Contingent Convertible Bonds: Payoff Structures, Agency Costs and Non-admissible Debt-to-Equity Swap*

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Abstract

This paper investigates three issues associated with equity-conversion and write-down/off contingent convertible (CoCo) bonds. First, we investigate and compare in detail the payoff structures of different bail-out/in schemes: no bail-out/in, government bail-out, equity-conversion CoCo bail-in and write-down / write-off CoCo bail-in. This establishes that the equityholders gain extra incremental “put-spread” or “condor-like” option structures at each step of the bail-out/in schemes in the order listed. Second, we investigate two elements of agency costs, namely the wealth-transfer and the value destruction problems. We show that these are aggravated under equity-conversion CoCo bail-ins, and are even higher under write-off CoCo bail-in for larger asset values. Finally, we establish CoCo bail-in as a non-admissible debt-to-equity swap (DES), and argue that the agency costs are worse than for the admissibleDES.

JEL Classification: D82; G21; G28; G32

Keywords: CoCo bond; bail-in; agency cost; incentives

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1. Introduction

The new financial regulation, namely Basel III, has had a strong impact on the nature of the banking business. Perhaps even more significant effect has been on the capital structure of the banks. Amongst the new Basel III features, the new style of subordinate debt stands out the most: the contingent convertible, or the CoCo bonds.¹ This is an intricate product which has become in vogue in a low yielding environment, as investors rush into high yield instruments, and banks take advantage of it by issuing a “cheap” (relative to the cost of equity of the banks) equity-like instruments that helps bolster the capital and leverage ratios.² However, the lack of standardisation in its characteristics, such as the equity conversion ratio, permanent or temporary write-downs/offs, high or low trigger and the embedded equity option (for equity-conversion CoCos), and its complex nature means that its impact on banks’ behaviour has not been well understood.³

The aim of this paper is to scrutinise in detail the characteristics of CoCo bond bail-in. CoCo bonds, initially termed “reverse convertible debentures” (RCDs), were first recommended by Flannery (2005). The idea was to counter a firm’s incentive to use tax-advantaged debt rather than equity, that also reduces the firm’s ability to take losses. Flannery argued that the issuance of RCDs would still maintain the tax advantage whilst reducing the latter risk. In more recent terminology the suggested structure was an equity-conversion CoCo bond with a market value trigger. In terms of post-trigger treatments there are two types of CoCo bonds: equity-conversion, and write-down or write-off bonds. In the former, upon trigger CoCo bonds are converted into common equity,⁴ whilst in the latter, bonds are either partially

¹ The European Banking Authority’s (EBA) Buffer Convertible Capital Securities Common Term Sheet (8 December 2011) defines “additional tier 1” (AT1) instruments as perpetual CoCo bonds with cancellable coupons.
⁴ Coffee (2010) suggests a conversion into preference shares with cumulative dividends and voting rights, for risk incentive reasons.
written down or wholly written off to cover the incurred loss. In this paper we investigate and compare both of these. For the trigger mechanism, broadly two types are suggested in the literature: an accounting ratio trigger and a market value trigger. Himmelberg and Tsyplakov (2011), Berg and Kaserer (2011) and Hilscher and Raviv (2014) are examples of the former. However Flannery (2014), amongst others, argues that “accounting measures trail economic developments when a firm encounters difficulties, and managers can manipulate accounting statements” (p235). Pennacchi (2010), Prescott (2011), Glasserman and Nouri (2012), Koziol and Lawrenz (2012) and Albul, Jaffee and Tchistyi (2013) are examples that adopt the latter. However, Sundaresan and Wang (2014) point out that a market trigger bail-in does not lead to a unique competitive equilibrium. The problem arises from the fact that the share price reflects both the current value of the firm (say below the CoCo trigger value) and the post-bail-in value of shares (which would then be above the trigger value). Many have sought solutions to this: Pennacchi (2010) by including CoCo bond values in the capital ratio’s numerator; Prescott (2011) by introducing a “sliding conversion rule”; Glasserman and Nouri (2012) argue that the multiple equilibria problem is a feature of discrete-time models; Albul, Jaffee and Tchistyi (2013) achieve unique equilibrium by placing the trigger directly on the asset value. However, market value trigger also suffers from the possibility of price manipulation; as suggested by Pennacchi, Vermaelen and Wolff (2014), “the financial industry justifies its objection to CoCos with market based triggers on the basis of... manipulation/death spiral fears.” (p550-1). In this paper we follow the common market practice and focus on accounting capital ratio trigger CoCos.6

The analysis in this paper is threefold. First, the payoffs to the stakeholders (the equityholders, the vanilla and CoCo bondholders, and the government in the case of the government

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6McDonald (2011) suggest a dual price trigger that depends on both the bank’s share price and the value of a market stock index.
bail-out) at the maturity of the bond are investigated in detail and compared in the following bail-out/in schemes: (i) no bail-out/in, (ii) government bail-out, (iii) equity-conversion CoCo bail-in, (iv) write-down CoCo bail-in, and (v) write-off CoCo bail-in. There is a minimum capital ratio that is set by the regulator, and in each case (except (i)), where possible the bail-out/in results in the common equity capital ratio being boosted up to the minimum ratio. We show a neat result that each step of the schemes in the listed order can be represented by a sale of an incremental “put-spread” or “condor-like” option structures\(^7\) from the bail-out/in providers to the equityholders. The original equityholders are therefore unambiguously better off in the order of the schemes listed. Evaluation before bond maturity, and bail-out by preference shares are also investigated as extensions. Berg and Kaserer (2011) undertake a similar exercise, but they consider extreme and stylised CoCo structures with immediate full conversion. Here we allow partial conversion, and moreover take into account the different scenarios for what happens when CoCo bonds are exhausted (i.e. the losses are larger than the face value of the CoCo bonds).

Second, we investigate the agency costs inherent in bail-out/in structures. Agency costs in banking was pointed out as far back as Jensen and Meckling (1976), who argued that the call option held by the equityholders would lead to asset substitution problem, resulting in excessive risk-taking and a “gambling-for-resurrection” in times of a financial crisis. Here we distinguish two elements of agency costs. The first is the wealth-transfer problem, where the equityholders have an incentive to take on riskier projects because of their positive vega\(^8\) of their long option positions (call option plus any incremental “put-spread” or “condor-like” options). A choice of a higher volatility of the projects’ values means higher option value, leading to wealth being transferred from the guarantors (CoCo bondholders or the government) to the equityholders.\(^9\)

\(^7\)These structures are described in the text.
\(^8\)Vega is the sensitivity of the option value with respect to an increase in the volatility of the underlying asset price. Thus where \(V\) is the value of the option and \(\sigma\) is the volatility, then \(Vega = \frac{\partial V}{\partial \sigma}\).
\(^9\)Basically, the holder of an option is then able to determine the volatility of the underlying asset. If this was possible in financial markets, then it would be an illegal market manipulation.
We compare the level of this agency cost by comparing the vega curves of each bail-out/in scheme. The second is the value destruction problem, where in a falling solvency scenario the equityholders are tempted to “gamble-for-ressurection”, i.e. sacrifice value for higher volatility. The temptation is higher, the more the potential gain from higher gamble offsets the firm value sacrificed. Therefore the level of this agency cost can be gauged by the ratio of delta$^{10}$ to vega, where the smaller the ratio, the higher the temptation. Three main results are obtained: (i) in no bail-out/in or government bail-out scenarios, both types of agency costs are worse the further the firm value falls towards insolvency; (ii) for asset values above the bail-in trigger point, the agency costs are unambiguously higher under equity-conversion CoCo bail-in than under no bail-out/in or government bail-out; and (iii) for higher asset values, the agency costs are still higher under write-off CoCo bail-in than under equity-conversion CoCo bail-in. The latter two are the unintended consequences of the deviation from absolute priority rules (DAPR). Under the absolute priority rule (APR), bondholders do not bear losses until equityholders have been wiped out. The new financial regulation advocates for the bondholders to assume losses (“bail-in”) on a going-concern basis (hence deviation from APR). We agree that bail-in should replace bail-out; however this analysis demonstrates that the embedded moral hazard problem in bail-out is not eliminated by replacing bail-out with bail-in, but is in fact aggravated.

Third, our analysis extends Moraux and Navatte’s (2009) work on “admissible” debt-to-equity swap (DES),$^{11}$ and show that the CoCo structures are non-admissible DES. This is because its terms of restructuring are pre-set in advance, preventing bondholders from seeking a swap that to some degree compensates them for the losses from forgiven debt. We argue that the agency costs are higher for CoCo bail-in compared to the traditional DES. Overall, our stress is on the point that, while by encouraging bail-in structures the regulator appears

$^{10}$Delta is the sensitivity of the option value with respect to an increase in the underlying asset price. Thus where $V$ is the value of the option and $S$ is the underlying asset price, then $\Delta = \frac{dV}{dS}$.

$^{11}$“Admissibility” is defined as where the conditions for DES is optimal for the bail-in providers, i.e. the bondholders.
to be tackling the moral hazard problem in the banking industry, it may in fact be increased as a result of the aggravated agency costs embedded in the bail-in structures.

There are much related work in the literature. In a pre-CoCo set-up, Eberhart and Senbet (1993) investigates the role of APR violation. They assume the wealth-transfer to be a constant proportion of the firm value, and argue that DAPR can reduce agency costs. In Flannery (2005) no DAPR is assumed, i.e. the equityholders continue to bear losses while the converted RCDs replenish the capital base. Pennacchi (2010) builds a model of a jump-diffusion process for asset return using Monte Carlo simulations. They investigate the bank’s risk-taking incentives, and find that “moral hazard is usually less than if it had issued an equivalent amount of subordinated debt” (p3). Himmelberg and Tsyplakov (2011) consider the dilution effect of a trigger and argue that the bank would have “strong incentives to avoid triggering conversion by preemptively de-leveraging and raising equity capital well before it becomes financially distressed” (p3), while for non-dilutive (write-off) CoCos it is incentivised to “burn” money. Calomiris and Herring (2013) also conclude that the threat of dilution gives the bank an incentive to reduce risk. Berg and Kaserer (2011) is perhaps the closest to our work here where they too investigate the vega. They consider “Convert-to-Steal (CoSt)” (write-off) and “Convert-to-Surrender (CoSu)” (immediate expropriation of equityholders) bonds and advocate the latter as a vega-reducing scheme. This is extended to a first-passage time framework in Berg and Kaserer (2015) to explore trigger before bond maturity. Hilscher and Raviv (2011) derive at a similar result under a different set-up (they price bonds as a set of barrier options\textsuperscript{12}), that for CoCo bonds with zero conversion ratio (“CoSt” in Berg and Kaserer) the equityholders have an incentive to increase risk, while for CoCo with conversion ratio equal to one (“CoSu”) they have an incentive to decrease risk. Then there is always an intermediate level of conversion ratio for which the incentives for equityholders to change asset risk are eliminated. Glasserman

\textsuperscript{12}Barrier options are options which can be “knocked-out” or “knocked-in” when the underlying asset price breaches a pre-determined barrier.
and Nouri (2012) and Albul, Jaffe and Tchistyi (2013) both price coupon-paying CoCo bonds, former using Black and Cox (1976) and the latter extending Leland (1994), but they do not discuss incentive issues. Finally, Koziol and Lawrenz (2012) focus on risk-taking incentives. They argue that debt financing exerts a disciplining effect on the decision-makers of the firm from the threat of losing control rights in bad states, and as “by construction, CoCo bonds postpone the transfer of complete control rights,... [they] may distort decision-makers’ incentives” (p91). In their model both default and trigger occur according to the level of cash flow, and a trigger results in “coupons default” that lowers the required level of cash flow before default (but there is no additional equity). Thus higher risk-taking is beneficial to the equityholders, as it increases the probability of a trigger that reduces the probability of default.\textsuperscript{13}

The paper is organised as follows. In Section 2, we analyse comprehensively the payoff structure of bail-out/in schemes. In Section 3 and 4, we investigate respectively the wealth-transfer and value destruction problems of the agency costs associated with these structures. In Section 5, we compare the CoCo bail-in structure with the traditional DES. Finally in Section 6, we give concluding remarks.

