Export and the Labor Market: a Dynamic Model with on-the-job Search

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December 3, 2012

Abstract

There is large evidence that firms attract workers from other firms; this is particularly true for firms that become exporters. Nevertheless, the recent literature in trade and labor did not focus on the role of job to job transitions. This paper develops a dynamic theoretical framework with on-the-job search to analyze the effect of a trade liberalization on unemployment and wage inequality. We propose a two-sector, two-factor dynamic general equilibrium model with homogeneous workers, who search also when they are employed, and heterogeneous firms that take forward looking entry and export decisions.

The main prediction of the model is that a decrease in the cost of export leads to an economy populated by fewer larger firms that are more productive and pay better wages; but this also determines more unemployment and it increases wage inequality. The dynamics of the economy is driven by firm entry and market clearing and it can be solved in exact form. Depending on the initial degree of trade openness, the transitional dynamics of unemployment and wage inequality can exhibit a non monotonic path that cannot be understood by looking only at the steady state comparison.

Many thanks to my advisors Gianmarco Ottaviano and Antonella Trigari, to Pol Antràs, Elhanan Helpman, Marc Melitz, Lucia Tajoli and to participants at International Economics seminars at Harvard University, University of Michigan in Ann Arbor, Winter Meeting of

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1 Introduction

There is no unique consensus on the effect of an increase in export participation on unemployment and wage inequality. This paper sheds light on this topic building a framework with job to job reallocation. This is a particularly prominent phenomenon in the data but it has not received enough attention in the theory of trade and labor markets. The recent labor literature provides evidence that worker job to job transitions represent the majority of worker reallocation flows. Nevertheless, the current literature on the effect of trade on the labor market is based on the mainstream approach by Diamond (1982) and (Mortensen and Pissarides, 1994), focusing on worker flows from unemployment to employment.

The following table shows that most of new hirings come from job to job transition, this is particularly true for exporters. The table refers to employer-employee data for 71 million workers and 3.75 million firms in Brazil and it covers the period 1990-2000.\footnote{A complete survey of this evidence is summarized in Muendler’s webpage \url{http://econ.ucsd.edu/muendler/html/brazil.html#brazdata}} The contribution of this paper is to address

<table>
<thead>
<tr>
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<th>New employees hired from employed workers</th>
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<tr>
<td>never exporter</td>
<td>71.1 %</td>
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<tr>
<td>new exporter</td>
<td>73.4 %</td>
</tr>
<tr>
<td>exporter</td>
<td>74.4 %</td>
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Table 1: Job to job transitions; source: Brazilian Ministry of Labor, confidential data collected by Marc Muendler, provided by Lorenzo Casaburi, Harvard University.

the effect of export participation on unemployment and wage inequality in a dynamic framework with on-the-job search. We solve for the dynamics of unemployment and wage inequality after an export shock; whereas most of the current studies develop static frameworks. The main prediction of the model
is that a policy that increases export participation leads to an economy populated by fewer larger firms that are more productive and pay better wages; but this also determines more unemployment and it increases wage inequality. The framework is highly tractable and it allows to solve for the exact dynamics of labor market variables. The path of unemployment rate, labor market tightness and the Gini index on the wage distributions are fully characterized. The source of worker reallocation and wage dispersion is the competition across firms in the labor market due to on-the-job search. Workers are all homogeneous and they randomly search among wage offers also when they are employed. As a consequence, firms face an additional competition channel, in the form of an increasing labor supply curve. The intuition is the following. The probability that a worker visits a vacancy is the same across firms. Unemployed workers accept all wage offers above the reservation wage but employed workers accept only wage offers that are better than their current employment status. Therefore, employers offering a better wage have a better acceptance rate per vacancy posted and lower separation per worker. More productive firms offer a higher wage, accepting to decrease their profit per worker in exchange for attracting more workers and maximize total profit. When there are exporters in the economy, the effect of on the job search on labor allocations is amplified. Following a policy that increases export participation, the economy experiences a reallocation of workers from many small firms paying a low wage to fewer bigger firms offering a better wage and serving both the domestic and foreign market. This mechanism causes the loss of employment and the increase of inequality. In the dynamic general equilibrium, firm entry and export decisions drive the demand of labor and employment allocation across firms. The entry of a firm in the domestic as well as in the export market requires a fixed amount of services, that are produced by the non tradable sector. The allocation of workers to the service is subject to the labor market transitional dynamics. Frictions in the labor market are responsible for the dynamics in the mass of employers in the economy and ultimately for the dynamics of the unemployment rate. Finally, as the worker reallocation across firms in the tradable sector is responsible for the increase in wage inequality, the reallocation of workers across sectors mitigates the rise in wage inequality in the entire economy.
The dynamic stochastic general equilibrium is characterized by a system of three linear first order difference equations in the mass of employers, the stock of services and the number of exporters. Policy analysis and dynamics can be easily worked out analytically in closed form solution. The solution of the dynamic system is exact and it exhibits saddle path stability.

The predictions of the model are consistent with the main findings in the empirical literature documenting the effect of trade liberalization on labor markets. Trefler (2004) documents the effects of trade openness on the labor market, in the case of a US-Canada free trade agreement. The increase in trade openness is associated with a loss of employment by 5% in total manufacturing; 12% for industries that were more affected by the liberalization. At the same time, productivity increases by 6% in total manufacturing; 15% for more impacted industries. Another example is Felbermayr et al. (2011b) where they show, on a sample of 20 OECD countries from 1983 to 2003, that once the business cycle components are taken out of the analysis then there is no evidence of an unemployment-increasing effect of trade openness.

Goldberg and Pavcnik (2007) provide large qualitative and quantitative evidence of the increase in inequality after trade liberalization reforms. Using data on Mexico, Colombia, Argentina, Brazil, Chile, India and Hong Kong from the 80s and 90s they show that all countries experienced an increase in wage inequality after trade liberalization policies. Moreover, there is evidence of a stabilization of wage inequality after a decade from the trade liberalization. A comprehensive survey on this topic is for example discussed in Bernard et al. (2011).

We model a trade liberalization policy as a fall in the sunk cost. Roberts and Tybout (1997) document the importance of sunk costs of entry in the export market. They also find that prior export experience increases the probability of exporting by 60% and that exporters are more productive, pay better wages and hire more workers. At the same time, the reallocation of workers to exporter firms determines a selection of less productive firms out of the market.

The paper can be framed in the “new trade theory” based on firm heterogeneity, forward looking entry and export decisions as in Melitz (2003), endogenous markup as in Melitz and Ottaviano (2008) and it is consistent
with a more broad set of findings. Bernard et al. (2007) provide extensive evidence of the dimensions through which exporters are “special”. They show that exporters pay a wage that is 6% larger than non exporters, they hire 97% more employees, they ship 108% more output and they are 11% more productive in terms of value added per worker. While exporters employ almost twice the number of workers than non exporters, non exporters have more than twice the probability of “death” than exporters have, given the same time span.

There is a recent and growing effort in the new trade theory to propose frameworks that can account for these empirical findings.

Felbermayr et al. (2011a) combine the Diamond, Mortensen and Pissarides modeling of the labor market with firm heterogeneity and selection into the export market, as in Melitz (2003). They show that following an increase in the extensive margin of trade unemployment should fall as a consequence of an increase in average productivity.

Helpman and Itskhoki (2010) develop a two sector framework in which trade liberalization is responsible for the reallocation of workers from a competitive non tradable sector to a tradable sector in which firms are heterogeneous and there are frictions in the labor market. Depending on the strength of this reallocation, overall unemployment may increase. In both these frameworks, as in most of the literature, there is no wage dispersion across firms of the tradable sector. Dealing with wage inequality requires instead a mechanism that is responsible for wage dispersion across firms.

Helpman et al. (2010) propose an argument based on match specific ability and capability of firms to invest in screening workers after they are matched. Workers have the same ex-ante expected earning across possible employers but they are paid differently depending on the employer with whom they are matched. In this framework, more productive firms pay higher wages and the status of being exporter increases the wage paid by a firm with a given productivity. The increase in trade openness increases wage inequality with respect to the autarky case. Moreover, the theoretical relationship between trade openness and wage inequality exhibits an inverted U shape.

This paper is related to at least other two strains of literature. Regarding labor with on the job search the model has a similar structure to Mortensen (2009). In this model contracts are not binding as in Stole and Zwiebel (1996),
and firms bargain with their employees and post wage offers on the market choosing the number of vacancies to keep employment at the desired level. Our paper extends this framework in general equilibrium where the derived demand of labor occurs as a result of the demand side of the goods market. Further extensions on the labor side of our framework could be to include counteroffers as in Postel-Vinay and Robin (2002) and pure wage posting as in Moscarini and Postel-Vinay (2012). Moreover our model satisfies the conditions for block recursivity under directed search as defined in Menzio and Shi (2010).

Dealing with industrial dynamics, the framework is a simplified version of Hopenhayn (1992a) and Hopenhayn (1992b) in a monopolistic competition setup. The model could also be extended to allow for endogenous innovation choice as in Burstein and Melitz (2011).

This framework reproduces all traditional gains from trade due to efficiency, but at the same time the theoretical implications we derive about unemployment and wage inequality make the evaluation of gains from trade richer. At the individual level homogeneous workers can be affected in opposite way by a trade liberalization, even if they are employed in the same tradable sector. At the aggregate level the economy experiences higher wages but more unemployment and more inequality.

The paper is organized as follows. The next section provides the discussion of the model. In section three we define and solve for the dynamic general equilibrium. In section four we discuss the consequences of fall in the fixed cost of export. Section five concludes.

2 Model

There are two sectors producing a consumption good and a durable good. In the consumption sector a continuum of single product monopolists supply varieties of a differentiated good. In the durable goods sector a continuum of perfectly competitive producers supply a composite good that is used as input in the consumption sector. Labor is used in both sectors as a variable factor of production. Workers are the owners of production inputs and they rent them to firms.

The consumption goods are treated as internationally tradable manufac-
turing goods. Instead, the durable good is a composite asset of non tradable inputs (such as land, infrastructure or utilities) and services that support firms in their hiring activity (such as agencies that match workers with firms). These resources have a value and depreciate according to their use in the production process of consumption goods. Each period agents demand consumption goods and hold stocks of the value of the durable input. In this sense, we refer to the durable good as capital.

2.1 Consumer preferences

Preferences are non homothetic\(^2\) and the marginal utility of income depends on the number of competitors in the market, first and second moments of their price distribution. Among several utility functions that can rationalize this structure we restrict our choice such that the marginal revenue is linear in price. This is a desirable property when firms bargain with workers over the match surplus.

Consumers allocate consumption over a continuum of varieties indexed by \(i \in \Omega (i)\):

\[
U \left( \{c_\omega (i)\}_{i \in \Omega (i)} \right) = \alpha \int_{i \in \Omega (i)} c_\omega (i) \, di - \frac{\gamma}{2} \int_{i \in \Omega (i)} c_\omega (i)^2 \, di
\]  

(1)

where \(c_\omega (i)\) is the consumption of variety \(i\) of agent \(\omega\). This utility function is a special case of Ottaviano et al. (2002) in which agents do not value aggregate consumption and they do not consume an outside good\(^3\). The demand system for each variety is linear. The parameter \(\alpha > 0\) is a demand shifter and the parameter \(\gamma > 0\) accounts for the sensitivity of demand to price. Condition \(c_\omega (i) \leq \alpha/\gamma \, \forall i\) guarantees positive and diminishing marginal returns.

