SEARCHING FOR EVIDENCE OF HIDDEN INFORMATION IN ESTIMATED DSGE MODELS

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Abstract. A growing literature in macroeconomics has examined the impact of hidden shocks, which would imply that agents in the economy have superior information to the econometrician. If so the structural model would imply a "nonfundamental" reduced form VARMA representation of the observables, with possible nonuniqueness of deep parameter estimates and shocks. We consider strategies for estimating DSGEs taking into account the existence of such models. Applying our analysis to the benchmark model of Smets and Wouters (2007) we conclude that while in the original sample of Smets and Wouters (2007) there is little evidence of hidden shocks, estimating the model in an updated sample the econometrician cannot discriminate between two alternative structural models, hence nonuniqueness is found.

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1. Introduction

In recent years a burgeoning literature has developed in which the deep parameters of dynamic stochastic general equilibrium (DSGE) models are estimated, rather than calibrated. Some models, notably that of Smets and Wouters (2007) - henceforth SW - have had a significant influence; as has the ready availability, and ease of use, of the estimation and modelling package Dynare,[1] which allows DSGE models to be estimated using Bayesian techniques.

Over the roughly the same period a (largely separate) literature has examined the role of *hidden shocks*, and their implications for empirical macroeconomic models. Fernandez-Villaverde et al. (2007) showed that in the ABCD representation of a rational expectations model then even in the "square case" in which the number of observables is equal to the number of structural shocks, then unless a particular parameter restriction (*Poor Man's Invertibility Condition*, henceforth PMI condition) is satisfied, a time series representation of the observables will not reveal the structural shocks. If this condition is not satisfied, agents in the economy must have superior information to the econonometrican who observes the economy. In this category fall the news shock and fiscal foresight models studied by Blanchard et al., Leeper et al. (2013) Beaudry and Portier (2004) and Forni et al. (2013b).

In this paper we draw out the link between the conditions for hidden shocks and the problem of estimating a DSGE model (usually, following the precedent of SW by Bayesian techniques, and also usually assuming the square case[2]). In such models, the ABCD coefficients are functions of a vector of deep parameters θ, and the econometrician's problem is to estimate the posterior distribution of θ.

A striking feature of the model estimated by SW is that it does not provide evidence of hidden shocks: that is, at least at posterior modes of θ, their estimates always imply an ABCD representation that satisfies

[1] See Adjemian et al. (2011).

[2] Thus as such models get bigger the econometrician must always assume a larger number of structural shocks.

the PMI condition[3]. In this paper we ask how robust this finding is for this particular model; but the core elements of our analysis are applicable to any estimated DSGE.

We show that, at least *ex ante*, we would have good reason to expect that deep parameter estimates might be non-unique, in the sense that for any history of the observables, there may be (possibly many) alternative local peaks of the likelihood associated with distinct sets of deep parameters[4]. Each of these alternative estimates of θ would imply a different ABCD representation, each with different hidden shocks. Furthermore we show that standard Bayesian techniques, with priors and starting values for θ derived at least in part from vector autoregressions estimated over initial training samples would be very unlikely to explore the regions of the parameter space associated with these alternative peaks. Standard techniques thus cannot truly reveal the full posterior for θ.

We illustrate the potential non-uniqness problem in the context of a simple model of the New Keynesian Phillips Curve, and propose new techniques to search for alternative peaks. Applying these techniques to the Smets-Wouters model, we do indeed find alternative deep parameter estimates. These imply hidden shocks, and thus that that the information set of agents in the model is superior to that of the econometrician. We show that the degree of information superiority (as captured by higher implied predictive R^2 values for the observables) implied by these alternative versions of the model is not, however, very marked. Indeed we suggest that if it were, this would in itself give grounds for scepticism. Furthermore we find that, if anything, the data appear to prefer SW's original estimates. Thus we conclude that, at least in this dataset, and for this model structure, there is little evidence in favour of hidden shocks.

[3]Yet Del Negro et al. (2007) found little evidence of noninvertibility as a VAR in the observables provides a good approximation of the VARMA reduced form of the model at Smets and Wouters parameters value

[4]Note that our analysis is quite distinct from the local identification problems discussed by, eg, Iskrev (2010). The problems we identify may exist even in a model that has no local identification problems.

We then update SW's original dataset using roughly ten more years of information and conclude that now the econometrician challenged with the task of estimating the model is not as lucky as Smets and Wouters. We find that two alternative structral models are equally supported by the data and informational implications provide little help in discriminating among the two.

Our analysis builds on a well-known (but arguably neglected) body of work on nonfundamentalness, most of which pre-dates the development of estimated DSGE models. The potential for non-uniqueness arises because in reduced form the set of observables usually has a finite order vector autoregressive moving average (VARMA) representation[5]. Lippi and Reichlin (1994), drawing on earlier work by Hansen and Sargent (1980) showed that for any VARMA process, there is a potentially large set of VARMA representations of the same order, all of which match the data equally well. The *structural* VARMA representation - in which the structural shocks are identical up to a scale factor to the predictive errors made by agents in the model - is just one element in this set. We note that there is an equivalence between Fernandez-Villaverde et al. (2007) PMI condition, and the requirement that the structural VARMA representation be fundamental. If this condition is not satisfied, and there are hidden shocks, the structural VARMA is nonfundamental.

In such cases the structural VARMA cannot be directly estimated; all likelihood-based inference must rely on its fundamental counterpart[6]. The problem for the econometrician is that different structural models, with different (hidden) shocks, and with deep parameters in quite different parts of the parameter space, may in principle map to a very similar (or - in the limiting just-identified case - identical) fundamental representation. Thus, we argue, the data alone may offer

[5]This feature always arises as long as the number of states exceeds the number of observables.

[6]In terms of the structural VARMA its fundamental conterpart has all roots of the MA polynomial outside the unit circle. In terms of the ABCD representation, the likelihood is derived via the Kalman Filter from the innovations representation of Fernandez-Villaverde et al. (2007).

us only limited ability to discriminate between competing models with different informational assumptions, on the basis of their fundamental representations.

The plan of the paper is as follows. In the next section, we outline the problem the econometrician is faced with and we put forward a simple example. In Section 3 we illustrate the role of the exogenous processes in rational expectation models and we show how this relates to nonfundamentalness. Applying Hansen-Sargent prediction formula to a New Keynesian Phillips curve, we show that fundamental representations of the driving variables may imply nonfundamental representations of forward-looking variables. Section 4 presents the results of the application on the model of SW. A final section concludes and outlines avenues for future research.

2. The Problem

The solution of a DSGE model is composed by a set of equilibrium conditions, which typically include expectational difference equations. Including the law of motion for the exogenous precesses it results in a system of the form [7]

$$\boldsymbol{\Psi_0}(\theta)\,\mathbf{z_t} = \Psi_1(\theta)\,E_t\mathbf{z_{t+1}} + \Psi_2(\theta)\,\mathbf{z_{t-1}} + \Psi_3(\theta)\,\mathbf{w_t}$$

where $\mathbf{z_t}$ is a $n \times 1$ vector of both endogenous observed and unobserved and exogenous variables whose dynamics depends upon $\mathbf{w_t}$ is an $n_w \times 1$ random vector of independent and identically distributed structural shocks. It should be noted that the matrix coefficients $\Psi_i, i = 1..3$ all depend upon the vector of deep parameters θ. The solution to this rational expectations system results in a state space system:

$$\mathbf{x_t} = \mathbf{A}(\theta)\,\mathbf{x_{t-1}} + \mathbf{B}(\theta)\,\mathbf{w_t} \tag{2.1}$$

$$\mathbf{y}_t = \mathbf{C}(\theta)\,\mathbf{x_{t-1}} + \mathbf{D}(\theta)\,\mathbf{w_t} \tag{2.2}$$

where $\mathbf{y_t}$ is a $n_y \times 1$ vector of observed variables and $\mathbf{x_t}$ a $n_x \times 1$ vector of endogenous and exogenous state variables some of which are

[7] See Sims (2002).

unobserved. In order to avoid singularity, the econometrician usually estimates the model employing as many observables as many shocks. Fernandez-Villaverde et al. (2007) refer to this as the *square case* ($n_y = n_w$), which we assume throughout in this paper.

Starting from the state and observation equations (2.1) and (2.2), we can derive the structural VARMA(p,q) representation for the observables, with autoregressive order[8] $p \leq n_x - n_y + 1$ and moving average order $q \leq p - 1$:.

$$\mathbf{\Phi}(L)\,\mathbf{y}_t = \gamma(L)\,\mathbf{D}\mathbf{w}_t \tag{2.3}$$

where $\mathbf{\Phi}(L) := \left(\mathbf{I_{n_y}}\gamma(L) - \mathbf{C}\,\mathrm{adj}\left(\mathbf{I_{n_x}} - \mathbf{\Gamma}z\right)\mathbf{B}\mathbf{D}^{-1}L\right)$, with $\gamma(L) := \det\left(\mathbf{I_{n_x}} - \mathbf{\Gamma}L\right)$ and $\mathbf{\Gamma}(\theta) := \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}$. Note that $\mathbf{\Phi}(L)$ and $\gamma(L)$ are, respectively, matrix and scalar lag polynomials in the lag operator, so that in this representation all elements of $\mathbf{y}_t$ have a common MA component. Also note that in the special case that the number of states is equal to the number of observables, $n_x = n_y \Rightarrow q = 0$, and hence both the states and the observables have a VAR(1) representation; but for most DSGE models q is nonzero.

The link between γ and the fundamentalness of the VARMA is straightforward. Fernandez-Villaverde et al. (2007) show that $\mathbf{w_t}$ is fundamental with respect to $\mathbf{y}_t$ if $\mathbf{\Gamma}$ has all its eigenvalues smaller than one in absolute value or, using their terminology, if the *Poor Man's Invertibility Condition* holds. Letting γ_i be the i-th element of γ, the set of eigenvalues of $\mathbf{\Gamma}$ (hereafter PMIC eigenvalues), then $1/\gamma_i$ is a root of $\gamma(z)$, the MA polynomial. A moving average is referred to as *nonfundamental* if the MA filter has a root inside the unit circle. Thus the PMI condition is simply a condition for fundamentalness.