2. Comparison of Structures

We first investigate in detail the payoff structures of the following bail-out/in schemes:

1. No bail-out/bail-in
2. Government bail-out
3. Equity-conversion CoCo bail-in with three different scenarios
4. Write-down and Write-off CoCo bail-in

In the no bail-out/bail-in case, the firm follows the absolute priority rule (APR) where the equityholders bear all the loss before the bondholders become the residual claimant once the firm becomes insolvent. With the government bail-out the APR is still followed, however the government injects capital to ensure that the minimum capital ratio is attained, which results in the bondholders’ position being guaranteed. With the equity-conversion CoCo bonds, the bail-in is triggered when the capital ratio is below a trigger level, in which case a necessary amount of the bond is converted into equity to attain the minimum capital ratio. This represents a deviation from absolute priority rule (DAPR). The write-down and write-off CoCo bail-in are the more extreme cases of DAPR, where the CoCo bonds are partially (write-down) or wholly (write-off) written down to cover the loss.

The three different scenarios in 3. arise from what happens once CoCo bonds are exhausted (the loss is larger than the face value of the CoCo bonds). More specifically, we consider the cases of: (i) no further bail-out/in, (ii) government bail-out, and (iii) forced bail-in of vanilla bonds. With the write-down and write-off CoCo bail-in, we simply assume forced bail-in of vanilla bonds once the CoCo bonds are exhausted.

2.1. Set-up

We consider a simple firm financed by common equity capital and discount bonds (vanilla or CoCo) with maturity $T$. The total face value of the bonds is $F$, which may include equity-conversion CoCo bond (face value $F_C$) or write-down/off CoCo bond (face value $F_W$). The face value of the plain vanilla bond is $F_B$. Therefore the firm can either have $F = F_B$ (no bail-out/in or government bail-out cases), $F = F_B + F_C$ (equity-conversion CoCo bond bail-in cases) or $F = F_B + F_W$ (write-down or write-off bond bail-in cases). The equity value at time 0 is $E_0$. The total asset value at time $T$ is $V_T$. All bail-outs / bail-ins trigger at the trigger capital ratio $\tau$. There exists a minimum capital ratio $\underline{E}$ set by the regulator, where $\underline{E} > \tau$. In all cases, where possible, when bailed-out/in the equity is boosted to this minimum capital
In the following analysis, for the numerical examples the following parameter values are used when relevant: $F = 90$, $F_C = 20$, $F_W = 20$, $\tau = 7\%$ and $E = 10\%$. The initial equity value is $E_0 = 20$ and the initial asset value is $V_0 = 110$.

### 2.2. Assumptions

For the purpose of this analysis, we make following two assumptions:

1. For the main body of this section, we review the payoff structure and the solvency of the firm at the bond maturity $T$.

2. Where government bail-out is required, this will be done by common equity.

Both of these assumptions are relaxed in Section 2.8, where the firm is reviewed at $t \leq T$ and preference share bail-out is considered.

### 2.3. No Bail-out/in

This is the standard case of absolute priority rule (APR), where at the bond maturity $T$ the initial losses are borne by the equityholders, and the bondholders become the residual claimant once the equityholders are wiped out. The table below outlines the payoffs to both the bondholders ($D_B$) and the equityholders ($E_E$) at time $T$ depending on the values of $V_T$:

<table>
<thead>
<tr>
<th>$V_T$</th>
<th>$[0, F)$</th>
<th>$[F, F + E_0)$</th>
<th>$[F + E_0, \infty]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_B$</td>
<td>$V_T$</td>
<td>$F$</td>
<td></td>
</tr>
<tr>
<td>$E_E$</td>
<td>$0$</td>
<td>$V_T - F$</td>
<td></td>
</tr>
<tr>
<td>Total Firm</td>
<td>$V_T$</td>
<td>$V_T$</td>
<td></td>
</tr>
<tr>
<td>Capital Ratio</td>
<td>$0$</td>
<td>$0, \frac{E_0}{F + E_0}$</td>
<td>$\frac{E_0}{F + E_0}, 1$</td>
</tr>
<tr>
<td>Notes</td>
<td>Capital wiped out, debt written down</td>
<td>Capital written down</td>
<td>Growth</td>
</tr>
</tbody>
</table>

For example for values of $V_T \in [F, F + E_0)$, the value of the firm is less than $V_0$ and hence the equityholders’ payoff $E_E = V_T - F$ is less than $E_0$. 

9
It is well established in the literature that the equityholders hold a long call option at strike price $F$, while the bondholders’ position is the bond minus a put option of the same strike price. Their payoffs are,

$$D_B^N = \min [V_T, F]$$  \hspace{1cm} (2)
$$E_E^N = \max [V_T - F, 0].$$

The superscript $N$ represents the case of no bail-out / bail-in. Fig.1 depicts these payoffs for the example with $F = 90$. The Black-Scholes-Merton valuation of the debt and equity holdings at time $t = 0$ are,$^{14}$

$$V_{DB}^N = F e^{-rT} - P (F)$$  \hspace{1cm} (3)
$$V_{EE}^N = C (F)$$

where $C (K)$ and $P (K)$ are the prices of call and put options with strike price $K$,

$$C (K) = V_0 N (d_1 (K)) - K e^{-rT} N (d_2 (K))$$
$$P (K) = -V_0 N (-d_1 (K)) + K e^{-rT} N (-d_2 (K))$$  \hspace{1cm} (4)

with $d_1 (K) = \frac{\ln (V_0 / K) + (r + \frac{\sigma^2}{2}) T}{\sigma \sqrt{T}}$, $d_2 (K) = d_1 (K) - \sigma \sqrt{T}$,

and $r$ is the risk-free rate, $T$ is the bond’s time to maturity and $\sigma$ is the asset volatility.

$^{14}$See for example Merton (1974).
2.4. Government Bail-out

Next consider the case of government bail-out. This is assumed to be triggered when the capital ratio is less than $\tau$. The bail-out occurs in the form of an injection of common shares $E_G$.\footnote{The case for preference share injection is explored in Section 2.8.} As a result the balance sheet is restored to the level where the minimum capital ratio $E$ is reattained. With the bondholders fully protected at their face value $F$, this would be $V = \frac{E}{1-E}$.

As with the no bail-out/in case, we investigate the stakeholders’ payoffs for different outcomes of $V_T$. For $V_T > F + E_0$, the balance sheet has expanded, while for $\frac{E}{1-E} \leq V_T < F + E_0$, the equityholders bear the loss according to the APR. In both cases, the bondholders receive their face value back and the equityholders receive the rest ($D_B = F$ and $E_E = V_T - F$).

If $V_T$ turns out to be less than $\frac{E}{1-E}$, then the capital ratio is below $\tau$ and the government bail-out is triggered. As stated, with the external capital injection of $E_G$ the balance sheet is restored to $\frac{E}{1-E}$, and the equity capital to $\frac{E}{1-E} F$. The original equityholders still bear all of the loss and thus $E_E = V_T - F$, while the government’s share of capital is $E_G = \frac{E}{1-E} F - (V_T - F) = \frac{E}{1-E} - V_T$, which equals the amount the balance sheet is boosted by.

For example when $V_T = 95$, with the loss $V_0 - V_T = 110 - 95 = 15$ wholly borne by the equityholders, $E_E$ is reduced to $E_0 - (V_0 - V_T) = 20 - 15 = 5$. Without a bail-out the capital ratio $\frac{E_E}{V_T} = \frac{5}{95} = 5.26\%$ is below the trigger level $\tau = 7\%$, and therefore the government injects common equity $E_G = \frac{E}{1-E} - V_T = \frac{90}{0.95} - 95 = 5$ to restore the balance sheet back up to $\frac{E}{1-E} = 100$ and the capital ratio to $\frac{E_E + E_G}{F (1-E)} = \frac{5 + 5}{100} = 10\% = E$. The bondholders are unaffected with $D_B = 90$.

For $V_T \leq F$, the original equityholders’ position is wiped out. The government continues to bail out the bondholders, with the taxpayers bearing the remaining loss.

The different scenarios of payoffs are summarised in the following table:
These payoffs can be summarised as,

\[ D_B^{BO} = F \]
\[ E_E^{BO} = \max [V_T - F, 0] \]
\[ E_G^{BO} = \left( E_{\tau} - \frac{F}{1-\tau} \right) \frac{F}{1-\tau} \chi_{V_T \leq \frac{F}{1-\tau}} + \left( \max \left[ \frac{F}{1-\tau} - V_T, 0 \right] - \max [F - V_T, 0] \right), \]

where \( \chi_{V_T \leq \frac{F}{1-\tau}} = \begin{cases} 
1 & \text{if } V_T \leq \frac{F}{1-\tau} \\
0 & \text{if } V_T > \frac{F}{1-\tau} 
\end{cases} \) is an indicator function. This represents the capital injection required to boost the capital ratio from \( \tau \) to \( E \). Fig.2 depicts the payoffs of the bondholders and the equityholders, the BSM valuation of which are,

\[ V_{DB}^{BO} = F e^{-rT} \]
\[ V_{E_k}^{BO} = C(F), \]
2.5. **Equity-conversion CoCo Bail-in**

Next we consider bail-in by equity-conversion contingent convertible (CoCo) bonds. As with the bail-out case, the bail-in is triggered when the capital ratio falls below $\tau$ to restore the ratio to the minimum capital ratio $E$. However in contrast to the government bail-out, there is no external capital injection and therefore the balance sheet remains depleted.

The pre-trigger scenarios are the same as before: when $V_T \geq F + E_0$, the balance sheet has expanded, while when $\frac{E}{1-\tau} \leq V_T < V_0$, the equityholders bear all of the loss. In both cases, therefore, $D_B = F$ and $E_E = V_T - F$.

For $V_T < \frac{E}{1-\tau}$ the CoCo would be triggered. Then,

- The equityholders take the loss up to $\tau V_T$.
- With the minimum capital ratio requirement of $E$, the CoCo bond is partially or wholly converted to make up the remaining required capital of $E_C = \left(\frac{E}{1-\tau}\right)V_T$.
- When there is enough CoCo bond to cover the loss, then $D_C = (1 - E) V_T - F_B$ (the total debt level minus the plain vanilla bond) of the CoCo bond is left unconverted. As a result the CoCo bondholders bear the loss equal to $F_C - (E_C + D_C) = F - (1 - \tau) V_T$. 
This would be the case when there is enough CoCo bond to cover the loss, i.e. \( D_C \geq 0 \iff V_T \geq \frac{F_B}{1-E}. \) To demonstrate, take the example of \( V_T = 80 \) where the firm loses 30. Without the bail-in the equityholders are wiped out. They bear the loss up to the trigger point, i.e. \( E_C = \tau V_T = 80 \times 7% = 5.6, \) implying a loss of \( E_0 - E_E = 20 - 5.6 = 14.4. \) The CoCo bond is partially converted to make up the shortfall for the minimum capital ratio, and therefore \( E_C = (E - \tau) V_T = (0.1 - 0.07) \times 80 = 2.4. \) This leaves \( D_C = (1 - E) V_T - F_B = (1 - 0.1) \times 80 - 70 = 2 \) of the CoCo bond unconverted, so the CoCo bondholders bear the loss of \( F_C - (D_C + E_C) = 20 - (2 + 2.4) = 15.6. \) The plain vanilla bondholders are unaffected.

For \( V_T < \frac{F_B}{1-E}, \) even with the whole conversion of the CoCo bond the minimum equity ratio cannot be attained. For example when \( V_T = 76 < \frac{F_B}{1-E} = \frac{70}{1-0.10} = 77.78, \) the firm loses \( V_0 - V_T = 110 - 76 = 34. \) As before the equityholders bear the loss up to \( E_E = \tau V_T = 76 \times 7% = 5.32, \) with a loss of \( E_0 - E_E = 20 - 5.32 = 14.68. \) The CoCo bond is converted in its entirety into \( E_C = V_T - (E_E + D_B) = 76 - (70 + 5.32) = 0.68 \) of equity, and therefore they bear the loss of \( F_C - E_C = 20 - 0.68 = 19.32. \) The capital ratio \( \frac{E_E + E_C}{V_T} = \frac{5.32 + 0.68}{96} = 7.89\% \) is now below the minimum capital ratio of 10%; however the firm is unable to attain this even with the full conversion. This would be the case as long as \( V_T \geq \frac{F_B}{1-E}, \) when \( E_C = (1 - \tau) V_T - F_B \geq 0. \)

For \( V_T < \frac{F_B}{1-E} \) the CoCo bond is wiped out, i.e. \( D_C = E_C = 0. \) There are now different scenarios that can be considered. We consider three of these. We could insist on the APR to be reinstated and write-down the equityholders’ capital \( E_E. \) This would be analogous to the no bail-out/in case in Section 2.3. Alternatively, as with the bail-out case in Section 2.4, we could assume that the government would step in to inject common equity. Finally, we could assume that the regulator will exercise its bail-in power to force conversion of necessary amount of plain vanilla debt, such that the minimum capital ratio is again reattained. This would correspond to a repeat of the equity conversion bail-in just described in this section.