Over a population of \(N\) consumers, the aggregate demand for each variety is given by:

\[
C_t (i) = \frac{N}{\gamma} (\alpha - E_\omega [\lambda (\omega)] p (i))
\]  

(2)

where \(E_\omega [\cdot]\) is the expectation operator over the space of agents \(\Omega (\omega)\), \(\lambda (\omega)\) is the marginal utility of income for agent \(\omega\) at time \(t\), \(p (i)\) is the price of variety

\(^2\)As Fieler (2011) argues “there is exhaustive evidence that the income elasticity of demand varies across goods and that this variation is economically significant”.

\(^3\)Income effect is restored as in Neary (2007).
and it is bounded above \( p(i) < \frac{\alpha}{E[\lambda(\omega)]} \equiv p_{\text{max}} \) for every variety that is sold in the market.

### 2.2 Technology

In both sectors labor is employed with constant returns to scale technology. In the capital sector productivity changes over time. In the consumption sector firms are heterogeneous, therefore each point in time we observe a distribution of productivity across firms.

Let \( \varphi \) be the productivity of labor in the service sector at time \( t \). Because of perfect competition, production in the service sector is equivalent to the output of \( E_{t}^{\text{serv}} > 0 \) homogeneous firms with productivity \( \varphi \) employing \( l_{t}^{\text{serv}} \) workers:

\[
A_{t} = \varphi E_{t}^{\text{serv}} l_{t}^{\text{serv}}
\]  

where \( A_{t} \) are the new units of service produced at time \( t \).

In the consumption sector, let \( c \in [c_{\underline{c}}, c_{\overline{c}}] \) be the random variable that describes the requirement in terms of labor per unit of output per time, with \( c_{\underline{c}} \) strictly positive finite values. Define \( \Phi(c) \) as the distribution of \( c \) over the support \( c \in [c_{\underline{c}}, c_{\overline{c}}] \). The production function in the consumption sector is:

\[
q(c) = \frac{l(c)}{c}
\]  

where \( l(c) \) is firm employment and \( q(c) \) is the output of a firm endowed with unit labor cost \( c \).

As in Hopenhayn (1992a), unit labor requirements are independent draws across firms. In addition we assume that all firms draw from the same distribution\(^{4}\). This choice increases the tractability of the model, at the cost of losing information about a single firm over time. Nevertheless, the primary concern of the present work is the effect of a trade shock on the unemployment rate and the wage distribution. As we will discuss later, firms do not commit

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\(^{4}\)A tractable way to avoid the assumption that all firms draw from the same distribution is to assume that idiosyncratic productivity follows a Markov process with draws that are independent across firms each point in time, that is exactly the specification in Hopenhayn (1992b). However, if we believe that there is no reason why workers should know about the firm’s idiosyncratic productivity process then the tractability and results of this framework will not be affected.
to binding contracts and workers can reallocate across firms when they are employed, other than move from unemployment to employment. Under this scenario, the assumption of independence will not drive our conclusions on unemployment rate and wage distribution.

2.3 Consumption sector: firm level variables

Let $w(c)$ be the wage paid by a firm endowed with unit labor cost $c$. The profit maximizing price $p(c)$ satisfies the equivalence between marginal cost $w(c)c$ and marginal revenue $2p(c) - \alpha \frac{E_{\omega} \lambda(\omega)}{E_{\omega} \lambda(\omega)}$:

$$p(c) = \frac{1}{2} \left( \frac{\alpha}{E_{\omega} \lambda(\omega)} + w(c)c \right)$$

As in Melitz and Ottaviano (2008), define $c_D$ as the maximum unit labor cost below which a firm faces positive demand, $p(c_D) = p_{\text{max}}$, such that $w(c_D)c_D = \alpha E_{\omega} \lambda(\omega)$. Then by (2) the firm’s optimal price is:

$$p(c) = \frac{1}{2} (w(c) c_D + w(c)c) \quad (5)$$

Equilibrium output and labor demand, revenue and variable profit (revenue minus variable cost) associated to the production in the domestic market are:

$$C^{\text{dom}}(c) = \frac{N}{2} \alpha \left( 1 - \frac{w(c)c}{w(c_D)c_D} \right) \quad (6)$$

$$l^{\text{dom}}(c) = \frac{N}{2} \gamma \left( 1 - \frac{w(c)c}{w(c_D)c_D} \right) c \quad (7)$$

$$r^{\text{dom}}(c) = \frac{N}{4} \alpha \gamma w(c_D)c_D \left[ 1 - \left( \frac{w(c)c}{w(c_D)c_D} \right)^2 \right] \quad (8)$$

$$\pi^{\text{dom}}(c) = \frac{N}{4} \alpha \gamma w(c_D)c_D \left( 1 - \frac{w(c)c}{w(c_D)c_D} \right)^2 \quad (9)$$

The domestic economy is small and opened to trade. Firms of the domestic economy that export face the same preference and technology structure but a competitive environment that is possibly different from the domestic one. All the informations exporters need to know about the foreign market consists of the expected marginal utility of income across foreign consumers $E_{\omega} [\lambda_t(\omega)]^*$ and the nominal bilateral exchange rate $e_t$, measured as number of foreign currency units per one unit of domestic currency.
We assume there is no variable cost associated to trade, exporters set the price according to producer currency pricing and the law of one price holds.\(^5\)

The optimal price of a good produced by a domestic firm endowed with productivity \(c\) is 
\[
p(c) = \frac{1}{2} \left( \frac{\alpha}{E_\omega [\lambda(\omega)]} + w(c) c \right)
\]
in domestic currency and 
\[
p^*(c) = \frac{1}{2} \left( \frac{\alpha}{E_\omega [\lambda(\omega)]^*} + ew(c) c \right)
\]
in foreign currency. The nominal bilateral exchange rate that satisfies the law of one price is equal to the ratio in the average marginal utilities of income in the two countries:
\[
E_\omega [\lambda(\omega)] = e E_\omega [\lambda(\omega)]^*
\]
Levels and fluctuations of the nominal bilateral exchange rate and average marginal utility of income in the foreign economy are exogenous. This paper does not investigate these sources of shocks. Still in a comparative statics exercise, this framework would imply that when the domestic currency loses value in terms of foreign currency, or the average marginal utility of income in the foreign economy decreases, then firms in the domestic economy face positive demand for a larger unit labor requirement:
\[
\frac{w(c_D)}{1/c_D} = \frac{\alpha}{E_\omega [\lambda(\omega)]} = \frac{\alpha}{e E_\omega [\lambda(\omega)]^*}
\]
The value of \(c_D\) is a sufficient statistics to describe the international competitiveness of domestic firms, as \(\frac{w(c_D)}{1/c_D}\) is the maximum unit labor cost below which a firm faces positive demand in the domestic market.

As a result of this scenario, the marginal revenue associated to sales of the same product in the domestic or foreign markets is the same. Demand, employment and profit associated to the foreign market are the same as in the domestic economy, but for the market size:
\[
C^{exp}(c) = \frac{N^*}{N} C^{dom}(c), \quad l^{exp}(c) = \frac{N^*}{N} l^{dom}(c), \quad \pi^{exp}(c) = \frac{N^*}{N} \pi^{dom}(c)
\]
where \(N^*\) is the population in the foreign economy.

This symmetric structure also implies that the value of a job is the same whether the worker is producing a good that will be sold in the domestic or in

\(^5\)Campa and Goldberg (2005) documented that the producer currency pricing is a more likely scenario, with respect to the local currency pricing, at least in the long run for many types of imported goods among 23 OECD countries from 1975 to 2003. Still they find evidence of incomplete pass through, ranging in the long run from 40% in US to 80% in Germany.
the foreign market. How does the value of a job in the domestic labor market depend on the destination goods market is a channel linking trade and labor that deserves investigation. To do so, in this framework we should relax at least one of the three assumptions that restrict the foreign transactions\(^6\).

2.3.1 Demand of labor at the firm level

Employment at the firm level is determined by the labor demand (7) in the domestic and export market (11):

\[
l^d_t(w) = \begin{cases} 
\frac{N \alpha \gamma}{2} \left(1 - \frac{w(c)}{w(c_D)} \right) \frac{c}{c_D}, & \text{if non exporter at time } t \\
\frac{N + N^*}{2} \frac{\alpha \gamma}{\gamma} \left(1 - \frac{w(c)}{w(c_D)} \right) \frac{c}{c_D}, & \text{if exporter at time } t 
\end{cases}
\]

The demand of labor is a continuous decreasing function of the wage. Notice that the unit labor cost for all firms that face a positive demand is lower than the unit labor cost for a firm with zero demand \(\frac{w(c)}{1/c} < \frac{w(c_D)}{1/c_D}\).

The main observation that will be crucial for the discussion is that exporters behave in the labor market as bigger employers, in the following sense. An exporter demands \((1 + \frac{N^*}{N})\) times the units of labor than it would do in case it was endowed with the same productivity but it did not have the chance to export. Let \(E_t\) be the number of employers that serve the domestic consumption sector and let \(x_t\) be the share of exporters out of those. Define an employer unit as an employer that demand labor to serve the domestic market, then the effective number of employer units in the demand side of the labor market is \((1 + x_t \frac{N^*}{N}) E_t\).

2.4 Fixed costs

In the consumption sector firms obey a structure of fixed costs. A firm purchases units of the capital service as fixed input employed in the activities of entry, production and export. The price of each unit of capital is one; hence capital is the numeraire.

Firms demand the services in the capital sector in order to adjust employment over time. Firms pay for the advertising vacancies, hiring and training

\(^6\)This is the research question for a new project that aims to address the correlation between labor market variables across countries that are trade partners. In this paper we do not address this question.
costs; in addition separations are costly to the extent that employment protection and unemployment insurance are enforced by law. The literature on search in the labor market treats these costs as match specific investments, Mortensen and Pissarides (1999). Under this assumption a two-tiers wage structure arises. Hiring costs are sunk in the case of renegotiations with current employees, but they decrease the value a firm attaches to a match with workers in the market. At the same time, firing costs affect continuation decisions; especially in case a penalty applies to separations that occur without renegotiations.

However in the context of this work, there are limitations to which that discussion can be performed. Firms change their productivity each period exogenously and contracts cannot commit firms and employees to a particular wage or employment level in the future. Therefore, there would not be a clear understanding of what a match specific hiring cost and firing cost mean. Workers are homogeneous and there is complete information, such that interpersonal comparison in a context of costless renegotiations would hardly be reconciled with a two wage structure. Most importantly, each period firms take decisions about enter a market or wait, stay or exit a market. These decisions bring the focus of the discussion on firm turnover, more than job turnover. Under these considerations, we treat fixed costs of employment adjustment as specific to the firm, not to the match.

Incumbent firms that adjust employment between two subsequent periods pay a regular price of $f_p$ units of capital service; where regular refers to the fact that the hiring / firing needs are an ordinary consequence of matching demand and production needs in the period. Firms that enter the domestic market for the first time require an exceptional hiring service in order to contact a stock of potential employees to start with in the next period. This is an irreversible investment, as it is specific to a firm in a particular point in time. The sunk cost of entry associated to this investment is $f_{ed}$. A firm that enters the export market for the first time demand more workers to satisfy the demand of a second market. Therefore also in this case, an exceptional hiring activity is

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7A more general discussion is developed in Coşar et al. (2010).
needed. The sunk cost of entry in the export market is $f_{ex}$. Just for the sake of simplicity, firms that exit the market do not pay any cost$^9$.