If the PMI condition is violated the VARMA (2.3) is nonfundamental, because $\gamma(z)$ has one or more (no) zeros within the unit circle. This must in turn imply that there are hidden shocks: the (Hilbert) space spanned by $\mathbf{y}^t := (y_1, \ldots, y_t)$ is a strict subspace of that spanned by $\mathbf{w}^t$. Thus nonfundamentalness implies that the information set of

[8]For derivation see Baxter et al. (2011), Proposition 4.

the agents is richer than that of the econometrician. Conversely, fundamentalness implies that, as $t \to \infty$, the econometrician can recover w^t from y^t, and thus has a common information set with the agents in the model.

The link between the structural VARMA (2.3) and the alternative representations of the data follows from the following Proposition.

Proposition 1. *Let s be the number of roots of $\gamma(z) = \prod_{i=1}^{s} -\gamma_i \left(z - \gamma_i^{-1}\right)$, then:*

1. For $j = 1, \ldots, 2^s - 1$ there are $2^s - 1$ $s-$dimensional vectors of dummy variables $\mathfrak{s}_j$ associated with just as many observationally equivalent VARMA representations of $\mathbf{y_t}$ alternative to (2.3)

$$\mathbf{\Phi}(L)\,\mathbf{y_t} = \xi^{(\mathbf{j})}(L)\,\eta_{\mathbf{t}}^{(\mathbf{j})} \tag{2.4}$$

$$\xi^{(\mathbf{j})}(z) = \gamma(z)\,\mathfrak{s}_j \bar{\mathcal{B}}(z,\gamma) \tag{2.5}$$

$$\eta_t^{(j)} = \tilde{\mathcal{B}}(z,\gamma)'\,\mathfrak{s}_j'\mathbf{D}\mathbf{w_t} \tag{2.6}$$

where

$$\bar{\mathcal{B}}(z,\gamma) = \begin{bmatrix} \mathcal{B}(z,\gamma_1) \\ \vdots \\ \mathcal{B}(z,\gamma_s) \end{bmatrix}, \qquad \tilde{\mathcal{B}}(z,\gamma) = \begin{bmatrix} \mathcal{B}(z,\gamma_1)^{-1} \\ \vdots \\ \mathcal{B}(z,\gamma_s)^{-1} \end{bmatrix}$$

$\mathcal{B}(z,\gamma_i) = \frac{z-\gamma_i}{1-\overline{\gamma}_i z}$ and $\mathbf{\Phi}(z) := \left(\mathbf{I_{n_y}}\gamma(z) - \mathbf{C}\,adj\left(\mathbf{I_{n_x}} - \mathbf{\Gamma} z\right)\mathbf{B}\mathbf{D}^{-1}z\right)$ is a $n_y \times n_y$ matrix polynomial which is the autoregressive filter common to all the VARMA representations and $\gamma = [\gamma_1, \ldots, \gamma_s]$. The representations (2.4) *are all of the same order of the VARMA* (2.3)*.*

2. Define $\varepsilon_{\mathbf{t}}$ the vector of residuals in the unique fundamental VARMA representation for $\mathbf{y_t}$. The structural VARMA representation (2.3) in fundamental if and only if $|\gamma_i| < 1\,\forall i = 1, \ldots, s$ and $\varepsilon_{\mathbf{t}}$ is equal to the structural shocks $\mathbf{w_t}$, otherwise the (2.3) is nonfundamental and the *unique fundamental VARMA representation is*

$$\mathbf{\Phi}(L)\,\mathbf{y_t} = \xi^{(\mathbf{k})}(L)\,\eta_{\mathbf{t}}^{(\mathbf{k})} \tag{2.7}$$

$$\mathfrak{s}_k = \mathbf{1}(|\gamma| > 1) \tag{2.8}$$

$$\varepsilon_{\mathbf{t}} = \eta_{\mathbf{t}}^{(\mathbf{k})} \tag{2.9}$$

where $\mathbf{1}(|\gamma| > 1)$ *is a vector indicator function which selects the roots of* $\gamma(z)$ *laying inside the complex unit disk.*

Proposition 1 is[9] by no means of any use for improving the estimation of a DSGE model. It is rather a threat because the econometrician wants to come up with the structural VARMA (2.3) rather than any of the other alternatives. The proposition does not put forward $2^s - 1$ nonfundamental VARMA representations of the observables but it just says that there are as many alternative representations, possibly including the fundamental one. In fact, if the (2.3) is nonfundamental, then the fundamental representation should be avoided because it will be one of the alternatives in Proposition 1.

According to the above proposition, there is a quite large number of representations associated with the same likelihood [10]. This does not mean that any of them can actually be attained in the estimation of the DSGE model, though. While the representation (2.3) directly arises from the solution of the model and therefore is always attainable, the alternative representations might not be attainable as they could not map to the deep parameters θ. Proposition 1 shows that the alternative representations are in fact obtained by flipping one or more root of the MA filter in the VARMA (2.3). But this does not imply that it is possible to reverse engineer θ from that representation as the restrictions imposed by the state space system (2.1)-(2.2) - and the

[9]Without loss of generality we assume that there are no complex roots. For c pairs of complex roots the the number of representations alternative to the structulral VARMA is $2^{s-c} - 1$. Also note that we are not considering the infinite number of *nonbasic* nonfundamental representations in which the AR and MA polynomials are of higher order than those in equation (2.3), as like Lippi and Reichlin (1994) we find unlikely that such representations arise from economic theory.

[10]For example, the results in Section 4 imply that SW has possibly $2^6 - 1 = 63$ alternative representations

dependency on θ of the coefficients of the reduced form - are ignored when the roots get flipped.

A possible misconception should be dispelled. If the econometrician knew the model, then she should not worry about the possible observational equivalence because an alternative VARMA representation with flipped roots involves a dynamic transformation of the shocks which compromises their structural meaning. Conversely, if the econometrician needs to estimate the model, she does not know what the structural model is and thus what the economic shocks are. The uncertainty in the estimation of the DSGE model involves both shocks and parameters as the structural shocks stem from the estimated model. As there are multiple representations of a VARMA process, just one of them - whether or not fundamental - is the structural one the econometrician aims to estimate. If the VARMA was just identified, then the nonuniqueness of structural shocks and deep parameters would translate into observational equivalence as the data would fail to prefer the representation (2.3) over those of Proposition 1. We suggest to investigate for that multiplicity with the reward that the overidentifying restrictions imposed by the DSGE on its VARMA representations should push the data to rule out many of those solutions, possibly all but one - the true structural model.

As far as modern DSGE models with a multitude of shocks, frictions and parameters are concerned, it becomes cumbersome to obtain analytic solutions for the VARMA representations and therefore it is not generally known how many peaks there are in its likelihood function. Furthermore, in such models the parameter space is large and it is impossible to explore it all. First, it is needed that the priors do not fail to embrace the regions of the parameter space where the peaks of the likelihood reside. Then, the MCMC posterior simulator should effectively explore the regions associated with positive prior probability.

The most commonly employed MCMC simulator in applied research is the Random-Walk Metropolis (RMW) algorithm whose computation has been described by An and Schorfheide (2007) as follows.

(1) A numerical optimization technique is employed to find a posterior mode $\tilde{\theta}$ by maximizing the logarithm of the posterior kernel which is defined as $\mathcal{K}(\theta|\mathbf{y_1},\ldots,\mathbf{y_T})$ - i.e. the likelihood times the prior
(2) Start a recursive rejection-sampling procedure for $i = 1, \ldots, n_{draws}$ from a proposal density chosen as $N\left(\theta_i,\, c^2\tilde{\Sigma}\right)$ where $\tilde{\Sigma}$ is the inverse of the Hessian computed in the initial value $\theta_0 = \tilde{\theta}$ and c is a scale parameter. The density above is known as a *jumping distribution* since it is updated at every iteration
(3) From the proposal density, θ^* is drawn and accepted according to the rule

$$\theta_{i+1} = \begin{cases} \theta^* & \text{with probability } \min\{1, r_i\} \\ \theta_i & \text{otherwise} \end{cases}$$

$$r_i = \frac{\mathcal{K}(\theta^*|\mathbf{y_1},\ldots,\mathbf{y_T})}{\mathcal{K}(\theta_i|\mathbf{y_1},\ldots,\mathbf{y_T})} \tag{2.10}$$

where r_i is the acceptance ratio of the proposal θ^*.

Step 2 is very important. In general the scale parameter is chosen in an efficient way targeting an acceptance rate approximately equal to 0.3 in order to calibrate the visits in the tail of the posterior distribution which under general assumptions is asymptotically Normal. This results in an "efficient exploration of the posterior distribution at least in the neighbourhood of $\tilde{\theta}$" (An and Schorfheide (2007), p.132).

This is not wise when the posterior has multiple modes. Supposing that the posterior is bimodal, whether the jumping distribution does effectively sample in a neighbourhood of a different mode than $\tilde{\theta}$ depends on the distance between the two modes and on the scale parameter. When such distance is large, the scale parameter needed to jump from a mode to the other might be very large and lead to a much smaller acceptance rate. Indeed, if the two modes are separated by a valley, an insufficiently large value of c would still yield $\tilde{\theta}$ because the posterior kernel calculated in the proposal will likely be equal to zero. The only chance left to the econometrician who suspect that the posterior is bimodal is to start step 1 from different regions of the parameter space in order to possibly find a mode which is not $\tilde{\theta}$, if any. This is a hard challenge as the number of parameters or modes in the posterior increases.

Example. We estimate the MA of order 2

$$z_t = (1 - \varpi_1 L)(1 - \varpi_2 L) \tag{2.11}$$

using a sample of 160 observations generated for $\varpi_1 = .5$ and $\varpi_2 = .25$, which is a fundamental data generating process. The number of observations is equal to the same sample size SW use to estimate their model. Without further conditions, in population $2^2 - 1 = 3$ modes $M = (\varpi_1, \varpi_2)$ are observationally equivalent to $M_1 = (.5, .25)$ which corresponds to the data generating process: $M_2 = (2, .25)$, $M_3 = (.5, 4)$, and $M_4 = (2, 4)$.

