Enforcement Missions: Targets vs Budgets

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Abstract

Enforcement of policy is typically delegated. What sort of mission should the head of an enforcement program be given? When there is more than one firm being regulated the firms’ decision problems – otherwise completely separate – become linked in a way that depends on that mission. Under some sorts of missions firms compete to avoid the attention of the enforcer by competitive reductions in the extent of their non-compliance, in others the interaction pushes in the opposite direction. We develop a general model that allows for the ordering of some typical classes of missions. We find that in plausible settings ‘target-driven’ missions (that set a hard enforcement target and flexible budget) achieve the same outcome at lower cost than ‘budget-driven’ ones (that fix the enforcement budget). Inspection of some fixed fraction of firms is never optimal.

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1 Introduction

The enforcement of government policy is typically delegated. At an aggregate level, for example, enforcement of environmental legislation is delegated to an environment agency, crime control is delegated to a police force, tax collection to a revenue service, and so on. At the intra-agency level the enforcement of a particular area of legislation (say, noise control) will be associated with a dedicated enforcement program.

We ask the following question: When establishing such enforcement programs what mission should be given to the program leader or ‘enforcer’? A variety of missions are in common use and our model will be flexible enough to embody any of them. For the purposes of discussion, however, we will focus attention on two types. The first type involves telling the enforcer to achieve a target level of compliance at least enforcement cost. We will refer to such missions as target-driven. The second type, which we will refer to as budget-driven, involves telling him to get compliance as high as possible subject to a budget constraint.

It is natural to suppose that the target-driven and budget-driven approaches are dual to one another and therefore that the choice between them does not matter. We show that in any setting involving more than one firm such a supposition is wrong. Mission matters.

The essence of the story we are going to tell is as follows: firms facing a common enforcer find themselves in a game not just with the enforcer, but with each other as well. The nature of the interaction among firms depends critically upon what the enforcer is trying to achieve (that is, his mission). This paper analyzes the impact of that strategic interaction on the outcomes and provides a basis for ranking alternative missions according to their efficacy. An important conclusion is that there should be ‘horses for courses’ – the best mission to assign in a given enforcement setting will depend in predictable ways upon the nature of the enforcement environment and technology. As such the paper generates practical policy

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1There is a substantial literature on the problems that arise when the interests of the principal and the enforcer (as agent) are imperfectly aligned. Gailmard (2002) and Hopenhayn and Lohmann (1996) are two examples amongst many. These are applications of well-understood principal-agency problems and we ignore them here. We assume, in other words, that incentives can be put in place to ensure that the enforcer pursue his mission diligently. In this sense our model fits into a second strand of the delegation literature, following Spulber and Besanko (1992), that regards delegation as strategic commitment.

2Formally, duality implies that if a budget-driven mission involving a budget $\beta$ leads to a realized compliance outcome $\tau$ then specifying $\tau$ as the target under a target-driven mission would result in realized enforcement costs $\beta$. The belief that such duality generally holds may explain why scholars setting up economic models of enforcement have paid relatively little attention to the mission assigned to the enforcer.
principles.

The central argument of our model can be motivated with an example.\(^3\) Consider a setting with many firms, each making a binary decision either to emit or not emit a unit of some forbidden pollutant. Emit corresponds with ‘violate’, not emit with ‘comply’. The enforcer observes a signal correlated with the aggregate level of emissions – say, has some ambient measure of pollution flows – and so knows how many firms have violated, but not which ones. Finding out – and rectifying it – requires a two-stage process of inspection and enforcement.\(^4\) First the enforcer visits firms sequentially. The visit reveals whether or not the firm is compliant. If a firm is non-compliant the enforcer then exerts some additional time/money/resource pursuing the matter – collecting evidence, litigating, administering a fine – and returning the firm to compliance.

Within this setting consider the implications of alternative missions. For instance, a ‘target-driven’ mission would ask the enforcer to achieve a specified level of compliance – say, ensure that no more than \(m\) firms are non-compliant – at the lowest enforcement cost. Here the enforcer visits firms in random sequence, putting violators back into compliance, until his compliance target is achieved. A decision by one firm (call it firm 1) to violate increases the chance that a violation by firm 2 will be detected and penalized.\(^5\) We say that non-compliance by firm 1 has a positive enforcement spillover on firm 2. Under standard assumptions this increased risk of detection makes non-compliance less attractive to firm 2, so that the compliance decisions of firms are strategic substitutes.

In contrast, a ‘budget-driven’ mission would tell the enforcer to minimize non-compliance within an assigned enforcement budget. In this case the agent visits firms in random sequence, pursuing those that it finds to be in violation, until its enforcement budget is exhausted. The higher the proportion of inspections that lead to enforcement activity, the lower the probability that any particular firm will be subject to inspection. A decision by firm 1 to violate therefore decreases the probability that a violation by firm 2 will be detected. We say that non-compliance by firm 1 has a negative enforcement spillover on firm 2. Under standard assumptions this reduced probability of detection makes non-compliance more attractive to firm 2, so that the compliance decisions of firms are strategic complements.

This is an example in which a switch in mission alters qualitatively the na-

\(^3\)The example does not precisely fit the analytic model presented later, but captures the spirit of what we are trying to do.

\(^4\)The separation of the monitoring activity from the enforcement activity is fairly common in the existing literature in this area, and makes obvious sense in many practical settings.

\(^5\)Suppose the target is to ensure that only \(m\) of, say, \(N\) firms are left non-compliant. If there are initially \(v\) violators then the probability that any given violator will be caught is \(\frac{v-m}{v}\), which is increasing in \(v\).
ture of the strategic interaction amongst the firms, even though the underlying enforcement context remains unchanged. In the second case each non-compliant firm benefits from ‘safety in numbers’. Others’ non-compliance means that they can be expected to absorb more enforcement resource, lowering the chance that the enforcer will get around to uncovering its wrongdoing. On the other hand, in the first example there is ‘danger in numbers’. The mission dictates that only a certain number of violators can be left in violation, so an increase in the number who choose initially to violate reduces the likelihood that any particular one of them will be one of the lucky ones.\(^6\)

The existing literature on enforcement has neglected this strategic interaction. It is universally assumed in the existing literature that the enforcement mission is fixed. It is also very common to assume that the enforcer is interacting with a single firm. Either assumption sidesteps the issues that we are explore here.\(^7\)

Of course, the precise incentives and interaction generated by alternative missions depends on the specifics of the enforcement setting. We begin a brief survey of enforcement objectives in the study of environmental policy. Section 2 develops a simple model to show how differences in enforcement spillovers under target-driven and budget-driven missions affect the regulatory outcome. Section 3 generalizes the argument and derives a criterion to rank alternative missions according to their efficacy in the presence of enforcement spillovers. Section 4 discusses the relevance of our findings for a wider class of missions and Section 5 concludes.

1.1 Enforcement Objectives in Theory and Practice

In setting up a theoretical model of enforcement the modeler must choose what objective function to give the enforcement agency in his model. The choice often

\(^6\)AOL-Autos has as its number one tip for avoiding a speeding ticket finding a ‘pack’ of speeding cars to travel in. “If you’re within a pack of cars all going 10 mph over the limit, you’ve automatically improved your odds of not being the one that gets pulled over for a speeding ticket, even though you’re all technically speeding. The cop has to pick one car; if you are in a pack of cars its less likely to be you.” (AOL Autos 2007).

\(^7\)The exceptions are notable. That compliance performance of one firm could affect the enforcement intensity brought to bear upon others has been noted by Lear and Maxwell (1998), but they do not consider the issue of alternative objective functions. Decker and Pope (2005) point to the potential for strategic complementarity between firms’ compliance behavior when the enforcement agency has a fixed budget. They provide empirical evidence from the US that the compliance behavior of firms is increasing in the compliance behavior of other firms in their sector. Erard and Feinstein (1994) characterize the interdependence of income reporting decisions in an income tax compliance/enforcement game. Our model develops these themes further, and emphasizes the fact that the strategic interaction between firms’ compliance choices is conditioned by the enforcement mission.
seems arbitrary and receives little discussion or motivation. Cohen (2000) and Firestone (2002, 2003) provide detailed discussions of the various assumptions scholars have made in this literature. Not surprisingly there has been a great diversity of practice.

One common assumption is that the enforcer acts to minimize aggregate non-compliance subject to a budget constraint (see, for examples Garvie and Keeler (1994), Hansen et al. (2006), and Jost (1997)).

Other authors have instead assumed that the enforcer minimizes the cost of achieving a pre-specified compliance rate (for examples Livernois and McKenna (1999), Mookherjee and Png (1992), Stranlund and Chavez (2000)). Amongst these, however, there are variations with regard to what is included in the cost function that the agency seeks to minimize. In Maxwell and Decker (2006) and Livernois and McKenna (1999) the agency is interested only in its own operational costs – the costs to it as an agency of monitoring, sanctioning and so on. Lear and Maxwell (1998) is similar in spirit, though they assume that penalty revenues are recycled into the agency budget and so the agency minimizes its expenditures net of penalty income. Harford and Harrington (1991) and Stranlund (2007), on the other hand, assume that the agency also takes account of the compliance cost burden on the regulated industry.