### 2.5 Labor market

Following the approach developed by Diamond (1982) and Mortensen and Pissarides (1994), the labor market is characterized by search and matching frictions. Workers are homogeneous and they search for better job offers also while they are employed as in Burdett and Mortensen (1998)$^{10}$. The search and matching process is random and time is discrete. When a worker-firm match occurs the two parties bargain over the match surplus as risk neutral agents$^{11}$. There is complete information and both parties know the distribution of wage offers in the economy.

Contracts are not binding in the sense of Stole and Zwiebel (1996), they only specify the current wage and they cannot be made contingent upon outside offers. In every negotiation there is a potentially infinite number of offers and firms move to the production stage only when i) an agreement is reached or ii) the firm and the employee separate$^{12}$. In this context, Stole and Zwiebel (1996) show that (i) the firm cares about the profit of the marginal worker in the current period (ii) prior unenforceable agreements cannot affect the outcome at any renegotiation round.

The labor market has a structure similar to Mortensen (2009). In this respect, the contribution of this work is to solve for labor market allocations within a general equilibrium approach. Each period, firms realize their unit labor requirement in the goods market and, given this information, they un-

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$^9$In alternative, the value of an exit would be negative instead of being zero. This does not affect the tractability of the model or the conclusions.

$^{10}$Postel-Vinay and Robin (2002) develop a structural model with workers and firms heterogeneity based on French panel data. They find that the extent to which worker individual characteristics explain wage differences lies close to 40% for high-skilled white collars, but it falls to be negligible as the observed skill level decreases.

$^{11}$The assumption upon which workers behave as risk neutral agents in the labor market is common to works that are our benchmark in this topic (see Helpman and Itskhioki (2010) for a discussion).

$^{12}$See Stole and Zwiebel (1996) footnotes 15 and 17 for a more detailed discussion on the stable bargaining outcome under the infinite number of negotiations, versus alternative specifications.
derstand the optimal employment level and the marginal profit they gain from a worker.

Nevertheless, the framework we described simplifies the discussion in several non trivial directions. First, technology exhibits constant returns on labor. Therefore, the marginal value of an employee does not depend on firm employment; as in the case with diminishing returns on labor\textsuperscript{13}. The optimal number of vacancies issued by a firm is implicitly determined as the number of vacancies that are needed to keep employment at the desired level.

Second, the reservation wage is fixed in a competitive outside sector. Firms in the capital sector pay workers their marginal value. At the same time, they do not have incentives to rise the wage above the reservation wage. As a consequence, the reservation wage in the economy is equal to the value of the marginal productivity of labor in the capital sector.

Third, in each period firms are endowed with i.i.d. draws of unit labor requirement. This implies that before the realization of productivity shocks all matches across firms have the same expected value. This assumption reduces uncertainty in the worker problem. In a more general formulation, the value of a match in a given period depends on the endogenous wage offer distributions in the next period, that is an infinite dimensional state variable. Moscarini and Postel-Vinay (2012) provide a detailed discussion of the problems that would arise in this context and they give sufficient conditions to solve the model Burdett and Mortensen (1998) out of the steady state. Menzio and Shi (2010) defines and proves the existence of a block recursive equilibrium in the case of directed search. An alternative approach that has been extensively used in macroeconomic models with heterogeneous agents is the approximate aggregation argument proposed by Krusell and Smith (1998). The first approach deals with a recursive equilibrium that depends on the aggregate state of the economy only through the aggregate state variables and not through the distribution of workers across employment status and wage. In the latter approach, the distribution of idiosyncratic income across agents is approximated by a finite number of moments, and agents solve their recursive maximization problem taking into account only

\textsuperscript{13}Notice that under diminishing returns an incentive to over employ arises in exchange for a lower wage paid to all workers.
moments of the actual distribution. Our characterization is consistent with the bargaining design in Stole and Zwiebel (1996), (the solution suggested in Moscarini and Postel-Vinay (2012) assumes pure wage posting) under random matching (Menzio and Shi (2010) assume directed search) and will allow us to determine the wage distribution endogenously.

Notice that despite the independence assumption, the strength of the on the job search argument does not vanishes. When firms hire or want to keep workers they are forced to compete on the current wage, since from the worker’s point of view the continuation value of employment is the same across employers. Moreover, since contracts are not binding, workers can freely engage in job to job reallocations. This feature decreases considerably the contribution of the independence assumption in driving our conclusions about the wage distribution; as the following argument shows.

Under this framework, the dynamic general equilibrium remains highly tractable. As in Burdett and Mortensen (1998) and Mortensen (2009) the equilibrium is characterized by a unique wage offer distribution that is a collection of wage offers across firms such that they are the outcome of mutual best response strategies. In addition, we can exploit the general equilibrium framework to characterize the demand side of the labor market and determine the wage offer distribution such that demand and supply of labor clear at the firm level each point in time.

2.5.1 Definitions

On the supply side of the labor market a mass of $N$ homogeneous workers randomly look for job offers. On the demand side, a mass of heterogeneous employers opens $V_t$ vacancies at time $t$. The labor market tightness at time $t$ is:

$$\theta_t = \frac{V_t}{N}$$

(13)

Workers and firms match randomly. New matches are formed according to a matching function homogeneous of degree one:

$$Matches (V_t, N) = V_t^{1/2}N^{1/2}$$
The probability that a single worker randomly finds a job offer over the $V_t$ vacancies is an increasing function of the labor market tightness:

$$m(\theta_t) = \frac{V_{1/2}^{1/2} N^{1/2}}{N} = \sqrt{\theta_t}$$

as the share of new matches over the entire workforce. The probability that a vacancy is randomly visited by a worker is:

$$\frac{m(\theta_t)}{\theta_t} = \frac{V_{1/2}^{1/2} N^{1/2}}{V_t} = \frac{V_{1/2}^{1/2} N^{1/2} N}{N V_t}$$

Each period an employed worker separates from the current match with a probability $\delta \in (0, m(\theta_t))$ due to an exogenous job destruction shock. The total number of unemployed workers at time $t$ is $u_t$. There is perfect information about the distribution of wage offers $F_t(w)$ and the distribution of employed workers across wages $G_t(w)$; the two cdf are defined over a compact wage support $[w, \overline{w}]$, for $w < \overline{w}$ positive real values. Let $R$ be the reservation wage in the economy, it is equal to the marginal productivity of labor in the capital sector. Since capital is the numeraire, $R = \varphi$.

### 2.5.2 Timing

Time is discrete. Each period is a sequence of four stages:

0. **Shock.** The uncertainty about unit labor costs realizes. Firms understand the optimal wage and employment level they will perform with during the current period.

1 **Adjustment.** Firms that enter the market compare the current employment level with the optimal employment level and they adjust their workforce. Intra-firm costless renegotiations take place based on the new optimal wage and employment levels. At the end of this stage, either incumbent firms are matched with the desired level of employment or they have open vacancies. Moreover there exist a number of workers who entered the period as employed and are waiting for a reallocation.

2 **Reallocation.** All firms face job to job transitions.

   - **Exogenous separation.** A share $\delta$ of total workers who entered the period as employed at the end of the period will be unemployed.
Search and Matching. All $N$ workers are randomly searching. With probability $m(\theta_t)/\theta_t$ a firm meets a worker. Each period an employed worker has a probability $1 - \delta$ to remain employed up to the next period, but he/she may change employer. During the period, an employed worker receives a wage offer from another employer with probability $m(\theta_t)$, that is the same probability at which an unemployed worker visits the same vacancy. With probability $1 - \delta - m(\theta_t) > 0$ an employed worker receives a wage offer from his current employer, at the new wage.

Job to job transitions. Employed workers accept wage offers that are better with respect to their current employment status. At the end of the period employed workers move to another employer with probability $m(\theta_t)\left[1 - F_t(w)\right]$, or he/she remains with the current employer with probability $\left[1 - m(\theta_t) + m(\theta_t)F_t(w)\right]$.

Outflows from unemployment. Unemployed workers accept all wage offers above or equal to the reservation wage. An unemployed worker becomes employed with probability $m(\theta_t)\left[1 - F_t(R)\right]$, otherwise with probability $1 - m(\theta_t) + m(\theta_t)F_t(R)$ stays unemployed.

Inflows to unemployment. Employed workers who separated and unemployed workers that did not find a match end the period as unemployed.

At the end of this stage all firms have the optimal level of employment according to their unit labor cost. Total outflows from unemployment is given by: $m(\theta_t)\left[1 - F_t(R)\right]u_t$. Total inflows into unemployment is given by: $\delta (N - u_t)$.

3. Production. Firms start production when they reach the optimal employment level. Over the period they issue a number of vacancies that is sufficient to keep their workforce at the desired level.

Figure 1 summarizes the timing of the labor market.
2.5.3 Workers

While a worker is employed he/she supplies one unit of labor and receives a wage \( w \). After the current period, with probability \( \delta \) he/she is unemployed. Conditional on not being unemployed because of the job destruction shock, with probability \( m(\theta_{t+1}) \) the worker receives an offer in the market, and with probability \( 1 - m(\theta_{t+1}) \) he/she will be matched with the same employer at the new wage \( w' \). Since contracts are not binding, the current wage is not an outside option for the worker and idiosyncratic uncertainty implies that \( w' \) is uncertain in the current period.

Let \( \Psi_t = \{\theta_t, F_t\} \) be the labor market tightness and the wage distribution in period \( t \), and let \( T_t \) be the operator mapping from \( \Psi_t \) to \( \Psi_{t+1} \). The value of employment is \( W (w; T_t (\Psi_t)) \):

\[
W (w; T_t (\Psi_t)) = w + \zeta \mathbb{E}_t \left\{ W (w', T_{t+1} (\Psi_{t+1})) + \delta \left[ U - W (w', T_{t+1} (\Psi_{t+1})) \right] + (1 - \delta) m (\theta_{t+1}) \int_{w'}^{w} [W (x) - W (w')] dF_{t+1} (x) \right\}
\]

where \( W (w) \) is the asset value of being employed at a wage \( w \), \( U (\cdot) \) is the asset value of unemployment, \( \zeta \in (0, 1) \) is the discount factor, \( \mathbb{E}_t [\cdot] \) is the expectation operator conditional on the information available at time \( t \).