We illustrate that the information available to the econometrician is crucial. Let us start with a just-identified case in which ϖ_2 is calibrated at its true value. In the top panel of Figure 1 we plot the posterior densities estimated with 500000 draws and a uniform prior distribution on the support [0, 2.5]. The density in solid line is the true bimodal density obtained conditioning on the true value of ϖ_2 with a modified RMW algorithm in which we replace the covariance matrix of the proposal distribution with an identity matrix in order to allow for lager jumps [11]. This reduces the acceptance rate turning the algorithm inefficient if no second mode was in the posterior, but in this bimodal case it is crucial to detect both modes. Detecting both M_1 and M_2 modes has been possible since, starting from one of the two found in step 1, we knew the size of the jump needed to reach the other mode. Things are different if no jump is imposed and the MCMC sampler used is the RWM algorithm described in the steps 1-3 above. Once the initial value given to the numerical optimizer in step 1 leads to a mode, the RMW above is unable to jump to the other mode. The dashed and the dashed and dotted lines - the densities estimated starting from an

[11]This can be done in Dynare with the command `mcmc_covariance_matrix`.

initial value of .5 and 2.49 respectively - are in fact both unimodal with modes close to the initial values. In the first case the only mode found is close to M_1, in the second case the mode is close to M_2.

Now suppose that the econometrician information set is inferior and rather than ϖ_2 she only knows a constraint between the two parameters so that she can back up ϖ_2 given ϖ_1 - i.e. an overidentified case. In the bottom panel of Figure 1 we analyse the effects of parameter restrictions on the posterior density of ϖ_1 estimated on the same sample still with 500000 draws from a uniform prior distribution on the support [0, 4] and using an identity matrix as the covariance of the proposal density. In different colours the posteriors estimated under different constraints are drawn as specified in the note. In any case, the covariance of the proposal distribution chosen allows to discover the bimodality. Nonetheless, as a consequence of the constraints, if those are at odds with the data, the modes are shifted as compared with their population counterparts M_1, M_2, M_3, and M_4 and the densities may change shape favouring a single mode. Conversely, when the constraint imposed is supported by the data, the estimated posterior has the two modes in the right place. This is the case of the black line which is the density estimated under $\varpi_2 = 4$ whose two modes are in fact around M_3 and M_4 and very similar to the black line in the top panel of the same figure.

Before carrying out our investigation in a modern medium-scale DSGE model, the analytical illustration in the next section discusses how rational expectation models produce fundamental and nonfundamental representations and how this relates to the specification of the exogenous processes. As in a medium-scale DSGE it becomes infeasible to choose enough starting points to carefully explore its large dimensional likelihood, we focus along the dimension of a few parameters governing the evolution of some exogenous processes.

3. Nonfundamentalness, rational expectations and exogenous processes: analytical illustrations

It is widely agreed that the specification of the exogenous processes in a DSGE model is very important in fitting the data. For example SW specify two ARMA $(1,1)$ processes for the exogenous variables hit by the price mark-up shock and the wage mark-up shock because it helps to capture high frequency components of the underlying aggregates. It is worth discussing how the property of an exogenous forcing process relate to those of the observed aggregate variable.

Of course, even an exogenous variable may follow itself a univariate nonfundamental process. If the estimation of a single ARMA for forecasting was concerned, there would be no need to think about nonfundamental processes. A nonfundamental representation cannot be used in forecasting because it needs the future of the target variable in order to fit better. Conversely, in the context of a DSGE model, any representation is possible because the story-telling in it may well generate a structural nonfundamental moving average.

Indeed, given the high nonlinearity in the mapping with the VARMA reduced form and the constraints imposed therein, it is not clear whether among the many alternative representations for the forcing process the maximum likelihood is attained in the region of the parameter space corresponding to a fundamental representation for the univariate forcing processes. Nevertheless, the nonfundamentalness of the forcing process does not necessarily imply the nonfundamentalness of the VARMA reduced form and vice-versa.

Let us write a New Keynesian Phillips Curve (NKPC) as

$$\pi_t = \beta \pi_{t+1|t} + \mu_t^{\pi} \tag{3.1}$$

where the Wold representation of the forcing process is $\mu_t^{\pi} = d(L)\,\varepsilon_t^{\pi}$ and ε_t^{π} is a structural disturbance. Given that the Wold residuals are the structural shock[12], the latter are fundamental to μ_t^{π}. Solving the

[12]Without loss of generality we are assuming that no orthogonal rotation is needed to identify the structural shocks.

(3.1) by forward iteration the rational expectations problem is

$$\pi_t = E_t \sum_{j\geq 0} \beta^j d(L)\, \varepsilon^{\pi}_{t+j} \tag{3.2}$$

$$= \left[\frac{Ld(L)}{L-\beta}\right]_+ \varepsilon^{\pi}_t \tag{3.3}$$

which using the prediction formula of Hansen and Sargent (1980) has the solution

$$\pi_t = \alpha(L)\, \varepsilon^{\pi}_t \tag{3.4}$$

$$\alpha(z) = \frac{Ld(z) - \beta d(\beta)}{z-\beta}$$

Even if ε^{π}_t is fundamental to μ^{π}_t it is not granted that it will also be fundamental with respect to the inflation. It will generally depend on the roots of the polynomial $\frac{Ld(L)-\beta d(\beta)}{L-\beta}$.

For example, setting $d(z) = (1-\psi z)/(1-\rho z)$ we have the same price mark-up forcing process of SW and the polynomial[13] $\alpha(z)$ has a zero in

$$\zeta \equiv \frac{1-\beta\psi}{\psi(1-\beta\rho)} \tag{3.5}$$

which lays into the complex unit disk for a non-negligible range of values for β, ρ and ψ. In Figure 2 we see the nonfundamental and fundamental parameter space generated by the couple of parameters of the exogenous forcing process (ρ, ψ) when the discount factor β is equal to 0.99 which is the value most commonly found in literature. This proves that fundamental forcing process does not necessarily imply a fundamental representation for the observables.

The Figure 2 also shows that the vice-versa holds as well i.e. that the structural ARMA for the inflation may be fundamental even if the MA representation for μ_t is nonfundamental. This can be seen from the top right of the figure where, for values of ψ above one corresponding to a nonfundamental forcing process and of ρ approximately larger than 0.5, the root ζ lays outside of the unit disk.

[13]Notice that $\alpha(z)$ does not involve negative powers of z because its numerator of has a zero in β.

In this example the structural ARMA representation (2.3) for the inflation is

$$\pi_t = \frac{1 - \zeta^{-1}L}{1 - \rho L}\varepsilon_t^\pi \tag{3.6}$$

not necessarily fundamental. If $|\zeta| > 1$ the generic fundamental representation

$$\pi_t = \frac{1 - \kappa L}{1 - \rho L}\tilde{\varepsilon}_t^\pi$$

with $\kappa \in (0, 1)$, corresponds to the structural (3.6), therefore $\tilde{\varepsilon}_t^\pi = \varepsilon_t^\pi$ and $\kappa = \zeta^{-1}$. As shown in Figure 3 nonuniqueness arises because for any estimate $\hat{\kappa}$ there exists two different values ψ_F and ψ_{NF} for the deep parameter ψ.

While in this paper we focus on complete information models, it is worth mentioning that structural nonfundamental representations arise directly from large classes of rational expectation models in which there is some degree of incomplete or imperfect information either on the side of the agents or on that of the econometrician. In incomplete or dispersed information models [14] nonfundamentalness generates business cycle fluctuations leading to different equilibria than the full information case. The difference with the nonfundamentalness in complete information frameworks is that under imperfect information the information set of the econometrician is not necessarily inferior to that of the agents, both being faced with some kind of signal extraction.

The news shocks (see Forni et al. (2014), Beaudry and Portier (2014) and Sims (2012) among others) and the fiscal foresight (see Leeper et al. (2013) among others) literature, while keeping the assumption of agents full information, imply that no mapping may exist between the observables and the structural shocks by modeling a lag scheme for the response of aggregates to shocks which are anticipated by the agents. Nevertheless, as Forni et al. (2013b) and Forni et al. (2013a) show, an econometrician that knows the model can still recover nonfundamental structural shocks from the present and past of the observables by taking a *dynamic* rotation of VAR residuals. Our paper is different because

[14] See Townsend (1983), Graham and Wright (2010) and Rondina and Walker (2012) among others.

we deal with the estimation rather than the evaluation of the DSGE model.

4. Application: Smets and Wouters

In the simple MA example in Section 2 the parameter space has dimension two and the data generating process is known so the econometrician can easily figure out where the modes should be and adjust the covariance matrix of the proposal density to allow for jumps of the appropriate size.

This not an option in a DSGE model in which the parameter space is high dimensional. Using the RWM algorithm the best the econometrician can do is to start the MCMC from a few different points of the parameter space and slice the likelihood functions along the dimensions that seems to be more empirically relevant for the fundamentalness of the VARMA representation for the observables. Given that alternative representations should differ for an approximate root flipping, we use the derivatives of the PMIC eigenvalues to gain intuition on that. Once the modes are found, their informational implications follow from the time series properties of the observables. In this section we follow this approach to study the nonfundamentalness in the popular medium-scale DSGE model of SW.

4.1. **Estimation.** While there is large consensus on the priors on most deep parameters, there is little help the econometrician can receive from economic theory on the parameters governing the evolution of the exogenous processes and their priors are not elicited in a subjective fashion. For such a subset of parameters data-based priors are commonly elicited. Standard practice is to train regressions on pre-sample data than the one used in the estimation of the DSGE and choose priors that allow to match some of the properties the observables exhibit in the presample, like e.g. their second moments [15]. SW estimate a VAR on the previous ten years of data to the estimation sample.

[15] see Del Negro and Schorfheide (2008)

Such a practice relies on the implicit assumption that the structural shocks are fundamental with respect to the variables in the training sample regression. Whether the set of regressors spans or not the space of the structural shocks is an empirical matter which, to the best of our knowledge, has not yet been investigated in the literature of estimated DSGE models. Of course, if the structural shocks are nonfundamental with respect to that data, the data-based priors elicited this way are biased to some extent. While the exploration of nonfundamentalness in pre-sample regressions goes beyond the goals of this paper, we urge practitioners to take this into account.