A third set of authors assume that the enforcement agency acts to maximize social welfare (Schmutzler and Goulder (1997), Franckx (2002)). A fourth ascribes political motives to the agency and assumes that enforcement decisions are made to maximize some sort of political support function (in the spirit of Peltzman (1976)).

There has been considerable debate amongst policy analysts as to what regulatory agencies try to achieve in practice. Agencies do not usually publish objective functions, and if they do the stated objective will often be too vague to interpret meaningfully. Furthermore, the model we develop in this paper is really best thought of as applying at the level of an individual enforcement program,

\[ \text{This implies focussing on the design of cost-effective (as opposed to cost-efficient) policy. In other cases, whilst no clear assumption is made about the objective function it is implicit from discussion that cost-effectiveness is what the authors have in mind. For example, Hentschel and Randall (2000) conduct simulations aimed at determining features of a monitoring and enforcement program that can reduce agency costs, drawing conclusions like “... it is possible to reduce agency costs significantly while maintaining a given industry-wide level of compliance (page 57).”} \]

\[ \text{The common motivation for such an assumption is that there is separation of responsibilities in a hierarchical regulatory regime, with the job of the enforcer being to enforce. Maxwell and Decker (2006: 428) for example note that “This implies that the regulator is not a social cost minimizer since he is not concerned with the firm’s compliance costs. This highlights the separation between the legislative and enforcement bodies of government.”} \]

\[ \text{In the United States the EPA is endowed with the following mission statement: “To Protect Human Health and the Environment.”} \]
and there is no particular reason to think that objectives will be the same across programs.

In the context of the EPA in particular Firestone (2003) concludes his careful institutional analysis by saying that “... for a number of reasons the social welfare maximization model may not fare well at EPA” and that to view it as “a violation-minimizing policeman is perhaps a more realistic behavioral model of EPA enforcement” (Firestone (2003: 130)).

Empirical attempts to use revealed preference methods to infer the EPA’s objective function from its enforcement actions have generated mixed results. Deily and Gray’s (1991) study of air monitoring, for example, found no evidence of a relationship between enforcement and plant abatement costs, which they interpreted as rejecting the social welfare maximization model.11 Gray and Deily (1996) found evidence in support of environmental harm minimization, with weaker evidence in favour of the political support model.

2 A Model With Enforcement Spillovers

An enforcer is appointed to control the level of some anti-social activity. To make things concrete, we will regard this activity as illegal emission of some pollutant but our model is general enough to capture almost any anti-social activity. There are \( N \) identical firms, each choosing its level of emissions simultaneously. Firm \( i \)'s choice – its emission in excess of the permissible limit – is given by the real-valued variable \( x_i \in [0, \hat{x}] \). Here \( x_i \) is a measure of the firm's non-compliance, where \( x_i = 0 \) denotes complete compliance and \( \hat{x} \) specifies some physical upper limit to the level of non-compliance.

The purpose of enforcement is to influence aggregate pollutant levels and therefore environmental quality. It does this in two ways. The threat of enforcement action has a deterrent effect on pollution choices \textit{ex ante}. It also generates reductions \textit{ex post} by enforced abatement activity that follows prosecution of violators.

Enforcement is costly so the regulator faces a familiar trade-off between pollution levels and enforcement cost. As we see below, this trade-off may be sensitive to the precise mission used to delegate enforcement. We assume that the assigned mission is common knowledge and that the enforcer pursues it diligently.

Each firm faces costs that depend upon its emissions according to some differ-

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11Results in a similar vein have been found in studies in other countries and contexts. For example, Dion et al’s (1998) of water pollution enforcement in Canada found that regulators did not consider compliance costs in making enforcement decisions.
entiable function $c(x_i)$, with standard features $c'(x_i) < 0$ and $c''(x_i) > 0$. So, other things equal, each firm would choose a high value of $x_i$.

However, non-compliance carries the risk of prosecution and penalty. A firm found to be in violation must pay a fine and may have to incur the cost of abatement. An operator who has illegally disposed of toxic wastes on a site, for example, may expect to have to clean up that site. It might also suffer reputational losses. The costs to the firm associated with these are captured in a single, composite penalty function $p(x)$. We assume that $p(0) = 0$: that is, a compliant firm faces no penalty. For non-compliant firms the penalty is positive and bounded, and increases convexly in the level of non-compliance in the relevant range. Thus, for $x > 0$ we have $p(x) > 0; p'(x) > 0$ and $p''(x) > 0$ in the range $(0, \hat{x})$.

The risk of enforcement action depends on the firm’s level of non-compliance, but also – as a novel feature of our model – on the compliance choices of other firms. Let $r(x_i, x_{-i})$ denote the likelihood of prosecution for the typical firm when it chooses $x_i$ and choices of the other $N - 1$ firms are denoted by the vector $x_{-i}$.

An individual firm’s choice involves a trade-off between the direct costs of compliance $c(x_i)$ and the expected penalty $r(x_i, x_{-i})p(x_i)$ associated with non-compliance. Firm $i$ chooses $x_i$ to minimize

$$V(x_i, x_{-i}) = c(x_i) + r(x_i, x_{-i})p(x_i). \quad (1)$$

Let $V_1$ denote the partial derivative of this function with respect to $x_i$. An interior minimum, if it exists, must satisfy the first-order condition

$$V_1(x_i, x_{-i}) \equiv c'(x_i) + p'(x_i)r(x_i, x_{-i}) + p(x_i)\frac{\partial r}{\partial x_i}(x_i, x_{-i}) = 0. \quad (2)$$

In this setting, the risk of prosecution varies with other firms’ choices $x_{-i}$. This implicitly defines a ‘reaction function’ $R_i(x_{-i})$, which captures firm $i$’s optimal response to $x_{-i}$. Given the strategic interaction in firms’ choices, a Nash equilibrium is defined, in the usual way, as an $N$-dimensional vector $x^*$ such that $x^*_i = R_i(x^*_{-i})$ for all $i$. By construction each firm’s choice is an optimal response to the others’ choices at this equilibrium.\(^{13}\)

\(^{12}\)The associated second-order condition for a minima is that $V_{11}(x_i, x_{-i}) > 0$: we restrict attention to cases where this is met.

\(^{13}\)For the moment we confine attention to symmetric equilibria. Define $V_{12} = \frac{\partial^2 V}{\partial x_i \partial x_{-i}}$, where $x_{-i}$ is the symmetric choice of all firms other than $i$. The equilibrium will be unique and ‘stable’ if the absolute value of the slope of the reaction function is less than 1. This requires that $|V_{12}| < |V_{11}|$ which corresponds with the standard stability assumption made in models of this sort. Whether firms’ choices are strategic substitutes or complements, in the sense of Bulow et al. (1985), depends on the sign of $V_{12}$, $V_{12} > 0$. Firms’ choices are strategic substitutes if $V_{12} > 0$, and strategic complements if $V_{12} < 0$. 

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2.1 Enforcement Technology

There are various ways in which we might sensibly model the process of enforcement, depending on the physical environment, the nature of emissions, the mechanics of the detection technology, the policy and legal architecture, and so on. Heyes (2000) and Cohen (2000) provide surveys of many of the issues. To illustrate our central argument, we adopt a particular approach for the rest of this Section. The framework is then generalized in the next Section.

For current purposes we assume a two-stage enforcement process of the sort common in the existing literature, and plausible in many practical settings. The first stage involves conducting inspections in order to identify non-compliant firms. The second stage involves pursuing and prosecuting these firms and returning them to compliance, to the extent possible, through abatement and cleanup operations. We assume that each instance of initial inspection costs enforcer $\phi_0$. The cost of pursuing and prosecuting a non-compliant firm is $\phi_1(x)$, where $\phi_1(0) = 0$ (compliant firms absorb no enforcement effort), $\phi_1'(x) > 0$ and $\phi_1''(x) < 0$. Without providing a micro-level description of the process of enforcement, we have in mind that the effort required to generate the evidence for and prosecute a large polluter will exceed that required for a small polluter.\(^{14}\) Total enforcement cost for a firm found to be at non-compliance level $x$ is, then, given by $k(x) = \phi_0 + \phi_1(x)$, with $k'(x) > 0$.

We consider enforcement settings in which the number of inspections varies with the aggregate level of non-compliance (or some signal of it). In an environmental setting this corresponds to the agency having some ambient measure of environmental quality, which depends on the cumulative emission of all firms (existing models where inspections are conditioned on an ambient measure include Franckx (2002) and Cabe and Herriges (1992)). The level of enforcement activity might then vary on the aggregate level of emissions, depending – as we elaborate below – on the mission assigned to the enforcer.\(^{15}\) On inspection, firms found to be non-compliant are prosecuted and forced to engage in costly clean-up activity.