Note in this case, any unsecured bond is inherently an equity-conversion CoCo bond.
### 2.5.1. Case 1: Bail-in-No-bail-out/in

In this case, for $F_B \leq V_T < \frac{F_B}{1-\tau}$ the equity $E_E$ is written-down, while for $V_T < F_B$, the bondholders become the residual claimants. In summary,

<table>
<thead>
<tr>
<th>$V_T$</th>
<th>$[0, F_B)$</th>
<th>$[F_B, \frac{F_B}{1-\tau}]$</th>
<th>$[\frac{F_B}{1-\tau}, F]$</th>
<th>$[\frac{F_B}{1-\tau}, F_E]$</th>
<th>$[\frac{F_B}{1-\tau}, F_E, F]$</th>
<th>$[F + E_0, \infty]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_B$</td>
<td>$V_T$</td>
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<td>$F_B$</td>
<td>$F_B$</td>
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<td>$D_C$</td>
<td>0</td>
<td>0</td>
<td>$\tau V_T$</td>
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<td>$V_T - F_B$</td>
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<td>$\tau V_T$</td>
<td>$\tau V_T$</td>
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<tr>
<td>$E_C$</td>
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<td>0</td>
<td>$(1-\tau)V_T - F_B$</td>
<td>$(1-\tau)V_T$</td>
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<td>0</td>
</tr>
<tr>
<td>Total Firm</td>
<td>$V_T$</td>
<td>$V_T$</td>
<td>$V_T$</td>
<td>$V_T$</td>
<td>$V_T$</td>
<td>$V_T$</td>
</tr>
<tr>
<td>Capital ratio</td>
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<td>$[0, \tau]$</td>
<td>$[\tau, \tau]$</td>
<td>$[\tau, \tau]$</td>
<td>$[\tau, \tau]$</td>
<td>$[\tau, \tau]$</td>
</tr>
</tbody>
</table>

| Notes | $E_E$ wiped out, debt-holders residual claimants | $E_E$ wholly triggered, $E$ unattainable | $E_E$ partially triggered | $E_E$ written down. Capital ratio $\geq \tau.$ |

The payoffs for bondholders, the original equityholders and the CoCo bondholders are, where the CoCo bondholders’ payoff is the total of their bond and equity positions:\[16\]

\[
D_{C}^{CN} = \min [V_T, F_B] \\
E_{C}^{CN} = \max [V_T - F, 0] + \left\{ (1 - \tau) \max \left[ \frac{F}{1-\tau} - V_T, 0 \right] - \max [F - V_T, 0] \right\} \\
D_{C}^{CN} + E_{C}^{CN} = F_C - (1 - \tau) \left( \max \left[ \frac{F}{1-\tau} - V_T, 0 \right] - \max \left[ \frac{F_B}{1-\tau} - V_T, 0 \right] \right).
\]

Fig.3 shows the bondholders’ and equityholders’ payoffs. The BSM valuation of these are,

\[\text{Notes: The CoCo bondholders’ respective positions in bond and equity are:} \]

\[
D_{C}^{CN} = (1 - E) \left( \max \left[ V_T - \frac{F_B}{1-\tau}, 0 \right] - \max \left[ V_T - \frac{F_B}{1-\tau}, 0 \right] \right) + \left( \frac{F_B}{1-\tau} \right) F \chi_{V_T \geq \frac{F_B}{1-\tau}} \\
E_{C}^{CN} = (1 - \tau) \max \left[ V_T - \frac{F_B}{1-\tau}, 0 \right] - (1 - E) \max \left[ V_T - \frac{F_B}{1-\tau}, 0 \right] - (\tau - \tau) \max \left[ V_T - \frac{F_B}{1-\tau}, 0 \right] \right).
\]
Figure 3: Equity-conversion Bail-in-No-Bail-out/in: $\tau = 7\%$, $E = 10\%$, $F_B = 90$ and $F_C = 20$

\[
\begin{align*}
V^C_{DB} &= F_B e^{-rT} - P(F_B) \\
V^C_{EE} &= C(F) + [(1 - \tau) P\left(\frac{F}{1-\tau}\right) - P(F)] - [(1 - \tau) P\left(\frac{F_B}{1-\tau}\right) - P(F_B)] \\
V^C_{EC} + V^C_{DC} &= F_C e^{-rT} - (1 - \tau) \left[P\left(\frac{F}{1-\tau}\right) - P\left(\frac{F_B}{1-\tau}\right)\right].
\end{align*}
\tag{11}
\]

Note, we recover $V^N_{DB}$ and $V^N_{EE}$ when $\tau = F_C = 0$. $V^C_{EE}$ derived in Eq.(11) differs from the expression for “Convert-to-surrender CoCo” in Berg and Kaserer (2011) in two ways. First, they assume 100% conversion of the CoCo bond when triggered. Here we allow partial conversion. Second, they assume the whole liability to be CoCo bonds, i.e. $F = F_C$, and therefore the equityholders are never wiped out for $V_T > 0$. Here our assumption of $F_C < F$ means that, once the CoCo bond is wiped out, the normal practice of APR resumes where the equityholders’ holdings are written down ahead of the vanilla bonds.

One way of viewing the CoCo bail-in effect is to regard the difference between $V^C_{EE}$ in Eq.(11) and $V^N_{EE}$ in Eq.(3) as the wealth-transfer induced by the introduction of deviation from absolute priority rule (DAPR). Diagrammatically, this is the area between the $E_E$ payoff in Fig.3 and the normal call option payoff in Fig.1. Eberhart and Senbet (1993) also investigate the role of APR violations in reducing agency conflicts between bondholders and sharehold-
ers. However they assume the wealth-transfer to be a constant proportion of the firm value, and argue that when the firm is in distress the negative vega of the assumed wealth-transfer partly offsets the positive vega of the equityholders’ position, hence mitigating the agency cost incentive. Here we are able to explicitly derive the amount of DAPR-induced wealth-transfer as $V_{E_E}^{CN} - V_{E_E}^N$:

$$V_{E_E}^{CN} - V_{E_E}^N = \left[ (1 - \tau) P\left(\frac{F}{1 - \tau}\right) - P(F) \right] - \left[ (1 - \tau) P\left(\frac{F_B}{1 - \tau}\right) - P(F_B) \right].$$ (12)

Intuitively, the equityholders’ payoff is improved by a bear spread-like protection, $(1 - \tau) P\left(\frac{F}{1 - \tau}\right) - P(F)$, which represents the DAPR induced by the introduction of the CoCo bond. The bull spread-like structure $\left[ (1 - \tau) P\left(\frac{F_B}{1 - \tau}\right) - P(F_B) \right]$ reinstates the APR once the CoCo bond is wiped out. Together they create a “condor-like” structure, which we will call the “CoCo condor”, depicted in Fig.4.\textsuperscript{17}

\textsuperscript{17}A condor is created by a combination of either a bull call spread with a bear call spread, or a bull put spread with a bear put spread. A bull call spread is formed by combining a long call option with a short call option of a higher strike price, such that the holder of the structure gains from a rise in the underlying asset price. In a bear call spread, the short call option has the lower strike price. Similarly for bull and bear put spreads.
2.5.2. Case 2: Bail-in-Bail-out

Here for $V_T < \frac{F_B}{1-\tau}$, with the capital ratio less than $\tau$, the government bail-out is triggered with an injection of common equity $E_G$. Analogous to before, this boosts the balance sheet to $\frac{F_B}{1-\tau}$ and the capital ratio to $E$. The equityholders are wiped out for $V_T < F_B$, at which point the taxpayers are required to bear any remaining loss. In summary,

<table>
<thead>
<tr>
<th>$V_T$</th>
<th>$[0,F_B)$</th>
<th>$[F_B, \frac{F_B}{1-\tau})$</th>
<th>$[\frac{F_B}{1-\tau}, F_B)$</th>
<th>$[F_B, \infty)$</th>
<th>$[F + E_0, \infty]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_B$</td>
<td>$F_B$</td>
<td>$F_B$</td>
<td>$F_B$</td>
<td>$F_B$</td>
<td>$F_B$</td>
</tr>
<tr>
<td>$D_C$</td>
<td>0</td>
<td>0</td>
<td>$V_T - F_B$</td>
<td>$\tau V_T$</td>
<td>$V_T$</td>
</tr>
<tr>
<td>$E_E$</td>
<td>0</td>
<td>$V_T - F_B$</td>
<td>$\tau V_T$</td>
<td>$V_T - F_B$</td>
<td>0</td>
</tr>
<tr>
<td>$E_C$</td>
<td>0</td>
<td>0</td>
<td>$(1-\tau) V_T$</td>
<td>$V_T - F_B$</td>
<td>0</td>
</tr>
<tr>
<td>$E_G$</td>
<td>$\frac{E}{1-\tau} F_B$</td>
<td>$\frac{E}{1-\tau} - V_T$</td>
<td>$\frac{E}{1-\tau}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total Firm</td>
<td>$\frac{F_B}{1-\tau}$</td>
<td>$\frac{F_B}{1-\tau}$</td>
<td>$V_T$</td>
<td>$V_T - F_B$</td>
<td>0</td>
</tr>
<tr>
<td>Tax payers</td>
<td>$-(F_B - V_T)$</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Capital ratio</td>
<td>$E$</td>
<td>$E$</td>
<td>$[\tau, E]$</td>
<td>$E$</td>
<td>$[\tau, \tau + E_0]$</td>
</tr>
</tbody>
</table>

Notes:
- $E_E$ wiped out. Govt injects $E_G$ to restore $E$.
- Taxpayers bear remaining loss.
- $E_E$ written down. Govt injects $E_G$ to restore $E$.
- CoCo wholly triggered, $E$ unattainable.
- CoCo partially triggered.
- $E_E$ written down. Capital ratio $\geq \tau$.

Growth

The payoffs for bondholders, equityholders, CoCo bondholders and the government are,

$$D_B^{CBO} = F_B$$

$$E_E^{CBO} = \max [V_T - F, 0] + \left\{ (1-\tau) \max \left[ \frac{E}{1-\tau} - V_T, 0 \right] - \max [F - V_T, 0] \right\}$$

$$D_C^{CBO} + E_C^{CBO} = F_C - (1-\tau) \left( \max \left[ \frac{E}{1-\tau} - V_T, 0 \right] - \max \left[ \frac{F_B}{1-\tau} - V_T, 0 \right] \right)$$

$$E_G^{CBO} = \left( \frac{E}{1-\tau} \right) \frac{F_B}{1-\tau} X_{V_T \leq \frac{F_B}{1-\tau}} + \left( \max \left[ \frac{F_B}{1-\tau} - V_T, 0 \right] - \max [F_B - V_T, 0] \right).$$
Figure 5: Equity-conversion Bail-in-Bail-out: $\tau = 7\%$, $E = 10\%$, $F_B = 90$ and $F_C = 20$

Fig. 5 shows the bondholders’ and equityholders’ payoffs. The equityholders’ payoff is the same as in Fig. 3, as discussed in Section 2.4. The vanilla bondholders’ position is guaranteed at $F$.

The BSM valuations for the original equity and bondholders are,\(^{18}\)

$$
\begin{align*}
V_{CBO}^{DB} & = F_B e^{-rT} \\
V_{CBO}^{EE} & = C(F) + \left[ (1 - \tau) P\left(\frac{E}{1+r}\right) - P(F) \right] - \left[ (1 - \tau) P\left(\frac{F_B}{1+r}\right) - P(F_B) \right] \\
V_{CBO}^{EC} + V_{CBO}^{DC} & = F_C e^{-rT} - (1 - \tau) \left[ P\left(\frac{F}{1+r}\right) - P\left(\frac{F_B}{1+r}\right) \right].
\end{align*}
$$

(15)

The equityholders again benefit from the CoCo condor in Eq.(12).

