ is given by

$$c_t^{-\gamma} = \beta (1 + i_{t+1}) \mathbb{E}_t \left[ c_{t+1}^{-\gamma} \frac{P_t}{P_{t+1}} \right].$$  \hspace{1cm} (15)

Combining this equation with the Euler equation for money, equation (3), gives the following expression for the “no-trade” – or shadow – nominal interest rate:

$$i_{t+1} = \chi \left( \frac{M}{P_t X_t} \right)^{-\zeta}, \text{ with } X_t = \mathbb{E}_t \left[ c_{t+1}^{-\gamma} \frac{P_t}{P_{t+1}} \right]^{-\frac{1}{\zeta}},$$  \hspace{1cm} (16)

where $M$ denotes the fixed supply of money. This shadow interest rate provides a useful measure to gauge the responsiveness of monetary policy.

Matters are somewhat more complicated when market are incomplete. In particular, since agents hold different levels of assets, and face different income profiles, their associated no-trade interest rates are individual-specific, and the overall stance of monetary policy is therefore less easy to assess. One possibly way out is to average each individual-specific shadow interest rate using the equilibrium cross-sectional distribution. However, since such an average would not take into account the varying purchasing power of agents at different parts of the distribution, this approach leads to similar issues as those of discounting profits with an average MRS. Thus, in order to derive an economy wide shadow interest rate with incomplete markets we take the following steps.

In any given period $t$, each agent makes a portfolio and consumption choice. Once this choice is made, an unanticipated (i.e. an event occurring with zero-probability) opportunity arises in which agents may shift (some of) their savings in money into interest bearing bonds. The interest accrued on bond holdings are lump-sum taxed back in the subsequent period, such that the continuation processes of both consumption and asset holdings are left intact. The equilibrium shadow interest rate is then such that aggregate bond purchases are zero.

More formally, let $B_{i,t+1}$ denote an agent’s optimal decision of purchasing bonds. The arbitrage condition between money and bond holdings is then

$$\mathbb{E}_t \left[ c_{i,t+1}^{-\gamma} \frac{P_t}{P_{t+1}} \right] + \chi \left( \frac{M_{i,t+1} - B_{i,t+1}}{P_t} \right)^{-\zeta} = (1 + i_{t+1}) \mathbb{E}_t \left[ c_{i,t+1}^{-\gamma} \frac{P_t}{P_{t+1}} \right].$$  \hspace{1cm} (17)

Solving for $B_{i,t+1}$ gives,

$$-B_{i,t+1} = \left( \frac{i_{t+1}}{\chi} \right)^{-\frac{1}{\zeta}} X_{i,t} P_t - M_{i,t+1}, \text{ with } X_{i,t} = \mathbb{E}_t \left[ c_{i,t+1}^{-\gamma} \frac{P_t}{P_{t+1}} \right]^{-\frac{1}{\zeta}}.$$  \hspace{1cm} (18)
As a consequence, the equilibrium shadow interest rate must satisfy

\[
\bar{M} = \left( \frac{i_{t+1}}{\chi} \right)^{-\frac{1}{\zeta}} \int X_{i,t} dF_i(e_i, q_i, M_i) P_t,
\]

or simply

\[
i_{t+1} = \chi \left( \frac{\bar{M}}{P_t \hat{X}_t} \right)^{-\zeta}, \quad \text{with} \quad \hat{X}_t = \int X_{i,t} dF_i(e_i, q_i, M_i).
\]

The left graph of figure 9 shows the normalized impulse response functions of the shadow nominal interest rates for the economy with and without complete markets. The right graph shows the corresponding real interest rates. There are two features of figure 9 that are worth emphasizing. First, in the economy without complete risk sharing the nominal interest rate declines throughout a recession, whereas it increases in the complete markets economy. Second, although the nominal interest rate does not fall by much, the real interest rate experiences a pronounced drop in the incomplete-markets economy. The reason for the reduction in the real rate is that the economy exhibits expected inflation throughout the transitional dynamics back to the unconditional steady state. By contrast, the real rate increases in the complete-markets economy, except for a small initial decrease.

Figure 9: Shadow interest rate

Notes. This figure plots the shadow nominal rate for the complete and incomplete-markets economy as defined in equations (16) and (20), respectively. The real interest rate is defined as \( \frac{1 + i_{t+1}}{\bar{M} [P_{t+1}/P_t]} \).

Contrast with the zero-lower-bound literature. To some extent, the modest response in the nominal interest rate raises the question whether the dynamics explored in this paper resembles, or
even mimics, those emphasized in the zero-lower-bound literature. It does not. To understand why, it is important to notice that the dynamics underlying the zero-lower-bound literature hinges on the idea that a negative demand shock – which is similar to the precautionary amplification mechanism in this paper – gives rise to a decline in expected inflation and quite often even deflation. With nominal interest rates at zero (or at some constant, unchanged, level), this decline in expected inflation raises real interest rates, which further propagates the initial shock to demand, and the process reinforces itself. This mechanism contrasts markedly to the one explored in this paper, in which a negative effect on demand – through the precautionary savings in liquid assets – gives rise to a persistent, but mean reverting, decline in the price level, which leads to a substantial rise in expected inflation. The rise in expected inflation lowers the real interest rate, which therefore alleviates some of the adverse consequences of the initial shock. In fact, the real interest rate falls to levels that are substantially below the nominal interest rate, whereas the opposite is true in the zero-lower-bound literature. Thus, the price level dynamics in this paper give rise to a countercyclical force that works in the opposite direction to that of the zero-lower-bound literature.

**Further monetary considerations.** Another issue that arises is to which extent there exist policy options available to the monetary authority that may mitigate the amplification mechanism further. In particular, the deflationary spiral explored in this paper emerges since agents want to hold more money. So why not just give the agents what they want? If the central bank could respond to changes in productivity instantaneously, and if the central bank could increase the money supply by “helicopter drops” of money in the hands of the right households, then the central bank could prevent the deflationary pressure on the price level and the ensuing upward pressure on real wages. Such helicopter drops of money are not part of the usual set of central bank instruments. The typical way for a central bank to increase liquidity in the economy is to purchase government bonds from banks. This increases the liquidity position of banks. If the additional liquidity induces banks to issue more loans, then bank deposits will increase. That is, money holdings of the non-financial private sector will increase. Note, however, that the liabilities of this sector must have increased by the same amount. It is possible that this combined increase of liquid assets and debt eases workers concerns about future unemployment, for example, because the loans are (perceived to be) long-term loans. If workers care about their net-liquidity position, however, then this monetary stimulus would not satisfy workers’ desire to hold more money balances, and there would still be downward pressure on the price level during recessions. This latter case would be especially relevant if loans cannot be rolled over if a worker becomes unemployed.