To formalize this, let $n(X) \leq N$ denote the number of inspections carried out when the aggregate level of non-compliance is expected to be $X = \sum_{i}^{N} x_i$ (for tractability, we treat $n$ as a continuous variable). Absent information on individ-

\(^{14}\)Penalties for large violators are typically bigger than those facing small violators and it may be that the court would set a higher standard on the quality of evidence that it requires, or that the larger violator would engage in more obfuscation than its smaller counterpart. Further, recall that $\phi_1$ incorporates the cost of putting the violator back into compliance and this might reasonably be thought to be increasing in the amount of ‘movement’ needed.

\(^{15}\)Importantly, we assume here that the risk of enforcement activity faced by an individual firm does not vary with its individual level of non-compliance. We discuss the implications of relaxing this assumption later.
ual firms’ behavior, firms are inspected at random so that the probability that a particular firm will be picked for inspection is $\frac{n(X)}{N}$. A randomly-inspected firm will display an average level of non-compliance $\overline{\pi} = X/N$, incurring enforcement (inspection and prosecution) cost $k(\overline{\pi})$, so that the expected total enforcement cost associated with $n$ inspections is $nk(\overline{\pi})$. Assuming that firms found to be non-compliant can be brought back to full compliance through abatement activity, this level of enforcement achieves outcome $X - n\overline{\pi}$.\footnote{It is important to distinguish between \textit{ex ante} non-compliance levels $x$ and the \textit{ex post} outcome that obtains after the enforcement program has run its course and a subset of violators have been pushed back into compliance by enforcement effort. The assumption that enforcement activity can restore firms to complete compliance within the reference period is extreme and made here only for simplicity. We can easily adapt our analysis to a setting where the restoration is partial, so that, say the outcome of enforcement activity would equal $X - n\gamma\overline{\pi}$, where $\gamma < 1$ is the fractional correction achieved.}

This setting captures the trade-off between enforcement expenditure and outcome. More inspections are costly but deliver greater compliance, both through the deterrent effect on the \textit{ex ante} compliance choices of firms and through the \textit{ex post} correction of detected violators. From a welfare perspective, the optimal choice along this trade-off depends on the cost and benefits of enforcement activity, but that is not the central focus of our analysis. Rather we examine the idea that when enforcement is delegated, the mission or objective assigned to the enforcer may \textit{in itself} affect the efficiency of enforcement action. To put it differently, regardless of where you want to be on the trade-off between enforcement costs and outcomes, some missions may be better in that they allow you to achieve any desired outcome at lower enforcement cost.

In this setting the relative efficacy of alternative missions depends on how the risk of enforcement action varies with the level of aggregate non-compliance, and the implied spillovers between the choices of firms. If the enforcement mission is such that $n(X)$ is increasing in $X$ – so that an increase in non-compliance by other firms increases the enforcement risk for all – we say that the enforcement spillover is positive. The case where $n(X)$ is decreasing in $X$ – so that an increase in non-compliance by other firms decreases enforcement risk – displays negative spillovers. We show how differences in missions imply differences in the direction of spillovers, and the implications for enforcement outcomes.

Clearly, there exists a wide range of models of compliance, with \textit{ad hoc} assumptions about the objective function of the enforcer, that are consistent with the above enforcement technology. To demonstrate how the regulatory mission affects the outcome, we consider two alternative missions. Our aim is to discover the characteristics of a “good” enforcement outcome.
2.2 Target-driven mission

Suppose, first, that the enforcer is asked to restrict aggregate emissions \( X \) to some level \( \tau > 0 \). Since each prosecution brings a typical firm back into compliance and reduces pollution, on average, by an amount \( \bar{x} \), the required reduction in aggregate pollution, \( X - \tau \), necessitates

\[
n^t(X; \tau) = \frac{X - \tau}{\bar{x}}
\]

inspections (we use the superscript \( t \) to denote that the target-setting mission is in play). Given any configuration of firms’ choices, the probability of inspection for any particular firm is

\[
r^t(x_i, x_{-i}; \tau) = \frac{X - \tau}{N\bar{x}}
\]

if \( X \geq \tau \), and zero otherwise. Given that firms are assumed to be identical, it is natural to focus on symmetric equilibria, though the arguments can be modified to cover the asymmetric case. With slight abuse of notation we now use \( r^t(x_i, x_{-i}) \) to denote the enforcement risk for a typical firm that chooses \( x_i \) and expects all other firms to make the symmetric choice \( x_{-i} \). Using \( r^t_1 \) and \( r^t_2 \) to denote the partial derivatives of this enforcement risk with respect to \( x_i \) and \( x_{-i} \) we have

\[
r^t_1 = \frac{\tau}{(N\bar{x})^2} > 0,
\]

and \( r^t_2 = (N - 1)r^t_1 > 0 \). The last relation describes the nature of the spillover for this mission. From the perspective of firm \( i \), an increase in emissions by other firms increases aggregate emissions. This compels the enforcer to increase the number of inspections in order to preserve the target \( \tau \). In terms of our earlier terminology, the enforcement spillover is positive.

Under easily-specified conditions each firm’s optimal choice is well-defined.\(^{17}\) The unique symmetric equilibrium under this mission can be represented as \((x^t, x^t)\), where \( x^t \) satisfies

\[
c'(x^t) + p'(x^t)r^t(x^t, x^t) + p(x^t)r^t_1(x^t, x^t) = 0.
\]

As we would expect, the equilibrium outcome varies with the assigned target \( \tau \). It is easy to verify that \( x^t \) is increasing in \( \tau \). In words, a less stringent target leads to an increase in the \textit{ex ante} rate of non-compliance.\(^{18}\)

\(^{17}\)For the optimal choice to be a minimum, it is sufficient that the elasticity of the penalty function exceeds \( \frac{1}{N} \) at the relevant point. Firms’ choices are strategic substitutes if the elasticity of the penalty function exceeds \( \frac{2}{N} \) at any symmetric outcome. Details are in Appendix B.

\(^{18}\)By design, the ex post level of emissions is \( \tau \).
2.3 Budget-driven mission

Consider an alternative mission in which the enforcer is given a fixed enforcement budget $\beta > 0$ and asked to get the level of emissions as low as possible within that budget.\(^{19}\)

Once again, we focus on symmetric equilibria where each firm ends up choosing the same level of non-compliance $x$. Given the average level of non-compliance $\bar{x} = x$ at this outcome, the assigned budget can finance at most

$$n^b(\bar{x}, \beta) = \frac{\beta}{k(\bar{x})}$$

inspections (and resulting pursuits/prosecutions). The implied probability of prosecution at this symmetric outcome is

$$r^b(x_i, x_{-i}) = \frac{\beta}{Nk(\bar{x})}.$$  \hspace{1cm} (8)

The partial derivatives of this function are given by

$$r^b_1 = -\frac{\beta k'(\bar{x})}{[Nk(\bar{x})]^2} < 0$$  \hspace{1cm} (9)

and $r^b_2 = (N-1)r^b_1 < 0$. Again this second term is of interest. An increase in non-compliance by other firms increases the average level of non-compliance. Since prosecution cost is assumed to be increasing in $\bar{x}$, the increased burden-per-inspection on the enforcer’s limited enforcement budget results in a reduced number of inspections. With this budget-driven mission, the enforcement spillover is negative.

Each firm’s choice can be described by a reaction function, which is well-behaved under moderate conditions. Firms’ choices display strategic complementarity if the elasticity of the penalty function is sufficiently large. The symmetric equilibrium under this mission, $(x^b, x^b)$, must satisfy

$$c'(x^b) + p'(x^b)r^b(x^b, x^b) + p(x^b)\frac{\partial r^b}{\partial x}(x^b, x^b) = 0.$$  \hspace{1cm} (10)

It is easy to check that $x^b(\beta)$ is decreasing in $\beta$, so that a higher enforcement budget achieves greater ex ante compliance.

\(^{19}\)To keep things interesting we assume that the budget is binding – in other words that inspecting all firms is not feasible. Formally, $\beta < Nk(0)$. 

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2.4 Comparing Missions

How do the equilibria under these two missions compare in terms of compliance decisions and enforcement expenditure? Note that outcomes \(x^t(\tau)\) and \(x^b(\beta)\) vary with the chosen target \(\tau\) and budget \(\beta\) respectively, so that any comparison makes sense only for suitably calibrated pairs of values of these parameters. One approach is to choose values of these parameters so that the two alternative missions are somehow similar in terms of their enforcement pressure. Such calibration is not straightforward because the enforcement pressure functions also varies with firms’ choices, which may differ across missions.