In SW model [16] the two exogenous processes for the price an wage markup shocks are modeled as ARMA$(1,1)$:

$$\eta_t^p = \rho_p \eta_{t-1}^p + \varepsilon_t^p + \mu_p \varepsilon_{t-1}^p \quad (4.1)$$

$$\eta_t^w = \rho_w \eta_{t-1}^w + \varepsilon_t^w + \mu_w \varepsilon_{t-1}^w \quad (4.2)$$

where ε_t^p, ε_t^w are the price and wage markup shocks and η_t^p, η_t^w are the exogenous variables. SW assign zero mass outside of the unit circle in the prior distribution of the two MA parameters μ_p and μ_w, we argue that the priors on these two parameters should not be limited on a range ensuring the fundamentalness of the forcing processes. As we discussed in Section 3, the properties of the exogenous variables do not directly imply those of the observables. So while we might find a mode out of the [0, 1] range implying the nonfundamentalness of the forcing process, the VARMA (2.3) could still be fundamental. Or vice-versa, a mode into the [0, 1] range would not grant the fundamentalness of the VARMA.

Imposing the fundamentalness of (4.1) and (4.2) implies that an information set containing the exogenous variables η_t^p and η_t^w is not inferior to one containing also the structural shocks ε_t^p and ε_t^w as the former span the same space of the latter. SW by imposing that priors on μ_p and μ_w are implicitly assuming that agents observing all the variables in the model can give up the information conveyed by the price

[16] See parameters description in appendix

and wage markup shocks with no information loss. So an incomplete information economy with two unobserved shocks would provide the same outcome.

Conversely, if the (4.1) and (4.2) are nonfundamental representations then a hierarchy of information sets can be outlined. At the bottom there is the information set of the econometrician who is bounded to observe only n_y observables. Superior to that there is the information set of the agents of an incomplete information economy in which ε_t^p and ε_t^w are unobserved and signal extraction is required to infer the shocks from the exogenous variables η_t^p and η_t^w. As spelled out by Fernandez-Villaverde et al. (2007) agents' Kalman filter , the largest information set is that of complete information agents including both the exogenous η_t^p and η_t^w and the shocks ε_t^p and ε_t^w.

As seen in the trivial MA example in Section 2, parameter constraints do not affect all the modes in a symmetric way. Due to those imposed by the equilibrium conditions of the DSGE to its VARMA reduced form, it might well be that the data prefers μ_p and μ_w out of the unit circle. Therefore, we do not use the prior beta distribution with mean 0.5 and standard deviation equal to 0.2 like SW do, because we want to allow for values of the two parameters greater than one. Nonetheless, motivated by the informational implications derived in the next subsection, we should not admit values extremely larger than one. For this reason, we use an asymmetric gamma distribution with a long right tail with most of its mass in [0, 1] and decreasing probability between 1 and 2. The mean of the two priors are chosen to be close to the posterior means found in SW under the beta prior[17], and the standard deviation is equal to 0.3. In Figure 4 our gamma and SW's beta priors are plotted together for comparison. The priors on remaining parameters are equal to those of SW as reported in Table1.

4.2. **Preliminary analysis.** We start estimating the model as SW do. In the second column of Table 1 we report the initial values provided

[17]SW posteriors for μ_w and μ_p are 0.74 and 0.88 respectively and we fix the prior means at 0.75 and 0.9.

to the numerical optimizer in step 1 of the RWM algorithm described in Section 2. We keep such initializations as given with the only exceptions of the two AR parameters for the price and wage markups whose initializations are denoted as $\rho_p^{(0)}$ and $\rho_w^{(0)}$. Using the same initializations as SW we get a vector of posterior modes for the deep parameters that we call θ^{SW} which is reported in the second column of Table 2. The influence of the gamma priors on the two MA parameters is limited, in fact the posterior modes θ^{SW} are very similar to those found by SW for all parameters, included μ_w and μ_p.

In the θ^{SW} point of the parameter space, the PMI condition holds true being all the elements of γ - which are reported in the first column of Table 3 - smaller than one in absolute value. Therefore the VARMA representation (2.3) at θ^{SW} values is fundamental. This is not totally surprising in the light of the results of Del Negro et al. (2007). In a DSGE-VAR approach they find that a VAR approximation of the structural model is sufficiently accurate which is impossible if the structural VARMA was not fundamental.

Different modes than θ^{SW} might still exist and possibly in a large number, corresponding to nonfundamental representations of the observables which cannot be obtained by simple root flipping because of the restrictions imposed by the DSGE on the coefficients of the VARMA representations. In a just-identified case the likelihood of any VARMA representation of a given process is exactly the same. Being the VARMA representations of a DSGE model overidentified, different modes might be associated with different values of the likelihood and its global maximum could be unique.

In the unfortunate case that the likelihood does not clearly prefer any representation over the others it is still possible to evaluate their informational implications. While all representations are consistent with the covariance structure of the observables, they do not fit the data equally well. In particular, the fundamental representation is the one that provides the worse fit. This is of no direct interest if the goal of the econometrician is forecasting because nonfundamental representations involve the future of the observables and therefore cannot be used to form predictions.

Nevertheless, in the context of a DSGE model the fit of the VARMA used to take the DSGE model to the data has a very useful interpretation. As discussed in Section (2), the nonfundamentalness of the VARMA representation 2.3 implies that the agents are endowed with a larger information set than that of the econometrician because the information conveyed by the observables is not sufficient to retrieve the structural shocks agents observe. We argue that it can be safely assumed that agents information cannot be too much superior than that the econometrician uses to estimate the model.

Therefore, according to reasonable informational assumptions, we can rule out representations involving too much nonfundamentalness. The more nonfundamental a representation of a given process is, the better it will fit as compared to its fundamental representation. We use the R squared as a measure of fit and, following Robertson and Wright (2012), the link between the fit of alternative representations of a ARMA process is as follows.

Proposition 2. *Let R_i^2 be the predictive R^2 of the equation for the i-th variable in the structural model (2.3) and $R_{i,F}^2$ the predictive R^2 for the same variable in the fundamental VARMA representation. Then*

$$R_i^2 \;=\; 1 - V(\gamma)\left(1 - R_{F,i}^2\right) \tag{4.3}$$

where $V(\check{\gamma}) = \prod_{\check{\gamma}_j \notin (0,1)} \gamma_j^{-2}$. Therefore, by construction, if the PMI condition holds $R_i^2 = R_{F,i}^2$, otherwise $R_i^2 > R_{F,i}^2$.

Proof. in Appendix □

The coefficients R^2 of the equations in (2.3) are calculated analytically starting from the state-space representation of the DSGE. From equation (2.1) it follows that $\Sigma = A\Sigma A' + BQB'$, where $\Sigma \equiv E\mathbf{x_t}\mathbf{x_t}'$ and $Q \equiv E\mathbf{w_t}\mathbf{w_t}'$. Therefore $vec(\Sigma) = (I - A \otimes A)^{-1}(B \otimes B)\, vec(Q)$ and using the observation equation (2.2) we have that the fit of the structural model for the i-th variable is given by

$$R_i^2 = \frac{P_i' C\Sigma C' P_i}{P_i' \left(C\Sigma C' + DQD'\right) P_i} \tag{4.4}$$

where P_i is a $n_y \times 1$ selection vector for variable i.

Applying the equation (4.4) the fit of the observed variables at θ^{SW} is in the first column of Table 4. Using formula (4.3) we calculate the implied fit of the nonfundamental representations obtained by flipping the elements of γ outside of the unit circle. Anyway, it is not generally true that a nonfundamental representation is consistent with the equilibrium conditions of the DSGE model. As the VARMA representations of a DSGE models are overidentified, root flipping procedures do not provide exact values but just an approximation.

Nonetheless, this is enough for deriving informational assumptions. It is reasonable to think that the information conveyed by the observables is not extremely poorer than that encompassed in agents information set. It is empirically found that we should not allow for too much nonfundamentalness, therefore we would not accept modal values implying that the roots of the VARMA representation (2.3) are too far from the complex unit disk.

For example, starting from θ^{SW} a nonfundamental representation obtained by flipping all its roots in $1/\gamma_1, \ldots, 1/\gamma_6$ implies that $V(\gamma) = 0.0049$ and its R^2 for the inflation is equal to an astonishing 0.9988 as opposed to $R^2_{F,\pi} = 0.7633$ of the fundamental representation. Should we get a posterior mode resembling that nonfundamental representation - i.e. a mode such that all the elements of γ are outside the unit circle and reasonably close to the reciprocals of γ^{SW}- we would wisely reject it. An alternative nonfundamental representation flipping only the two largest roots $1/\gamma_6$ and $1/\gamma_5$ gives a still amazing fit given by $R^2_\pi = 0.9860$. Finally, flipping the closest root to the unit circle we get the worst fitting nonfundamental representation - i.e. $R^2_\pi = 0.7740$. Again, in a Bayesian spirit, it is left to the prior belief of the econometrician to allow for a sensible range of $R^2_\pi \in [0.7740,\ 0.9988]$.

We now have everything in order to search for other modes in the posterior density of the parameters of the DSGE model. First, we expect to find such modes by exploring a range of different initializations along certain dimensions of the parameter space. Second, evaluating their informational implications we can accept or reject any of those modes. We are just left with the task of finding the points in the parameter space that are more likely to lead us to more modes.

As the parameter space is highly dimensional, this is not a trivial task and we tackle it by analysing the derivatives of γ with respect to the deep parameters θ. In Table 5 we report the average of the numerically estimated derivatives across the elements of γ evaluated in a neighbourhood of θ^{SW}. In this point of the parameter space, the parameters of the ARMA shocks are the most important. Based on this evidence, in the next section we show MCMC results for the initializations of such parameters. Other influential parameters are α (capital share), λ (habit formation in consumption), ξ_p(price stickiness), ξ_w(wage stickiness) and r_y (output coefficient in the Taylor rule). The analytical derivatives of the moments of the observables and impulse responses calculated by Iskrev (2010) are very similar to those of Table 5.