2.5.3. Case 3: Bail-in-Bail-in

Finally in this case, when even with the CoCo bond wholly wiped out the capital ratio is otherwise below $\tau$, the vanilla bondholders bail-in the equityholders by forced conversion to reattain the minimum capital ratio $E$. Then the equityholders lose up to $\tau V_T$, the CoCo bond is wiped out ($D_C = E_C = 0$) thus bearing the loss of $F_C$, and $F_B - (1 - E) V_T = F_B - D_B$ of vanilla bond is converted to $E_B = (E - \tau) V_T$ of equity. The bondholders thus bear the loss of

\(^{18}\)The CoCo bondholders’ respective positions in bond and equity are the same as in the Bail-in-No-Bail-out/in case, i.e. $D_C^{CBO} = D_C^{CN}$ and $E_C^{CBO} = E_C^{CN}$.
\[ F_B - (1 - E) V_T - [(E - \tau) V_T] = F_B - (1 - \tau) V_T. \] Consider as an example \( V_T = 60 \), when the firm loses 50. Then \( E_C = \tau V_T = 60 \times 7\% = 4.2 \), so the equityholders lose 20 - 4.2 = 15.8. The CoCo bondholders lose their entire position of \( F_C = 20 \). Further, \( F_B - (1 - E) V_T = 70 - 0.9 \times 60 = 16 \) of the vanilla bond is converted to \( E_B = (E - \tau) V_T = (0.1 - 0.07) \times 60 = 1.8 \) of equity, and therefore they bear the loss of 16 - 1.8 = 14.2.

In summary,

\[
\begin{array}{|c|c|c|c|c|}
\hline
V_T & 0, \frac{F_B}{1-\tau} & \frac{F_B}{1-\tau}, \frac{F_B}{1-\tau} & \frac{F_B}{1-\tau}, F + E_0 & [F + E_0, \infty] \\
\hline
D_B & (1-E)V_T & F_B & F_B & F_B \\
\hline
D_C & 0 & 0 & (1-E)V_T-F_B & F_C \\
\hline
E_E & \tau V_T & \tau V_T & \tau V_T & \tau V_T-F \\
\hline
E_C & 0 & (1-\tau)V_T-F_B & (E-\tau)V_T & 0 \\
\hline
E_B & (E-\tau)V_T & 0 & 0 & 0 \\
\hline
\end{array}
\]

(16)

The payoffs are,\(^{19}\)

\[
\begin{align*}
D_B^{CBI} + E_B^{CBI} &= \min \left[(1-\tau) V_T, F_B \right] \\
E_E^{CBI} &= \max \left[V_T - F, 0 \right] + \left\{ (1-\tau) \max \left[ \frac{F}{1-\tau} - V_T, 0 \right] - \max \left[ F - V_T, 0 \right] \right\} \\
D_C^{CBI} + E_C^{CBI} &= F_C - (1-\tau) \left\{ \max \left[ \frac{F}{1-\tau} - V_T, 0 \right] - \max \left[ F_B - V_T, 0 \right] \right\}.
\end{align*}
\] (18)

Fig.6 shows the bondholders’ and equityholders’ payoffs. The BSM valuation of these are,

\(^{19}\)The CoCo bondholders’ positions in bond and equity are the same as in the Bail-in-No-Bail-out/in case, i.e. \( D_B^{CBI} = D_C^{CBI} \) and \( E_C^{CBI} = E_C^{CBI} \). The vanilla bondholders’ respective positions in bond and equity are:

\[
\begin{align*}
D_B^{CBI} &= F_B - (1 - E) \max \left[ \frac{F_B}{1-\tau} - V_T, 0 \right] - \left( \frac{E-\tau}{1-\tau} \right) F_B \chi_{V_T \leq \frac{F_B}{1-\tau}} \\
E_B^{CBI} &= (E - \tau) \left( \frac{F_B}{1-\tau} \chi_{V_T \leq \frac{F_B}{1-\tau}} - \max \left[ \frac{F_B}{1-\tau} - V_T, 0 \right] \right).
\end{align*}
\] (17)
Figure 6: Equity-conversion Bail-in-Bail-in: $\tau = 7\%$, $E = 10\%$, $F_B = 90$ and $F_C = 20$

\[ V_{DB}^{CBI} + V_{EB}^{CBI} = F_B e^{-rT} - (1 - \tau) P \left( \frac{F_B}{1 - \tau} \right) \]
\[ V_{EE}^{CBI} = C (F) + \left[ (1 - \tau) P \left( \frac{F}{1 - \tau} \right) - P (F) \right] \]
\[ V_{DC}^{CBI} + V_{EC}^{CBI} = F_C e^{-rT} - (1 - \tau) \left[ P \left( \frac{F}{1 - \tau} \right) - P \left( \frac{F_B}{1 - \tau} \right) \right]. \]  

Note that compared with Eq.(11), the equityholders now benefit from a bear spread-like protection $\left(1 - \tau\right) P \left( \frac{F_B}{1 - \tau} \right) - P (F)$, rather than the CoCo condor in Eq.(12). This is due to the forced bail-in by the vanilla bond, that corresponds to a continued DAPR even after the CoCo bond is exhausted.

2.6. Write-down/off CoCo Bail-in

Finally, we consider bail-in by write-down / write-off CoCo bonds. Write-down bonds are only partially written-down when the trigger occurs, while the write-off bonds are immediately written-off in its entirety to cover the loss. There are unknowns as to what happens when these bonds are triggered. For the write-down bond, it is unclear how the firm reattains the minimum capital ratio $E$ after the trigger at $\tau$. Here we assume a bail-in by the write-down bond holders (“write-down-bail-in”) such that $(E - \tau) V_T$ of the write-down bond is converted one-to-one to a contingent capital reserve (CCR). For the write-off bond, it is unclear what
happens to the remainder of the written-off bond when the write-off more than covers the firm’s loss. Here, again, it is assumed that the net amount becomes a CCR.

In both cases, the outcomes for $V_T \geq \frac{F}{1-F}$ when there is no trigger are equivalent to the equity-conversion CoCo bail-in outcomes. We therefore consider the outcomes for $V_T < \frac{F}{1-F}$ for each type of bond, respectively.

2.6.1. Write-down-Bail-in

For write-down bonds the bond is written-off only partially. For example for $V_T = 80$, when the firm loses 30, the equityholders bear the loss of $E_0 - \tau V_T = 20 - 80 \times 0.07 = 14.4$. The write-down bond is triggered to cover the remaining loss of $30 - 14.4 = 15.6$. As it is, $D_W = 20 - 15.6 = 4.4$ would then remain untriggered. However, at this point the capital ratio is $\tau$, below the minimum required ratio of $E$. It is unclear what would happen in this case. Here, for the purpose of the comparison with the equity-conversion CoCo case, we assume a bail-in by the write-down bond that results in a 1-to-1 conversion of the required amount of the bond into common equity to make up the difference to $EV_T$. Thus in this example $(E - \tau) V_T = (0.1 - 0.07) \times 80 = 2.4$ is converted into a contingent capital reserve $E_{CCR}$. This leaves $D_W = 20 - 15.6 - 2.4 = 2$ as the unconverted write-down bond. Note this also equals $(1 - E) V_T - F_B = (1 - 0.1) \times 80 - 70 = 2$.

This bail-in scenario would be the case as long as $(1 - E) V_T > F_B \iff V_T > \frac{F_B}{1-E}$. For values of $V_T$ below this the minimum capital ratio $E$ is not attainable even with a full write-down of the write-down bond. In this case we assume a forced bail-in by the vanilla bondholders, as with the equity-conversion CoCo bond bail-in-bail-in case above. Thus for $V_T = 70$, when the firm loses 40, then $E_E = \tau V = 70 \times 7\% = 4.9$ so the equityholders lose $E_0 - E_E = 20 - 4.9 = 15.1$. The write-off bondholders lose all of their 20. This still leaves $40 - 15.1 - 20 = 4.9$ of the loss to be covered, which is written down by the plain vanilla bondholders. There is further equity-conversion bail-in of $E_B = (E - \tau) V_T = 70 \times (0.1 - 0.07) = 2.1$ to attain the required
capital ratio $E$. As a result, the vanilla bondholders are left with $D_B = 70 - 4.9 - 2.1 = 63$ of the bond.

In summary,

<table>
<thead>
<tr>
<th>$V_T$</th>
<th>$0, \frac{\tau_i}{\tau_i + \beta}$</th>
<th>$\frac{\tau_i}{\tau_i + \beta}$, $\frac{\tau_i}{\tau_i + \beta}$</th>
<th>$\frac{\tau_i}{\tau_i + \beta}$, $\frac{\tau_i}{\tau_i + \beta}$</th>
<th>$\frac{\tau_i}{\tau_i + \beta}, V_0$</th>
<th>$[V_0, \infty]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_B$</td>
<td>$(1-E)V_T$</td>
<td>$F_B$</td>
<td>$F_B$</td>
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<td>$F_B$</td>
</tr>
<tr>
<td>$D_W$</td>
<td>0</td>
<td>0</td>
<td>$(1-E)V_T - F_B$</td>
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</tr>
<tr>
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</tr>
<tr>
<td>$E_{CCR}$</td>
<td>0</td>
<td>$(1-\tau)V_T - F_B$</td>
<td>$(E-\tau)V_T$</td>
<td>0</td>
<td></td>
</tr>
<tr>
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<td>0</td>
<td></td>
</tr>
<tr>
<td>Total Firm</td>
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<td>$V_T$</td>
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</tbody>
</table>

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<tr>
<th>Capital ratio</th>
<th>$E$</th>
<th>$[\tau, E]$</th>
<th>$E$</th>
<th>$[\tau, \frac{E}{\tau + \beta}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notes</td>
<td>Forced bail-in by vanilla bondholders</td>
<td>WD bond wholly written down. $E$ unattainable.</td>
<td>WD bond written down to cover loss and converted to attain $E$.</td>
<td>$E_E$ written down. Capital ratio $\geq \tau$.</td>
</tr>
</tbody>
</table>

It is unclear who claims the contingent capital reserve $E_{CCR}$. Importantly, if $E_{CCR}$ remains with the write-down bondholders, then the above outcomes are identical to the equity-conversion bail-in-bail-in case in Section 2.5.3. We therefore assume here that the claim for $E_{CCR}$ is transferred to the equityholders. In this case the payoffs can be summarised as,$^{209}$

$$D_B^{WDWI} + E_B^{WDWI} = \min [(1-\tau)V_T, F_B]$$

$$E_E^{WDWI} + E_{CCR}^{WDWI} = V_T - F + \left( \frac{E-\tau}{\tau - \beta} \right) F x_{V_T \leq \frac{F}{\tau - \beta}} + (1-E) \left( \max \left[ \frac{F}{\tau - \beta} - V_T, 0 \right] \right) - \left[ F x_{V_T \leq \frac{F}{\tau - \beta}} - V_T, 0 \right] + (1-\tau) \max \left[ \frac{F}{\tau - \beta} - V_T, 0 \right]$$

$$D_W^{WDWI} = F_w - \left( \frac{E-\tau}{\tau - \beta} \right) F x_{V_T \leq \frac{F}{\tau - \beta}} - (1-E) \left( \max \left[ \frac{F}{\tau - \beta} - V_T, 0 \right] - \max \left[ \frac{F}{\tau - \beta} - V_T, 0 \right] \right).$$

$^{209}$As already noted this write-down-bail-in case is the redistribution of the equity-conversion bail-in-bail-in case, where the equity-converted portion of the CoCo bond $E_{CCR}^{WB}$ is now allocated to the equityholders as the contingent capital reserve $E_{CCR}^{WDWI}$. Therefore it is no surprise that $E_E^{WDWI} = E_E^{WB}$, $E_{CCR}^{WDWI} = E_{CCR}^{WB}$, $D_B^{WDWI} = D_B^{WB}$ and $E_B^{WDWI} = E_B^{WB}$.
Figure 7: Write-down-Bail-in: \( \tau = 7\% \), \( E = 10\% \), \( F_B = 90 \) and \( F_W = 20 \).