Consider an arbitrary budget-driven mission \((b, \beta)\) and an arbitrary target-driven one \((t, \tau)\). Suppose under the former mission, the symmetric outcome \(x^b(\beta)\) obtains with \(n^b(x^b, \beta)\) inspections. We seek the following calibration: does there exist a value of \(\tau\) (call it \(\tau^*\)) which, if set under target-driven mission \((t, \tau)\) results in precisely \(n^b(x^b, \beta)\) prosecutions when firms choose \(x^b(\beta)\)? For ranges of \((x^b, \beta)\) where such \(\tau^*\) can be found, we have a functional relationship \(\tau^*(\beta)\), with

\[
r^t(x^b, \tau^*(\beta)) \equiv r^b(x^b, \beta).
\]

(11)

The central question is, how does the outcome \(x^t(\tau^*)\) under mission \((t, \tau^*(\beta))\) compare with that under mission \((b, \beta)\)? We have the following proposition.

**Proposition 1** Let \(x^b(\beta)\) denote the (symmetric) outcome under budget-driven mission \((b, \beta)\). The same outcome can be achieved at lower enforcement cost under an appropriately calibrated target-driven mission \((t, \tau^*)\).

A formal proof is provided in the Appendix, but a comparison of equations (6) and (10) is revealing. The two missions differ in the nature of the enforcement externality. The target-driven mission generates a positive enforcement spillover which serves to enhance the incentive impact of any particular level of enforcement pressure. A budget-driven mission dilutes incentives, so ends up with higher realized enforcement costs for any particular compliance outcome.

We will return to discuss of the implications and limitations of this result once we have considered the case of heterogenous firms, and developed the more general model.

2.5 Heterogeneous firms

While for the purposes of exposition we have thus far concentrated on symmetric outcomes, the machinery developed here can be adapted to richer cases. Symmetric
outcomes may be the natural focal points when firms are identical, but heterogeneity amongst firms may result in asymmetric responses. Extending our setting to the case where firms have heterogenous costs, it is possible to show that Proposition 1 is sustained - even when firms’ choices differ, target-driven missions achieve better outcomes at lower cost compared to budget-driven missions.

Imagine that firms differ in their cost functions, so that firm $i$ has cost function $\theta_i c(x_i)$. Firms with relatively low values of $\theta_i$ display lower marginal cost of abatement, so can viewed as being ‘cleaner’. Firm $i$ then chooses $x_i$ to minimize

$$V(x_i, x_{-i}) = \theta_i c(x_i) + p(x_i)r(x_i, x_{-i}).$$

Any interior minimum must satisfy the first-order condition

$$V_i(x_i, x_{-i}) \equiv \theta_i c'(x_i) + p'(x_i)r(x_i, x_{-i}) + p(x_i)\frac{\partial r}{\partial x_i}(x_i, x_{-i}) = 0,$$

with $V_{ii}(x_i, x_{-i}) > 0$ being a sufficient condition for a minima. Under these conditions, there exists a well-defined reaction function $R_i(x_{-i})$ that represents firm $i$’s optimal response to the choices made by other firms and a Nash equilibrium is defined in the usual manner. With asymmetric choices, we cannot rule out the possibility of ‘corner solutions’ for some firms at the equilibrium.

Once again, our aim is to compare the enforcement outcome and costs under budget-driven and target-driven missions. Denoting the equilibrium outcomes as $x^b$ and $x^t$, we would like to compare a typical firm’s choices, $x^b_i$ and $x^t_i$ under the two alternative missions, calibrated so that the enforcement pressure is the same for some configuration of firms’ choices.

**Proposition 1A** (Generalization of Proposition 1, with heterogeneous firms). Let $x^b(\beta)$ denote the outcome under budget-driven mission $(b, \beta)$. An appropriately calibrated budget-driven mission $(t, \tau^*)$ can achieve a (weakly) more compliant outcome for all firms at lower cost.

### 3 A Generalization

While the comparison in the previous section illustrates the significance of enforcement spillovers for outcome, it is limited by the specificity of the missions and the particularities of the enforcement setting. Our aim in this Section is to establish as a general result what we have noted by example – namely that a ‘good’ mission, in any particular setting will be one that generated positive enforcement spillovers.

As above, we consider an enforcer with a mission to control the level of some antisocial activity. There are $N$ identical firms and each firm’s non-compliance choice
is denoted by a real-valued variable \( x_i \in [0, \hat{x}] \). Aggregate non-compliance is given by the \( N \)-dimensional vector \( \mathbf{x} = \{x_1, x_2, \ldots, x_N\} \). The purpose of enforcement is to influence \( \mathbf{x} \).

The enforcer is given a mission to pursue. We consider missions of the form \((M, \mu)\), where \( M \) describes any broad objective and \( \mu \) is a real-valued parameter associated with that objective. To relate this to the missions compared in the previous section, \( M \) might refer to ‘maximize compliance with given enforcement budget’, or to ‘minimize enforcement cost of achieving some target level of compliance’, with \( \mu \) the assigned target level or allocated budget. Going beyond those missions, \( M \) might call for inspecting a fixed fraction \( \mu \) of the population of firms. The form allows a relatively rich class of missions, including hybrid versions of target-driven / budget-driven missions, with a parameter capturing the ‘softness’ of the budget constraint.

As before, firms face a choice between spending on compliance and the risk of being penalized. The enforcement environment faced by each firm can be described by an enforcement pressure function, which captures the probability that non-compliance will be detected and penalized. The form of this function depends on the mission (as well as the behavior of other firms). For firm \( i \), write the enforcement pressure function under mission \((M, \mu)\) as:

\[
r^M_i(x_i, \mathbf{x}_{-i}; \mu).
\]

Enforcement pressure on a firm depends on its own choice \( x_i \) but also on \( \mathbf{x}_{-i} \), the vector of choices made by the \( N - 1 \) other firms. We make no prior assumption about the effect of changes in \( \mathbf{x}_{-i} \), as this can differ across missions. A mission generates negative spillovers if the enforcement pressure on firm \( i \) is decreasing in another firm’s – call it firm \( j \) – level of non-compliance:

\[
\frac{\partial r^M_i}{\partial x_j} < 0.
\]

The opposite sign describes a positive spillover.

We make some simplifying assumptions for tractability. One, that all firms face identical enforcement environment, allowing us to drop the firm-specific subscript (so that \( r^M_i = r^M_j = r^M \)). Two, that the effect of individual compliance choices on the enforcement pressure function is symmetric across firms, so that \( \frac{\partial r^M}{\partial x_i} = \frac{\partial r^M}{\partial x_j} \) for all \( i \) and \( j \). The latter assumption is natural in environments where, as in the previous section, individual choices affect enforcement intensity through the aggregate level of non-compliance. We outline the implications of alternative assumptions in the next section. Lastly, the enforcement pressure function is assumed to be smooth and differentiable.
First consider an individual firm’s choices in such enforcement environments. The firm aims to maximize expected profits, given by a function of the form

$$\pi(x_i, r^M(x_i, x_{-i})),$$  \hspace{1cm} (12)

As greater enforcement intensity is associated with higher expected value of financial penalties, we assume this function is decreasing in its second argument, $r$. Each firm’s profit varies with other firms’ choices due to the assumed enforcement spillovers.\footnote{In order to focus on the enforcement spillover, we abstract from any other linkages between firms. We do not, for example, consider the possibility that firms might interact in an imperfectly competitive product market such that they might have incentive to ‘raise rivals costs’ (Salop and Scheffman, 1983).}

To study the strategic interaction in firms’ compliance choices, we make the standard assumption that firms choose their own compliance choices taking other firms’ choices as given and consider symmetric Nash equilibria in the level of non-compliance. Define $W(x_i, x_{-i})$ as the profit function for the typical firm when it chooses $x_i$ and all other firms make a symmetric choice $x_{-i}$. As defined, $W$ is a function of just two arguments, the firm’s own choice $x_i$ and the symmetric choice $x_{-i}$ made by other firms. We consider a firm’s profit-maximizing choice of $x_i$ given any arbitrary $x_{-i}$, focusing on environments in which the optimal choice is interior. Let $W_1$ and $W_2$ denote the partial derivatives of the profit function with respect to these arguments. An interior solution must satisfy the first-order condition:

$$W_1(x_i, x_{-i}) = \frac{\partial \pi}{\partial x_i} + \frac{\partial \pi}{\partial r} \frac{\partial r^M}{\partial x_i} = 0.$$  \hspace{1cm} (13)

If $W_{11}$ is negative at this solution, the solution characterizes firm $i$’s best response to $x_{-i}$. The optimal choice defines the firm’s reaction function. As the enforcement spillover is sensitive to the enforcement mission $(M, \mu)$, so is the reaction function: we write $R_i^M(x_{-i}; \mu)$. A Nash equilibrium is given by $\{x_1^M, x_2^M, \ldots, x_N^M\}$ where

$$x_i^M(\mu) = R_i^M(x_{-i}^M, \mu) \quad \text{for all } i.$$  \hspace{1cm} (14)

The superscript $M$ highlights the feature that equilibrium outcome varies with the enforcement mission.