Although, there are in theory monetary policies that undo the deflationary pressure, this paper

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See, for example, Eggertsson and Woodford (2003), Christiano, Eichenbaum, and Rebelo (2011), and Eggertsson and Krugman (2012).
focuses – in the spirit of the New Keynesian literature – on a monetary policy which is reasonably accommodative, but which is not sufficiently aggressive and/or sophisticated to entirely neutralize all nominal frictions.

8 Concluding comments

The properties of our model depend crucially on whether the deflationary mechanism is sufficiently powerful. If it is not powerful enough, then the model properties are close to the outcomes of a representative-agent version of the model. In particular, the presence of nominal sticky wages would then *dampen* the effects of productivity shocks, and an increase in the replacement rate would *decrease* the average employment rate. If the deflationary mechanism is strong enough, however, then our model predicts the opposite. In so far as the conditions that affect the strength of the deflationary mechanism vary across time and place, one can also expect business cycle properties to vary across time and place. The same is true for the effects of changes in unemployment insurance. Whether the deflationary mechanism is operative or not may depend on relatively small changes. For example, the mechanism is quantitatively very important when the replacement rate is equal to its benchmark value of 50%, but not when the replacement rate exceeds 60%. The message is that even if one is confident that a particular model describes the data well, it may still be difficult to predict business cycle behavior and the consequences of policy changes.
A Further empirical motivation

In this appendix, we provide more empirical motivation for our model and the underlying assumptions. First, we review some evidence in support of our assumption that nominal wages do not respond one-for-one to price changes. Second, we discuss the inability of individuals to insure themselves against unemployment spells. Lastly, we discuss whether savings respond to an increase in idiosyncratic uncertainty.

Nominal wage stickiness and inflation. There are many papers that document that nominal wages are sticky.\footnote{See, for example, Dickens, Goette, Groshen, Holden, Messina, Schweitzer, Turunen, and Ward (2007), Druant, Fabiani, Kezdi, Lamo, Martins, and Sabbatini (2009), Barattieri, Basu, and Gottschalk (2010), Daly, Hobijn, and Lucking (2012), and Daly and Hobijn (2013).} However, what is important for our paper is the question of to which extent nominal wages adjust to aggregate shocks and, in particular, to changes in the aggregate price level. Druant, Fabiani, Kezdi, Lamo, Martins, and Sabbatini (2009) provide survey evidence for a sample of European firms with a focus on the wages of the firms’ main occupational groups; these would not change for reasons such as promotion. Another attractive feature of this study is that it explicitly investigates whether nominal wages adjust to inflation or not. In their survey, only 29.7% of Eurozone firms indicate that they have an internal policy of taking inflation into account when setting wages, and only half of these firms do so by using automatic indexation. Moreover, most firms that take inflation into account are backward looking. Both findings imply that real wages increase (or decrease by less) when inflation rates fall.

Papers that document nominal wage rigidity typically highlight the importance of downward nominal wage rigidity. Suppose there is downward, but no upward nominal wage rigidity. Does this imply that all nominal wages respond fully to changes in aggregate prices as long as aggregate prices increase? The answer is no. The reason is that firms are heterogeneous and a fraction of firms can still be constrained by the inability to adjust nominal wages downward. In fact, downward nominal wage rigidity is supported by the empirical finding that the distribution of firms’ nominal wage changes has a large mass-point at zero.\footnote{See Barattieri, Basu, and Gottschalk (2010), Dickens, Goette, Groshen, Holden, Messina, Schweitzer, Turunen, and Ward (2007), Daly, Hobijn, and Lucking (2012), and Daly and Hobijn (2013).}

The fraction of firms that is affected by this constraint would increase if the aggregate price level increases by less. In fact, Daly, Hobijn, and Lucking (2012) document that the fraction of US workers with a constant nominal wage increased from 11.2% in 2007 to 16% in 2011, whereas the fraction of workers facing a reduction in nominal wages was roughly unchanged.\footnote{Similarly, at http://nadaesgratis.es/?p=39350, Marcel Jansen documents that from 2008 to 2013 there was a massive increase in the fraction of Spanish workers with no change in the nominal wage. There is some increase in the fraction of workers with a decrease in the nominal wage, but this increase is small relative to the increase in the spike of the histogram at constant nominal wages.} This indicates that there is upward pressure on real wages when the inflation rate falls, even if it remains positive and nominal wages are only rigid downward.

Inability to insure against unemployment risk An important feature of our model is that workers are poorly insured against unemployment risk. That is, that consumption decreases considerably following a displacement. Using Swedish data, Kolsrud, Landais, Nilsson, and Spinnewijn (2015) document that...
expenditures on consumption goods drop sharply during the first year of an unemployment spell, after which they settle down at 34% below the pre-displacement level. This sharp fall is remarkable given that Sweden has a quite generous unemployment benefits program. As is discussed in section 4, one reason is that the amount of assets workers hold at the start of an unemployment spell is low. Another reason is that average borrowing actually decreases during observed unemployment spells.

Using US data Stephens Jr. (2004), Saporta-Eksten (2014), Aguiar and Hurst (2005), Chodorow-Reich and Karabarbounis (2015) provide empirical support for substantial drops in consumption follow job loss, even when expenditures on durables are not included.80 Using Canadian survey data, Browning and Crossley (2001) find that workers that have been unemployed for six months report that their total consumption expenditures level during the last month is 14% below consumption in the month before unemployment.

Savings and idiosyncratic uncertainty The idea that idiosyncratic uncertainty plays an important role in the savings decisions of individuals has a rich history in the economics literature. From a theoretical point of view Kimball (1992) shows that idiosyncratic uncertainty increases savings when the third-order derivative of the utility function with respect to consumption is positive and/or the agent faces borrowing constraints. Moreover, idiosyncratic uncertainty regarding unemployment is more important in recessions which are characterized by a prolonged downturn and an increase in the average duration of unemployment spells. Krueger, Cramer, and Cho (2014) document that during the recent recession the number of long-term unemployed increased in Canada, France, Italy, Sweden, the UK, and the US. The only case in which they found a decrease is Germany. The results are particularly striking for the US. During the recent recession, the amount of workers who were out of work for more than half a year relative to all unemployed workers reached a peak of 45%, whereas the highest peak observed in previous recessions was about 25%.

Several papers have provided empirical support for the hypothesis that increases in idiosyncratic uncertainty increases savings. Using 1992-98 data from the British Household Panel Survey (BHPS), Benito (2004) finds that an individual whose level of idiosyncratic uncertainty would move from the bottom to the top of the cross-sectional distribution reduces consumption by 11%. An interesting aspect of this study is that the result holds both for a measure of idiosyncratic uncertainty based on an individuals’ own perceptions as well as on an econometric specification.81 Further empirical evidence for this relationship during the recent downturn can be found in Alan, Crossley, and Low (2012). They argue that the observed sharp rise in the

80 Using the four 1992-1996 waves of the Health and Retirement Study (HRS), Stephens Jr. (2004) finds that annual food consumption is 16% lower when a worker reports that he/she is no longer working for the employer of the previous wave either because of a layoff, business closure, or business relocation, that is, the worker was displaced between two waves. Similar results are found using the Panel Study of Income Dynamics (PSID). Using the 1999-2009 biannual waves of the PSID, Saporta-Eksten (2014) finds that job loss leads to a drop in total consumption of 17%. About half of this loss occurs before job loss and the other half around job loss. The drop before job loss suggests that either the worker anticipated the layoff or labor income was already under pressure. Moreover, this drop in consumption is very persistent and is only slightly less than 17% six years after displacement. Using data for food and services, Chodorow-Reich and Karabarbounis (2015) find that the consumption level of workers that are unemployed for a full year is 21% below the consumption level of employed workers. Using scanner data for food consumption, Aguiar and Hurst (2005) report a drop of 19%.