Unemployed workers gain the value of home production \( v_h \geq 0 \) and may receive or not a wage offer with the same probability than employed workers.
The value of unemployment is:

\[ U (T_t (\Psi_t)) = v_h + \zeta E_t \left\{ U + m (\theta_{t+1}) \int_R^w [W(x) - U] dF_{t+1}(x) + m (\theta_{t+1}) \int_w^w [W(x) - U] dF_{t+1}(x) \right\} \] (17)

The option value of considering further options is included in the value of being employed, therefore during a bargaining, with the current employer, the worker surplus is simply the difference between the value of being employed and unemployed:

\[ W(w; T_t (\Psi_t), T_{t+1} (\Psi_t)) - U (T_{t+1}) = w - v_h + \zeta E_t \left\{ (1 - \delta) [W(w') - U] + (1 - \delta) m (\theta_{t+1}) \int_w^w [W(x) - W(w')] dF_{t+1}(x) - m (\theta_{t+1}) \int_R^w [W(x) - U] dF_{t+1}(x) + m (\theta_{t+1}) \int_w^w [W(x) - U] dF_{t+1}(x) \right\} \] (18)

A reservation policy holds between employment and unemployment. The reservation wage, is \( R : W(R_t; T_t (\Psi_t)) - U (T_t (\Psi_t)) = 0 \):

\[ R = v_h - \zeta E_t \left\{ (1 - \delta) [W(w') - U] + (1 - \delta) m (\theta_{t+1}) \int_w^w [W(x) - W(w')] dF_{t+1}(x) - m (\theta_{t+1}) \int_R^w [W(x) - U] dF_{t+1}(x) + m (\theta_{t+1}) \int_w^w [W(x) - U] dF_{t+1}(x) \right\} \] (18)

The reservation wage is equal to the value of home production minus the continuation value of the expected worker surplus of being employed versus unemployed. The worker surplus is simply the difference between wage and reservation wage:

\[ W(w; T_t (\Psi_t)) - U (T_t (\Psi_t)) = w - R \] (19)

A reservation policy holds for job to job transactions. Each point in time, the optimal policy for unemployed workers is:

\[ \Upsilon_u (w, R) = \begin{cases} 
"accept" & \text{if } w \geq R \\
"keep searching" & \text{if } w < R 
\end{cases} \] (20)

where the equality sign holds as a convention.

In the case of employed workers, notice that the continuation values of employment does not depend on the current wage because of i.i.d. unit labor cost across employers period by period. Therefore a reservation policy holds also for job to job transitions, when the worker who received a wage offer chooses between staying with the same employer or changing employer. The
optimal policy for an employed worker at a wage $w \geq R$ that receives an alternative offer $\tilde{w}$ is simply:

$$
\Upsilon_e (w, \tilde{w}) = \begin{cases} 
"stay" & \text{if } w \geq \tilde{w} \\
"change" & \text{if } w < \tilde{w} 
\end{cases}
$$

(21)

where the equality sign holds as a convention\textsuperscript{14}.

\subsection*{2.5.4 Firms}

Firms cannot write binding contracts. The value of filling a vacancy for a firm is equal to the marginal profit per worker in the current period:

$$
J_t (w, c) = \frac{p_t (w, c)}{c} - w
$$

(22)

The value of filling a vacancy is the same regardless if the worker is already employed in the firm or not. This is due the assumption that the adjustment cost of employment is fixed. When a firm negotiates with a worker in the market the cost of employment adjustment is sunk, it is not specific to the particular match.

\subsection*{2.5.5 Employment flows}

The measurement of employment flows refers to the value they attain in the production stage and they maintain throughout the rest of the period.

\textit{Unemployment.} In a given period inflows from unemployment to employment are due to unemployed workers who match with a firm offering at least $R$, that is $m (\theta_t) [1 - F_t (R)] u_t$. Outflows from employment to unemployment are $\delta (N - u_t)$. In each point in time we define the equilibrium unemployment level such that flows in and out of employment are equalized:

$$
u_t \frac{N}{\delta} = \frac{\delta}{\delta + m (\theta_t) [1 - F_t (R)]}
$$

(23)

\textit{Wage distribution.} Consider the break down of employment flows by wage. In each point in time $m (\theta_t) u_t [F_t (w) - F_t (R)]$ unemployed workers move into

\textsuperscript{14}Optimality is due to the fact that any sudden deviation from (20) or (21) depending on the current employment status, would imply a strictly lower current value and the same continuation value.
employment accepting an offer at a wage $w$ or lower, $\delta G_t(w) (N - u_t)$ workers who are hired at a wage $w$ or lower separate because of exogenous job destruction, $m(\theta_t) [1 - F_t(w)] G_t(w) (N - u_t)$ move to an other job because they receive a better offer. The distribution of effective wages that satisfies the balance in employment flows is:

$$G_t(w) = \frac{\delta}{1 - F_t(R)} \frac{F_t(w) - F_t(R)}{\delta + m(\theta_t) [1 - F_t(w)]}$$

(24)

where we used the fact that $\frac{m}{(N - u_t)} = \frac{\delta}{m(\theta_t)[1 - F_t(R)]}$ by (23).

**Separation rate.** Firms and workers separate either because of a job destruction shock or because the worker finds a job offer that is better than the worker’s current wage. The separation rate for a firm that pays a wage $w$ is the sum of the two components:

$$s_t(w) = \delta + m(\theta_t) (1 - F_t(w))$$

(25)

**Hiring rate.** With probability $m(\theta_t) / \theta_t$ a firm meets a worker, the match is formed if the worker accepts the wage offer. With probability $\frac{u_t N}{N}$ the worker is unemployed, otherwise he/she is employed at a given wage not lower than the reservation wage. Offering a wage $w \geq R$ the firm will hire unemployed workers or employed workers with a current wage that is lower than $w$. The hiring rate per issued vacancy at a given wage $w$ is given by:

$$h_t(w) = \frac{m(\theta_t)}{\theta_t} \left[ \frac{u_t N}{N} + \left( \frac{N - u_t}{N} \right) G_t(w) \right]$$

(26)

**Employment.** When workers search for a job in the consumption sector, on the demand side of the labor market there are $(1 + x_t N^*) E_t$ employer units. For an arbitrarily small $\varepsilon > 0$, the measure of workers employed at a wage lower or equal to $w$ is $[G_t(w) - G_t(w - \varepsilon)] (N - u_t)$ and the measure of employer units offering a wage in the same interval is $[F_t(w) - F_t(w - \varepsilon)] (1 + x_t N^*) E_t$. As in Burdett and Mortensen (1998), the number of workers who are employed in an employer unit that offers a wage $w$ is given by the following limit:

$$\text{emp}_t(w) = \lim_{\varepsilon \to 0} \frac{[G_t(w) - G_t(w - \varepsilon)] (N - u_t)}{[F_t(w) - F_t(w - \varepsilon)] (1 + x_t N^*) E_t}$$

leading to the employment function:

$$\text{emp}_t(w) = \frac{\delta m(\theta_t)}{(\delta + m(\theta_t) [1 - F_t(w)])^2 (1 + x_t N^*) E_t}$$

(27)
Figure 2 shows the employment function at the firm level for a given parameter set and when the wage offer distribution is assumed to be uniform\textsuperscript{15}.

![Employment function](image)

**Figure 2: Employment function for a uniform distribution of wage offers.**

*Vacancies.* During the production stage, each employer issues the number of vacancies that is needed in order to keep employment constant. Let $v_t$ be the number of vacancies issued by an employer unit at time $t$, then the balance in hiring and separations implies $h_t(w) v_t = s_t(w) emp_t(w)$. Vacancies per employer unit level are given by:

$$v_t = \frac{\theta_t N}{\left(1 + x_t \frac{N^*}{N}\right) E_t}$$

(28)

The number of vacancies per employer unit during the production stage does not depend on idiosyncratic characteristics. The reason for this result is that a firm paying a relatively high wage has a relatively high employment and hiring success and a relatively low separation rate. The opposite is true for a firm paying a relatively low wage. The two firms have a different optimal level of employment and they open vacancies to restore that employment level given their different hiring and separation rate. The level of open vacancies is a flow variable that acts as a buffer between these firm specific tensions; but the gap the vacancies will cover would be the same, because of random matching.

### 2.5.6 Supply of labor at the firm level

A firm coincides with an employer unit only in the case it serves the domestic market. An exporter is an employer $\left(1 + \frac{N^*}{N}\right)$ times bigger the employer unit

\textsuperscript{15}The parameter values in the example are $\delta = 0.05$, $m(\theta_t) = 0.30$, $x_t = 0.20$, $N = 100$, $N^* = 20$, $E_t = 100$ and the wage offer distribution is uniform with support $w \in [1, 1.5]$. 

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and it offers the same wage. The labor supply at the firm level is:

$$I_t^* (w) = \begin{cases} 
emp_t (w) , & \text{if non exporter at time } t \\
\left(1 + \frac{N^*}{N}\right) emp_t (w) , & \text{if exporter at time } t \end{cases}$$ (29)

The labor supply is a positive, continuous, strictly increasing function of the wage, for $F_t (w) > 0$. The number of vacancies issued by a firm during the production stage is proportional to the employment level:

$$\vartheta_t = \begin{cases} 
v_t , & \text{if non exporter at time } t \\
\left(1 + \frac{N^*}{N}\right) v_t , & \text{if exporter at time } t \end{cases}$$ (30)

2.5.7 Wage bargaining

Two different bargaining processes occur simultaneously: renegotiation of the firm with current employees and job to job reallocation. Only firms that decided to stay in the market in the current period participate in the labor market. In the bargaining firms choose a wage that is consistent with optimal pricing and ultimately with the entry/exit choice they made.

Workers are homogeneous, they observe each other, then costless renegotiation will rule out the possibility of intra-firm wage dispersion is possible\textsuperscript{16}. When a firm matches with a worker the cost of advertising the vacancy has been already paid. Under this scenario the firm will offer one wage in a given period, both to inside workers and outside workers, who can be employed or unemployed\textsuperscript{17}. Workers choose according to the policy rules (20) and (21).

Negotiations between firms and workers follow the extension of Rubinstein (1982) discussed in Stole and Zwiebel (1996). Costless alternate bargaining takes place under complete information and non binding contracts. Under this approach the wage determination rule is the (generalized) Nash bargaining solution with threat points the value of a vacant job for the firm

\textsuperscript{16}In line with Mortensen (2009) “interpersonal comparison of earning of observably identical workers provide a reason for a “non-response” policy. Offer matching induces intra-firm wage dispersion, which can be costly to the extent that equity norms are important. Also dispersion of this form violates anti-discrimination laws when workers are interchangeable within the firm”; as it is our case.

\textsuperscript{17}Notice that when a firm match with a worker the cost of advertising the vacancy has been already paid.
(no revenue) and the value of unemployment for the worker (reservation wage). A firm endowed with unit labor cost $c$ at time $t$ offers a wage:

$$w = \arg \max_w (w - R)^\varpi \left( \frac{p_t(w)}{c} - w \right)^{1-\varpi}$$

where $\varpi \in (0, 1)$ is the bargaining power of the workers, and we made explicit that the price is a function of the wage.

It must be noticed that agents bargain over the nominal surplus of the match. In fact, if they were trading over the output of the match (as it is the case in most of the literature) the optimal real wage would be a function of the reservation wage relative to the price of the firm variety and the comparison of real wages would significantly differ from the comparison of nominal wages; that is a primary concern of the present work. This approach is not neutral since the price itself depends on the wage:

$$\varpi \left( \frac{p_t(w)}{c} - w \right) = (1 - \varpi) \left( 1 - \frac{1}{c} \frac{\partial p}{\partial w} \right) (w - R)$$

Price is linear in the wage by (5). The optimal wage is a convex combination of the value of the marginal productivity of labor and the reservation wage:

$$w = \left( \frac{\varpi}{\varpi + \frac{1}{2} (1 - \varpi)} \right) \frac{p(w_t)}{c} + \left( \frac{1}{2} (1 - \varpi) \right) R$$

Imposing the optimal pricing equation (5) and solving for $c = c_D$, $w(c_D) = R$ and the general expression for the wage equation simplifies to:

$$w = \varpi \left( R \frac{c_D}{c} \right) + (1 - \varpi) R \quad (31)$$

Firms with low unit labor costs relative to cutoff $c_D$ pay better wages. Everything else being constant the larger is the reservation wage the higher is the wage. In the reminder of the paper we consider the particular solution of the wage equation that satisfies the Nash axiom of symmetric bargaining:

$$w(c) = \frac{R}{2} \left( 1 + \frac{c_D}{c} \right) \quad (32)$$

as it is a benchmark of comparison with previous work in our subject.