Given the assumed interiority of the optimal choices at the symmetric equilibrium, we have, for all firms,

$$W_1(x_i^M, x_{-i}^M) = 0.$$  \hspace{1cm} (15)

Our proposed task of comparing equilibrium outcomes under alternative missions is easiest in environments with unique equilibria. A sufficient condition for
uniqueness is that the absolute value of slope of firms’ reaction function is less than unity at any symmetric equilibrium. Formally, if we define $W_{12}$ to be the second-order cross-partial of the function $W$, then $|W_{12}| < |W_{11}|$ is sufficient to ensure uniqueness of the equilibrium, and we assume that this condition holds.

Lastly, within a particular mission, the equilibrium outcome is sensitive to the choice of enforcement parameter $\mu$. Implicit differentiation of the set of first-order conditions suggests that the symmetric outcome $x^M$ is increasing (decreasing) in $\mu$ if and only if $W_{1\mu}$ is positive (negative).

### 3.1 Comparing outcomes under alternative missions

We say that two missions are equivalent in terms of their enforcement pressure if the implied risk of being penalized is equal under the two missions. To formalize this, consider any two missions, denoted as $(A, \alpha)$ and $(B, \beta)$. The enforcement pressure under these missions depends parametrically on $\alpha$ and $\beta$, and also varies with firms’ choices $x$. We ask if, for a given configuration of firms choices, $x$, there exist values $\alpha$ and $\beta$ such that the enforcement pressure functions are equi-valued,\(^{21}\) and propose the following definition.

**Definition 1** The enforcement pressure under two missions $(A, \alpha)$ and $(B, \beta)$ is equivalent for some profile of firms’ choices, $x$, if

$$r^A(x, \alpha) = r^B(x, \beta).$$

We aim to compare outcomes under alternative missions that are equivalent in terms of their enforcement pressure but differ in their enforcement spillover. To elaborate, let $x^A(\alpha)$ denote the unique equilibrium outcome under mission $(A, \alpha)$. Consider another mission $(B, \beta)$, where by suitable choice of parameter value, $r^A(x^A, \alpha) = r^B(x^A, \beta)$. Now if $x^B(\beta)$ is the unique equilibrium under mission $(B, \beta)$, how does outcome $x^B(\beta)$ compare with $x^A(\alpha)$? Indeed, as we consider only symmetric equilibria, each outcome can be characterized by the choice of the typical firm under that mission. Our question reduces to: how do we rank $x^A(\alpha)$ and $x^B(\beta)$?

If the equilibrium outcome is sensitive to enforcement spillovers, it should not surprise us that missions that differ in enforcement spillovers generate distinct outcomes even when the enforcement pressure is equivalent. Our aim, then, is to

---

\(^{21}\)Of course, two arbitrarily chosen missions could differ so much that such equivalence never holds, regardless of the values of $\alpha$, $\beta$ and $x$. We confine attention to mission-pairs that are not inconsistent in this sense.
examine if outcomes vary with the nature of the spillover in a systematic fashion. We have the following proposition.

**Proposition 2** Consider two missions \((A, \alpha)\) and \((B, \beta)\) with unique symmetric outcomes \(x^A(\alpha)\) and \(x^B(\beta)\). If these missions are equivalent in terms of enforcement pressure at outcome \(x^A\), then if

\[
\frac{\partial r^A(x^A, \alpha)}{\partial x_{-i}} > \frac{\partial r^B(x^A, \beta)}{\partial x_{-i}} \tag{16}
\]

it must be that \(x^B(\beta) > x^A(\alpha)\).

A formal proof of this proposition is in the Appendix.\(^{22}\) The proposition says that, relatively speaking, if a mission generates strong (positive) enforcement spillover, it serves to enhance the compliance incentives associated with a given level of enforcement pressure. In the comparison described in the previous section, target-driven missions, which generate positive spillovers, induced more compliance than budget-driven alternatives with negative spillovers. Proposition 2 allows for greater generality: it is not restricted to the case where missions under comparison generate spillovers of differing signs. It is the relative ordering of enforcement spillovers that is key in determining the relative efficacy of alternative enforcement missions. As long as enforcement spillovers can be ranked, so can the enforcement outcome: any given level of spending on enforcement will generate a correspondingly higher level of compliance through missions that have stronger enforcement spillovers.

Alternatively we can fix performance for the purpose of comparison. Corollary 1 highlights the fact that the expected enforcement cost of achieving a particular enforcement outcome is lower for missions that induce stronger positive enforcement spillovers.

**Corollary 1** Consider two missions \((A, \alpha)\) and \((B, \beta)\) that satisfy the inequality in (16). Then any given outcome \(x\) can be achieved at lower enforcement cost under mission \(A\) than under mission \(B\).

\(^{22}\)Note that the proposition requires us to compare the value of the derivatives only at a specific points \((x^A, \alpha)\) and \((x^A, \beta)\) respectively. This provides the weakest necessary condition for the proposition to hold. In the preceding examples one of these derivatives was positive and the other negative, so the required inequality held everywhere.
4 Discussion

Proposition 2 and its corollary allow us to assess the efficacy of a rich class of enforcement settings. In general the size and sign of spillovers will depend upon the combination of the fundamental elements of the enforcement setting, and the mission according to which the agency embedded in that setting behaves. This provides for the notion that particular missions may be particularly suited in particular contexts.

Consider, for instance, a mission that calls for an inspection of an exogenously-fixed fraction of firms, so that \( r(x) = \mu \). By design such missions imply no enforcement spillovers, as \( \frac{\partial r}{\partial x} = 0 \). Our argument suggests that this mission will be dominated by missions that create positive enforcement spillovers, and yet will dominate missions that create negative spillovers. To relate this to the missions discussed earlier, while fixed inspections do not create the positive enforcement spillovers generated by target-driven missions, they are still better than budget-driven missions that dilute compliance incentives through negative spillovers.

In practice, enforcers are often given multiple, and potentially conflicting, objectives. For purposes of presentation we have concentrated our attentions on two concrete missions – (a) hitting a particular environmental target with certainty and, (b) sticking to a particular budget with certainty. In reality we could imagine neither constraint being absolute, but rather each being ‘soft’ to varying degrees. As we have noted the general version of the model presented in the last Section is rich enough to handle such ‘hybrid’ missions. You might for example think of minimizing some loss function that attaches weight to ‘missing’ the environmental target but also to going over budget, with relative weight \( \mu \) on the former. More complex interactions between the budget and target objectives might suggest other solutions. Of course no single model can capture all possible complexities, but in each case the principle that an assessment would depend on the enforcement spillovers generated would be sustained.

In general, various elements of the enforcement setting, combined with the mission, will serve to determine the size and sign of the spillovers. Are inspections sequential? What is the order of moves between the agency and firms, and amongst firms? Is it inspection that is costly, or does enforcement against a firm shown to be non-compliant absorb extra resource? Is inspecting a non-compliant firm more costly than inspecting a compliant one? Does the agency have access to a measure of aggregate compliance rates in the population (such as an ambient measure of pollution in an environmental setting) before deciding on the intensity with which

\[23\] Recall that missions took the form \((M, \mu)\), where \(M\) describes any broad objective and \(\mu\) is a real-valued parameter associated with that objective.
to progress a firm-by-firm inspection/enforcement programme? But amongst this wide set of ways in which particular enforcement settings might vary the analysis here allows us to understand the principles according to which particular combinations of missions and enforcement settings can be evaluated – the basis on which we can distinguish ‘good’ ones from less good ones.

In the model above regulatory pressure on a particular firm depended only upon aggregate (industry-wide) emissions, but the model can be extended to allow it to in addition be sensitive to own emissions. We can think of plausible settings in which ‘large’ and ‘small’ violators co-exist, and in which the probability of detection varies positively with size of violation – in other words where enforcement is conditioned on firms’ relative non-compliance. In an Appendix we develop a version of our model in which the regulatory pressure function takes the form

\[ r(x, y; M) = \left( \frac{x}{x + y} \right)^{nM} \]

where the regulatory ‘effort’ \( n^M \) exerted under mission \( M \) is divided amongst the (in this case, two) violators in proportion to their chosen levels of non-compliance, \( x \) and \( y \).\(^{24}\) The earlier results are shown to carry over into such settings, the compliance incentives facing any particular firm still being sensitive to the choices of others in qualitatively similar directions. In particular the the target-based mission is better than the budget-based one and – it can be seen from the analysis – for the same reason, namely the sign/size of the enforcement spillover that is generated.