81 Although the sign is correct, the results based on individuals’ own perceptions are not significant.
savings ratio of the UK private sector is driven by increases in uncertainty, rather than other explanations such as tightening of credit standards. In line with the mechanism emphasized in this paper, Carroll (1992) argues that employment uncertainty is especially important because unemployment spells are the reason for the most drastic fluctuations in household income. In addition, Carroll (1992) provides empirical evidence to support the view that the fear of unemployment leads to an increased desire to save even when controlling for expected income growth.

B Data Sources

B.1 Data used for calibration and empirical motivation

- Eurozone GDP implicit price deflators are from the Federal Reserve Economic Data (FRED). Data are seasonally adjusted. Here the Eurozone consists of the 18 countries that were members in 2014.

- Eurozone private sector hourly earnings are from OECD.STATExtracts (MEI). The target series for hourly earnings correspond to seasonally adjusted average total earnings paid per employed person per hour, including overtime pay and regularly recurring cash supplements. Data are seasonally adjusted.

- Unit labor costs are from OECD.STATExtracts. Data are for the total economy. Unit labour costs are calculated as the ratio of total labour costs to real output. Data are seasonally adjusted.

- Average unemployment rate: Average unemployment rate for the four large Eurozone economies, France, Germany, Italy, and Spain. Data is from OECD.STATExtracts (ALFS).

- Average unemployment duration: Average unemployment duration for Europe is from OECD.StatExtracts. This is annual data. The data series for Europe is used because no data for the Eurozone is available, nor data for the big Eurozone countries. Starting in 1992, separate data is given for Europe, the European Union with 21 countries, and the European Union with 28 countries, and the series are quite similar over this sample period.

B.2 Data used for model evaluation

In the paper, we report some summary statistics for France, Germany, and Italy. Price indices, Share prices, and unit labor costs are from data.oecd.org. Employment and GDP data are from stats.oecd.org. Data series end in the first quarter of 2015 for the employment series, in the third quarter of 2015 for the GDP series, and in the second quarter of 2015 for the others. The CPI and Share Price index start in the first quarter of 1960 for all three countries. The PPI and unit labor cost series start in the first quarter of 2005 for France, the first quarter of 1995 for Germany, and the first quarter of 2000 for Italy.
C Equivalence with standard matching framework.

In the standard matching framework, new firms are created by “entrepreneurs” who post vacancies, $\tilde{v}_t$, at a cost equal to $\kappa$ per vacancy. The number of vacancies is pinned down by a free-entry condition. In the description of the model above, such additional agents are not introduced. Instead, creation of new firms is carried out by investors wanting to increase their equity holdings.

Although, the “story” we tell is somewhat different, our equations can be shown to be identical to those of the standard matching model. The free-entry condition in the standard matching model is given by

$$\kappa = \frac{\tilde{h}_t}{\tilde{v}_t F_t},$$ \hspace{1cm} (21)

where

$$\tilde{h}_t = \tilde{\psi} \tilde{v}_t u_t^{1-\eta}. \hspace{1cm} (22)$$

Each vacancy leads to the creation of $\tilde{h}_t / \tilde{v}_t$ new firms, which can be sold to households at price $J_t$.

Equilibrium in the equity market requires that the net demand for equity by households is equal to the supply of new equity by entrepreneurs, that is

$$\int_i (q_i (e_i, q_i, M_i; s_i)) \eta (1 - \delta) q_i dF_t (e_i, q_i, M_i) = \tilde{\psi} \tilde{v}_t u_t^{1-\eta}. \hspace{1cm} (23)$$

Using equations (21) and (22), this equation can be rewritten as

$$\int_i (q_i (e_i, q_i, M_i; s_i)) \eta (1 - \delta) q_i dF_t (e_i, q_i, M_i) = \psi \kappa \eta \tilde{v}_t u_t^{1-\eta}. \hspace{1cm} (24)$$

This is equivalent to equation (13) if

$$\tilde{\psi} = \psi \kappa^{\eta}. \hspace{1cm} (25)$$

It only remains to establish that the number of new jobs created is the same in the two setups, that is,

$$h_t = \tilde{h}_t \hspace{1cm} (26)$$

or

$$\psi \tilde{v}_t u_t^{1-\eta} = \tilde{\psi} \tilde{v}_t u_t^{1-\eta}. \hspace{1cm} (27)$$

From equations (21) and (22), we get that

$$\tilde{v}_t = \left(\frac{\tilde{\psi} J_t}{\kappa F_t}\right)^{1/(1-\eta)} u_t. \hspace{1cm} (28)$$
Substituting this expression for $\tilde{v}_t$ and the expression from equation (11) for $v_t$ into equation (27) gives indeed that $h_t = \tilde{h}_t$. Moreover, the total amount spent on creating new firms in our representation, $v_t$, is equal to the number of vacancies times the posting cost in the traditional representation, $\kappa \tilde{v}_t$.

The focus of this paper is on the effect of negative shocks on the savings and investment behavior of agents in the economy when markets are incomplete. We think that our way of telling the story behind the equations has the following two advantages. First, there is only one type of investor, namely, the household and there are no additional investors such as zombie entrepreneurs (poor souls who get no positive benefits out of fulfilling a crucial role in the economy). Second, all agents have access to investment in the same two assets, namely equity and the liquid asset, whereas in the standard labor market model there are households and entrepreneurs and they have different investment opportunities.

D Consumption and portfolio decisions: Additional results

In this appendix, we provide some more detailed information regarding the agents’ consumption, investment, and portfolio decisions. Figure 10 displays the average post-displacement change in consumption. As discussed in the main text, the model captures the drop in consumption during the first year following a displacement, but misses that consumption stops falling after the first year. Figure 11, however, documents that most unemployment spells do not exceed one year.

Figure 10: Evolution of consumption drop over the unemployment spell.

Notes. The black line illustrates the average path of consumption of an individual that becomes unemployed in period 1, conditional on being in an expansion at the time of displacement. The grey line illustrates the equivalent path conditional on being in a recession.

82 One could argue that entrepreneurs are part of the household, but with heterogeneous households the question arises which households they belong to.
Figure 11: Distribution of the unemployed.

Notes. The black bars measure the fraction of unemployed at various durations conditional on being in an expansion at the time of displacement. The grey bars provide the corresponding measure conditional on being in a recession.