Notice that for an hiring firm in the consumption sector it is never optimal to offer a wage equal to the reservation wage or lower. That would be the case if and only if the firm was endowed with a unit labor cost $c_D$ but in that case
it would not demand workers since it does not face a positive demand. As a consequence, the minimum wage in the consumption sector is strictly larger than the reservation wage. The service sector employs labor from the subset of unemployed workers only and all workers employed at the reservation wage belong to the service sector.

2.6 Value of an open firm

Firms face a static maximization problem each period. They maximize profit over the choice of price $p_t \in [0, p_{\text{max}}]$ and exporter status $x_{s_t} = \{0, 1\}$, subject to the demand, the wage determination rule due to the bargaining with employees and the employment function. Each period a firm is forced to exit with probability $\delta$ otherwise it continues to operate. The value of operating profit is:

$$\Pi^o_t (c) = \max_{p_t \in [0, p_{\text{max}}], \ x_{s_t} = \{0, 1\}} \pi_t (c) - f_p + x_{s_t} \pi^*_t (c) + (1 - \delta) \mathbb{E}_{t,c} \{\Pi^o_{t+1} (c')\}$$

where $\mathbb{E}_{t,c}$ is the expectation operator conditional on the information at time $t$ and the set of possible future realizations of unit labor requirement $c'$ given the current realization $c$.

Firms make forward looking entry decisions facing uncertainty about their future unit labor requirement. There is not aggregate uncertainty. The expected discounted value of a firm endowed with unit labor requirement $c$ is:

$$\Pi_t (c) = \max \{0, \Pi^o_t (c)\}$$

where $\Pi^o_t (c)$ is the expected discounted value of a firm endowed with unit labor requirement $c$ that is open at time $t$.

2.7 Entry and export decisions

There is a large (unbounded) number of potential entrants into the domestic market. In order to enter the domestic market firms make an irreversible investment of $f_{ed}$ and they become ready for production in the upcoming period.

Free entry. New firms draw their unit labor requirement from a distribution $\Phi (c)$, that is invariant over time. In any period with positive entry of firms,
free entry requires that:

$$f_{ed} = \int \Pi_{t+1}(c) \, d\Phi(c)$$

we restrict the discussion to this scenario.

**Profit of the marginal entrant.** We assume that new entrants must be willing to operate in the first period after the entry investment, otherwise they do not have the chance to obtain market shares in future periods. Let $\pi^\text{entry}_t(c)$ be the profit of a new entrant at time $t$ endowed with unit labor requirement $c \in (0, c_D]$. Then the profit of the marginal entrant satisfies:

$$\pi^\text{entry}_t(c_{in}) - f_p = 0 \implies \Pi_{t+1}^p(c) < 0, \forall t$$

New entrants that in the first period draw a unit labor requirement $c > c_{in}$ immediately exit, they do not participate neither in the labor market nor in the consumption good market. Notice that also incumbent firms do not have an incentive to participate in the labor market for $c > c_{in}$.

**Entry condition.** There is no aggregate uncertainty and we assume that firms do not engage in technological investment. As a result, all firms draw unit labor requirements from the same distribution $\Phi(c)$ each period. Each period $t$ positive entry requires that the expected profit conditional on a successful entry is equal to the sunk cost of entry::

$$\int_0^{c_{in}} \Pi_t^p(c) \, d\Phi(c) = [1 - \Phi(c_{in})] f_{ed}$$

The unit labor requirement cutoff $c_{in}$ is determined as the unique solution of the entry condition.

**Export condition.** There are not fixed costs associated to export. The decision to export for the first time is taken with knowledge about the current unit labor cost. Once a firm makes the investment it is able to serve the export market starting from the current period. Therefore, the decision on export is not dynamic. A firm pays the sunk cost of export the first time exporting in the export market leads to positive profit associated to export sales. After this investment has been made, the firm learned how to serve the foreign market and it will export in each period it will be profitable to do so. The unit labor requirement cutoff $c_{x,new}$ is determined as:

$$c_{x,new} = \arg \max_{c \in (0, c_{in}]} \{xs_t(c)\} = 1$$

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We rule out by assumption the scenario in which either the export market becomes exceptionally profitable once and then ends to be attractive, or the sunk cost of export is exceptionally high such that the net continuation value from export is positive only for producers that always export.

2.7.1 Characterization of the free entry condition

The distribution of unit labor requirement is an inverse Pareto truncated in the support \([c, \bar{c}]\) with shape parameter \(\rho > 2\). The cdf and pdf take the form:

\[
\Phi (c) = \frac{c^\rho - \bar{c}^\rho}{\bar{c}^\rho - \rho^\rho} \quad \text{and} \quad \phi (c) = \frac{\rho c^{\rho-1}}{\bar{c}^\rho - \rho^\rho}
\]

where without loss of generality \(\bar{c} < c_D\). Under this parametrization the export cutoff is a continuous increasing function of the incumbent cutoff \(c_{in}\) and a decreasing function of the sunk cost of export \(fex\):

\[
c_{new, x} = c_D - \left(\frac{\bar{c}^\rho - c_{in}^\rho}{\bar{c}^\rho - \rho^\rho} c_D f_{ex} \right)^{\frac{1}{\rho}}
\]

where \(f_{ex} < \frac{N^* \alpha R}{\gamma c_D} \left(\frac{c_D - \bar{c}}{c_D - \rho^\rho}\right)^2\) guarantees that \(c_{new, x} > c\). Autarky is possible, that is there exists a sunk cost of export large enough that \(c_{new, x} = c\); the unit labor requirement is \(c_{in}^{aut}\) such that \(\Theta R c_D N I (c_{in}^{aut}) - f_p = [1 - \Phi (c_{in})] f_{ed}\) and 

\[
f_{ex}^{aut} = \frac{N^* \alpha R}{\gamma c_D} \left(\frac{\rho^\rho - c_{in}^\rho}{\bar{c}^\rho - \rho^\rho}\right)^{-1} (c_D - \bar{c})^2.
\]

In order to compute the expected profit it is convenient to define the operator \(I (\cdot) : [c, \bar{c}] \to \mathbb{R}\) that consists of the integral function:

\[
I (x) = \int_x^\bar{c} (c_D - c)^2 c^{\rho-1} dc
\]

\[
= c_D^2 \int_x^\bar{c} c^{\rho-1} dc - 2 c_D \int_x^\bar{c} c^\rho dc + \int_x^\bar{c} c^{\rho+1} dc
\]

\[
= \frac{c_D^2}{\rho} (x^\rho - \bar{c}^\rho) - \frac{2 c_D}{\rho+1} (x^{\rho+1} - \bar{c}^{\rho+1}) + \frac{1}{\rho+2} (x^{\rho+2} - \bar{c}^{\rho+2})
\]

The integral function \(I (x)\) is continuous and differentiable in the compact support \([c, \bar{c}]\), with positive first derivative \(I' (x) = x^{\rho-1} (c_D - x)^2 > 0\). Collect the parameters that describe preferences and technology \(\Theta = \frac{1}{16 \gamma} \left(\frac{\alpha}{\bar{c}}\right)^\rho\). The expected profit among incumbent firms is given by:

\[
\Pi = \frac{\Theta R}{c_D} (NI (c_{in}) + N^* I (c_{new, x})) - f_p
\]
Equation (34) characterizes the MPC condition. The free entry condition:

$$\Pi = \frac{\sigma^o - \sigma^o_{in}}{\sigma^o - \sigma^o_{ed}} f_{ed}$$

(35)

Provided that the parametrization of the fixed cost structure is not trivial, such that $c_{new,x} > c$, then there exists an intersection between (34) and (35) and it is unique.

### 2.8 Service sector: production and accumulation

Firms need knowledge and information flows in order to run the business within a context of regulation compliance, contract enforcement and frictions in the resource markets. In real life the value of this service can be measured as the expenditure due to support functions. In this framework such service is modeled as a durable input that is produced in a service sector, accumulates over time and it is used in the consumption sector as fixed input.

The endowment of service in the economy is subject to a time-to-build lag. As in Ottaviano (2011), new units of asset that are produced at time $t$ are available from time $t + 1$ on. The supply of service follows an accumulation law given by:

$$S_{t+1} = (1 - \delta_s) S_t + A_t$$

(36)

where $\delta_s \in (0, 1)$ is the depreciation rate.

The demand for additional units of service is due to the entry of new firms. Firms that take the entry decision at time $t$ rent $f_{ed}$ units of service. A share $\eta \in (0, 1)$ of these units are new and will be available when the firm is ready for production, at the beginning of period $t + 1$. Let $M_{Et}$ be the mass of entrants that pay the entry cost at time $t$. Then the demand of additional units of capital is:

$$A_t = \eta f_{ed} M_{Et}$$

(37)

The production of units of capital takes place in a competitive sector. Workers are compensated with the value of the marginal productivity of labor. Because of perfect competition, the value of each firm-worker match in the capital sector does not generate profit.
3 Equilibrium

For a given mass of employers in the consumption sector $E_t$ a general equilibrium consists of:

(i) a vector of unit labor requirement cutoffs $c_{iin} > c_{new,x}$

(ii) a vector of output prices and wages $\{p_t(c), w_t(c)\}_{c=x}^c$, consumption good allocations $\{C_{dom}^i(c), C_{exp}^i(c)\}_{c=x}^c$ and employment allocations $\{l_t(c)\}_{c=x}^c$

(iii) a wage offer distribution $F_t(w)$, a wage distribution $G_t(w)$ and a wage support $[w, \bar{w}]$

(iv) a labor market tightness $\theta_t$, a probability of finding a job $m(\theta_t)$ and an unemployment rate $u_t/N$

such that:

a) free entry conditions are satisfied

b) the demand and supply of consumption goods and labor clear at the firm level

3.1 Wage offer distribution

Entry and exit of firms and labor market adjustment happen simultaneously. Firms draw their unit labor cost, bargain with inside and outside workers. If an equilibrium wage offer distribution does exist it satisfies the clearing in demand and supply of labor at the firm level.

Employment at the firm level is determined by the labor demand (7) in the domestic and export market (11) and the wage equation (32):

$$ l_t(w) = \begin{cases} \frac{N \alpha}{2} R_{CD} \frac{(w-R)}{(2w-R)^2}, & \text{if non exporter} \\ \left(1 + \frac{N^*}{N}\right) \left(\frac{N \alpha}{2} R_{CD} \frac{(w-R)}{(2w-R)^2}\right), & \text{if exporter} \end{cases} $$

(38)

Notice that employment goes to zero as $w \to R$. Since a minimum employment level is needed to break even with fixed cost structure, we expect the distribution of wage offers in the consumption sector be defined above a wage $w > R$. Employment is increasing in the wage up to a maximum where $w = \frac{3}{2} R$. For
larger wages, firms would hire less workers and pay more all their employees, given the same demand and technology. Clearly this outcome cannot be part of an optimal plan by any firm. We expect the wage offer distribution in the consumption sector to be defined in the support \( w \in \left[ w, \frac{3}{2} R \right] \).