There is now a discontinuous jump in the payoff for the equityholders of \( \left( \frac{E - \tau}{1 - \tau} \right) \) \( F_{V_T \leq \frac{E}{1 - \tau}} \), due to the transfer of the CCR to them. This is also demonstrated in Fig.7. The BSM valuation of these are,

\[
V_{DB}^{WDBI} + V_{EB}^{WDBI} = F_B e^{-rT} - (1 - \tau) P \left( \frac{F_B}{1 - \tau} \right)
\]
\[
V_{EE}^{WDBI} + V_{ECCR}^{WDBI} = C(F) + \left[ (1 - \tau) P \left( \frac{E}{1 - \tau} \right) - P(F) \right] + \left( E - \tau \right) \left[ \frac{F}{1 - \tau} B_P \left( \frac{E}{1 - \tau} \right) - P \left( \frac{F}{1 - \tau} \right) \right] - (1 - E) P \left( \frac{F_B}{1 - E} \right) + (1 - \tau) P \left( \frac{F_B}{1 - E} \right)
\]
\[
V_{DW}^{WDBI} = F_w e^{-rT} - \left( \frac{E - \tau}{1 - \tau} \right) F_B P \left( \frac{F}{1 - \tau} \right) - (1 - E) \left[ P \left( \frac{F}{1 - \tau} \right) - P \left( \frac{F_B}{1 - E} \right) \right]
\]

where \( B_C(K) \) and \( B_P(K) \) are the price of the binary call and put options with unit payout at strike \( K \),

\[
B_C(K) = e^{-rT} N(\sigma_2(K))
\]
\[
B_P(K) = e^{-rT} N(-\sigma_2(K)).
\]

2.6.2. Write-off-Bail-in

The write-off bond differs from the write-down bond above in that the entire bond is written-off at once for values of \( V_T < \frac{F}{1 - \tau} \). Then upon trigger immediately, \( D_W = 0 \). Any remainder net of the loss is then assumed to be added to the firm’s capital as the contingency capital reserve.
Consider again the example $V_T = 80$ when the firm loses 30. As before $E_E = \tau V_T = 80 \times 7\% = 5.6$ and so the equityholders bear the loss of $E_0 - E_E = 20 - 5.6 = 14.4$, and the write-off bond is triggered to cover the rest of the loss. Of $F_W = 20$, $30 - 14.4 = 15.6$ is required to write-off this loss, while the remaining $20 - 15.6 = 4.4$ is added to the equity capital as $E_{CCR}$. The capital ratio $E_{E+E_{CCR}} = \frac{V_T - F_B}{V_T} = \frac{80-70}{80} = 12.5\%$ is now above the minimum ratio $E$. This would be true for values of $V_T$ for which $\frac{V_T - F_B}{V_T} \geq E \iff V_T \geq \frac{F_B}{1-E}$. For $V_T$ below this level, we again assume forced bail-in by the vanilla bondholders, as was the case for both the equity-conversion bail-in-bail-in and the write-down-bail-in.

In summary then,

<table>
<thead>
<tr>
<th>$V_T$</th>
<th>$0, \frac{E_B}{1-\tau}$</th>
<th>$\frac{F_B}{1-\tau}, \frac{F_B}{1-\tau}$</th>
<th>$\frac{\tau}{1-\tau}, V_0$</th>
<th>$[V_0, \infty]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_B$</td>
<td>$(1-\tau)V_T$</td>
<td>$F_B$</td>
<td>$F_B$</td>
<td>$F_W$</td>
</tr>
<tr>
<td>$D_W$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$V_T - F$</td>
</tr>
<tr>
<td>$E_E$</td>
<td>$\tau V_T$</td>
<td>$\tau V_T$</td>
<td>$\tau V_T$</td>
<td>$\frac{E_B}{1-\tau}$</td>
</tr>
<tr>
<td>$E_{CCR}$</td>
<td>$0$</td>
<td>$(1-\tau)V_T - F_B$</td>
<td>$(1-\tau)V_T - F_B$</td>
<td>0</td>
</tr>
<tr>
<td>$E_B$</td>
<td>$(E-\tau)V_T$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total Firm</strong></td>
<td>$V_T$</td>
<td>$V_T$</td>
<td>$V_T$</td>
<td>$V_T$</td>
</tr>
<tr>
<td><strong>Capital ratio</strong></td>
<td>$E$</td>
<td>$[\tau, E]$</td>
<td>$E$</td>
<td>$[\tau, \frac{E_B}{1+\tau E_B}]$</td>
</tr>
<tr>
<td><strong>Notes</strong></td>
<td>Forced bail-in by vanilla bondholders</td>
<td>$E$ unattainable even with the CCR.</td>
<td>WO bond triggered. Remainder net of loss added as CCR. $E$ not breached.</td>
<td>$E_E$ written down. Capital ratio $\geq \tau$.</td>
</tr>
</tbody>
</table>

As in the write-down-bail-in case, here we assume that the contingent capital reserve is transferred to the equityholders, although in reality it is unclear who would lay the claim to
this capital. Then the payoffs can be summarised as,\(^{21}\)

\[
\begin{align*}
D_B^{WOBI} + E_B^{WOBI} &= \min \left[(1 - \tau) V_T, F_B\right] \\
E_E^{WOBI} + E_{CCR}^{WOBI} &= V_T - F + F_W X_{V_T \leq \frac{F}{1-\tau}} + (1 - \tau) \max \left[\frac{F_B}{1-\tau} - V_T, 0\right] \\
D_W^{WOBI} &= F_W X_{V_T > \frac{F}{1-\tau}}
\end{align*}
\]

Fig. 8 shows the bondholders’ and equityholders’ payoffs. The BSM valuation of the debt and equity holdings at time \(t = 0\) are,

\[
\begin{align*}
V_{DB}^{WOBI} + V_{EB}^{WOBI} &= F_B e^{-rF} - (1 - \tau) P \left(\frac{F_B}{1-\tau}\right) \\
V_{EE}^{WOBI} + V_{ECCR}^{WOBI} &= C(F) + F_W B_P \left(\frac{F}{1-\tau}\right) + \left[(1 - \tau) P \left(\frac{F}{1-\tau}\right) - P\left(F\right)\right] - \left[P\left(\frac{F}{1-\tau}\right) - P\left(F_B\right)\right] \\
V_{DW}^{WOBI} &= F_W B_C \left(\frac{F}{1-\tau}\right)
\end{align*}
\]

\(^{(25)}\)

The equityholders’ position \(V_{EE}^{WOBI} + V_{ECCR}^{WOBI}\) in Eq. (27) differs from the expression for “Convert-to-steal CoCo” in Berg and Kaserer (2011), in that here the trigger point is at \(\frac{F}{1-\tau}\), and there

\(^{21}\)For vanilla bondholders the payoffs are the same as in the equity-conversion bail-in-bail-in case, i.e. \(D_B^{WOBI} = D_E^{WOBI}\) and \(E_B^{WOBI} = E_E^{WOBI}\). For the equityholders, \(E_E^{WOBI} = E_E^{WOBI}\) and

\[
E^{WOBI}_{CCR} = (1 - \tau) \left(\max \left[V_T - \frac{F_B}{1-\tau}, 0\right] - \max \left[V_T - \frac{F}{1-\tau}, 0\right]\right) - F_W X_{V_T \geq \frac{F}{1-\tau}}.
\]
Figure 9: Write-off Condor: Write-off-Bail-in

is a forced bail-in by the vanilla bondholders at $\frac{F_B}{1-\tau}$.

Analogous to the CoCo bail-in analysis, the difference between $V_{E_E}^{WOBI} + V_{E_C}^{WOBI}$ in Eq.(27) and $V_{E_E}^{N}$ in Eq.(3) represents the wealth-transfer induced by the introduction of DAPR:

\[
\left( V_{E_E}^{WOBI} + V_{E_C}^{WOBI} \right) - V_{E_E}^{N} = \sum B \left( \frac{F}{1-\tau} \right) + \left[ (1-\tau) P \left( \frac{F}{1-\tau} \right) - P (F) \right] - \left[ P \left( \frac{F_B}{1-\tau} \right) - P \left( \frac{F_B}{1-\tau} \right) \right].
\]  

(28)

This we call a Write-off condor, shown in Fig.9.
2.7. Analysis

The BSM valuations of the equityholders’ positions in Eqs.(3), (7), (11), (15), (19), (22) and (27) are summarised below, but with \( C(F) \) replaced with \( V_0 - F e^{-rT} + P(F) \):

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Valuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Bail-out/in</td>
<td>( V_0 - F e^{-rT} + P(F) )</td>
</tr>
<tr>
<td>Bail-out</td>
<td>( V_0 - F e^{-rT} + P(F) )</td>
</tr>
<tr>
<td>Bail-in-No-bail-out/in</td>
<td>( V_0 - F e^{-rT} + (1 - \tau) P \left( \frac{F}{1 - \tau} \right) - \left[ (1 - \tau) P \left( \frac{F_B}{1 - \tau} \right) - P(F_B) \right] )</td>
</tr>
<tr>
<td>Bail-in-Bail-out</td>
<td>( V_0 - F e^{-rT} + (1 - \tau) P \left( \frac{F}{1 - \tau} \right) - \left[ (1 - \tau) P \left( \frac{F_B}{1 - \tau} \right) - P(F_B) \right] )</td>
</tr>
<tr>
<td>Bail-in-Bail-in</td>
<td>( V_0 - F e^{-rT} + (1 - \tau) P \left( \frac{F}{1 - \tau} \right) )</td>
</tr>
<tr>
<td>Write-down-Bail-in</td>
<td>( V_0 - F e^{-rT} + (1 - \tau) P \left( \frac{F}{1 - \tau} \right) + (E - \tau) \left[ \frac{F_B}{1 - \tau} B_P \left( \frac{F}{1 - \tau} \right) - P \left( \frac{F}{1 - \tau} \right) \right] )</td>
</tr>
<tr>
<td>Write-off-Bail-in</td>
<td>( V_0 - F e^{-rT} + \left[ F_W B_P \left( \frac{F}{1 - \tau} \right) + (1 - \tau) P \left( \frac{F_B}{1 - \tau} \right) \right] ).</td>
</tr>
</tbody>
</table>

Writing these in this way clarifies the protection each scheme offers to the equityholders. For example in both the no-bail-out/in and bail-out cases the equityholders are protected by the put option with strike price \( F \). On the other hand with equity-conversion bail-in-bail-in, the equityholders’ protection is by \( 1 - \tau \) of a put option with a higher strike price \( \frac{F}{1 - \tau} \)

These protections are plotted respectively in Figs 10 and 11. They clearly demonstrate the increasing protection for the equityholders in the order of (i) no bail-out/in and bail-out, (ii) bail-in-no-bail-out/in and bail-in-bail-out, (iii) bail-in-bail-in, (iv) write-down-bail-in, and (v) write-off-bail-in. In other words at each step, there is an extra incremental “put-spread” or “condor-like” option structure inherently sold by the bail-out / bail-in providers to the equityholders. These lead to increasing agency costs, as discussed in Sections 3 and 4.
Figure 10: Equityholders’ Protections for No Bail-out/in, Government Bail-out and Equity-conversion Bail-in

Figure 11: Equityholders’ Protections for Write-down-Bail-in and Write-off-Bail-in
2.8. Extensions

2.8.1. Valuation before Bond Maturity

As discussed in Section 2.4, the government bail-out provides no benefit to equityholders at bond maturity. This is not the case before maturity $t < T$, where the bail-out enables the firm to continue operating as going-concern in cases where the firm would otherwise become gone-concern. This provides the equityholders with a strictly positive time value of the continuing option, which is the benefit of the bail-out to the equityholders.