Figure 12 displays the complete cumulative distribution function of the value of assets at the beginning of an unemployment spell relative to the average net-income loss. As discussed in the main text, agents in the bottom of the wealth distribution are substantially richer than their real world counterparts, even if we focus on gross assets.

Figure 12: Financial assets at the beginning of an unemployment spell.
Portfolio composition and cash-on-hand levels. Figure 13 presents a scatter plot of the liquid asset’s share in the agents’ investment portfolios (y-axis) and the beginning-of-period cash-on-hand levels (x-axis). Although the pattern is somewhat intricate, the figure can be characterized reasonably well as follows. First, the fraction invested in money is higher at lower cash-on-hand levels. Second, conditional on the cash-on-hand level, this fraction also increases when an agent becomes unemployed. Third, conditional on the cash-on-hand level and employment status, this fraction increases when the economy enters a recession. These three properties imply that the portfolio share invested in money increases during a recession. Without large enough increases in money portfolio shares, aggregate demand for money would decrease during recessions, like it does in the representative-agent model. This is because the total amount of funds carried over into the next period decreases during recessions, which in turn implies that the value of agents’ portfolios is lower.

![Figure 13: Portfolio shares in liquid asset.](image)

Notes. This figure displays the fraction of financial assets invested in the liquid asset as a function of beginning-of-period cash on hand for workers of the indicated employment status and for both outcomes of aggregate productivity

Which forces explain the observed patterns? The first is that the transaction benefits of money are subject to diminishing returns. As a consequence, agents whose total demand for financial assets is high tend to invest a smaller fraction in money. This explains why the fraction invested in money is generally lower for agents with higher cash-on-hand levels. The second driving force is that money is less risky than equity. Therefore, agents whose total demand for financial assets is high relative to their non-asset income invest a larger fraction in money. For a given cash-on-hand level, this explains why the fraction invested in money increases when a worker becomes unemployed, and why the fraction increases when the economy enters a recession. The third driving force explains the non-monotonicity. If the amount invested is substantial relative to the agent’s non-asset income, then equity is not appealing because of equity’s risky returns. However, if
the amount invested is small, then risk has only second-order consequences, whereas the higher return on equity has first-order benefits.

![Figure 14: Post displacement equity holdings.](image)

**Notes.** The black line illustrates the average path for equity holdings of an individual that becomes unemployed in period 1, conditional on being in an expansion at the time of displacement. The grey line illustrates the equivalent path conditional on being in a recession.

![Figure 15: Post displacement money holdings.](image)

**Notes.** The black line illustrates the average path for money holdings of an individual that becomes unemployed in period 1, conditional on being in an expansion at the time of displacement. The grey line illustrates the equivalent path conditional on being in a recession.
Financial assets during unemployment spells. Figures 14 and 15 document how the demand for equity and money holdings behave following a job displacement. The remarkable feature is that the demand for money actually increases during the first couple periods of an unemployment spell.

E Replacement rate increases: Additional results

Aggregate transition dynamics. Figure 16 displays the time paths for employment when the economy moves from an expansion to a recession and back to an expansion. It plots the series when the replacement rate remains unchanged throughout, and when it unexpectedly and permanently increases to 0.55 at the onset of a recession. The results above made it clear that this increase in the replacement rate leads to smaller fluctuations and, if wage setting is not affected, also to a higher average employment rate. Consequently, employment should drop by less if $\mu$ is increased at the start of a recession. The same turns out to be true if the increase in $\mu$ is associated with an increase in wages. That is, the negative effect of the induced increase in wages on average employment is smaller than the dampening effect of the increase in $\mu$ on business cycle fluctuations.

![Figure 16: Switch to higher $\mu$ at the start of the recession.](image)

*Notes.* This figure compares the benchmark time path of beginning-of-period employment with the time path when $\mu$ increases (unexpectedly and permanently) to 0.55, both when $\omega_0$ does and when $\omega_0$ does not adjust upwards.

When the economy leaves the recessionary phase, however, the recovery is at some point dampened by the higher unemployment benefits, irrespective of whether $\mu$ affects wages.\(^{83}\) The result that higher unemployment benefits can be harmful for a recovery is consistent with the results in Hagedorn, Karahan,

\(^{83}\)How soon this happens does depend on whether the level of $\mu$ affects wages.
Manovskii, and Mitman (2015), who argue that the extension of unemployment benefits in the US increased unemployment in 2011 – when the US recovery had started – by 2.5 percentage points.\footnote{Amaral and Ice (2014), in contrast, argue that the extension in benefits only had a minor impact and that part of the increase in the unemployment rate was due to a reduction in the number of unemployed leaving the labor force.}

**Unemployment benefits and unemployment duration.** As discussed above, it is not clear from empirical studies whether changes in unemployment benefits affect wages. There is much more empirical support for the hypothesis that more generous benefits increase unemployment duration (see, for instance, Le Barbanchon (2012) for an overview). Several of these studies identify the effect of unemployment benefits on unemployment duration by considering changes in benefits that affect workers differently. These results may, thus, not be relevant for our general equilibrium experiment in which everybody is affected by the same increase in the replacement rate. If a large share of the unemployed search less intensely, then this provides improved opportunities of finding a job for those who actively search.\footnote{Lalive, Landais, and Zweimller (2015) argue that these externalities are quantitatively important.}

In response to a 10% increase in the replacement rate, $\mu$, from 0.5 to 0.55, our framework generates an increase in average unemployment duration of 1.7% when wages respond to the increase in $\mu$.\footnote{Our framework can also generate an increase in average unemployment duration following an increase in $\mu$ when wages are not affected by changes in $\mu$, but only when $\mu$ is above 0.6.} Krueger and Meyer (2002) report that 0.5 is not an unreasonable rough summary of empirical estimates for the elasticity of unemployment duration with respect to unemployment benefits, but estimates vary. So even though search intensity is constant in our model and an increase in unemployment insurance leads to a sharp decrease in unemployment duration during recessions, our model can still explain a substantial part of the observed relationship between unemployment benefits and unemployment durations.\footnote{The empirical literature focuses on changes in UI benefits on individual workers and changes in search effort are thought to be behind changes in unemployment duration. In our model, search effort is constant and the increase in unemployment duration is due to a reduction in the job creation, either because wages increase or because precautionary savings decrease.}

## F Idiosyncratic labor income risk and demand for risky assets

Here we prove that an increase in idiosyncratic risk increases the demand for equity when – as is the case in typical macroeconomic models – the wage rate and the return on investment are affected by the same factor.