A firm that offers a wage \( w \) attracts a labor supply given by (29). The only candidate correspondence \( F_t(w) \) such that demand and supply clear at the firm level is:

\[
F_t(w) = 1 - \left( \frac{2\delta}{m(\theta_t)^2} R c_D \left( 1 + x_t \frac{N^*}{N} \right) \frac{(2w - R)^2}{(w - R)} \right)^{\frac{1}{2}} + \frac{\delta}{m(\theta_t)} \]

The candidate function \( F_t(w) \) is monotonic increasing in the wage over the entire support \( w \in \left[ w, \frac{3}{2} R \right] \). Imposing the condition \( F_t \left( \frac{3}{2} R \right) = 1 \), the value of the probability of finding a job, given the number of employers in the consumption sector is given by:

\[
m(\theta_t) = \frac{\delta^{\alpha} c_D \left( 1 + x_t \frac{N^*}{N} \right) E_t}{16} \tag{39}\]

The probability of finding a job is monotonically increasing in the number of employers in the market and in the number of exporters. The expression for the cdf of the wage offer distribution in the consumption sector simplifies to:

\[
F_t(w) = 1 - \left( \frac{(2w - R)^2}{(w - R)} \frac{2}{R} \right)^{\frac{1}{2}} - 4 \frac{\delta^{\alpha} c_D \left( 1 + x_t \frac{N^*}{N} \right) E_t}{2^{\alpha} c_D \left( 1 + x_t \frac{N^*}{N} \right) E_t} \]

Finally, there exists a wage level \( \inf \{ w_t \} \in \left( R, \frac{3}{2} R \right) \) such that \( F_t \left( \inf \{ w_t \} \right) = 0 \). It can be shown that \( \inf \{ w_t \} \) is increasing in the reservation wage \( R \) and it decreases with the number of firms in the consumption sector \( E_t^{18} \). Moreover for a given share of offers due to the capital sector, \( F_t(R) > 0 \) the minimum wage that is actually offered in the consumption sector is larger than \( \inf \{ w_t \} \),

\[18\text{The wage such that } F_t \left( \inf \{ w_t \} \right) = 0 \text{ is the lower solution of a quadratic equation:}
\]

\[
\inf \{ w_t \} = \frac{R}{8} \left( \frac{1}{2} \left( 4 + \frac{\delta^{\alpha} c_D E_t}{4} \right)^2 + 4 \right)
\]

\[
- \frac{1}{2} \left( 4 + \frac{\delta^{\alpha} c_D E_t}{4} \right)^2 \left( \frac{1}{2} \left( 4 + \frac{\delta^{\alpha} c_D E_t}{4} \right)^2 - 8 \right)
\]

where the choice of the lower solution delivers is due to the constraint that \( w_t \) must be bounded for any \( E_t > 0 \).
$w > \inf \{ w_i \}$. The wage offer distribution in the entire economy is defined by interval:

$$F_t(w) = \begin{cases} 
F_t(R) & \text{for } w < w \frac{1}{2} \\
1 - \left( \left( \frac{(2w - R)^2}{(w - R)} \right)^\frac{3}{2} - 4 \right) \frac{4}{\delta \gamma c_D (1 + x_t N^*) E_t} & \text{for } w \geq w \frac{1}{2} 
\end{cases}$$

(40)

where $F_t(R) \leq F_t(w)$. The minimum wage in the consumption sector is a monotonic decreasing function of the cutoff $c_{in}$:

$$w = \frac{R}{2} \left( 1 + \frac{c_D}{c_{in}} \right)$$

(41)

A firm endowed with a given unit labor requirement $c \in [c, \bar{c}]$ offers a wage that satisfies the bargaining equilibrium according to the wage equation (32). At that levels of wage and unit labor requirement the demand of labor by the firm is determined in the consumption goods market and it is given by (12). In the space wage and employment the two conditions are orthogonal straight lines parametric to the value of unit labor requirement $c \in [c, \bar{c}]$. The intersection uniquely identifies the point that associates unit labor requirement and employment at the firm level\(^{19}\).

Figure 3: Labor demand and supply clearing at the firm level.

Figure 3 illustrates the clearing of labor demand and supply for two firms as an example and the implied equilibrium allocation for the entire population of firms. The wage offer distribution $F_t(w)$ is the unique correspondence $[w, \bar{w}] \rightarrow [F_t(R), 1]$ that satisfies all the intersections for $c \in [c, \bar{c}] \subset \left[ \frac{c_D}{\delta \gamma}, c_D \right]$.

\(^{19}\)The parameter values for the supply of labor in the example are $\delta = 0.05$, $m(\theta_t) = 0.30$, $x_t = 0.20$, $N = 100$, $N^* = 20$, $E_t = 100$. The parameter values for the demand of labor in the example are $\frac{c_D}{\gamma} = 1$, $w(c_D) = 1$, $c_D = 1$. The unit labor requirements are $c = 0.6, 0.8$, the corresponding wages after the bargaining are $w = 1.3, 1.125$, the corresponding demands of labor are $l \simeq 6, 4$.  

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3.1.1 Exporter behavior in the labor market

Consider the evidence on exporters that has been documented by Bernard et al. (2007) and we quoted in the introduction. Two remarks are striking. First, if the increase in employment was due to the wage premium then we should assume an incredibly elastic labor supply. This scenario would be even more not realistic because it should be measured to some extent at the right of the wage support, as exporters pay larger wages. Second, the correlations between wage and employment, and between productivity and employment give a limited information respect to the breakdown between exporters and non exporters. Instead output and employment are strongly linearly correlated, to the extent that when we control for employment we also capture most of the difference between exporters and non exporters in terms of output. This evidence suggests that when a firm becomes exporter hires more workers without a necessary increase of the wage; then, because of this, it produces more output with the new workers that are almost as much as productive than the old ones.

This evidence can hardly be reconciled with frameworks in which the production function shows diminishing returns on labor, the cost of posting a vacancy is match specific and the number of vacancies is optimally chosen to maximize the expected value of hiring. In this context, firm level employment adjusts as a continuous function of the wage. Nevertheless it has to be discontinuous with respect to productivity and the discontinuity occurs at the cutoff productivity level of the marginal exporter; (as a discussion and an exhaustive example of this approach see figure 1 in Helpman et al. (2010)).

The framework developed in this work captures this evidence. The demand supply equilibrium in the consumption good market requires that an exporter employs \((1 + N^*/N)\) times workers than in the case it only served the domestic economy, by (11). Still the value of the match with a worker does not change with the exporter status, therefore an exporter firm will not offer a higher wage because it is an exporter. The exporter status and the relative high wage are correlated as a result of a relatively high productivity but one does not imply the other. The only channel through which an exporter can match its demand for labor is behaving as an employer \((1 + N^*/N)\) times bigger in terms of employment, with respect to the case he/she was serving the domestic market.
The segment of the labor supply curve that is relevant for non exporters shifts down when there are exporters in the economy and the opposite is true for the labor supply of exporters.

Figure 4: Employment at the firm level, in closed (dash) and open economy (solid).

Figure 4 illustrates this argument as it describes the firm labor supply function in open economy (bold line) with respect to the closed economy equilibrium, (dash line), for a possible new exporter. The jump occurs at the wage of the marginal new exporter and the labor supply function is right continuous as the cdf $F_t(w)$ does.

### 3.2 Wage distribution

Combining the expression for the wage distribution (24) and the probability of matching (39). We obtain a closed form equation for the cdf of the wage distribution given the mass of employers:

$$G_t(w) = \left( \frac{N}{N-u_t} \right) 4 \left( \frac{(w-R)}{(2w-R)^2} \right)^{\frac{1}{2}} \frac{R}{N-u_t}$$

for $w \in \left[w, \frac{3}{2}R\right]$. The density of the wage distribution follows by right differentiation of (42) in $\left[w, \frac{3}{2}R\right]$:

$$g_t(w) = \frac{\sqrt{2} \sqrt{R} (3R-2w)}{(2w-R)^2 (w-R)^{\frac{3}{2}}} \left( \frac{N}{N-u_t} \right)$$

Figures (5) show the shape of the cdg and the density of the wage distribution. The two functions are parametric to the value of unemployment rate. An
increase in the unemployment rate tilts down the cdf whereas the density function tilts up. Notice that $G_t (w) > 0$ because of employment in the service sector.

Figures 5: Cumulative density function of the wage distribution in the consumption sector (left), and density of the wage distribution (right).

The first moment of the wage distribution over employed workers in the consumption sector is increasing in the minimum wage $w$:

$$
\mu_{\text{cons}} = \frac{1}{1 - G_t (w)} \int_{w}^{\frac{3}{2}R} wg_t (w) \, dw
$$

(44)

The Gini coefficient on the wage distribution of employed workers in the consumption sector is given by:

$$
gini_{\text{cons}} = 1 - \frac{1}{\mu_{\text{cons}}} \int_{w}^{\frac{3}{2}R} (1 - G_t (w))^2 \, dw
$$

(45)

The Leibniz rule and the monotonicity of the average wage are sufficient to show that (45) is a monotonic increasing function of the minimum wage $w$.

Computing the two measures over the entire population of workers we obtain:

$$
\mu_t = RG_t (w) + \mu_{\text{cons}} (1 - G_t (w))
$$

(46)

$$
gini_t = 1 - \frac{1}{\mu_t} \left[ (1 - G_t (w))^2 \frac{1}{2} R + \int_{w}^{\frac{3}{2}R} (1 - G_t (w))^2 \, dw \right]
$$

(47)

The change in average wage and wage inequality over employed workers due to an increase in $w$ is necessarily of smaller magnitude than the change in the subset of workers employed in the consumption sector only. In principle, if we allow for a very large reallocation of workers across sectors then the effect of a change in the minimum wage $w$ on overall average wage and wage inequality can be offset or even reversed.
3.3 Short run equilibrium

A short run equilibrium consists of objects (i)-(iv) of a general equilibrium and in addition it requires:

(v) a pair of laws of motion for the predetermined state variables \((S_{t+1}, E_{t+1})\), and for the measures of new entrants \(M_{Et}\) and exporters \(X_t\)

such that conditions (a), (b) are satisfied and:

(c) the service market and the labor market clear at the aggregate level

3.3.1 Selection

Let \(M_{Et}\) be the mass of firms that decide at time \(t\) to pay the sunk cost and enter the market in the next period. Out of the entire population of current incumbents \(E_t\) and new entrants \(M_{Et}\) only a share \(\Phi (c_{in})\) decides to be incumbent in the next period:

\[
E_{t+1} = \Phi (c_{in}) (E_t + M_{Et})
\] (48)

Firms that were exporters in the previous period, \(X_{t-1}\) and in the current period stay in the market continue to export. We refer to this subgroup as old exporters: \(X_t^{old} = \Phi (c_{in}) X_{t-1}\). Firms that were not exporters in the previous period and in the current period are endowed with a unit labor cost \(c \leq c_{new,x}\) are new exporters: \(X_t^{new} = \Phi (c_{new,x}) (M_{Et-1} + E_{t-1} - X_{t-1})\). The total mass of exporters is the sum of the two components:

\[
X_t = \Phi (c_{new,x}) E_{t} + [\Phi (c_{in}) - \Phi (c_{new,x})] X_{t-1}
\] (49)

3.3.2 Service market clearing

The total demand of services at time \(t\) is given by: \(S_t = f_p E_t + f_{ex} X_t^{new} + (1 - \eta) f_{ed} M_{Et}\). The mass of entrants follows by market clearing in the service sector:

\[
M_{Et} = \left[ \frac{1}{(1 - \eta) f_{ed}} \right] S_t - \left[ \frac{f_p + f_{ex} \Phi (c_{new,x})}{(1 - \eta) f_{ed}} \right] E_t + \left[ \frac{f_{ex} \Phi (c_{new,x})}{(1 - \eta) f_{ed}} \right] X_{t-1}
\] (50)

The interpretation of equation (50) will be crucial for our results. A large stock service allows the entry of new firms. But the allocation of service to
new entrants is challenged by the allocation of service to current incumbent firms. A large population of incumbent firms, and out of those a large number of exporters, decrease the stock of service available for new entrants.