To demonstrate this, consider an inspection by the regulator at time $t < T$. Assume that in the case of no bail-out/in the firm is closed down if its capital ratio is below the minimum equity ratio $E$, i.e. $V_t < \frac{F}{E}$. In this case the value of the equityholders’ position at $t$ is,

$$V_{EE}^N = \begin{cases} \max [V_t - F, 0] & \text{if } V_t < \frac{F}{E}, \\ C (V_t, F, r, \sigma, T - t) & \text{if } V_t \geq \frac{F}{E} \end{cases},$$

where $C (S, K, r, \sigma, s)$ is the price of a call option given by (4), with the price of the underlying asset $S$, strike price $K$, continuously compounding interest rate $r$, volatility $\sigma$ and the time to maturity $s$. The payoff reflects the fact that when the capital ratio is below $E$ and the firm is forced to close, the equityholders are left with the intrinsic value of the call option. This is not the case when there is government bail-out:

$$V_{EE}^{BO} = \begin{cases} \max[V_t - F, 0] C \left(\frac{F}{E}, F, r, \sigma, T - t\right) & \text{if } V_t < \frac{F}{E}, \\ C (V_t, F, r, \sigma, T - t) & \text{if } V_t \geq \frac{F}{E} \end{cases}.$$

Upon inspection, if the capital ratio is less than $E$ the government injects common equity $E_G$ to boost the asset value to $\frac{F}{E}$. The total equity $E_E + E_G$ is then $\frac{E + E_G}{E} F$. The market value of this total equity is $C \left(\frac{F}{E}, F, r, \sigma, T - t\right)$, with the equityholders holding a share $\frac{\max[V_t - F, 0]}{\frac{F}{E}}$.
of it. This represents the dilution resulting from the common equity capital injection. Now $V_{BO}^E > V_{E}^N$ unambiguously as,

$$\frac{\max [V_t - F, 0]}{E} C \left( \frac{F}{1 - E}, F, r, \sigma, T - t \right) > \max [V_t - F, 0]$$

(32)

$$\Leftrightarrow C \left( \frac{F}{1 - E}, F, r, \sigma, T - t \right) > \frac{E}{1 - E} F,$$

where $\frac{E}{1 - E} F = \frac{E}{1 - E} - F$ is the intrinsic value of $C \left( \frac{E}{1 - E}, F, r, \sigma, T - t \right)$. This clearly illustrates the equityholders’ benefit from the government bail-out, which is their share $\frac{\max \{V_t - F, 0\}}{\frac{E}{1 - E} F}$ of the time value of the continuing call option $C \left( \frac{E}{1 - E}, F, r, \sigma, T - t \right) - \frac{E}{1 - E} F$.

2.8.2. Preference Share

So far the government bail-out has been assumed to be conducted by an injection of common equity only. Here we extend this to include preference shares injection. We assume a minimum common equity floor $EC < E$, such that the government’s preference shares $EP$ are utilised to attain the minimum capital ratio $E$, while the government’s common equity bail-out $EG$ is used to maintain $EC$. The former kicks in if the common equity ratio is below $\tau$, with $EP$ boosting the asset value to $\frac{E}{1 - E}$ and the total equity to $\frac{E}{1 - E} F$ as before. Then $EP = \frac{E}{1 - E} F - (V_t - F) = \frac{1}{1 - E} F - V_t$. The latter kicks in if the common equity ratio even after the preference share injection is below the minimum common equity ratio $EC$, which occurs when $\frac{V_t - F}{1 - E} < EC \Leftrightarrow V_t < \frac{1 - EC}{1 - E} F$. Then the equityholders’ values at $t < T$ are,

$$V_{BO}^{EC} = \begin{cases} 
\frac{\max \{V_t - F, 0\}}{\frac{E}{1 - E}} C \left( \frac{F}{1 - E}, 1 - \frac{EC}{1 - E} F, r, \sigma, T - t \right) & \text{if } V_t < \frac{1 - EC}{1 - E} F \\
C \left( \frac{E}{1 - E}, 1 - \frac{EC}{1 - E} F - V_t, r, \sigma, T - t \right) & \text{if } \frac{1 - EC}{1 - E} F \leq V_t < \frac{F}{1 - E} \\
C (V_t, F, r, \sigma, T - t) & \text{if } V_t \geq \frac{F}{1 - E} 
\end{cases}$$

(33)
When there is no trigger \((V_t \geq \frac{F}{1-E_c})\), the equityholders’ value is the same as under no bail-out/in. When there is just the preference shares injection \((\frac{1-E+ E_c}{1-E} F \leq V_t < \frac{F}{1-E})\), then the equityholders’ position remains undiluted, but their claim at bond maturity \(T\) is now on the asset value \(V_T\) minus the sum of the bond face value \(F\) and the preference shares principal \(\frac{F}{1-E} - V_t\). The strike price of the call option is therefore \(F + \frac{F}{1-E} - V_t = \frac{2-E}{1-E} F - V_t\). Finally when there is also common equity injection \((V_t < \frac{1-E+ E_c}{1-E} F)\), then the equityholders’ share of equity is diluted to \(\max[V_t-F,0] \), where \(\frac{E_c}{1-E} F\) is the total common equity after bail-out. Their claim at \(T\) is on \(V_T - F\) minus the maximum preference share injection of \(\frac{E-E_c}{1-E} F\), and therefore the strike price of the call option is \(F + \frac{E-E_c}{1-E} F = \frac{1-E_c}{1-E} F\).

Using preference shares instead of common shares in order to attain the minimum equity ratio \(E\) has two opposing effects on the equityholders’ position. The positive effect is that of no or less dilution. Specifically, compared to Eq.(31), in Eq.(33) we can see that when \(\frac{1-E+ E_c}{1-E} F \leq V_t < \frac{F}{1-E}\) there is no dilution (only preference shares are injected), while when \(V_t < \frac{1-E+ E_c}{1-E} F\), the dilution is smaller (there is less common equity injected). The negative effect is that of reduced claim on the asset due to higher ranking of the preference shares. This is reflected in the higher strike price of the call options in Eq.(33) (note, \(\frac{2-E}{1-E} F - V_t > F\) for \(V_t < \frac{F}{1-E}\), which reduces the option value. Fig.12 shows that when \(F = 90, E_c = 10\%\), \(E_c = 5\%, r = 5\%, \sigma = 20\%\) and \(T - t = 0.5\), the positive effect of smaller dilution outweighs the negative effect of smaller claim. Indeed, it can be shown that this is always the case:

**Proposition 1** \(V_{E_c}^{BO}\) is unambiguously larger with preference shares than without.

**Proof.** Note when \(E_c = E\), the curves coincide in Fig.12. Therefore it suffices to show that the gap between the two curves at \(V_t = \frac{1-E+ E_c}{1-E} F\) (the kink of the preference shares curve)
increases as $E_C$ decreases, or

$$\frac{\partial}{\partial E_C} \left[ C \left( \frac{F}{1 - E} \frac{1 - E_C}{1 - E} F, r, \sigma, T - t \right) - \frac{E_C}{E} C \left( \frac{F}{1 - E}, F, r, \sigma, T - t \right) \right] < 0$$

$$\Leftrightarrow \frac{F}{1 - E} e^{-r(T-t)} N \left( d_2 \left( \frac{F}{1 - E} \frac{1 - E_C}{1 - E} F, r, \sigma, T - t \right) \right) < \frac{1}{E} C \left( \frac{F}{1 - E}, F, r, \sigma, T - t \right).$$

(34)

But $N \left( d_2 \left( \frac{F}{1 - E} \frac{1 - E_C}{1 - E} F, r, \sigma, T - t \right) \right) < N \left( d_2 \left( \frac{F}{1 - E}, F, r, \sigma, T - t \right) \right)$, and so it suffices to show that,

$$\frac{E}{1 - E} e^{-r(T-t)} N \left( d_2 \left( \frac{F}{1 - E}, F, r, \sigma, T - t \right) \right) < C \left( \frac{F}{1 - E}, F, r, \sigma, T - t \right)$$

$$\Leftrightarrow \frac{E}{1 - E} e^{-r(T-t)} N \left( d_2 (.) \right) < \frac{F}{1 - E} N \left( d_1 (.) \right) - Fe^{-r(T-t)} N \left( d_2 (.) \right)$$

$$\Leftrightarrow e^{-r(T-t)} N \left( d_2 (.) \right) < N \left( d_1 (.) \right).$$

(35)

This is true as $N \left( d_2 (.) \right) < N \left( d_1 (.) \right)$, since $d_1 = d_2 + \sqrt{T - t}$. Therefore as $E_C$ decreases below $E$, the equityholders are unambiguously better off with preference shares bail-out. ■

Figure 12: $V_{EE}^{BO}$ with and without preference shares
3. Agency Cost: Wealth-Transfer Problem

Having established the details of the different bail-out / in structures, we now investigate the agency costs associated with the over-investment problems in these structures. We distinguish two types of such agency costs:

1. **Wealth-transfer problem.** This is when the equityholders have an incentive for higher risk-taking, normally represented by the vega of their option position.

2. **Value-destruction.** Eberhart and Senbet (1993) state, “Risk-shifting can enhance equity value even when higher risk projects are of lower value, implying that investment decisions can be distorted away from firm value maximisation.” When negative NPV projects are still beneficial to the equityholders (due to their convex payoff and the project’s higher volatility), the reduction in the firm’s total value represents this type of agency cost.

We investigate these in turn in this section and the next. For the purpose of the technical analyses, we assume \( r > \frac{\sigma^2}{2} \) for the remainder of the paper.

As common in the literature (e.g. Eberhart and Senbet, 1993; Berg and Kaserer, 2011), we investigate the vega of the equityholder position as a measure of their incentive to take on riskier projects. The vega for each of the above cases are,

\[
\begin{align*}
Vega_{EE}^{N} &= Vega_{EE}^{BO} = V_0\sqrt{T}N'(d_1(F)) \\
Vega_{EE}^{CN} &= Vega_{EE}^{CBO} = V_0\sqrt{T}\left[(1 - \tau)N'(d_1\left(\frac{F}{1 - \tau}\right)) - (1 - \tau)N'(d_1\left(\frac{F_B}{1 - \tau}\right)) + N'(d_1(F_B))\right] \\
Vega_{EE}^{CBI} &= (1 - \tau)V_0\sqrt{T}N'(d_1\left(\frac{F}{1 - \tau}\right)) \\
Vega_{EE}^{WBI} &= V_0\sqrt{T}\left[(1 - E)N'(d_1\left(\frac{F}{1 - \tau}\right)) - (1 - E)N'(d_1\left(\frac{F_B}{1 - \tau}\right)) + (1 - \tau)N'(d_1\left(\frac{F_B}{1 - \tau}\right))\right] \\
&\quad + \frac{1}{\delta}d_1\left(\frac{F}{1 - \tau}\right)\left(E - \tau\right)e^{-rT}N'(d_2\left(\frac{F}{1 - \tau}\right)) \\
Vega_{EE}^{WOB1} &= \frac{1}{\delta}d_1\left(\frac{F}{1 - \tau}\right)F_1e^{-rT}N'(d_2\left(\frac{F}{1 - \tau}\right)) + (1 - \tau)V_0\sqrt{T}N'(d_1\left(\frac{F_B}{1 - \tau}\right)),
\end{align*}
\]
Figure 13: Vega vs $V_0$ when $F = 90$, $F_C = F_W = 20$, $\tau = 7\%$, and $E = 10\%$

where $N'(d_1(K)) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(d_1(K))^2}{2}}$ for strike price $K$. These are depicted in Fig.13. The graph compares the incentives for the equityholders to take on riskier projects at different values of $V_0$ above $F$ between the five structures. We make the following observations:

**Proposition 2** With no bail-out/in or government bail-out, the incentive for higher risk taking increases as the firm’s asset value falls towards the critical value $F$.

In Fig.13 the critical value for no bail-out/in is $F = 90$. This basically states that the vega of a call option increases as the option approaches at-the-money (ATM). More technically,

**Proof.** The vega curve for no bail-out/in and government bail-out, $Vega^N_{EE}$ and $Vega^BO_{EE}$ in Eq.(36), is quasi-concave as $e^{-x^2}$ is a quasi-concave function and $d_1(F)$ is a monotonically increasing function of $V_0$. Its maximum occurs at,

$$\frac{\partial Vega^N_{EE}}{\partial V_0} = -\frac{1}{\sigma} d_2(F) N'(d_1(F)) = 0 \iff V_0 = Fe^{-(r-\frac{\sigma^2}{2})T}. \quad (37)$$
Proposition 3 For the asset value above the trigger point, the risk-taking incentive is higher with equity-conversion CoCo bail-in than under no bail-out/in or the government bail-out.

Proof. We show this for the case of bail-in-bail-in, which would be true if \( \text{Vega}^{\text{CBI}}_{EE} > \text{Vega}^{\text{BO}}_{EE} \) for \( V_0 > \frac{F}{1-\tau} \), or

\[
(1 - \tau) V_0 \sqrt{T} N' \left( d_1 \left( \frac{F}{1-\tau} \right) \right) > V_0 \sqrt{T} N' \left( d_1 \left( F \right) \right) \quad \text{for} \quad V_0 > \frac{F}{1-\tau}.
\] (38)

This is proved in Appendix A. Note Fig.13 also depicts that this is true for the bail-in-no-bail-in/out and bail-in-bail-out cases, or \( \text{Vega}^{\text{CN}}_{EE} = \text{Vega}^{\text{CBI}}_{EE} > \text{Vega}^{\text{BO}}_{EE} \) for \( V_0 > \frac{F}{1-\tau} = 96.77 \).\textsuperscript{22}

\[\blacksquare\]

Proposition 4 For higher asset values the risk-taking incentive is higher with the write-off CoCo bond than with the equity-conversion CoCo bond.