\[
\max_{c_1, c_2, i, b, q} \quad \frac{c_1^{1-\gamma} - 1}{1 - \gamma} + \beta \mathbb{E} \left[ \frac{c_2^{1-\gamma} - 1}{1 - \gamma} \right]
\]

s.t.
\[
c_1 + p_q q + p_i b = y_1, \quad (29)
\]
\[
c_{2,i} = q \tilde{y}_q + b + \tilde{y}_i (1 + \sigma \eta_i), \quad (30)
\]

where $\tilde{\eta}_i$ is an idiosyncratic component that is i.i.d. distributed.
The Euler equations are given by

\[ p_b c_1^{-\gamma} = \beta \mathbb{E} \left[ c_{2i}^{-\gamma} \right], \quad (31) \]
\[ p_q c_1^{-\gamma} = \beta \mathbb{E} \left[ c_{2j}^{-\gamma} \tilde{y}_q \right]. \quad (32) \]

Kimball (1992) considers the case in which the common component of labor income, \( \tilde{y}_l \), is constant and, thus, not correlated with the return of the risky investment. He shows that an increase in idiosyncratic uncertainty leads to a decrease in the amount in invested in the risky asset even though total savings increase. Here we make the assumption that labor income and the return on the risky investment are correlated, because average labor income, \( \bar{y}_l \), is correlated with the return on the risky investment. In particular, we assume that

\[ \tilde{y}_q = \alpha \bar{y}, \quad (33) \]
\[ \tilde{y}_l = (1 - \alpha) \bar{y}. \quad (34) \]

Also,

\[ \mathbb{E} [\bar{\tilde{h}}] = 0, \mathbb{E} [\bar{\tilde{h}}^2] = 1, \quad (35) \]
\[ \mathbb{E}[\bar{\tilde{y}}_q] = 1. \quad (36) \]

**Proposition.** Suppose that the random variables satisfy equations (33) through (36) and agent’s choices are determined by equations (29) through (32). Let \( \bar{b} \) and \( \bar{q} \) denote the values for \( b \) and \( q \) when \( \sigma_\eta = \bar{\sigma}_\eta \). Prices are such that \( \bar{b} = 0 \). Let \( \hat{b} \) and \( \hat{q} \) denote the values for \( b \) and \( q \) when \( \sigma_\eta = \hat{\sigma}_\eta \). If

\[ \hat{\sigma}_\eta > \bar{\sigma}_\eta \]

then

\[ \hat{b} = \bar{b} = 0, \quad (38) \]
\[ \hat{q} > \bar{q}. \quad (39) \]

**Proof.** Since \( \bar{b} \) and \( \bar{q} \) satisfy the agent’s first-order conditions and \( \bar{b} = 0 \), we know that the following two equations hold:

\[ p_b (y_1 - p_a \bar{q})^{-\gamma} = \beta \mathbb{E} \left[ (\bar{q}\bar{y}_q + \tilde{y}_l (1 + \bar{\sigma}_\eta \bar{\tilde{h}}))^{-\gamma} \right], \quad (40) \]
\[ p_q (y_1 - p_a \bar{q})^{-\gamma} = \beta \mathbb{E} \left[ (\bar{q}\bar{y}_q + \tilde{y}_l (1 + \bar{\sigma}_\eta \bar{\tilde{h}}))^{-\gamma} \bar{\tilde{y}}_q \right]. \quad (41) \]
Using equations (33) and (34) and the fact that \( \bar{\eta}_i \) is an idiosyncratic random variable and, thus, not correlated with \( \bar{y} \), we can rewrite these two equations as

\[
\begin{align*}
\hat{p}_b \left( y_1 - p_b \bar{q} \right)^{\gamma} &= \beta \mathbb{E} \left[ (\bar{q} \alpha + (1 - \alpha) (1 + \bar{\sigma}_i \bar{\eta}_i))^{-\gamma} \right] \mathbb{E}[\bar{y}^{-\gamma}], \\
\hat{p}_q \left( y_1 - p_q \bar{q} \right)^{\gamma} &= \beta \mathbb{E} \left[ (\bar{q} \alpha + (1 - \alpha) (1 + \bar{\sigma}_i \bar{\eta}_i))^{-\gamma} \right] \mathbb{E}[\alpha y^{1-\gamma}].
\end{align*}
\]

(42) Combining gives

\[ \hat{p}_b = \hat{p}_q \frac{\mathbb{E}[\bar{y}^{-\gamma}]}{\mathbb{E}[\alpha y^{1-\gamma}]).} \]

\[ \hat{b} \text{ and } \hat{q} \text{ satisfy the following two equations:} \]

\[
\begin{align*}
\hat{p}_b \left( y_1 - p_q \hat{q} - \hat{p}_b \hat{b} \right)^{\gamma} &= \beta \mathbb{E} \left[ (\hat{q} \alpha + \hat{b} + \bar{y}_i (1 + \bar{\sigma}_i \bar{\eta}_i))^{-\gamma} \right], \\
\hat{p}_q \left( y_1 - p_q \hat{q} - \hat{p}_b \hat{b} \right)^{\gamma} &= \beta \mathbb{E} \left[ (\hat{q} \alpha + \hat{b} + \bar{y}_i (1 + \bar{\sigma}_i \bar{\eta}_i))^{-\gamma} \right].
\end{align*}
\]

(45) (46) To check whether \( \hat{b} = 0 \) is also the solution when \( \sigma_\eta = \bar{\sigma}_i \), we substitute \( \hat{b} = 0 \) in the two equations above and check whether both equations would give the same solution for \( \hat{q} \). Substituting \( \hat{b} = 0 \) gives

\[
\begin{align*}
\hat{p}_b \left( y_1 - p_q \hat{q} \right)^{\gamma} &= \beta \mathbb{E} \left[ (\hat{q} \alpha + \bar{y}_i (1 + \bar{\sigma}_i \bar{\eta}_i))^{-\gamma} \right], \\
\hat{p}_q \left( y_1 - p_q \hat{q} \right)^{\gamma} &= \beta \mathbb{E} \left[ (\hat{q} \alpha + \bar{y}_i (1 + \bar{\sigma}_i \bar{\eta}_i))^{-\gamma} \right].
\end{align*}
\]

(47) (48) which can be rewritten as

\[
\begin{align*}
\hat{p}_b \left( y_1 - p_q \hat{q} \right)^{\gamma} &= \beta \mathbb{E} \left[ (\hat{q} \alpha + (1 - \alpha) (1 + \bar{\sigma}_i \bar{\eta}_i))^{-\gamma} \right] \mathbb{E}[\bar{y}^{-\gamma}], \\
\hat{p}_q \left( y_1 - p_q \hat{q} \right)^{\gamma} &= \beta \mathbb{E} \left[ (\hat{q} \alpha + (1 - \alpha) (1 + \bar{\sigma}_i \bar{\eta}_i))^{-\gamma} \right] \mathbb{E}[\alpha y^{1-\gamma}].
\end{align*}
\]

(49) (50) If we use equation (44), we get that both equations are identical and, thus, would give the same solution for \( \hat{q} \).

It remains to show that \( \hat{q} > \bar{q} \). An increase in \( \sigma_\eta \) means that the right-hand side of both Euler equations increases. If \( q \) would decrease, then the right-hand sides would increase further and the left-hand sides would decrease, which clearly could not lead to a solution.