We may also consider settings in which enforcement activity is conditioned on an initial round of self-reporting by firms (for example, Livernois and McKenna, 1999). The ambiguity of the direction of the externality – and its sensitivity to the enforcer’s mission – has been noted in the context of a model of tax reporting and verification by Heyes (2001). He notes,

“In Erard and Feinstein (1994) the tax agency is assumed to have a fixed monitoring budget. Optimal policy involves concentrating verification on low-income reports, which have a greater chance of being under-reports. An increase in the proportion of honest taxpayers reduces the fraction of low income reports and makes any such report

\(^{24}\)Readers will recognise this as a ‘contest’ or imperfectly-discriminating auction approach to modelling rewards (in this case the negative rewards associated with enforcement attention) on the basis of relative performance. A tournament specification could also have been used. A mission implicitly embodies a particular structure of expected pay-offs, sensitive to a firm’s own performance but possibly also to the performance of others, and so puts regulated parties in a pseudo-tournament or pseudo-contest situation. We expect the intuition developed in our model to be robust to a variety of other ways of specifying the model.
more likely to be audited, so an increase in the proportion of taxpayers who are honest has the effect of encouraging dishonest taxpayers to cheat by less. The honest impose a form of (negative) externality on the dishonest.” (Heyes (2001: 227))

In contrast, in Heyes’ (2001) own model the agency chooses an optimal level of resource to devote to verification. That level is decreasing in the number of dishonest in the population (since a reduction in the propensity to dishonest reduces the likelihood that an inspection will score a ‘hit’) such that “... the presence of an additional honest firm induces an incremental cut in monitoring intensity which advantages the dishonest. Growth in the propensity to honesty in the population will cause the equilibrium behavior of the dishonest to get worse.” (Heyes (2001: 227)).

When characterizing strategic interaction it is natural to think in terms of strategic complementarity or substitutability, in the sense introduced by Bulow et. al. (1985), so it is natural to ask how they fit in with the analysis and results here. Strictly speaking, strategic complementarity and spillover are not the same thing. Spillovers describe interactions in payoffs, while strategic complementarity refers to interactions in strategies. Mathematically the difference is straightforward: spillovers refer to the sign of the partial derivative of one firm’s objective function with respect to a rival’s choice, while strategic complementarity is determined by the sign of the second cross-partial derivative of the objective function. In the particular cases that we have explored – analyzing enforcement/compliance games underpinned by stylized inspection ‘technologies’ of various different types – negative (positive) spillovers invariably go together with the non-compliance game played between firms being one in strategic complements (substitutes). It is intuitive why this should typically be the case, and whilst we cannot rule out the possibility of the perverse pairing it is straightforward to develop conditions that ensure a correspondence between the two.²⁵

5 Conclusions

We have shown that outcomes – actual patterns of compliance achieved – depend not only on the level of enforcement expenditure but also on the specific mission given to the enforcer. Different missions can generate qualitatively different types of strategic interaction amongst regulated parties. Those that generate positive enforcement spillovers are preferable to those that generate negative – or positive

²⁵Appendix B does so for the examples discussed in Section 2
but smaller – spillovers. In plausible settings this suggests a preference for target-driven enforcement missions over budget-driven ones.

We acknowledge that a more general mechanism design formulation could be used to explore the characteristics of an ‘optimal’ mission, whilst not (for reasons of tractability) going down that route. Besides, there are a variety of practical reasons why pursuit of social welfare is unlikely to be the task (see Firestone (2003) for some careful discussion) with which a particular agency or enforcement program is endowed. In the case of the EPA in particular there is revealed preference evidence (outlined earlier) to other objectives. The theoretical analysis presented here can provide a further efficiency rationale in favor of other missions.

While we have explored strategic linkages through the mission, other features of enforcement regimes might generate linkages too. Heather Eckert at Alberta University is using GIS methods to investigate spatial correlations in enforcement inspection patterns. One stylized story to hold in mind there is that an inspector who has reason to drive to locale Y to visit some firm may have a tendency to visit other firms nearby “whilst he is in the neighborhood” (Eckert, personal correspondence).

The spirit of our enquiry suggests a more fundamental mechanism design problem: the issue of an optimal mission, and indeed whether delegation of enforcement activity is optimal.\textsuperscript{26} We do not address this larger problem in this paper, taking as given that most enforcement activity is delegated to specialist agencies.

The extent to which better-designed missions can improve the outcome will, of course, depend upon the setting. It is reasonable to conjecture that the benefits will be greatest where the number of regulated parties is comparatively small. Indeed the strategic interaction matters less as the number of firms becomes large (or as each firm becomes ‘small’ in the formal sense) – the type of mission matters more in oligopolistic than more competitive sectors. This may further the case for compartmentalizing the activities of enforcement scrutiny to a more local level.\textsuperscript{27}

Our analysis offers a new rationale for a broad preference for target-driven missions over budget-driven ones in public governance. There are, as ever, caveats. There are potential weaknesses in target-driven approaches that our framework is insufficiently rich to pick-up. For instance, there may be scope for ‘drift’ between

\textsuperscript{26} Though it is reasonable to think that in the sorts of setting we are considering delegation is inevitable – the President cannot police every section of highway and every effluent pipe on his own!

\textsuperscript{27} ‘Local’ could refer to the usual geographical notion or to, for example, a tighter delineation of enforcement activities by industry or activity. The debate about the appropriate boundaries to place around the activities of the various enforcement agencies (state versus federal, for example) has been particularly keen in the US and EU.
targets that are measurable/contractible and the true objective of governing performance. Much of the public criticism of the so-called ‘targets culture’ in public governance in the UK rely, for example, on drift between intermediate and ultimate targets. It reflects that in many public service settings true outputs are comparatively difficult or expensive to measure. Similarly, delegation of a fixed budget may have other benefits – especially in the presence of agency issues between regulator and politician – that do not feature here.
Bibliography


Appendix: Proofs of Propositions 1 and 2

Proof of Proposition 1: Consider the symmetric equilibrium $x^b(\beta)$ under the budget-driven mission $(b, \beta)$, with each firm choosing $x^b(\beta)$. By construction, the number of inspections $n^b(x^b, \beta)$ is such that

$$n^b(x^b, \beta)k(x^b(\beta)) \equiv \beta. \quad (A.1)$$

Ex-post, after prosecuted firms are brought back into compliance, aggregate pollution falls to $[N - n^b(x^b, \beta)]x^b(\beta)$.

Consider $\tau^* (\beta)$, the target level which under a target-driven mission results in precisely $n^b(x^b, \beta)$ inspections, or that

$$r^t(x^b, \tau^*(\beta)) \equiv r^b(x^b, \beta), \quad (A.2)$$

and let $x^t(\tau^*)$ be the symmetric equilibrium under mission $(t, \tau^*(\beta))$, with each firm choosing $x^t(\tau^*)$.

To compare the outcome and enforcement cost under these two missions, consider the first-order condition (6) with (10), setting $\tau = \tau^*(\beta)$ in the former case. Equation (6) for the equilibrium under mission $(t, \tau^*)$ can be written as

$$c'(x^t(\tau^*)) + p'(x^t(\tau^*))r^t(x^t(\tau^*), \tau^*) + p(x^t(\tau^*))\frac{\partial r^t}{\partial x}(x^t(\tau^*), \tau^*) = 0. \quad (A.3)$$

For the equilibrium under mission $(b, \beta)$ the following must hold:

$$c'(x^b) + p'(x^b)r^b(x^b, \beta) + p(x^b)\frac{\partial r^b}{\partial x}(x^b) = 0. \quad (A.4)$$

Given that $p(x) > 0$, and that $\frac{\partial r^t}{\partial x} > 0 > \frac{\partial r^b}{\partial x}$, these conditions are both satisfied only if

$$c'(x^b) + p'(x^b)r^b(x^b, \beta) > c'(x^t(\tau^*)) + p'(x^t(\tau^*))r^t(x^t(\tau^*), \tau^*). \quad (A.5)$$

By calibration $r^b(x^b, \beta) = r^t(x^b, \tau^*)$, so that the last inequality requires

$$c'(x^b) + p'(x^b)r^t(x^b, \tau^*) > c'(x^t(\tau^*)) + p'(x^t(\tau^*))r^t(x^t(\tau^*), \tau^*). \quad (A.6)$$

Recall that $r^t$ is increasing in $x$ (see (5) in the text), and both $c'$ and $p'$ are increasing (as we assumed $c'' > 0$ and $p'' > 0$). The above inequality can hold.
only if \( x^t(\tau^*) < x^b \). In words, the calibrated target-driven mission generates better compliance.