Proof. For this we require \( \text{Vega}^{\text{WOB}}_{EE} > \text{Vega}^{\text{CBI}}_{EE} \) for sufficiently large \( V_0 \). It suffices to show that

\[
\frac{1}{\sigma} d_1 \left( \frac{F}{1-\tau} \right) F \text{We}^{-rT} N' \left( d_2 \left( \frac{F}{1-\tau} \right) \right) > (1 - \tau) V_0 \sqrt{T} N' \left( d_1 \left( \frac{F}{1-\tau} \right) \right) \quad \text{for large} \quad V_0.
\]

\[\text{Note from}\] \textsuperscript{22} Using Appendix A one can also prove that \( \text{Vega}^{\text{CBI}}_{EE} > \text{Vega}^{\text{CN}}_{EE} = \text{Vega}^{\text{CBO}}_{EE} \) for \( V_0 > \frac{F}{1-\tau} \).
the property of Black-Scholes option pricing model that \( V_0 N'(d_1(K)) = Ke^{-rT} N'(d_2(K)) \) for strike price \( K \). Then we require,

\[
\frac{1}{\sigma} d_1 \left( \frac{F}{1 - \tau} \right) \frac{F_W}{F} (1 - \tau) V_0 N' \left( d_1 \left( \frac{F}{1 - \tau} \right) \right) > (1 - \tau) V_0 \sqrt{T} N' \left( d_1 \left( \frac{F}{1 - \tau} \right) \right)
\]

\[
\Leftrightarrow d_1 \left( \frac{F}{1 - \tau} \right) > \frac{F}{F_W} \sigma \sqrt{T}
\]

\[
\Leftrightarrow V_0 > \frac{F}{1 - \tau} e^{-\left[ r - \left( \frac{F}{\tau} - \frac{1}{2} \right) \sigma^2 \right] T}.
\]

Thus \( Vega^{\text{WOB}}_{EE} \) is unambiguously larger than \( Vega^{\text{CB}}_{EE} \) for \( V_0 \) higher than \( \frac{F}{1 - \tau} e^{-\left[ r - \left( \frac{F}{\tau} - \frac{1}{2} \right) \sigma^2 \right] T} \).

\[37\]

For our numerical example, the expression on the right-hand side of Eq.(39) equals 102.25. The result of Proposition 4 can be checked in Fig.13. It is also clear in the diagram that \( Vega^{\text{WOB}}_{EE} \) is smaller than \( Vega^{\text{CB}}_{EE} \) for \( V_0 \) closer to the CoCo trigger point \( \left( \frac{F}{\tau} = 96.77 \right) \); indeed the vega for write-off is lower even than for no bail-out/in case. This reflects the shape of the vega curve for the binary put option in Eq.(27), which takes a negative value when the option is deep in-the-money. This means that to the left of the trigger point of the write-off condor structure depicted in Fig.9, the vega of the option structure rapidly decreases, and takes a negative value when the holders of the write-off condor (the equityholders) benefit from lower volatility.

To conclude, using the detailed analysis of the bail-out/in structures outlined in Section 2, in this section we have been able to establish that, in comparison to the no bail-out/in or government bail-out cases, the equity-conversion CoCo bond exacerbates the wealth-transfer element of the agency cost for all firm values above the CoCo trigger point, and that the effect is even larger for the write-off CoCo bonds for larger values of \( V_0 \). In option trading terms, this is analogous to the holder of an option having the right to determine the volatility of the underlying asset price. In financial markets, this would be an illegal manipulation.
4. Value Destruction

Value destruction agency cost occurs when the equityholders do not follow value maximisation for the firm. This is a principal-agent problem where the interest of the decision makers (the equityholders) does not align with that of the firm.

To investigate this, let there be a discrete set of projects defined by their expected outcome $E[V_T^i]$ and the return volatility $\sigma^i$. Let the market price of risk be $\lambda$. Then the present value of each project is,

$$V_0^i = e^{-r^i T} E[V_T^i]$$

where $r^i$ is the required rate of return of project $i$ and $r^f$ is the risk-free rate. Under value maximisation the firm would choose project $m$ such that,

$$V_0^m = \max_i \{ V_0^i \} .$$

On the other hand, under no bail-out the equityholders choose project $m^N$ such that,

$$V_0^{m^N} = \max_i \{ V_E^N (V_0^i) \} , \text{ where } V_E^N (V_0^i) = C (V_0^i, F, \sigma^i)$$

with $C(.)$ as given in Eq.(3), where the arguments now specify the underlying asset value, the strike price and the volatility. When $m^N \neq m$, $V_0^{m^N} < V_0^m$, and hence there is value destruction.

The value destruction problem arises from the fact that the firm value is determined as the expected present value (Eq.(40)) and does not depend on the asset volatility beyond its effect on the required rate of return $r^i$, while for the equityholders their value increases with higher $\sigma$ (positive vega of their call option position). Value destruction results when the reduction in the equityholders’ value due to the lower choice of $V_0^i$ (the delta effect), is more than offset by
the increase in the value due to the higher volatility (the vega effect). The degree of this effect can therefore be represented by the relative size of the two, which we denote $\eta$:

$$\eta = \frac{\Delta}{Vega}.$$  \hfill (43)

The smaller the $\eta$ of the structure, the more likely that there will be value destruction.

The delta of the equityholders’ positions for each bail-out/in structure are, respectively,

$$\Delta_{EE}^N = \Delta_{EE}^{BO} = N(d_1(F))$$

$$\Delta_{EE}^{CN} = \Delta_{EE}^{CBO} = (1 - \tau) N\left(d_1\left(\frac{F}{1-\tau}\right)\right) - (1 - \tau) N\left(d_1\left(\frac{F_B}{1-\tau}\right)\right) + N\left(d_1\left(F_B\right)\right)$$

$$\Delta_{EE}^{CRI} = \tau + (1 - \tau) N\left(d_1\left(\frac{F}{1-\tau}\right)\right)$$

$$\Delta_{EE}^{WDBI} = \tau - (E - \tau) \frac{e^{-\frac{\tau T}{2}}}{\sqrt{\pi}} N'\left(-d_2\left(\frac{F}{1-\tau}\right)\right) + (1 - E) N\left(d_1\left(\frac{F}{1-\tau}\right)\right)$$

$$- (1 - E) N\left(d_1\left(\frac{F_B}{1-\tau}\right)\right) + (1 - \tau) N\left(d_1\left(\frac{F_B}{1-\tau}\right)\right)$$

$$\Delta_{EE}^{WBOI} = \tau - \frac{F e^{-\tau T}}{\sqrt{\pi} V_0 \sigma \sqrt{T}} N'\left(-d_2\left(\frac{F}{1-\tau}\right)\right) + (1 - \tau) N\left(d_1\left(\frac{F}{1-\tau}\right)\right) + (1 - \tau) N\left(d_1\left(\frac{F_B}{1-\tau}\right)\right).$$  \hfill (44)

These are depicted on Fig.14.

We now make the following observations:

**Proposition 5** With no bail-out/in or government bail-out, the value destruction is more likely as the firm’s asset value falls towards the critical value $F$.

**Proof.** We already know from Eq.(44) that $\Delta_{EE}^N = \Delta_{EE}^{BO} = N(d_1(F))$. Then,

$$\frac{\partial \Delta_{EE}^N}{\partial V_0} = \frac{\partial \Delta_{EE}^{BO}}{\partial V_0} = \frac{1}{V_0 \sigma \sqrt{T}} N'(d_1(F)) > 0.$$  \hfill (45)

Therefore the deltas decrease as $V_0$ decreases. We also have established in Proposition 2 that $Vega_{EE}^N$ and $Vega_{EE}^{BO}$ increase as $V_0$ falls towards the critical value $F$. Therefore $\eta$ is unambiguously decreasing for falling $V_0$ above $F$, indicating a higher likelihood of value
Figure 14: Delta vs $V_0$ when $F = 90$, $F_C = F_W = 20$, $\tau = 7\%$ and $E = 10\%$

destruction.

**Proposition 6** For the asset value above the trigger point, the value destruction is more likely with equity-conversion CoCo bail-in-bail-in than under no bail-out/in or the government bail-out.

**Proof.** First we show that $\Delta \text{CBI}^{\text{CE}} < \Delta \text{BO}^{\text{CE}}$ for the required range of $V_0$, or

\[
N(d_1(F)) < \tau + (1 - \tau) N\left(\frac{F}{1 - \tau}\right)
\]

\[
\Leftrightarrow N(-d_1(F)) < (1 - \tau) N\left(-d_1\left(\frac{F}{1 - \tau}\right)\right).
\]  

(46)
To show this, consider the following derivative:

\[
\frac{\partial}{\partial V_0} \left[ N(-d_1(F)) - (1 - \tau) N\left(-d_1\left(\frac{F}{1-\tau}\right)\right) \right] = -\frac{1}{V_0 \sigma \sqrt{T}} \left[ N'(-d_1(F)) - (1 - \tau) N'\left(-d_1\left(\frac{F}{1-\tau}\right)\right) \right].
\] (47)

As \(N'(-d_1(.)) = N'(d_1(.))\), we know from Eq.(38) that this is positive for \(V_0 > \frac{F}{1-\tau}\). Also,

\[
\lim_{V_0 \to \infty} \left[ N(-d_1(F)) - (1 - \tau) N\left(-d_1\left(\frac{F}{1-\tau}\right)\right) \right] = 0
\] (48)

as the limit for both terms are zero. This means that \(N(-d_1(F)) - (1 - \tau) N\left(-d_1\left(\frac{F}{1-\tau}\right)\right)\) approaches 0 from below as \(V_0\) increases from \(\frac{F}{1-\tau}\), proving that \(\Delta E^{CB1}_{E_E} < \Delta E^{BO}_{E_E}\) for \(V_0 > \frac{F}{1-\tau}\). We also know from Proposition 3 that \(\text{Vega}^{CB1}_{E_E} > \text{Vega}^{BO}_{E_E}\) for \(V_0 > \frac{F}{1-\tau}\). Together this implies that \(\eta\) is unambiguously lower for equity-conversion bail-in-bail-in than for no bail-out/in or government bail-out. This indicates a higher likelihood of value destruction for the former. ■

Figs. 13 and 14 suggest that this is also true for the remaining equity-conversion CoCo bail-in cases, namely the bail-in-no-bail-out/in and the bail-in-bail-out.

**Proposition 7** For higher asset values the value destruction is more likely with the write-off CoCo bond than with the equity-conversion bail-in-bail-in case.

**Proof.** Again compare the deltas. \(\Delta E^{WOBI}_{E_E} < \Delta E^{CB1}_{E_E}\) if,

\[
-\frac{F_{WE} e^{-rT}}{V_0 \sigma \sqrt{T}} N'\left(-d_2\left(\frac{F}{1-\tau}\right)\right) + (1 - \tau) N\left(d_1\left(\frac{F_B}{1-\tau}\right)\right) - (1 - \tau) N\left(d_1\left(\frac{F}{1-\tau}\right)\right) < 0.
\] (49)

Appendix B proves that this is true for \(V_0 > \frac{F}{1-\tau}\). We already know from Proposition 4 that \(\text{Vega}^{WOBI}_{E_E} > \text{Vega}^{CB1}_{E_E}\) for sufficiently large \(V_0\). Together this implies that \(\eta\) is lower for the
write-off-bail-in for sufficiently large $V_0$ than for the equity-conversion bail-in-bail-in, indicating a higher likelihood of value destruction.

Figs. 13 and 14 suggest that this is also true for the write-down-bail-in case.

To conclude, not only do introduction of equity-conversion or write-down/off CoCo bonds increase the incentive for wealth-transfer by increasing the vega of the equityholders’ position, as shown in Section 3, in this section we have established that it also increases the incentive for value destruction by decreasing the delta, hence aggravating the delta-vega ratio $\eta$. Closer to the trigger point, this suggests a higher temptation to attempt “gamble-for-resurrection”, where the equityholders sacrifice firm value for high risk strategies, in the hope for a positive outcome.