**G Solution algorithm**

**G.1 Solution algorithm for representative-agent model**

**G.1.1 Algorithm**

To solve the representative-agent models, we use a standard projection method, which solves for \( q_t \) and \( P_t \) on a grid and approximates the outcomes in-between gridpoints with piecewise linear interpolation.
G.1.2 Accuracy

Petrosky-Nadeau and Zhang (2013) consider a search and matching model with a representative agent. They show that it is not a trivial exercise to solve this model accurately, even though fluctuations are limited. Our representative-agent model is even simpler than the one considered in Petrosky-Nadeau and Zhang (2013). Nevertheless, we document here that both the linear and the log-linear perturbation solution are clearly not accurate. We also document that the projection solution is accurate.

![Graphs showing accuracy of different solution methods.](image)

**Figure 17:** Accuracy representative-agent solution.

*Notes.* These graphs plot the time series for the employment rate generated with the indicated solution method and the exact solution according to the Euler equation when the approximation is only used to evaluate next period’s choices.

To establish accuracy, we use the dynamic Euler-equation errors described in Den Haan (2010). The test compares simulated time series generated by the numerical solution for the policy rules with alternative time series. The alternative time paths are calculated using the exact equations of the model in each period; the
approximation is not used, except when evaluating next period’s choices inside the expectations operator. This test is similar to the standard Euler equation test, but reveals better whether (small) errors accumulate over time. If a numerical solution is accurate, then the two procedures generate very similar time paths.

Figure 17 displays part of the generated time series and clearly documents that the linear perturbation solution has a substantial systematic error, whereas our projection solution does not. Table 2 provides a more complete picture.

<table>
<thead>
<tr>
<th></th>
<th>projection</th>
<th>linear perturbation</th>
<th>log-linear perturbation</th>
</tr>
</thead>
<tbody>
<tr>
<td>average error (%)</td>
<td>0.84 \times 10^{-5}</td>
<td>1.01</td>
<td>0.43</td>
</tr>
<tr>
<td>maximum error (%)</td>
<td>0.28 \times 10^{-4}</td>
<td>1.76</td>
<td>0.74</td>
</tr>
<tr>
<td>average unemployment rate (%)</td>
<td>11.5</td>
<td>10.7</td>
<td>10.7</td>
</tr>
<tr>
<td>standard deviation employment</td>
<td>2.91</td>
<td>2.63</td>
<td>2.87</td>
</tr>
</tbody>
</table>

Notes: These results are based on a sample of 100,000 observations.

G.2 Solution algorithm for heterogeneous-agent model

In appendix G.2.1, we document how we solve the individual problem taking as given perceived laws of motion for prices and aggregate state variables. In appendix G.2.2, we document how to generate time series for the variables of this economy, including the complete cross-sectional distribution, taking the individual policy rules as given. The simulation is needed to update the laws of motion for the aggregate variables and to characterize the properties of the model. We make a particularly strong effort in ensuring that markets clear exactly such that there is no “leakage” during the simulation. This is important since simulations play a key role in finding the numerical solution and in characterizing model properties.\textsuperscript{88}

G.2.1 Solving for individual policy functions

When solving for the individual policy functions, aggregate laws of motion as specified in appendix G.2.2 are taken as given. Let $\tilde{x}_i$ denote an individual’s cash on hand at the perceived prices. That is,

$$
\tilde{x}_i = e_i (1 - \tau) \frac{\bar{W}}{P} + (1 - e_i) \mu (1 - \tau) \frac{\bar{W}}{P} + q_i \left( \frac{\bar{D}}{P} + (1 - \delta) \frac{\bar{J}}{P} \right) + \frac{M_i}{P}.
$$

\textsuperscript{88}If the equilibrium does not hold exactly, then the extent to which there is a disequilibrium is likely to accumulate over time, unless the inaccuracy would happen to be exactly zero on average. Such accumulation is problematic, since long time series are needed to obtain accurate representations of model properties.
Individual policy functions for equity, \( q'_i = q(x_i, e_i, q, z) \), and money, \( M'_i = M(x_i, e_i, q, z) \), are obtained by iteration:

(i) Using initial guesses for \( q'_i \) and \( M'_i \), a policy function for consumption can be calculated from the agent’s budget constraint:

\[
c(x_i, e_i, q, z) = x_i - q'_i J + M'_i.
\]

(ii) Conditional on the realizations of the aggregate shock and the agent’s employment state, cash on hand and consumption in the next period can be calculated:

\[
\tilde{x}'(e', z') = e' (1 - \tau') \tilde{W}' \frac{\tilde{p}'}{\tilde{p}'} + (1 - e') \mu (1 - \tau') \tilde{W}' \frac{\tilde{D}' + (1 - \delta) \tilde{J}' \tilde{p}'}{\tilde{p}'},
\]

\[
c'(e', z') = c(\tilde{x}'(e', z'), e', q', z').
\]

(iii) Using the individual and aggregate transition probabilities, the expectations \( \mathbb{E} \left[ c^{\gamma - \gamma} \tilde{D}' + (1 - \delta) \tilde{J}' \tilde{p} \right] \) and \( \mathbb{E} \left[ c^{\gamma - \gamma} \tilde{p} \right] \), in the first-order conditions (3) and (4) can be calculated. Then, the first-order condition for equity holdings gives an updated guess for consumption of agents holding positive amounts of equity:

\[
c^{\text{new}}(x_i, e_i, q, z) = \beta \mathbb{E} \left[ c^{\gamma - \gamma} \tilde{D}' + (1 - \delta) \tilde{J}' \tilde{p} \right]^{-\frac{1}{\gamma}}.
\]

The first-order condition for money gives an updated policy function for money:

\[
M^{\text{new}}(x_i, e_i, q, z) = \tilde{p}^{\frac{1}{\gamma}} \left( c^{\text{new}}(x_i, e_i, q, z)^{-\gamma} - \beta \mathbb{E} \left[ c^{\gamma - \gamma} \tilde{p} \right] \right)^{-\frac{1}{\gamma}}.
\]

The budget constraint in the current period gives the updated policy function for equity:

\[
q^{\text{new}}(x_i, e_i, q, z) = \max \left( 0, \frac{x_i \tilde{p} - c^{\text{new}}(x_i, e_i, q, z) \tilde{p} - M^{\text{new}}(x_i, e_i, q, z)}{J} \right).
\]

For agents with a binding short-sale constraint, updated policy functions for consumption and money are instead calculated using only the first-order condition for money and the budget constraint:

\[
c^{\text{new,constraint}}(x_i, e_i, q, z) = \left( \beta \mathbb{E} \left[ c^{\gamma - \gamma} \tilde{p} \right] + \chi \left( \frac{M'_i \tilde{p}}{\tilde{p}} \right)^{-\frac{1}{\gamma}} \right)^{-\frac{1}{\gamma}},
\]

\[
M^{\text{new,constraint}}(x_i, e_i, q, z) = x_i \tilde{p} - c^{\text{new,constraint}}(x_i, e_i, q, z) \tilde{p}.
\]

(iv) A weighted average of the initial guesses and the new policy functions is used to update the initial guesses. The procedure is repeated from step (i) until the differences between initial and updated
policy functions become sufficiently small.