Production of new units of capital is driven uniquely by the entry of new firms: \( A_t = \eta f_{ed} M_{Et} \). This condition together with the accumulation law of service stock (36) characterizes the law of motion for the stock of service in the economy:

\[
S_{t+1} = (1 - \delta_s) S_t + \eta f_{ed} M_{Et} \quad (51)
\]

### 3.3.3 Dynamic system

Combining (50) with (48) and (51), the dynamics of the economy is described by the following system of linear difference equations:

\[
\begin{bmatrix}
    E_{t+1} \\
    S_{t+1} \\
    X_t
\end{bmatrix} =
\begin{bmatrix}
    k_{EE} & k_{ES} & k_{EX} \\
    k_{SE} & k_{SS} & k_{SX} \\
    \Phi (c_{new,x}) & 0 & \Phi (c_{in}) - \Phi (c_{new,x})
\end{bmatrix}
\begin{bmatrix}
    E_t \\
    S_t \\
    X_{t-1}
\end{bmatrix} \quad (52)
\]

where the coefficients are:

- \( k_{EE} = \Phi (c_{in}) \left( 1 - f_p + \frac{\Phi (c_{new,x})}{\Phi (c_{in})} f_{ex} \right) > 0 \)
- \( k_{ES} = \frac{\Phi (c_{in})}{(1 - \eta) f_{ed}} > 0 \)
- \( k_{EX} = \frac{\Phi (c_{new,x}) f_{ex}}{f_{ed}} > 0 \)
- \( k_{SE} = \frac{n}{1 - \eta} \left( f_p + \frac{\Phi (c_{new,x})}{\Phi (c_{in})} f_{ex} \right) > 0 \)
- \( k_{SS} = \frac{1}{1 - \eta} - \delta_s > 0 \)
- \( k_{SX} = \frac{n}{1 - \eta} f_{ex} \Phi (c_{new,x}) > 0 \)

Eigenvalues are real and positive. The system exhibits saddle path stability, where the unstable eigenvalue is associated to the mass of employers in the market.

#### 3.3.4 Labor market clearing

Because of perfect competition, the mass of firms in the service sector is undetermined. Nevertheless, employment in the service sector must be equal to the share of total employment that is not allocated to the consumption sector. Labor market clearing at the aggregate level implies: \( E_t^{serv} = G_t (w) (N - u_t) \). By imposing (23) and (39) we obtain the share of total workforce that is employed in the service sector:

\[
\frac{G_t (w) (N - u_t)}{N} = 4 \left( \frac{(w - R)}{(2w - R)^2} \right)^\frac{1}{2} - \frac{u_t}{N} \quad (53)
\]
The share of unemployed workers is given by (23) and (14):

$$u_t = \frac{16}{16 + \frac{\alpha c_D}{\gamma} \left(1 - F_t(R)\right) \left(1 + x_t N^* \frac{N}{N^*}\right) E_t}$$  \hspace{1cm} (54)

The share of total workforce that is employed in the consumption sector results from labor market clearing:

$$\left(1 - G_t(w)\right) \left(N - u_t\right) = 1 - 4 \left(\frac{w - R}{(2w - R)^2 R}\right)^{\frac{1}{2}}$$  \hspace{1cm} (55)

Notice that employment in the consumption sector is decreasing in $w$ and it does not depend on the number of employers. Instead the share of unemployment and employees in the capital sector adjusts as the number of employers and exporters change.

The supply and demand of labor in the service sector clear each point in time:

$$\frac{16 N}{16 + \frac{\alpha c_D}{\gamma} \left(1 - F_t(R)\right) \left(1 + x_t N^* \frac{N}{N^*}\right) E_t} = 4 \left(\frac{(2w - R)^2 R}{(w - R)^2 R}\right)^{\frac{1}{2}} N - \frac{\eta f_{ed}}{\varphi} M_{Et}$$

Notice that $4 \left(\frac{(2w - R)^2 R}{(w - R)^2 R}\right)^{\frac{1}{2}} = \frac{16}{16 + \frac{\alpha c_D}{\gamma} \left(1 - F_t(R)\right) \left(1 + x_t N^* \frac{N}{N^*}\right) E_t}$. Therefore $F_t(R) = F(w)$ if and only if $\eta M_{Et} = 0$, otherwise $F_t(R) < F(w)$. In an equilibrium with positive entry the wage offer distribution is discontinuous to the left of $w$. For a given predetermined value $E_t$ and an equilibrium mass of new entrants $M_{Et} > 0$ and exporters $X_t = x_t E_t$ there exists one possible value of $F_t(R)$ such that (3.3.4) is satisfied. The left hand side of (3.3.4) is increasing in $F_t(R)$ and it shifts down with an increase in $X_t$. The right hand side is constant in $F_t(R)$, it shifts up with an increase in $F_t(w)$ and down with an increase in the mass of new entrants $M_{Et}$.

$$1 - F_t(R) = \frac{16 \left(1 - 4 \left(\frac{(2w - R)^2 R}{(w - R)^2 R}\right)^{\frac{1}{2}} + \frac{\eta f_{ed} M_{Et}}{\varphi N}\right)}{\frac{\alpha c_D}{\gamma} \left(1 + x_t N^* \frac{N}{N^*}\right) E_t}$$  \hspace{1cm} (56)

**3.4 Deterministic steady state**

As the mass of employers evolves over time two forces arise. First, on the job search drives the reallocation of workers across firms in the consumption
sector. Second, the demand of labor in the capital sector follows entry and exit of firms and it determines flows of workers moving between employment in the capital sector and unemployment.

The labor market clearing shows that the share of employment in the consumption sector over total workforce does not change with the mass of employers in the sector, $E_L$. An adjustment in the minimum wage in the consumption sector leads to an immediate adjustment in total employment in the sector, through selection. From the first period following the shock the dynamics is driven by the reallocation of workers from the population of unemployed and employment in the service sector. This process continues as long as workers have an incentive to look for a job in the service sector.

### 3.4.1 Harris-Todaro long run equilibrium

Workers are risk neutral, they have complete information and match randomly with employers. Under this context, the Harris and Todaro (1970) condition must hold in the long run. The movement of workers searching for a job leads to a steady state in which the expected earning of being employed in the two sectors is the same:

$$E_T = \frac{1}{2} \left( \frac{w - R}{w - R} \right)^{-\frac{1}{2}} \frac{\mu_{cons}^T}{R}$$

By substitution of (44) and (42) is it possible to show that employment in the service sector is a decreasing function of the minimum wage in the consumption sector $w$.

Given employment in the service sector, the mass of new entrants in steady state is determined through (37),

$$M_{ET} = \frac{\varphi E^T_T \mu_{serv}^T}{\eta_{fed}}$$

and the steady state stock of service, the mass of employers in the consumption sector, and the mass of exporters are:

$$S_T = \frac{\varphi E^T_T \mu_{serv}^T}{\delta_s}$$

$$E_T = \left( \frac{\Phi (c_m)}{1 - \Phi (c_m)} \right) M_{ET}$$

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\[ X_T = \frac{\Phi(c_{new,x})}{1 - (\Phi(c_{in}) - \Phi(c_{new,x}))} E_T \]  

The vector \( \{E_T, S_T, X_T\} \) is the deterministic steady state of the dynamic system that consists of the three linear difference equations \((48), (49), (51)\) once we substitute for \((50)^{20}\).

4 Consequences of an export shock

We define an export shock as an exogenous fall in the sunk cost of export \( f_{ex} \) everything else being constant. This section describes the effect of an unanticipated export shock.

Efficiency. A fall in the fixed cost of export allows more firms to become exporters, the cutoff \( c_{new,x} \) shifts to the right everything else being constant. In the space defined by the expected profit as a function of the cutoff of incumbent firms, conditions \((35)\) and \((34)\) identify the new equilibrium value of the cutoff:

**Proposition (1).** *A fall in the sunk cost of export \( f_{ex} \) determines a more severe selection, through a lower unit labor requirement \( c_{in} \).*

![Figure 6: A fall in the sunk cost of export increases efficiency.](image)

Firm turnover and exporter share. A fall in the unit labor requirement \( c_{in} \) implies a lower share of incumbent firms that stay in the market the following period, when \((35)\) and \((34)\) are satisfied. As a consequence a larger share of incumbent firms exports for the first time.

---

\(^{20}\)There exists a necessary structural relationship between the depreciation rate and the rate of use of new units of service in the entry process:

\[
\frac{\eta(1+\delta_s)}{\delta_s} = \left( \frac{f_{ex} + f_{ed} \Phi(c_{new,x}) / \Phi(c_{in})}{f_{ed}} \right) - \left( \frac{\Phi(c_{new,x})^2}{1 - (\Phi(c_{in}) - \Phi(c_{new,x}))} f_{ed} \right) \left( \frac{\Phi(c_{in})}{1 - \Phi(c_{in})} \right).
\]

This condition is immediately reached once the steady state values of \( \Phi(c_{in}) \) and \( \Phi(c_{new,x}) \) are determined.
Proposition (2). Following the fall in the fixed cost of export we observe a higher turnover of firms and a larger share of firms are new exporters.

![Graph showing phi(c) vs c](image)

Figure 7: A fall in the sunk cost of export increases firm turnover and exporters share.

Figure 7 shows the change in the population of firms. After the export shock, the cutoff below which firms become exporters shifts to the right, the cutoff above which firms exit shifts to the left.

Wages and prices. The wage determination rule in the consumption sector implies that a lower cutoff $c_{in}$ leads to an increase of the minimum wage in the consumption sector. The maximum wage does not depend on $c_{in}$, as well as the wage in the consumption sector. Combining the price equation (5) and the wage equation (32), the gain in efficiency due to the decrease in $c_{in}$ is responsible for an increase in the real wage, both in the consumption sector and in the capital sector.

Proposition (3). After the decrease in the fixed cost of export, prices fall, the minimum wage in the consumption sector increases while the maximum wage does not change.

Proposition (3) establishes that there are gains from trade in real terms both for employed workers in the consumption sector and in the service sector. For employed workers in the consumption sector the gain is also in nominal terms. Nevertheless there are workers who become unemployed because the export shock increased selection of incumbent firms. The next section discusses the three effects that are responsible for a clear understanding of the impact of export shock on labor market variables.
4.1 Selection, job destruction, job creation

Three effects are crucial to assess the change in unemployment rate and wage inequality following an export shock. Proposition (1) refers to a selection effect, a decrease in the cutoff $c_m$. From the point of view of the labor market, proposition (2) identifies a job destruction effect, as the increase in firm turnover. At the same time, the larger proportion of employers that become exporters and the demand of service to finance new entrants are responsible for a job creation effect.