4.3. **Results.** Out of the four parameters of the ARMA processes (4.1) - (4.2) for the price and wage markup shocks, we found that the initilizations of the two AR parameters lead to other posterior modes than θ^{SW}. In Table 2 three more modes are reported in the last three columns. With respect to the initialization of SW leading to θ^{SW}, the mode θ^* is found by just setting $\rho_p^{(0)} = .6$ rather than 0.8692, the mode $\bar{\theta}$ is found with $\rho_w^{(0)} = 0.6$ rather than 0.9546, finally the mode $\theta^\dagger$ is estimated by shifting both initializations[18].

The posterior modes of the MA parameters are directly affected by those initializations. When an AR parameter is not initialized at SW's values the posterior of the corresponding MA parameter peaks beyond one, so the forcing process becomes nonfundamental. So θ^* implies

[18]Note that such initializations are not the only ones providing the modes above. There is quite a large range of values of $\rho_w^{(0)}$ and $\rho_p^{(0)}$ leading to θ^*, $\bar{\theta}$ and $\theta^\dagger$.

that the equation (4.1) is nonfundamental, while in $\bar{\theta}$ the (4.2) is nonfundamental and in $\theta^{\dagger}$ the (4.1) and (4.2) are both nonfundamental. The wage indexation parameter ι_w seems to be very affected by the fundamentalness of the wage markup process, being close to 0.6 in the case of θ^{SW} and θ^{*} and close to 0.3 in $\theta^{\dagger}$ and $\bar{\theta}$. Other parameters displaying noteworthy differences across the different modes are the price indexation ι_p and, again, the price and wage stickiness ξ_p and ξ_w.

At those modal values, the PMIC eigenvalues are reported in Table 3. As expected, the other modes found are all nonfundamental VARMA representations being at least one element of γ greater than one in absolute value, and are by and large close to a root flipping solution of a VARMA representation at θ^{SW} values. Any of the other modes is in a neighbourhood of a root flipping solution of θ^{SW}. In particular, θ^{*} does approximately flip[19] γ_5, $\bar{\theta}$ seems to flip γ_4 and $\theta^{\dagger}$ both γ_5 and γ_4. As the corresponding roots in $1/\gamma_5$ and $1/\gamma_4$ are not too far from the unit circle we would not expect such root flipping to have too strong informational implications. In fact, from Table 4 the fit of those representations do not seem unacceptably better than their fundamental counterparts which are calculated reversing the formula (4.3).

At this point, we need to evaluate how the posterior modes found are supported by the data. In Table 6 we report the results of Bayesian model comparison methods. The mode θ^{SW} is clearly preferred by the data so no nonuniqueness arise and we conclude that the estimation of SW does most likely correspond to the global maximum of the likelihood function. This is therefore a case in which the overidentifying restrictions do allow to confidently distinguish the structural VARMA (2.3) from any of the other representations in Proposition 1.

However, it may not be the case. We repeated the estimation of the model on the sample 1966:1 - 2014:4 updating the original estimation sample of SW which spans 1966:1 to 2004:4 and has been used to produce the results above. In Table 7 the posterior modes θ^{SW} and θ^{*} are found exactly in the same way as in the Table 2 relative to

[19]Of course, even the PMIC eigenvalues are flipped being the moduli of the reciprocal of MA roots.

SW's sample. Comparing the two tables most parameters remained quite stable, the price stickiness, the wage indexation, the steady state elasticity of the capital adjustment cost, and the AR parameter for the risk premium shock rose, while the habit in consumption, and the coefficients r_y and r_π in the Taylor rule decreased. Also the PMIC eigenvalues in Table 8 are similar to those calculated in SW sample (Table 3). Comparing those calculated at θ^{SW} it seems that just one eigenvalue changes of a respectable amount decreasing from 0.5204 to 0.3508. Given these results it is not surprising that the informational implications do not change too much as shown in Table 9. Nonetheless, now the likelihood function does not discriminate between θ^{SW} and θ^*. In Table 10 the posterior model probabilities associated with the two modes are almost the same being slightly more likely the mode θ^*. As both modes are roughly equally supported by the data, either the deep parameters and the structural shocks are nonunique and there is one of the VARMA representations of Proposition 1 which is observationally equivalent to the structural model (2.3). Therefore, in this sample, the econometrician cannot determine which one of the two alternatives is the structural model.

In both the estimation samples above, all the nonfundamental representations are associated with at least a nonfundamental exogenous ARMA process[20]. As discussed in Section 3, the properties of the exogenous processes do not directly imply those of the reduced form of the DSGE model. We now show that the structural VARMA of SW model can actually be nonfundamental even if both ARMA exogenous processes are fundamental. To do so we now use SW original beta priors on μ_w and μ_p with mean 0.5 and standard deviation 0.2 with bounds in [0, 1]. The modes θ^{SW} and θ^* in Table 11 are found using the same initializations used for the gamma priors. Form Table 12 we see that now θ^* leads to a nonfundamental VARMA even if the posterior modes of μ_p implies a fundamental price markup exogenous process. The value of the PMIC eigenvalues under SW priors is very similar to that under

[20]In both estimation samples (see Tables 2 and 7), the μ_p is greater than one in θ^*. μ_w is greater than one in $\bar{\theta}$ and in $\theta^\dagger$ both $\mu_w$and μ_p are greater than one.

the gamma priors in Table 3. The same result, which for sake of brevity is not presented here, has been obtained even in the updated sample.

Finally, we investigate in the two models how the price markup shock propagates in the economy. One of the biggest challenges in modern DSGE modeling is to improve the endogenous propagation of the structural shocks. As Chari et al. (2008) criticize, a scenario in which the exogenous processes explain most of the aggregate fluctuation is likely to reflect misspecified endogenous propagation and favour for a non-structural interpretation of some shocks. In Figure (5) the solid black line are the responses of output, hours worked, inflation and nominal interest rate to the price markup shock in the nonfundamental model obtained at θ^* values, while the black dashed lines are the responses in the fundamental model at θ^{SW} values.

In order to assess the relevance of the exogenous propagation of the price markup shock we proceed as follows. The state equation (2.1) can be partitioned as

$$\mathbf{x_t} = \begin{pmatrix} \mathbf{x_t^e} \\ \mathbf{x_t^x} \end{pmatrix} = \begin{pmatrix} \mathbf{A_e} & \mathbf{A_{ex}} \\ \underset{n_x \times n_y}{\mathbf{0}} & \mathbf{A_x} \end{pmatrix} \mathbf{x_{t-1}} + \begin{pmatrix} \mathbf{B_e} \\ I \end{pmatrix} \mathbf{w_t} \tag{4.5}$$

where $\mathbf{x_t^e}$ and $\mathbf{x_t^x}$ are respectively the endogenous and exogenous state variables. The dynamics of the endogenous is

$$\mathbf{x_t^e} = (I - \mathbf{A_e} L)^{-1} \left(\mathbf{B_e} + \mathbf{A_{ex}} (I - \mathbf{A_x} L)^{-1} L \right) \mathbf{w_t} \tag{4.6}$$

and its exogenous propagation is given by the matrix $\mathbf{A_x}$. The exercise we propose is to observe the impulse response functions of key macroeconomic aggregates to the price markup shock in a model with the endogenous propagation of the fundamental model and the exogenous propagation of the nonfundamental model. Such responses are the red dashed and dotted lines in Figure 5. As those resemble the responses of the nonfundamental model we conclude that most of the propagation of the price markup shock is exogenous.

5. Conclusions, discussion and future research

As VARMA processes admit a number of alternative representations, the overidentified VARMA representation of the observables of a DSGE model may be nonunique. Generally, multiple posterior modes should arise from the Bayesian estimation of the model in an approximation of root flipping solutions. If, following the standard approach, the RWM algorithm is used to simulate the posterior density then the initialization of the parameters plays an important role determining the mode to be found. This is particularly true when the modes are reasonably spaced out in the parameter space. Ignoring the potential multiplicity of posterior modes may lead to the estimation of a non structural model or to observational equivalence between the structural model and a model in a neighbourhood of its root flipping solutions.

This paper provides an approach to find posterior modes based on the derivatives of the roots of the VARMA representations with respect to the deep parameters. From the estimated modes, we also show how to derive informational implications which are useful to accept or reject any of them. This is important as the number of modes found may be large. Finally, Bayesian model comparison is employed to address whether the modes found are observationally equivalent.

We apply our method to the model of Smets and Wouters (2007). We conclude that SW results are accurate as no nonuniqueness arise in their estimation sample. However, we also show that when the model is estimated on an updated sample, with roughly ten more years of quarterly data, observational equivalence is found. Our result is robust with respect to the prior distributions and to the properties of the exogenous processes. In particular we show both analytically and empirically that while nonfundamental exogenous processes cannot be ruled out, those are not necessary for obtaining nonfundamentalness in the VARMA representation of SW's model. Indeed, in Section 3 we also illustrate that this is not even sufficient. We finally show that the parameters of the forcing processes are important as the exogenous propagation of the shocks is often in large part exogenous.

Our suggestion for applied research is to explore well the parameter space in response to the multimodality we suspect to be endemic in DSGE models. Apart from our method, we encourage more research in alternative - and better suited in face of multimodality - algorithms like that of Chib and Ramamurthy (2010) and that of Herbst and Schorfheide (2014).

The more informative the observables are, the more likely the structural model is nonfundamental with respect to the information those encompass. Testing for the information generated by the observables has yet been proposed by Forni and Gambetti (2011). This opens the possibility to variable selection approaches like that of Canova et al. (2014). If the model is fundamental, then structural VAR techniques are suitable for the evaluation of the model allowing the econometrician to recover the structural shocks. Otherwise, for the evaluation of non-fundamental structures the information set of the econometrician need to be enlarged. In this sense, the structural factor model approach of Forni et al. (2009) seems promising.