G.2.2 Simulation and solving for laws of motion of key aggregate variables

The perceived laws of motion for the real stock price, $\tilde{J}/\tilde{P}$ and the price level, $\tilde{P}$, are given by the following two polynomials (using a total of 12 coefficients):

\[
\ln \frac{\tilde{J}}{\tilde{P}} = a_0(z) + a_1(z) \ln q + a_2(z) (\ln q)^2, \tag{56}
\]

\[
\ln \tilde{P} = b_0(z) + b_1(z) \ln q + b_2(z) (\ln q)^2. \tag{57}
\]

Note that $q$ is not only the level of employment, but also the number of firms, and the aggregate amount of equity shares held. We only use the first moment of the distribution of equity holdings, as in Krusell and Smith (1997), but we use a nonlinear function.\(^89\) To update the coefficients of this law of motion, we run a regression using simulated data. In this appendix, we describe how to simulate this economy taking the policy rules of the individual agents as given. We start by describing the general idea and then turn to the particulars.

**General idea of the simulation part of the algorithm.** Policy functions are typically functions of the state variables, that is, functions of predetermined endogenous variables and exogenous random variables. These functions incorporate the effect that prices have on agents’ choices, but this formulation does not allow for prices to adjust if equilibrium does not hold exactly when choices of the individuals are aggregated. If we used the true policy functions, then the equilibrium would hold exactly by definition. Unfortunately, this will not be true for numerical approximations, not even for very accurate ones. Since long simulations are needed, errors accumulate, driving supply and demand further apart, unless these errors happen to be exactly zero on average. Our simulation procedure is such that equilibrium does hold exactly. The cost of achieving this is that actual prices, $J$ and $P$, will be different from perceived prices, $\tilde{J}$ and $\tilde{P}$ and some of the actual individual choices will be different from those according to the original policy functions.\(^90\) These are errors too, but there is no reason that these will accumulate. In fact, we will document that perceived prices are close to actual prices in appendix G.2.3.

**Preliminaries.** To simulate this economy, we need laws of motions for perceived prices, $\tilde{J}(q,z)$ and $\tilde{P}(q,z)$, as well as individual policy functions, $q_i^t = q(\tilde{x}_i, e_i, q, z)$ and $M_i^t = M(\tilde{x}_i, e_i, q, z)$. At the beginning of each period, we would also need the joint distribution of employment status, $e_i$, and cash on hand, $x_i$. This distribution is given by $\psi(\tilde{x}_i, e_i)$, where the tilde indicates that cash on hand is evaluated at perceived prices.

The distribution is such that,

\[
\int_{e_i} \int_{x_i} \tilde{x}_i d\psi_i = zq + (1 - \delta)q \frac{\tilde{J}}{\tilde{P}} + \frac{M}{\tilde{P}}, \tag{58}
\]

\(^{89}\)Note that the first-moment of money holdings is constant, since money supply is constant.

\(^{90}\)Throughout this appendix, perceived variables have a tilde and actual outcomes do not.
where the dependence of prices on the aggregate state variables has been suppressed. Below, we discuss how we construct a histogram for the cross-sectional distribution each period and show that this property is satisfied. We do not specify a joint distribution of equity and money holdings. As discussed below, we do know each agent’s level of beginning-of-period equity holdings, \( q_i \), and money holdings, \( m_i \).

A household’s cash-on-hand level is given by

\[
\tilde{x}_i = e_i (1 - \tau) \frac{\bar{W}}{P} + (1 - e_i) \mu (1 - \tau) \frac{\bar{W}}{P} + q_i \left( \frac{\bar{D}}{P} + (1 - \delta) \frac{\bar{J}}{P} \right) + \frac{M_i}{P},
\]

and the household can spend this on consumption and asset purchases, that is,

\[
\bar{x}_i = c_i + q_i' \frac{\bar{J}}{P} + \frac{M_i'}{P},
\]

The government has a balanced budget each period, that is,

\[
\tau = \mu \frac{1 - q}{q + \mu (1 - q)}.
\]

Even if the numerical solutions for \( q_i' \), \( M_i' \), \( \bar{J} \), and \( \bar{P} \) are very accurate, it is unlikely that equilibrium is exactly satisfied if we aggregate \( q_i' \) and \( M_i' \) across agents. To impose equilibrium exactly, we modify the numerical approximations for equity and money holdings such that they are no longer completely pinned down by exogenous random variables and predetermined variables, but instead depend directly – to at least some extent – on prices. In the remainder of this section, we explain how we do this and how we solve for equilibrium prices.

**Modification and imposing equilibrium.** To impose equilibrium we adjust \( q_i', M_i', \bar{J}, \) and \( \bar{P} \). The equilibrium outcomes are denoted by \( q_{i, +1}, M_{i, +1}, \bar{J}, \) and \( \bar{P} \). The individual’s demand for assets is modified as follows:

\[
q_{i, +1} = \frac{\bar{J}}{\bar{P}} \frac{\bar{P}}{P} q_i',
\]

\[
M_{i, +1} = \frac{P}{P} M_i'.
\]

We will first discuss how equilibrium prices are determined and then discuss why this is a sensible modification. An important accuracy criterion is that this modification of the policy functions is small, that is, actual and perceived laws of motions are very similar.

We solve for the actual law of motion for employment, \( q_{+1} \), the number of new firms created, \( h \), the amount spent on creating new firms in real terms, \( v = hJ/P \), the market clearing asset price, \( J \), and the market prices for equity and money. The policy functions \( q(\tilde{x}, \epsilon, q, z) \) and \( M(\tilde{x}, \epsilon, q, z) \) do depend on prices, but this dependence is captured by the aggregate state variables.

As explained above, it is important to do a modification like this to ensure that equilibrium holds exactly, even if the solution is very accurate and the modification small.
clearing price level, \( P \), from the following equations:\(^\text{93}\)

\[
q_{t+1} = (1 - \delta) q_t + h, \\
h = \psi \upsilon^n (1 - q_t)^{1 - \eta}, \\
\upsilon = h J / P.
\]

\[
h = \int \int (q_{i,t+1}(\bar{x}_i, e_i, q, z) - (1 - \delta) q_t) d \psi_i
\]

\[
\bar{M} = \int \int M_{i,t+1}(\bar{x}_i, e_i, q, z) d \psi_i = \int \int \frac{P}{\bar{P}} M(\bar{x}_i, e_i, q, z) d \psi_i.
\]

In particular, the distribution satisfies

\[
\int \int q_{i,t+1} d \psi_i = q_{t+1}.
\]

**Logic behind the modification.** Recall that \( q(\bar{x}_i, e_i, q, z) \) and \( m(\bar{x}_i, e_i, q, z) \) are derived using perceived prices, \( \tilde{J}(q, z) \) and \( \tilde{P}(q, z) \). Now suppose that – in a particular period – aggregation of \( q(\bar{x}_i, e_i, q, z) \) indicates that the demand for equity exceeds the supply for equity. This indicates that \( \tilde{J}(q, z) \) is too low in that period. By exactly imposing equilibrium, we increase the asset price and lower the demand for equity. Note that our modification is such that any possible misperception on prices does not affect the real amount each agent spends, but only the number of assets bought.