In order to discuss the dynamics of model variables after a fall in the sunk cost of export we use a parametrization of the fixed cost structure that is consistent with the structural estimation presented in Coşar et al. (2010). The depreciation rate $\delta_s$ and the relative size of the export market $N^*/N$ are chosen such that during the experiment the unemployment rate varies in the range (4.5%, 6%) and the share of exporters varies between (14%, 20%); as these ranges are common to the findings of the empirical literature we quoted in the introduction.

Figure 8 shows the dynamics of the employers in the consumption sector, stock of service and exporters, $\{E_t, S_t, X_t\}$, after an unanticipated fall of 5% in the fixed cost of export $f_{ex}$. The first point in the picture is the initial steady state value, the time length is 24 periods, as we think a period as a quarter.

At the impact, forward looking decisions determines both a decrease in the number of employers (due to selection) and an increase in the number of exporters. In so doing, we observe the effect of propositions (1) and (2). The stock of service decreases for two reasons. First, there is a reallocation of workers to the consumption sector. Second, the increase in firm turnover at the impact decreases the demand for service.

After the shock, the larger turnover induced by the more severe selection is responsible for the decrease in the number of employers. The number of exporters increases because of the persistence that characterizes the exporter status. The probability that a firm becomes a new exporter is the same each period after the shock, but those firms that were exporting in the previous period continue to export in the following periods. The additional demand of service due to new entrants drives the production of service in the economy.
Figure 8: Dynamics

after the shock to the new steady state, given the allocation of workers in the two sector at has been implied by the labor market equilibrium.

Figure 9 describes the dynamics of the unemployment rate $u_t/N$ and the Gini index on the wage distribution $G_t(w)$, across all workers employed in both sectors.

At the impact, the selection effect is the only mechanism driving the dynamics. The unanticipated cut in the sunk cost of export forces the exit of firms. The minimum and average wage in the consumption sector reach the new, higher, steady state level. There is worker reallocation within the consumption sector, from low productive firms that exit the market to new exporters. Employment in the consumption sector reaches immediately the new steady state level, lower than in the initial steady state.

Employment in the service sector does not satisfies the long run equilibrium (57), at the impact. The dynamics of employment in the service sector is the result of two effects. First, immediately with the export shock and the change in the minimum wage in the consumption sector, there is a disproportionate decrease of the share of wage offers due to the service sector that are accepted and there is more endogenous separation in the service sector. Second, from the first period after the shock, the entry of new firms occurs at a larger turnover. The demand of new units of service increases because the number
of new entrants per period increases. Combining the two effects we know that employment in the service sector can only reach the new steady state "from below". That means that the decrease in employment in the service sector is larger than the difference in the two steady state levels. As a consequence the unemployment rate overshoots at the impact and then it converges to the new steady state following the demand of labor in the service sector that adjust with the entry of new firms.

The change of wage inequality results from the reallocation of workers within the consumption sector and from unemployment to employment in the service sector. Wage inequality increases in the consumption sector because of the reallocation of workers from many low productive firms to few and large new exporters. Workers employed in the service sector are all paid the same wage and this homogeneity decreases the overall level of wage inequality, but it does not reverse the rate of change. In order to understand the path of wage inequality over time, consider that the persist ency in the export status is a driver of wage inequality per se. In facts as the share of exporters increases, ceteris paribus the wage inequality in the consumption sector continues to
increase and it leads overall wage inequality in the economy.

![Figure 10: Correlation between inequality and exporter share.](image)

The reallocation of workers from unemployment to the service sector, after the export shock plays a role in explaining the concavity of the qualitative relationship between wage inequality and share of exporters. In fact as the economy reaches the new steady state the share of employed workers in the service sector increases with respect to immediately after the export shock. In so doing, the reallocation of workers to the service sector smooths the increase in inequality due to the increase in the share of exporters.

5 Conclusion

This paper develops a two-sector two-factor dynamic general equilibrium model with on the job search and firm entry/exit. Under this framework, we analyze the effect of a reduction of the sunk cost of export on the dynamics of unemployment and wage inequality.

An unanticipated permanent decrease of the sunk cost of export determines an increase of firm turnover and in the share of incumbent firms that become exporters. In the labor market, workers reallocate from low productive firms
to more productive firms that became new exporters. As a result, in the (tradable) consumption sector, minimum wage, average wage and employment per firm increase. Nevertheless total employment in the consumption sector decreases, because of the selection of less efficient firms out of the market. The purpose of the service sector is to produce inputs for the firms in the consumption sector. Therefore, a decrease in employment and number of employers in the consumption sector also implies a decrease of employment in the service sector. The comparison of the two steady states before and after a fall in the sunk cost of export leads to the conclusion that unemployment and wage inequality rise after the export shock.

Nevertheless the transitional dynamics to this new steady state contains richer policy implications. Following the fall in the sunk cost of export the unemployment rate overshoots, that is, it reaches a higher level than the long run equilibrium. The level of employment in the consumption sector adjust immediately, but employment in the service sector is too low immediately after the export shock. The demand of service due to the entry of new firms leads the adjustment of unemployment over time to the new steady state level. Wage inequality smoothly adjusts to a new higher level. The driver for the increase in inequality is the reallocation of workers across firms to new exporters. Instead the reallocation of workers from unemployment to employment in the service sector decreases the rate the speed of the rise in inequality. As a result the model predict a positive correlation between the share of exporters and wage inequality but with a concave shape.

Appendix

Robustness check: wage posting and unbounded productivity distribution

The purpose of this section is to assess the change in the wage distribution following an export shock under a different wage determination mechanism. We derive the wage offer distribution under pure wage posting. As in Burdett and Mortensen (1998) firms post take it or leave offers at a wage that is a
solution of the following problem:

\[ w = \arg \max_{w \geq R} \left( \frac{p_t(w)}{c} - w \right) \text{emp}_t(w) \]

Firms take as given the demand of consumption goods \( \frac{p}{c} = \frac{1}{2} \left( R \frac{c_D}{c} + w \right) \), \( \frac{\partial p}{\partial w} = \frac{1}{2}c \) then \( \left( \frac{p(w)}{c} - w \right) = \frac{1}{2} \left( R \frac{c_D}{c} - w \right) \) and the employment function that is implied by the search equilibrium: \( \text{emp}_t(w) \). The necessary first order condition for an interior solution is

\[-\frac{1}{2} \text{emp}_t(w) + \frac{1}{2} \left( R \frac{c_D}{c} - w \right) \text{emp}_t'(w) = 0,\]

and it can be written as:

\[
\left( R \frac{c_D}{c} - w \right) \left[ \frac{2m(\theta_t) F_t'(w)}{\delta + m(\theta_t) [1 - F_t(w)]} \right] = 1
\]

The sufficient second order condition is:

\[-\frac{3}{2} \text{emp}_t'(w) + \left( R \frac{c_D}{c} - w \right) \text{emp}_t''(w) < 0,\]

therefore the concavity of the employment function is sufficient to guarantee the optimality of an interior solution. Impose the unique mapping between the distribution of wage offers and the distribution of unit labor requirements \( F_t(w(c)) = 1 - \Phi(c) \). For the sake of comparison with the framework in Burdett and Mortensen (1998) define \( J(a) \) as the distribution of idiosyncratic productivity \( a = R \frac{c_D}{c} \) as it is implied by \( \Phi(c) \). Then \( \frac{\partial F_t(w(c))}{\partial w} = (\frac{\partial w}{\partial a})^{-1} \frac{\partial J(a)}{\partial a} \).

The wage is a solution of the differential equation:

\[
\frac{\partial w}{\partial a} = \frac{2m(\theta_t) J'(a)}{\delta + m(\theta_t) [1 - J(a)]} (a - w(a))
\]

The wage offer distribution satisfies the equality \( \left( \frac{p_t(w(c))}{c} - w(c) \right) \text{emp}_t(w(c)) = \pi_t(c) \) for all wages, where the variable profit is determined in (9) under the assumption that the wage paid by a firm endowed with unit labor requirement \( c_D \) is the reservation wage. The cdf and density of the wage offer distribution are:

\[
F_t(w) = 1 - \left( \frac{\delta}{m(\theta_t) \left( 1 + x_t \frac{N^*}{N} \right) E_t \alpha/\gamma R \frac{c_D}{c} (a - w)} \right) ^{\frac{2}{\gamma}} + \frac{\delta}{m(\theta_t)}
\]

\[
F_t'(w) = -\frac{a^2}{R c_D} \left( \frac{1}{a - w} \right) ^{\frac{2}{\gamma}} \frac{\delta}{\alpha/\gamma m(\theta_t) \left( 1 + x_t \frac{N^*}{N} \right) E_t}
\]

Since \( \frac{\partial w}{\partial a} F_t(w(c)) = J'(a) \) then the wage productivity profile is given by:

\[
\frac{2 \delta}{\delta + m(\theta_t) [1 - J(a)]} \frac{1}{\alpha/\gamma (1 + x_t \frac{N^*}{N} \ E_t)} \left( \frac{2 \delta}{\delta + m(\theta_t) [1 - J(a)]} \frac{1}{\alpha/\gamma (1 + x_t \frac{N^*}{N} \ E_t)} \right)^{\frac{2}{\gamma}}
\]
Assume that \( 1 - J(a) = \left( \frac{a}{n} \right)^\rho \) then

\[
 w(a) = a - \left( \frac{2\delta \alpha}{\delta a^2 + m(\theta_t) a^{2-\rho} \alpha \gamma} \left( \frac{1}{1 + \frac{x_t}{e^t}} \right) \right)^2
\]

Under the restriction \( a > \left( \frac{m(\theta_t)(\rho-2)}{\gamma^2} \right)^{\frac{1}{\rho}} \), the wage is a monotonic increasing function of idiosyncratic productivity \( a \). The cdf of the wage distribution is:

\[
 G_t(w(a)) = \frac{\delta}{1 - F_t(R)} \frac{J(a) - F_t(R)}{\delta + m(\theta_t) [1 - J(a)]}
\]

For \( \rho \to 2 \) and \( c^4 \to 0 \) then \( w(a) \simeq a \). The cdf and the density of the wage distribution are:

\[
 G_t(w) = \frac{\delta}{1 - F_t(R)} \frac{1 - F_t(R) - \left( \frac{w}{w} \right)^2}{\delta + m(\theta_t) \left( \frac{w}{w} \right)^2}
\]

\[
 g_t(w) = \frac{2\delta w^2 w}{(m(\theta_t) w^2 + \delta w^2)^2} \left( \frac{\delta + (1 - F_t(R)) m(\theta_t)}{1 - F_t(R)} \right)
\]

where \( G_t(w) \to 1 \) as \( w \simeq a \to \infty \). A numerical analysis can provide a full comparison of this scenario with the one that has been described in the paper.

![Figure 11: Change in the wage distribution due to an increase in the probability of finding a job.](image)

Nevertheless for a given \( F_t(R) \) if the export shock is associated to an increase in the probability of finding a job then the response of the cdf following to an export shock is the same than under wage bargaining; in the following sense. The average wage increases, the cdf after the export shock is dominated
by the cdf before the export shock. The scenario in which the probability of finding a job increases after an export shock arises when the increase in the share of exporters more than compensates the fall in the number of employers in the market (14).

References