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TABLE 1. Prior distributions

parameter	initval	LB	UB	pdf	mean	variance
σ_a	0.4618	0.01	3	INV. GAMMA	0.1	2
σ_b	0.1818513	0.025	5	INV. GAMMA	0.1	2
σ_g	0.6090	0.01	3	INV. GAMMA	0.1	2
σ_I	0.46017	0.01	3	INV. GAMMA	0.1	2
σ_r	0.2397	0.01	3	INV. GAMMA	0.1	2
σ_p	0.1455	0.01	3	INV. GAMMA	0.1	2
σ_w	0.2089	0.01	3	INV. GAMMA	0.1	2
ρ_a	.9676	.01	.9999	BETA	0.5	0.2
ρ_b	.2703	.01	.9999	BETA	0.5	0.2
ρ_g	.9930	.01	.9999	BETA	0.5	0.2
ρ_I	.5724	.01	.9999	BETA	0.5	0.2
ρ_r	.3	.01	.9999	BETA	0.5	0.2
ρ_p	$\rho_p^{(0)}$	.01	.9999	BETA	0.5	0.2
ρ_w	$\rho_w^{(0)}$	.001	.9999	BETA	0.5	0.2
μ_p	.7652			GAMMA	0.75	0.3
μ_w	.8936			GAMMA	0.9	0.3
ϕ	6.3325	2	15	NORMAL	4	1.5
σ_c	1.2312	0.25	3	NORMAL	1.50	0.375
h	0.7205	0.001	0.99	BETA	0.7	0.1
ξ_w	0.7937	0.3	0.95	BETA	0.5	0.1
σ_l	2.8401	0.25	10	NORMAL	2	0.75
ξ_p	0.7813	0.5	0.95	BETA	0.5	0.10
ι_w	0.4425	0.01	0.99	BETA	0.5	0.15
ι_p	0.3291	0.01	0.99	BETA	0.5	0.15
ψ	0.2648	0.01	1	BETA	0.5	0.15
ϕ_p	1.4672	1.0	3	NORMAL	1.25	0.125
r_π	1.7985	1.0	3	NORMAL	1.5	0.25
ρ	0.8258	0.5	0.975	BETA	0.75	0.10
r_y	0.0893	0.001	0.5	NORMAL	0.125	0.05
$r_{\Delta y}$	0.2239	0.001	0.5	NORMAL	0.125	0.05
$\overline{\pi}$	0.7	0.1	2.0	GAMMA	0.625	0.1
$100(\beta^{-1}-1)$	0.7420	0.01	2.0	GAMMA	0.25	0.1
$\bar{l}$	1.2918	-10.0	10.0	NORMAL	0.0	2.0
$100\,(\overline{\gamma}-1)$	0.3982	0.1	0.8	NORMAL	0.4	0.10
ρ_{ga}	0.05	0.01	2.0	NORMAL	0.5	0.25
α	0.24	0.01	1.0	NORMAL	0.3	0.05

Note: initval are the parameter initializations, LB and UB the upper and lower bounds respectively.

TABLE 2. Posterior distributions (1966:1-2004:4)

	θ^{SW}	θ^{*}	$\bar{\theta}$	$\theta^{\dagger}$
ρ_a	0.962	0.962	0.971	0.97
ρ_b	0.188	0.226	0.165	0.206
ρ_g	0.975	0.973	0.974	0.974
ρ_I	0.695	0.689	0.7	0.705
ρ_r	0.117	0.0981	0.103	0.0914
ρ_p	0.918	0.944	0.963	0.968
ρ_w	0.978	0.982	0.986	0.985
μ_p	0.765	1.03	0.775	1.06
μ_w	0.922	0.942	1.01	1.01
ϕ	5.5	4.95	5.34	4.9
σ_c	1.43	1.45	1.51	1.53
λ	0.703	0.685	0.707	0.688
ξ_w	0.768	0.792	0.837	0.836
σ_l	2.0	2.14	2.27	2.24
ξ_p	0.656	0.674	0.537	0.593
ι_w	0.594	0.609	0.354	0.291
ι_p	0.221	0.3	0.284	0.395
ψ	0.551	0.531	0.599	0.555
ϕ_p	1.61	1.6	1.54	1.55
r_π	2.01	1.98	2.04	2.01
ρ	0.818	0.831	0.833	0.845
r_y	0.0921	0.102	0.106	0.11
$r_{\Delta y}$	0.224	0.235	0.227	0.234
$\overline{\pi}$	0.749	0.712	0.704	0.689
$100\left(\beta^{-1}-1\right)$	0.144	0.143	0.141	0.138
$\bar{l}$	0.824	0.895	1.2	1.11
$100\left(\overline{\gamma}-1\right)$	0.435	0.439	0.434	0.438
ρ_{ga}	0.524	0.532	0.536	0.541
α	0.192	0.196	0.196	0.201
σ_a	0.453	0.454	0.459	0.457
σ_b	0.241	0.232	0.243	0.232
σ_g	0.521	0.521	0.517	0.518
σ_I	0.461	0.476	0.469	0.473
σ_r	0.238	0.237	0.235	0.236
σ_p	0.141	0.212	0.158	0.216
σ_w	0.253	0.255	0.3	0.301

Note: posterior distributions in the estimation sample 1966:1-2004:4. Priors and initial values are those in Table (1). The modes in this table are found under the following initial values for the price and markup AR parameters: $\rho_p^{(0)}\left(\theta^{SW}\right)=0.8692$, $\rho_w^{(0)}\left(\theta^{SW}\right)=0.9546$, $\rho_p^{(0)}\left(\theta^{*}\right)=0.6$, $\rho_w^{(0)}\left(\theta^{*}\right)=0.9546$, $\rho_p^{(0)}\left(\bar{\theta}\right)=0.8692$, $\rho_w^{(0)}\left(\bar{\theta}\right)=0.6$ and $\rho_p^{(0)}\left(\theta^{\dagger}\right)=0.6$, $\rho_w^{(0)}\left(\theta^{\dagger}\right)=0.6$.

TABLE 3. Moduli of PMIC eigs (1966:1-2004:4)

	θ^{SW}	θ^{*}	$\bar{\theta}$	$\theta^{\dagger}$
γ_1	0.9771	1.804	1.266	1.666
γ_2	0.9652	0.9769	0.977	1.309
γ_3	0.8318	0.9666	0.9663	0.9765
γ_4	0.7078	0.8191	0.8245	0.9684
γ_5	0.5204	0.763	0.4964	0.8134
γ_6	0.4675	0.4801	0.4694	0.4663

TABLE 4. R-squareds (1966:1-2004:4)

	θ^{SW}	θ^{*}		$\bar{\theta}$		$\theta^{\dagger}$	
	$R_i^2 = R_{F,i}^2$	R_i^2	$R_{F,i}^2$	R_i^2	$R_{F,i}^2$	R_i^2	$R_{F,i}^2$
l_t	0.9624	0.9626	0.8782	0.9694	0.9510	0.9694	0.8547
r_t	0.8637	0.8774	0.6011	0.8875	0.8197	0.8996	0.5223
π_t	0.7633	0.8984	0.6694	0.8137	0.7015	0.9299	0.6664
y_t	0.9807	0.9808	0.9375	0.9844	0.9750	0.9843	0.9255
c_t	0.9913	0.9916	0.9726	0.9937	0.9900	0.9936	0.9698
I_t	0.9784	0.9784	0.9297	0.9782	0.9651	0.9795	0.9024
w_t	0.9696	0.9782	0.9292	0.9762	0.9619	0.9819	0.9137

Table 5. Derivatives of γ wrt deep parameters

θ	$\partial\gamma/\partial\theta$
α	-0.396
σ_c	-0.0883
ϕ_p	-0.0076
ϕ	0.0137
λ	1.06
ξ_w	-0.703
σ_l	-0.0505
ξ_p	-0.487
ι_w	0.0293
ι_p	0.0761
r_π	0.0685
ρ	-0.193
r_y	-0.518
r_{Δ_y}	-0.127
ρ_a	-0.0193
ρ_b	-0.0096
ρ_g	-0.0266
ρ_i	-0.018
ρ_r	-0.0103
ρ_p	-1.54
ρ_w	-1.73
μ_p	1.51
μ_w	2.12
$100\,(\overline{\gamma}-1)$	-0.0135
$100\,(\beta^{-1}-1)$	0.0294

Note: nonzero derivatives after rounding to 4 decimal digits

Table 6. Model comparison (1966:1-2004:4)

	θ^{SW}	θ^{*}	$\bar{\theta}$	$\theta^{\dagger}$
Priors	0.25	0.25	0.25	0.25
Log Marginal Density	-921.42	-923.855	-930.2	-931.383
Bayes Ratio	1.0	0.087596	0.000154	0.000047
Posterior Model Probability	0.919289	0.080526	0.000141	0.000043

TABLE 7. Posteriors (1966:1-2014:2)

	θ^{SW}	θ^{*}
ρ_a	0.978	0.976
ρ_b	0.5	0.516
ρ_g	0.974	0.97
ρ_I	0.803	0.796
ρ_r	0.198	0.173
ρ_p	0.996	0.995
ρ_w	0.979	0.98
μ_p	0.891	1.04
μ_w	0.967	0.969
ϕ	4.38	4.19
σ_c	1.97	1.89
λ	0.529	0.529
ξ_w	0.86	0.864
σ_l	1.79	2.03
ξ_p	0.748	0.786
ι_w	0.696	0.665
ι_p	0.232	0.268
ψ	0.704	0.718
ϕ_p	1.65	1.6
r_π	1.71	1.65
ρ	0.803	0.821
r_y	0.0522	0.0556
$r_{\Delta y}$	0.231	0.242
$\overline{\pi}$	0.759	0.736
$100\left(\beta^{-1}-1\right)$	0.116	0.119
$\bar{l}$	0.775	0.108
$100\left(\overline{\gamma}-1\right)$	0.38	0.376
ρ_{ga}	0.485	0.494
α	0.196	0.198
σ_a	0.469	0.472
σ_b	0.16	0.158
σ_g	0.487	0.484
σ_I	0.38	0.387
σ_r	0.231	0.23
σ_p	0.11	0.199
σ_w	0.38	0.374

Note: posterior distributions in the estimation sample 1966:1-2014:2. Priors and initial values are those in Table (1). The modes in this table are found under the following initial values for the price and markup AR parameters: $\rho_p^{(0)}\left(\theta^{SW}\right)=0.8692$, $\rho_w^{(0)}\left(\theta^{SW}\right)=0.9546$, $\rho_p^{(0)}\left(\theta^{*}\right)=0.6$, $\rho_w^{(0)}\left(\theta^{*}\right)=0.9546$.