Further, if \( x^t(\tau^*) < x^b \), then \( n^t(x^t(\tau^*),\tau^*) < n^t(x^b,\tau^*) \equiv n^b(x^b,\beta) \). Also as \( k(x) \) is increasing, we must have
\[
n^t(x^t(\tau^*))k(x^t(\tau^*)) < n^b(x^b,\beta)k(x^b) = \beta. \tag{A.7}
\]
In words, the target-driven mission achieves greater compliance at lower enforcement cost. Given that higher enforcement budgets can only deliver better outcomes, our claim goes through. \( \diamond \)

**Proof of Proposition 2**: As \( x^A(\alpha) = (x^A_i(\alpha), x^A_{-i}(\alpha)) \) denotes the symmetric equilibrium under mission \((A,\alpha)\), \( x^A_i \) must be the optimal response to the \( x^A_{-i} \). If so, \( x^A_i \) satisfies the first-order condition
\[
W^A_i(x^A,\alpha) \equiv \frac{\partial \pi}{\partial x^A_i}(x^A_i) + \frac{\partial \pi}{\partial r^A}(r^A)\frac{\partial r^A}{\partial x^A_i}(x^A,\alpha) = 0. \tag{A.8}
\]
From the assumed equivalence of regulatory pressure for the missions at \( x^A \)
\[
r^A(x^A,\alpha) = r^B(x^A,\beta). \tag{A.9}
\]
From the assumed ordering of spillovers, and recalling that the externality is symmetric,
\[
\frac{\partial r^A}{\partial x^A_i}(x^A,\alpha) > \frac{\partial r^B}{\partial x^A_i}(x^A,\beta). \tag{A.10}
\]
Given that profit is decreasing in regulatory pressure, \( (A.8) \), \( (A.9) \) and \( (A.10) \) together imply that
\[
\frac{\partial \pi}{\partial x^A_i}(x^A_i) + \frac{\partial \pi}{\partial r^A}(r^B)\frac{\partial r^B}{\partial x^A_i}(x^A,\beta) \equiv W^B_i(x^A,\beta) > 0. \tag{A.11}
\]
Also, as \( x^B(\beta) \) is the equilibrium under mission \((B,\beta)\),
\[
W^B_i(x^B,\beta) = 0. \tag{A.12}
\]
Relations \( (A.11) \) and \( (A.12) \) compare the value of the function \( W^B_i \) at two distinct points, \( x^A \) and \( x^B \). This function takes the value zero at \( x^B \) and is positive at \( x^A \), so its total differential at \( x^B \) must be positive for \( dx_i = x^A_i - x^B_i \) and \( dx_{-i} = x^A_{-i} - x^B_{-i} \). If so
\[
W^B_{11}dx_i + W^B_{12}dx_{-i} > 0. \tag{A.13}
\]
The proof of the proposition is by contradiction. If \( x^A(\alpha) > x^B(\beta) \), we have \( dx_i = x^A_i - x^B_i \) and \( dx_{-i} = x^A_{-i} - x^B_{-i} \) both positive and, by symmetry of the two Nash equilibria, equal. Recall that \( W_{11} \) is negative and that \( |W_{12}| < |W_{11}| \) by Assumption 1, so that the total differential is necessarily negative, contradicting \( (A.13) \). \( \diamond \)
B Appendix – probably not for publication

B.1 Details of formal arguments in Section 2

Firm $i$ chooses $x_i \in [0, \hat{x}]$ to minimize the continuous function

$$V(x_i, x_{-i}) = c(x_i) + p(x_i)r(x_i, x_{-i}).$$

The solution is interior for a given $x_{-i}$ as long as $\lim_{x_i \to 0} V_1(x_i, x_{-i}) < 0$ and $\lim_{x_i \to \hat{x}} V_1(x_i, x_{-i}) > 0$, where $V_1$ is the partial derivative with respect to $x_i$. A sufficient condition for the first inequality is that $c'(0) + p'(0) < 0$. In what follows, we assume this to hold. An interior minimum must satisfy the first-order condition

$$V_1(x_i, x_{-i}) = c'(x_i) + p'(x_i)r(x_i, x_{-i}) + p(x_i)\frac{\partial r}{\partial x_i}(x_i, x_{-i}) = 0.$$

As $c' < 0$ the first-order condition requires that $p'r + pr_1 > 0$ at the optimum where $r_1$ is the partial derivative of $r$ with respect to $x_i$. If $r_1 > 0$ this requirement is straightforward. If $r_1 < 0$, we require that

$$\frac{p'}{p} > -\frac{r_1}{r},$$

or that the elasticity of the penalty function exceed the (absolute value of) elasticity of the regulatory pressure function with respect to a firm’s own choice. A sufficient condition for this critical point to be a minima is that the second derivative $V_{11}(x_i, x_{-i})$ be positive. Formally,

$$V_{11} = c'' + pr_{11} + 2p'r + rp'' > 0.$$

As long as these conditions are satisfied, there exists a well-defined reaction function $R_i(x_{-i})$ that represents firm $i$’s optimal response to the choices made by other firms. For simplicity we confine attention to the symmetric, with the subscript 2 denoting the partial derivative with respect to $x_{-i}$, the symmetric choice of all firms other than $i$. Define

$$V_{12} = pr_{12} + p'r_2.$$

The slope of a firm’s reaction function (to the symmetric choice of others) is

$$\frac{dR_i}{dx_{-i}} = -\frac{V_{12}}{V_{11}}.$$

As $V_{11} > 0$ at the minima, the reaction function is upward sloping (a case of strategic complementarity) if $V_{12} < 0$, and downward sloping (strategic substitutes) if $V_{12} > 0$.

At a symmetric Nash equilibrium $x^* = \{x^*, x^*, \ldots x^*\}$, we have $V_1(x^*, x^*) = 0$. The equilibrium will be unique and ‘stable’ if the absolute value of the slope of the reaction function is less than 1. For this we require that $|V_{12}| < |V_{11}|$. 

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B.1.1 Target-driven mission

For this case

\[ r^t(x_i, x_{-i}; \tau) = \frac{X - \tau}{N^T} \]

if \( X \geq \tau \), and zero otherwise. Recall that firm \( i \) minimizes \( c(x_i) + p(x_i)r^t(x_i, x_{-i}) \).

Consider any positive target \( \tau < N\hat{x} \). If aggregate pollution is within the target \( \tau \), the probability of inspection is zero, and with \( c' < 0 \) there is an incentive for every firm to pollute more. In the aggregate, emissions must rise to exceed the target. Abatement activity on prosecution then restores pollution to the target level. This establishes interiority of the optimum.

The first-order condition for the interior optimum is

\[ c' + r^t p' + pr_1^t = 0, \]

where the partial derivative of the enforcement pressure function is (for \( \tau < X \))

\[ r_1^t = \frac{\tau}{(N^T)^2} > 0. \]

A sufficient second-order condition for a minimum is that

\[ V_{11} = c'' + r^t p'' + pr_{11}^t + 2r_1^t p' > 0, \quad \text{where } r_{11}^t = \frac{-2\tau}{(N^T)^3} < 0. \]

As

\[ pr_{11}^t + 2r_1^t p' = \frac{2\tau}{(N^T)^2} \frac{p'}{p} \left[ \frac{p'}{p - \frac{1}{N}} - \frac{1}{N} \right], \]

this holds, at least in the neighborhood of the symmetric equilibrium, if the elasticity of the penalty function is not smaller than \( \frac{1}{N} \) at that point. We assume this to be the case. With symmetric choices

\[ V_{12} = r_{12} p + r_2 p' = \frac{(N - 1)\tau}{(N^T)^2} \frac{p'}{p} \left[ \frac{p'}{p - \frac{1}{N}} - \frac{2}{N} \right], \]

using the facts that \( r_2^t = (N - 1)r_1^t > 0 \) and \( r_{12} = (N - 1)r_{11}^t \). Thus, \( V_{12} \) is positive – a case of strategic substitutes – as long as the elasticity of the penalty function is greater than \( \frac{2}{N} \).

A symmetric equilibrium under this mission is given by \((x^t, x^t)\), where

\[ c'(x^t) + p'(x^t)r^t(x^t, x^t) + p(x^t) \frac{\partial r^t}{\partial x} (x^t, x^t) = 0. \]
The requirement for uniqueness and stability, that $|V_{11}| > |V_{12}|$, amounts to

$$c'' + p\frac{-2\tau}{(N\bar{x})^3} + 2\frac{\tau}{(N\bar{x})^2}p' + (1 - \frac{\tau}{N\bar{x}})p'' > p\frac{-2(N-1)\tau}{(N\bar{x})^3} + \frac{(N-1)\tau}{(N\bar{x})^2}p'. $$

This holds if $c'' + (1 - \frac{\tau}{N\bar{x}})p''$ is sufficiently large relative to the elasticity of the penalty function.