Throughout this section, the value of cash on hand that is used as the argument of the policy functions is constructed using **perceived** prices. In principle, the equilibrium prices that have been obtained could be used to update the definition of cash on hand and one could iterate on this until convergence. This would make the simulation more expensive. Moreover, our converged solutions are such that perceived and actual prices are close to each other, which means that this iterative procedure would not add much.

**Equilibrium in the goods market.** It remains to show that our modification is such that the goods market is in equilibrium as well. That is, Walras’ law is not wrecked by our modification.

From the budget constraint we get that actual resources of agent \( i \) are equal to

\[
x_i = e_i (1 - \tau) \frac{W}{P} + (1 - e_i) \mu (1 - \tau) \frac{W}{P} + \left( \frac{D}{P} + (1 - \delta) \frac{J}{P} \right) q_i + \frac{M_i}{P}
\]

and actual expenditures are equal to

\[
x_i = c_i + \frac{J}{P} q_{i+1} + \frac{M_{i+1}}{P}.
\]

\(^{93}\)Recall that we define variables slightly different and \( v \) is not the number of vacancies, but the amount spent on creating new firms.
The value of $c_i$ adjusts to ensure this equation holds. Aggregation gives

$$x = zq + \frac{J}{P} (1 - \delta) q + \frac{M}{P}$$

and

$$x = c + \frac{J}{P} \int \int q_{i,+1} d\psi_i + \int M_{i,+1} d\psi_i + \frac{J}{P} q_{i,-1} + \frac{M}{P}.$$

Equation (72) uses the definition of dividends together with equation (69). Equation (73) follows from the construction of $J$ and $P$.

Since

$$\frac{J}{P} q' - \frac{J}{P} (1 - \delta) q = v,$$

we get

$$zq = c + v,$$

which means that we have goods market clearing in each and every time period.

**Implementation.** To simulate the economy, we use the “non-stochastic simulation method” developed in Young (2010). This procedure characterizes the cross-sectional distribution of agents’ characteristics with a histogram. This procedure would be computer intensive if we characterized the cross-sectional distribution of both equity and bond holdings. Instead, we just characterize the cross-sectional distribution of cash-on-hand for the employed and unemployed. Let $\psi(\bar{x}_{i,-1},e_{i,-1})$ denote last period’s cross-sectional distribution of the cash-on-hand level and employment status. The objective is to calculate $\psi(\bar{x}_{i},e_{i})$.

(i) As discussed above, given $\psi(\bar{x}_{i,-1},e_{i,-1})$ and the policy functions, we can calculate last period’s equilibrium outcome for the total number of firms (jobs) carried into the current period, $q$; the job-finding rate, $h_{-1}/(1 - q_{-1})$; last period’s prices, $J_{-1}$ and $P_{-1}$; and for each individual the equilibrium asset holdings brought into the current period, $q_i(\bar{x}_{i,-1},e_{i,-1})$ and $M_i(\bar{x}_{i,-1},e_{i,-1})$.

(ii) Current employment, $q$, together with the current technology shock, $z$, allows us to calculate perceived prices $\bar{J}$ and $\bar{P}$.

(iii) Using the perceived prices together with the asset holdings $q_i$ and $M_i$, we calculate perceived cash on hand conditional on last-period’s cash-on-hand level and both the past and the present employment status. That is,

$$\bar{x}(e_i,\bar{x}_{i,-1},e_{i,-1}) = e_i (1 - \tau) \bar{W} + (1 - e_i) \mu (1 - \tau) \bar{W}$$

$$+ q_i(\bar{x}_{i,-1},e_{i,-1}) \left( \frac{\bar{D}}{P} + (1 - \delta) \frac{\bar{J}}{P} \right) + M(\bar{x}_{i,-1},e_{i,-1}) \frac{M(\bar{x}_{i,-1},e_{i,-1})}{P}.$$
Using last period’s distribution \( \psi(\tilde{x}_i, e_{i,-1}) \) together with last-period’s transition probabilities, we can calculate the joint distribution of current perceived cash on hand, \( \tilde{x}_i \), past employment status, and present employment status, \( \psi(\tilde{x}_i, e_i, e_{i,-1}) \).

Next, we retrieve the current period’s distribution as

\[
\psi(\tilde{x}_i, 1) = \hat{\psi}(\tilde{x}_i, 1, 1) + \hat{\psi}(\tilde{x}_i, 1, 0),
\]

\[
\psi(\tilde{x}_i, 0) = \hat{\psi}(\tilde{x}_i, 0, 1) + \hat{\psi}(\tilde{x}_i, 0, 0).\]

Even though we never explicitly calculate a multi-dimensional histogram, in each period we do have information on the joint cross-sectional distribution of cash on hand at perceived prices and asset holdings.

Details. Our wage-setting rule (7), contains \( \bar{P} \), an indicator for the average price level. For convenience, we use the average between the long-run expansion and the long-run recession value. Since it is a constant, it could be combined with the scaling factor, \( \omega_0 \). The properties of the algorithm are improved by including \( \bar{P} \). If a term like \( \bar{P} \) would not be included, then average wages would change across iteration steps. Moreover, without such a term, then recalibrating \( \omega_0 \) would be more involved, for example, if one compares the case with and the case without aggregate uncertainty. We use a simulation of 2,000 observations to estimate the coefficients of the laws of motion for aggregate variables. The first 200 observations are dropped to ensure the results are not affected by the specification of the initial state. The histogram that we use to track the cross-sectional distribution has 2,000 grid points. Statistics reported in the main text that are obtained by simulation are from a sample of 100,000 observations.

G.2.3 Accuracy

Conditional on perceived laws of motion for the price level and the employment rate, individual policy rules can be solved accurately using common numerical tools even though the presence of a portfolio problem makes the individual optimization problem a bit more complex than the standard setup in heterogeneous-agent models. The key measure of accuracy is, therefore, whether the perceived laws of motion for the price level and the employment rate coincide with the corresponding laws of motion that are implied by the individual policy rules and market clearing. Figure 18 shows that both perceived laws of motion track the implied market clearing outcome very closely.

\footnote{This actually is a good approximation of the average price level.}
Figure 18: Accuracy heterogeneous-agent solution.

Notes. These graphs plot for the indicated variable the timeseries according to the perceived law of motion (used to solve for the individual policy rules) and the actual outcomes consistent with market clearing.
References