TABLE 8. Moduli of PMIC eigenvalues (1966:1-2014:2)

	θ^{SW}	θ^*
γ_1	0.9761	1.999
γ_2	0.9705	0.9767
γ_3	0.872	0.9698
γ_4	0.7743	0.8729
γ_5	0.462	0.771
γ_6	0.3508	0.3326

TABLE 9. R-squareds (1966:1-2014:2)

	θ^{SW}	θ^*	
	$R_i^2 = R_{F,i}^2$	R_i^2	$R_{F,i}^2$
l_t	0.9856	0.9813	0.9251
r_t	0.9532	0.9490	0.7965
π_t	0.9234	0.9673	0.8692
y_t	0.9960	0.9943	0.9772
c_t	0.9977	0.9966	0.9865
I_t	0.9958	0.9944	0.9777
w_t	0.9982	0.9977	0.9910

TABLE 10. Model comparison (1966:1-2014:2)

	θ^{SW}	θ^*
Priors	0.5	0.5
Log Marginal Density	-1173.08	-1172.92
Bayes Ratio	1.0	1.17178
Posterior Model Probability	0.460451	0.539549

TABLE 11. Posteriors (1966:1-2004:4) - SW priors

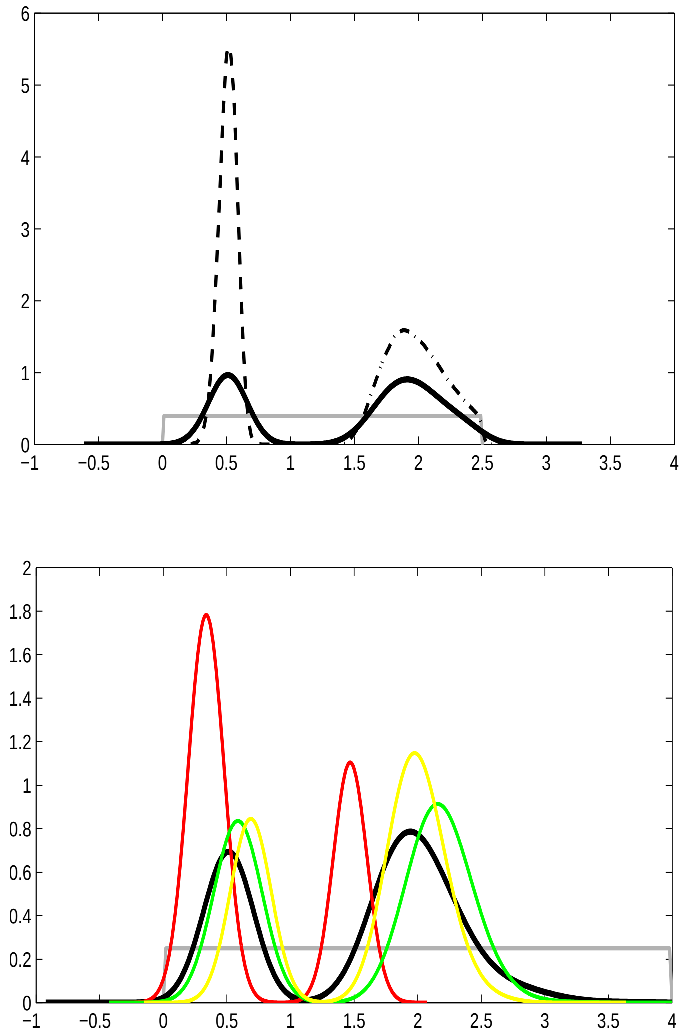
	θ^{SW}	θ^{*}
ρ_a	0.961	0.959
ρ_b	0.183	0.219
ρ_g	0.976	0.977
ρ_I	0.703	0.701
ρ_r	0.123	0.0989
ρ_p	0.908	0.883
ρ_w	0.974	0.973
μ_p	0.744	0.972
μ_w	0.893	0.91
ϕ	5.49	5.22
σ_c	1.42	1.45
λ	0.706	0.694
ξ_w	0.734	0.775
σ_l	1.87	2.0
ξ_p	0.654	0.747
ι_w	0.598	0.558
ι_p	0.219	0.333
ψ	0.545	0.479
ϕ_p	1.61	1.61
r_π	2.02	1.98
ρ	0.815	0.834
r_y	0.0881	0.0974
$r_{\Delta y}$	0.222	0.233
$\overline{\pi}$	0.765	0.742
$100\left(\beta^{-1}-1\right)$	0.144	0.143
$\bar{l}$	0.726	0.764
$100\left(\overline{\gamma}-1\right)$	0.434	0.439
ρ_{ga}	0.523	0.528
α	0.191	0.194
σ_a	0.453	0.455
σ_b	0.242	0.234
σ_g	0.521	0.521
σ_I	0.455	0.461
σ_r	0.239	0.238
σ_p	0.14	0.213
σ_w	0.247	0.236

Note: posterior distributions in the estimation sample 1966:1-2004:4. Apart from μ_w and μ_p, the priors and initial values are those in Table 1. The priors for μ_w and μ_p are both beta with mean 0.5 and standard deviaton 0.2 and with [0.01, 0.9999] bounds and initializations as in Table 1. The modes in this table are found under the following initial values for the price and markup AR parameters: $\rho_p^{(0)}\left(\theta^{SW}\right)=0.8692$, $\rho_w^{(0)}\left(\theta^{SW}\right)=0.9546$, $\rho_p^{(0)}\left(\theta^{*}\right)=0.6$, $\rho_w^{(0)}\left(\theta^{*}\right)=0.9546$.

TABLE 12. Moduli of PMIC eigenvalues (SW priors)

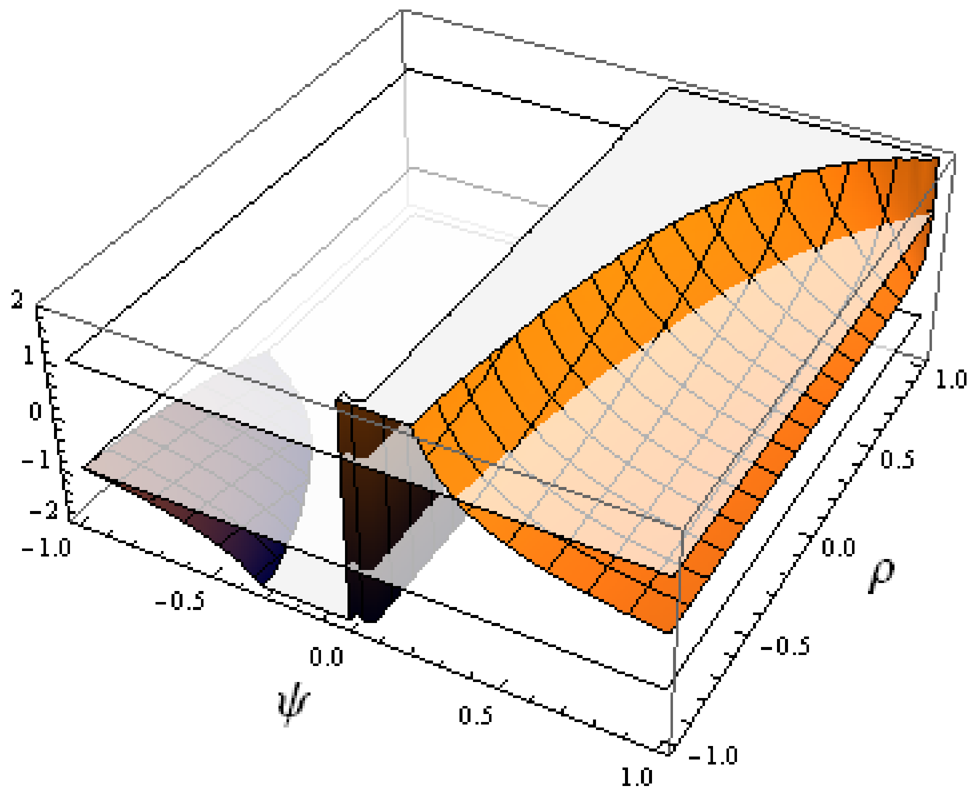
	θ^{SW}	θ^{*}
γ_1	0.977	1.661
γ_2	0.9648	0.9764
γ_3	0.8338	0.9675
γ_4	0.6386	0.8269
γ_5	0.5337	0.6734
γ_6	0.4574	0.5036

FIGURE 1. Posterior density of ϖ_1



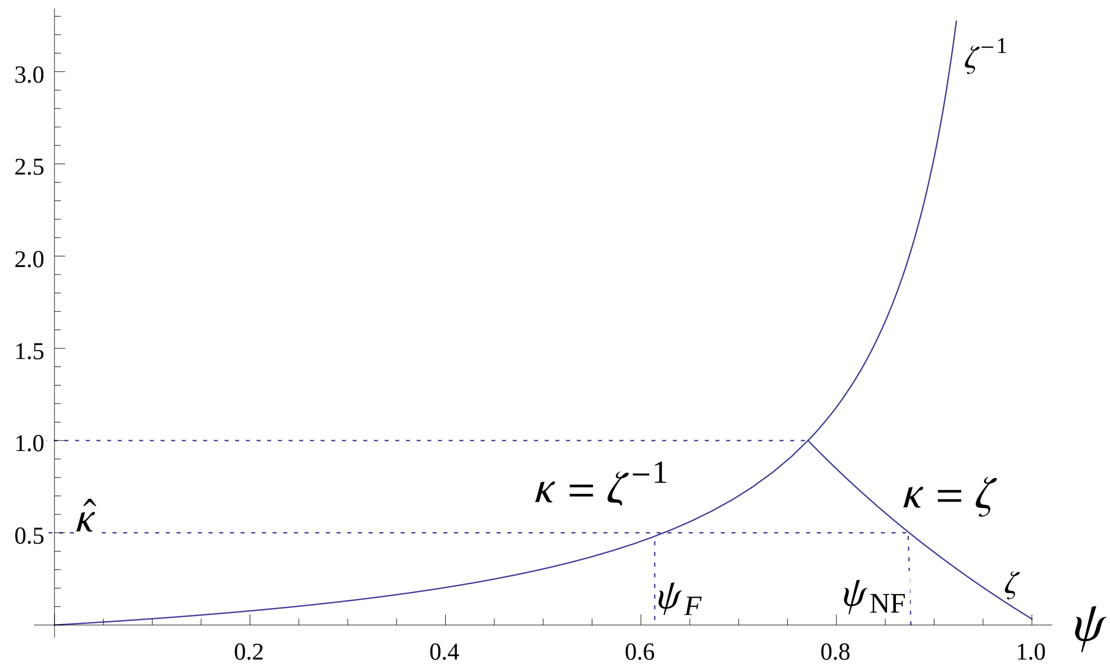
Note: in the top panel the density of ϖ_1 estimated on a sample of 160 observations using a RMW algorithm with 500000 draws. The dashed and the dotted and dashed lines are the posterior densities using 0.5 and 2.49 respectively as initial values for the numerical optimization in step 1 of the RMW algorithm. The solid line is the posterior density obtained using an identity matrix as proposal density. In the bottom panel the density of ϖ_1 estimated on the same sample an identity matrix as proposal density under the following constraints: $\varpi_2 = 1 + (\varpi_1 - 1)$ (green line), $\varpi_2 = 1 - 2(\varpi_1 - 1)$ (red line), $\varpi_2 = 1 + 3(\varpi_1 - 1)$ (yellow line),$\varpi_2 = 4$ (black line).