It is easy to check that the equilibrium level of non-compliance is increasing in $\tau$. A semi-formal proof runs as follows.\footnote{A formal proof requires us to solve $N$ equations, each representing the total differential of a first-order condition. This is straightforward to check but tedious to report.} The shift in a reaction function $R_i(x_i, \tau)$ when $\tau$ changes is given by

$$\frac{dR_i(\tau)}{d\tau} = -\frac{V_{1\tau}}{V_{11}}. $$

We have $V_{11} > 0$, so that the $R_i(\tau)$ is increasing in $\tau$ if and only if $V_{1\tau}$ is negative.

$$V_{1\tau} = p' r_\tau + pr_{1\tau} = \frac{1}{N\bar{x} \bar{x}} \left[ -\frac{p'}{p} + \frac{1}{N} \right]. $$

$V_{1\tau}$ is negative if the elasticity of the penalty function exceeds $\frac{1}{N}$, which we have assumed above. An increase in $\tau$ shifts the reaction function outwards. At any symmetric equilibrium $(x^t(\tau), x^t(\tau))$, it must be that $x^t(\tau)$ is increasing in $\tau$.

### B.1.2 Budget-driven mission

For this case

$$r^b(x_i, x_{i-1}, \beta) = \frac{\beta}{Nk(\bar{x})}. $$

so that

$$r^b_1 = -\frac{\beta k'(\bar{x})}{[Nk(\bar{x})]^2} < 0, $$

and

$$r^b_{11} = -\frac{\beta [kk'' - 2(\bar{k}')^2]}{(Nk(\bar{x}))^3} > 0. $$

Firm $i$ minimizes

$$c(x_i) + p(x_i)r^b(x_i, x_{i-1}). $$

As long as we assume $c'(0) + p'(0) < 0$, a firm’s optimal choice is bounded away from zero. The first-order condition for the an interior optimum is

$$c' + r^b p' + pr^b_1 = 0. $$
As \( c' < 0 \), at the optimum we require \( r^b p' + pr^b > 0 \), which implies that
\[
\frac{p'}{p} > \frac{1}{N} k',
\]
or that the elasticity of the penalty function exceeds \( \frac{1}{N} \) times the elasticity of the average enforcement cost function. A sufficient second order condition is that
\[
V_{11} = c'' + p''r^b + pr^b + 2 r^bp' > 0.
\]
This holds if, say, \( c'' + rp'' \) is large enough.

The firm’s choices are strategic complements if the elasticity of the penalty function is sufficiently high. With symmetric choices
\[
V_{12} = r_1^bp + r_2^bp'
\]
must be negative. Using the facts that \( r_2^b = (N-1)r_1^b \) and \( r_1^b = (N-1)r_{11}^b > 0 \) we require
\[
\frac{p'}{p} > 2 \frac{k'}{Nk} - \frac{k''}{Nk'}.
\]
We assume this to be the case.

The symmetric equilibrium under this mission, \( \{x^b, x^b, \ldots x^b\} \), must satisfy
\[
c'(x^b) + p'(x^b)r^b(x^b, x^b) + p(x^b) \frac{\partial r^b}{\partial x}(x^b, x^b) = 0.
\]
The requirement that \( |V_{11}| > |V_{12}| \) amounts to
\[
c'' + p''r^b + 2 r^bp' + r_{11}^bp > r_2^bp + r_{12}^bp'
\]
or
\[
c'' + p''r^b > (N-3)r_1^bp + (N-2)r_{11}^bp'.
\]
Finally, \( R_1^i(\beta) \) is decreasing in \( \beta \) if and only if \( V_{1i} \) is positive.
\[
V_{1i} = p'r_{\beta} + pr_{1i} = \frac{p}{Nk} \left[ \frac{p'}{p} - \frac{1}{N} k' \right],
\]
which is positive by earlier assumptions. An increase in \( \beta \) shifts each reaction function inwards, so that at symmetric equilibria \( x^b \) is decreasing in \( \beta \).
Imagine that firms differ in their cost functions, so that firm \( i \) has cost function \( \theta_i c(x_i) \). Firms with relatively low values of \( \theta_i \) display lower marginal cost of abatement, so can viewed as being 'cleaner'. Firm \( i \) then chooses \( x_i \) to minimize

\[
V(x_i, x_{-i}) = \theta_i c(x_i) + p(x_i) r(x_i, x_{-i}).
\]

Any interior minimum must satisfy the first-order condition

\[
V_i(x_i, x_{-i}) \equiv \theta_i c'(x_i) + p'(x_i) r(x_i, x_{-i}) + p(x_i) \frac{\partial r}{\partial x_i}(x_i, x_{-i}) = 0,
\]

with \( V_{ii}(x_i, x_{-i}) > 0 \) being a sufficient condition for a minima. As long as these conditions are satisfied, there exists a well-defined reaction function \( R_i(x_{-i}) \) that represents firm \( i \)'s optimal response to the choices made by other firms. At the Nash equilibrium \( x^* = \{x_i^*\}_i \), we have \( x_i^* = R_i(x_{-i}^*) \) for all \( i \).

Note that, with asymmetry, it is harder to rule out corner solutions. For instance, it may be optimal for some firm, with low abatement costs, to choose zero levels of non-compliance.

Once again, our aim is to compare the enforcement outcome and costs under budget-driven and target-driven missions. Denoting the equilibrium outcomes as \( x^b \) and \( x^t \), we would like to compare a typical firm’s choices, \( x^b_i \) and \( x^t_i \) under the two alternative missions, calibrated so that the enforcement pressure is the same for some configuration of firms’ choices. Allowing for asymmetry, we must compare vectors of choices.

**Proposition 1A (Generalization of Proposition 1, with heterogeneous firms).** Let \( x^b(\beta) \) denote the outcome under budget-driven mission \((b, \beta)\). An appropriately calibrated budget-driven mission \((t, \tau^*)\) can achieve a (weakly) more compliant outcome for all firms at lower cost.

**Proof.** For given \( x^b \) and \( \beta \), let \( \tau^*(\beta) \) be such that

\[
r^t(x^t, \tau^*(\beta)) \equiv r^b(x^b, \beta).
\]

This calibration differs slightly from the one used for Proposition 1, but delivers the result. As for Proposition 1, we compare the first-order conditions that must hold at the equilibria. For firm \( i \), in the budget-driven case with budget \( \beta \), this requires

\[
\theta_i c'(x^b_i(\beta)) + p'(x^b_i(\beta)) r^b(x^b, \beta) + p(x^b_i(\beta)) \frac{\partial r^b}{\partial x_i}(x^b, \beta) = 0.
\]
Under the calibrated target-driven mission \((t, \tau^*(\beta))\), we require
\[
\theta_i c'(x^t_i(\tau^*)) + p'(x^t_i(\tau^*)) r^t(x^t_i, \tau^*) + p(x^t_i(\tau^*)) \frac{\partial r^t}{\partial x^t_i}(x^t_i, \tau^*) = 0. \tag{B.3}
\]
As \(p(x) > 0\) and \(\frac{\partial r^t}{\partial x^t_i} > 0 > \frac{\partial b}{\partial x^t_i}\), these conditions are both satisfied only if
\[
\theta_i c'(x^b_i(\beta)) + p'(x^b_i(\beta)) r^b(x^b_i, \beta) > \theta_i c'(x^t_i(\tau^*)) + p'(x^t_i(\tau^*)) r^t(x^t_i, \tau^*). \tag{B.4}
\]
By calibration (B.1), \(r^b(x^b_i, \beta) = r^t(x^t_i, \tau^*)\), so that the last inequality requires
\[
\theta_i c'(x^b_i(\beta)) + p'(x^b_i(\beta)) r^t(x^t_i, \tau^*) > \theta_i c'(x^t_i(\tau^*)) + p'(x^t_i(\tau^*)) r^t(x^t_i, \tau^*). \tag{B.5}
\]
Since \(c'\) and \(p'\) are both increasing in \(x_i\) (recall that, by assumption, \(c'' > 0\) and \(p'' > 0\)), the above inequality can hold only if \(x^t_i(\tau^*) < x^b_i\). Each firm chooses a lower level of emissions under a target-driven mission calibrated as above.

Since \(x^b_i(\beta) > x^t_i(\tau^*)\) for all \(i\) and as there is strategic complementarity in firms’ choices under the budget-driven mission, we have \(x^b_i(\beta) > x^t_i(\tau^*(\beta))\), so that overall level of non-compliance is necessarily higher under the budget-driven mission.

Further, if \(x^t_i(\tau^*) < x^b_i\) for all \(i\), the ordering holds for the average levels of non-compliance under the two missions – we have \(\bar{x}^t(\tau^*) < \bar{x}^b\). Since \(k(x)\) is increasing in \(x\), we must have \(k(\bar{x}^t(\tau^*)) < k(\bar{x}^b)\). Then, since \(n^t(x^t_i(\tau^*), \tau^*) = n^b(x^b_i, \beta)\) by our assumed calibration,
\[
n^t(x^t_i(\tau^*)) k(\bar{x}^t(\tau^*)) < n^b(x^b_i, \beta) k(\bar{x}^b) = \beta. \tag{B.6}
\]
In words, the target-driven mission achieves greater compliance at lower enforcement cost. Given that higher enforcement budgets can only deliver better outcomes, our claim goes through. \(\diamond\)