FIGURE 2. Nonfundamentalness NKPC example: value of the root ζ



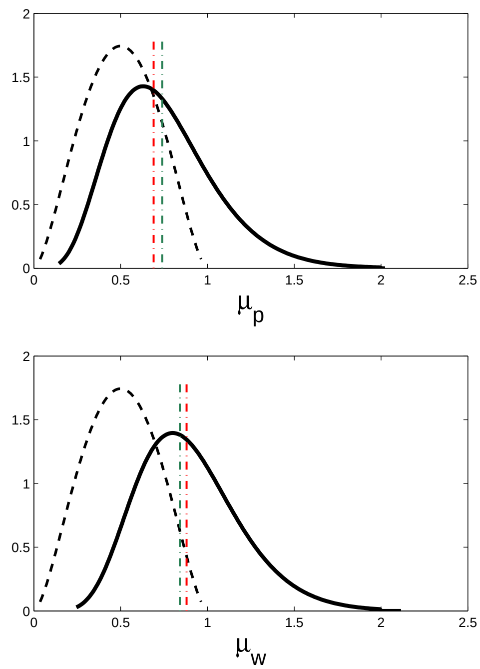
Note: value of $\zeta = (1 - \beta\psi) / (1 - \beta\rho)\psi$ the root of the ARMA representation for inflation for $\beta = .99$. The transparent planes are the borders of the nonfundamental region $[-1, 1]$ for ζ.

FIGURE 3. Nonfundamentalness NKPC example: nonuniqueness



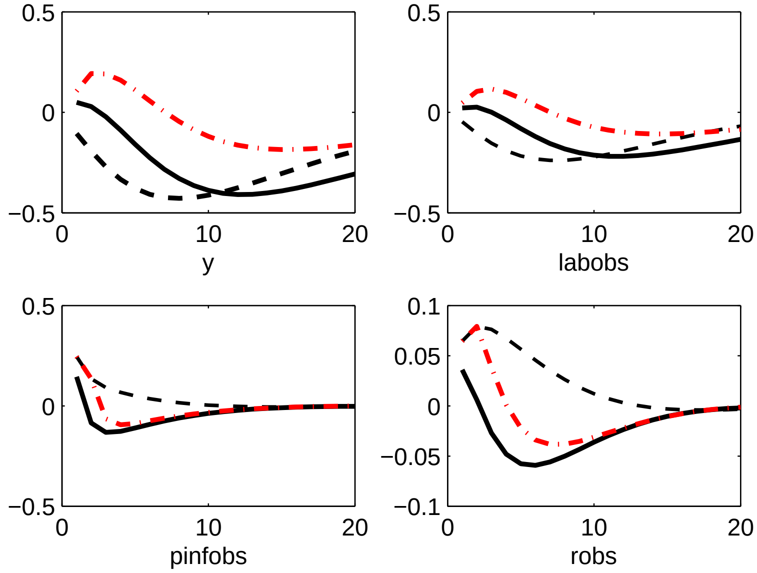
Note: value of the MA parameter κ as a function of $\zeta = (1 - \beta\psi)/(1 - \beta\rho)\psi$ the root of the ARMA representation for inflation for $\beta = 0.99$ and $\rho = 0.7$.

FIGURE 4. Priors MA coefficients



Note: the black solid lines are the gamma prior distributions in Table 1, the black dashed lines are the beta priors used by SW with mean 0.5 and standard deviation 0.2. The red and the green dashed and dotted lines are SW's posterior modes and means respectively.

FIGURE 5. Propagation of the price markup shock



Note: the black solid lines are the impulse responses of output, hours worked, inflation and nominal interest rate to the price markup shock in the nonfundamental model obtained at θ^* values , the black dashed lines are the responses in the fundamental model at θ^{SW} values, the red dashed and dotted lines are impulse responses obtained in a model with the exogenous propagation of the former and the endogenous propagation of the latter.

Appendix

Derivation of the VARMA representation (2.3). Suppose that $\mathbf{D}$ is nonsingular, from equation (2.2) we get $\mathbf{w_t} = \mathbf{D}^{-1}(\mathbf{y_t} - \mathbf{C}\mathbf{x_{t-1}})$. Substitute into the (2.1), and rearranging, the mapping between the states and the observables is $(\mathbf{I_{n_x}} - \mathbf{\Gamma}L)\,\mathbf{x_t} = \mathbf{B}\mathbf{D}^{-1}\mathbf{y_t}$. Define $\gamma(L) :=$ $\det(\mathbf{I_{n_x}} - \mathbf{\Gamma}L)$, then $\gamma(L)\,\mathbf{x_t} = \operatorname{adj}(\mathbf{I_{n_x}} - \mathbf{\Gamma}L)\,\mathbf{B}\mathbf{D}^{-1}\mathbf{y_t}$ and plugging into the observation equation we get the VARMA

$$\left(\mathbf{I_{n_y}}\gamma(L) - \mathbf{C}\,adj\,(\mathbf{I_{n_x}} - \mathbf{\Gamma}L)\,\mathbf{B}\mathbf{D}^{-1}L\right)\mathbf{y}_t = \gamma(L)\,\mathbf{D}\mathbf{w_t}$$

Proof of Proposition 2.

Proof. Define the scalar polynomial $\phi(z) \equiv \det(\mathbf{\Phi}(z))$ and re-write the VARMA (2.3) as

$$\begin{pmatrix} \phi(L) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \phi(L) \end{pmatrix}\mathbf{y}_t = \mathbf{\Xi}(L)\,\gamma(L)\,\mathbf{w_t}$$

where $\mathbf{\Xi}(z) \equiv \operatorname{adj}(\mathbf{\Phi}(z))\,\mathbf{D}$ is an $n \times n$ matrix polynomial. Then the VARMA for the i-th element of $\mathbf{y}_t$ is

$$\begin{aligned} \phi(L)\,y_{it} &= \mathbf{\Xi}_i(L)\,\gamma(L)\,\mathbf{w_t} \\ &= \gamma(L)\left(\mathbf{\Xi}_{i1}(L)\,\mathbf{w}_{1t} + \ldots + \mathbf{\Xi}_{in}(L)\,\mathbf{w}_{nt}\right) \end{aligned}$$

where $\mathbf{\Xi}_i(z)$ is the i-th row of $\mathbf{\Xi}(z)$. Define $\tilde{y}_{it}^j \equiv \gamma(L)\,\Xi_{ij}(L)\,w_{jt}$, then $\phi(L)\,y_{it} = \sum_{j=1}^n \tilde{y}_{it}^j$ is a decomposition of $\phi(L)\,y_{it}$ in independent components each following an MA process. The formula (5) in Robertson and Wright (2012) can be applied on each independent component

$$R^2\left(\tilde{y}_i^j\right) = 1 - V(\gamma)\left(1 - R_F^2\left(\tilde{y}_i^j\right)\right)$$

Given the additivity of the above formula we have that

$$R_i^2 = 1 - V(\gamma)\left(1 - R_{F,i}^2\right)$$

which follows from the fact that the term $\gamma(z)$ is common to every row of the (2.3) and therefore $V(\gamma)$ does not depend on i. This completes the proof of the formula (4.3).

The proof of the inequality result $R_i^2 > R_{F,i}^2$ is trivial. Suppose that it does not hold and there is an i such that $R_i^2 = 1 - V(\gamma)\left(1 - R_{i,F}^2\right) < R_{F,i}^2$. Collecting terms and rearranging the previous expression gives $R_{F,i}^2 > 1$, which is impossible. $\square$

Parameter description.

θ	Description
δ	depreciation rate of capital
ϕ_w	1+ mark-up in wage setting
g_y	steady state exogenous spending-output ratio
ε_p	curvature of the Kimball goods market aggregator
ε_w	curvature of the Kimball labour market aggregator
α	share of capital in production
γ	steady state growth rate
β	households discount factor
σ_c	relative risk aversion
ϕ_p	1+ share of fixed costs
ψ	steady state elasticity of the capital adjustment cost function
λ	external habit formation in consumption
ξ_w	degree of wage stickiness
σ_l	elasticity of labour supply w.r.t. real wage
ξ_p	degree of price stickiness
ι_w	wage indexation
ι_p	price indexation
r_π	inflation coeff. (Taylor rule)
ρ	AR for lagged interest rate (Taylor rule)
r_y	output gap coeff. (Taylor rule)
$r_{\Delta y}$	output gap growth coeff.(Taylor rule)
ρ_g	government expenditure shock AR coeff.
ρ_{ga}	TFP effect on net exports
ρ_b	risk premium shock AR coeff.
ρ_i	investment shock AR coeff.
ρ_a	TFP shock AR coeff.
ρ_p	price mark-up shock AR coeff.
ρ_w	wage mark-up shock AR coeff.
μ_p	price mark-up shock MA coeff.
μ_w	wage mark-up shock MA coeff.
ρ_r	monetary shock AR coeff.