5. CoCo Bond as Non-admissible Debt-to-Equity Swap

In the above sections we concluded that the equity-conversion CoCo and write-down/off bond bail-in structures have inherently higher agency costs than under no bail-out/in. However, in reality banks are rarely allowed to become insolvent, with restructuring occurring long before the asset value is allowed to fall below $F$. Here we consider one of those possibilities, the debt-to-equity swap (DES), and compare this with the CoCo bail-in structure.

We begin with a detailed investigation of the DES structure. The framework is an extension of Moraux and Navatte (2009) (MN here onwards). As before a firm is financed by equity and a zero coupon bond with maturity $T$ and face value $F$. Under normal APR the payoffs at $T$ for the debt and equityholders are similar to Eq.(2), with an additional feature of bankruptcy costs of a proportion $1 - \beta \in (0, 1]$ of the asset value $V_T$. Then according to MN, the bondholders’
payoff when there is no DES is,

\[ D_B^N = \begin{cases} F, & \text{if } V_T \geq F \\ \beta V_T, & \text{if } V_T < F \end{cases} \quad (50) \]

The existence of the bankruptcy cost means that there is a gain from restructuring, which the stakeholders can share. In a DES the bondholders rescue the equityholders by: (i) extending the existing debt by further \( s \) years to \( S = T + s \) at rate \( r \), and (ii) forgiving an amount \( A \in [0, F] \) of the debt while receiving a proportion \( \theta \in [0,1] \) of the firm’s equity in exchange.

As extreme examples, \( \theta = 0 \) with \( A > 0 \) means that the bondholders forgive part or all of the debt with no equity in return, while \( \theta = 1 \) means that they expropriate current equityholders.

As with MN we assume that bondholders control the financial restructuring but with no intent to take over the firm, i.e. \( \theta < 1 \). In contrast to MN, which assumes that the DES only kicks in when \( V_T < F \), we assume that DES is enforced by the regulator at the point of non-viability (PONV), which reflects the reality more. This level is further assumed to be the same as the bail-out/in trigger point \( \tau \) in previous sections. Hence DES is implemented when \( V_T \leq \frac{F}{1-r} \). Post-DES, at the new bond maturity \( S \) the face value of the debt is \( (F - A)e^{rs} \).\(^{23}\)

For \( V_T > \frac{F}{1-r} \) when there is no DES, the debt is assumed to roll over to \( S \) with the new face value \( Fe^{rS} \). For simplicity, we assume there to be no debt restructuring at \( S \) irrespective of the asset value \( V_S \).\(^{24}\) The table below summarises the stakeholders’ holdings at \( S \) for different scenarios:

\(^{23}\)This differs from MN, who seem to assume that the repayment for the remaining debt \( F - A \) is simply postponed until \( S \) with zero interest cost. The problem with this assumption is that, ceteris paribus, the debtholders would always want an immediate redemption.

\(^{24}\)This can be an extension to this analysis. Especially in the case that \( V_T \geq \frac{F}{1-r} \) at \( T \), it seems reasonable to allow DES at \( S \) when \( V_S < \frac{Fe^{rS}}{1-r} \), even if we rule out repeated restructuring for the case of \( V_T < \frac{F}{1-r} \) and \( V_S < \frac{(F-A)e^{rS}}{1-r} \).
The expected present value at time $T$ of the stakeholders’ payoff at time $S$ can now be calculated. When there is no DES at $T$, these are,

$$
V_{B_T}^{NoDES} = e^{-rs} \widehat{E}_T \left[ Fe^{rs} \chi_{V_S \geq Fe^{rs}} + \beta V_S \chi_{V_S < Fe^{rs}} \right] \\
= \beta V_T N \left( -d_1 (Fe^{rs}) \right) + FN \left( d_2 (Fe^{rs}) \right)
$$

Similarly when DES is triggered,

$$
V_{B_T}^{DES} = e^{-rs} \widehat{E}_T \left[ (F - A) e^{rs} + \theta [V_S - (F - A) e^{rs}] \chi_{V_S \geq (F - A)e^{rs}} + \beta V_S \chi_{V_S < (F - A)e^{rs}} \right] \\
= \beta V_T + (\theta - \beta) V_T N \left( d_1 ((F - A) e^{rs}) \right) + (1 - \theta) \left( F - A \right) N \left( d_2 ((F - A) e^{rs}) \right)
$$

Note as a special case, when $s \to \infty$, $V_{B_T}^{DES} = \beta V_T + (\theta - \beta) V_T$, i.e. for a very long term investment the creditors are incited to swap debt for equity only when $\theta > \beta$.
the bondholders controlling the financial restructuring, their problem is that of maximising their wealth with the choice of \((A, \theta, s)\). The equityholders are always better off with the DES for \(\theta < 1\), as they will receive a strictly positive claim instead of the zero value that would result from bankruptcy.

First consider the socially optimal outcome when DES is triggered (i.e. \(V_T < \frac{F}{1-\gamma}\)). This is where the total present value of the firm is maximised:

\[
\max_{A,s} e^{-rs} \hat{E}_T [V_S] = \beta V_T + (1 - \beta) V_T N(d_1 ((F - A)e^{-rs})).
\]  (54)

When there is no DES, then the firm is dissolved, losing bankruptcy costs. This is the first term \(\beta V_T\). The second term is the net gain of restructuring of not incurring the bankruptcy costs, which happens with the probability \(N(d_1 ((F - A)e^{-rs}))\). In DES this gain is shared between the stakeholders. Eq.(54) is monotonically increasing in \(A\) for any given \(s\), and thus the first-best is attained at \(A^* = F\). This is intuitive: a strictly positive value of \(F - A\) implies a strictly positive probability of insolvency at the new maturity \(S\); given the non-zero bankruptcy costs \(1 - \beta\), this reduces the present value of the firm. Hence the firm value is maximised at \(A = \beta\) where the future insolvency probability is zero. Now comparing Eq.(54) with \(V^D_{DES}\) in Eq.(53) reveals that the socially optimal outcome can be implemented by the choice \(\theta^* = 1\). This is again intuitive; setting \(\theta^* = 1\) aligns the interests of the decision-maker (in the case of a DES, the bondholders) with that of the total firm. Thus any equilibrium outcome with \(\theta^* \in (0,1)\) would be second-best. However,

**Proposition 8** When the bondholders have the full bargaining power, their optimal strategy is the full takeover of the firm, \((A, \theta) = (F, 1)\).
Proof. Consider the following derivatives of $V_{B_T}^{DES}$:

$$\frac{\partial V_{B_T}^{DES}}{\partial \theta} = V_T N \left( d_1 \left( (F - A) e^{rs} \right) \right) - (F - A) N \left( d_2 \left( (F - A) e^{rs} \right) \right) > 0 \ \forall \theta \ \forall A \tag{55}$$

$$\frac{\partial^2 V_{B_T}^{DES}}{\partial \theta^2} = \frac{1 - \theta}{\sigma \sqrt{\theta}} N' \left( d_2 \left( (F - A) e^{rs} \right) \right) - (1 - \theta) N \left( d_2 \left( (F - A) e^{rs} \right) \right).$$

The monotonicity of $\frac{\partial V_{B_T}^{DES}}{\partial \theta}$ is not surprising; whilst $A$ determines the future default probability, and hence the present value of the firm, $\theta$ simply determines the stakeholders’ shares of it. Therefore for any levels of $A$ the bondholders would prefer the full transfer $\theta^* (A) = 1$. The second derivative in Eq.(55) suggests that there are two opposing effects of an increase in $A$ on the value of $V_{B_T}^{DES}$: an increase in $A$ increases the total value of the firm by decreasing the probability of future default, but it also decreases the face value of the bond holding. Now suppose that the optimal outcome is $(A^*, \theta^*)$ with $A^* < F$ and $\theta^* < 1$. However this outcome is not stable, as we know that $\theta^* (A) = 1 \ \forall A$. At $\theta = 1$, $\frac{\partial V_{B_T}^{DES}}{\partial A} > 0$ unambiguously, and the bondholders will optimally choose $A = F$ where $V_{B_T}^{DES} = V_T$. This is the only stable outcome.

Thus for equityholders there is a simple trade-off between $A$ and $\theta$. Their ICs, drawn on a graph, show the indifference curves. Consider the following derivatives of $V_{E_T}^{DES}$:

$$\frac{\partial V_{E_T}^{DES}}{\partial \theta} = - \left[ V_T N \left( d_1 \left( (F - A) e^{rs} \right) \right) - (F - A) N \left( d_2 \left( (F - A) e^{rs} \right) \right) \right] < 0 \ \forall \theta \ \forall A \tag{56}$$

$$\frac{\partial^2 V_{E_T}^{DES}}{\partial \theta^2} = (1 - \theta) N \left( d_2 \left( (F - A) e^{rs} \right) \right) > 0 \ \forall \theta \ \forall A.$$

Thus for equityholders there is a simple trade-off between $A$ and $\theta$. Their ICs, drawn on a

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25Similarly, $\frac{\partial V_{E_T}^{DES}}{\partial \theta} < 0 \ \forall \theta \ \forall A$ implies that the equityholders would always prefer $\theta^* (A) = 0$. 

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Figure 15: Indifference Curves for Equityholders and Bondholders: $V_T = 70$, $F = 90$, $r = 6\%$, $\sigma = 20\%$, $s = 2$ and $\beta = 0.8$

$\theta - A$ plane, will therefore be upward-sloping. The ICs of the bondholders are downward-sloping for lower values of $A$ where $\frac{\partial V_{DES}^{B_T}}{\partial A} > 0$, while they turn upward-sloping for larger values of $A$ when $\frac{\partial V_{DES}^{B_T}}{\partial A} < 0$. Examples of these ICs are depicted in Fig.15, where the solid curves are the bondholders’ ICs and the dashed curves are those of the equityholders. The equityholders are better off with lower ICs, while the bondholders are better off with higher ICs. The turning point in the bondholders’ ICs is where they can attain the highest IC given the value of $\theta$, i.e. where the value of their holding is maximised for that level of $\theta$.26 In all cases the tangency points of the ICs are shown to be at $A = 90$, i.e. at $A = F$. There is a continuum of such Pareto efficient equilibria, and the choice of the final outcome depends on the relative bargaining power of the stakeholders. With full bargaining power the bondholders optimise at $V_{B_T}^{DES} = V_T$, attained by the choices $(A^*, \theta^*) = (F, 1)$, as derived in Proposition 8. On the other hand if the equityholders had the full bargaining power, it would result in $V_{B_T}^{DES}$ being driven down to the bondholders’ outside option, which is default without DES when they would receive $\beta V_T$. Note that this minimum value for the bondholders ensures that

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26 For example for $\theta = 0.1$, the maximum value for the bondholders is that of the IC tangent to the horizontal line $\theta = 0.1$. On the diagram this is shown to be $V_{B_T}^{DES} = 57.27$. 

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\( \theta^* \geq \beta \) for all cases of DES.

In reality, however, we observe partial forgiveness \( A < F \). In order to achieve this we introduce a cost term \( C(\theta) \) for the bondholders of higher control of the firm, with \( C' > 0 \), \( C'' > 0 \), \( C(0) = 0 \) and \( \lim_{\theta \to 1} C(\theta) = \infty \). This reflects the bondholders’ reluctance to take over the firm. This may be because they lack the company and industry expertise and the know-how of shareholders, or that their mandate to invest in low yielding, low volatile instruments may mean that they would be forced to sell equity position. For our analysis here we let

\[
C(\theta) = \frac{k\theta}{1-\theta}
\]

for some constant \( k > 0 \). We will see below that the inclusion of the cost term results in the bondholders choosing an outcome that is less than the socially optimal. The bondholders’ value under DES is now,

\[
V_{B_T}^{DES} = \beta V_T + (\theta - \beta) V_T N (d_1 ((F - A) e^{rs})) + (1 - \theta) (F - A) N (d_2 ((F - A) e^{rs})) - \frac{k\theta}{1-\theta}.
\]

(57)

Now we define the admissibility of a DES under full bondholder bargaining power as follows:\(^{27}\)

**Definition 9 (DES Admissibility)** A DES structure is admissible if the parameters \((A^*, \theta^*, s^*)\)

\(^{27}\)In their definition of admissibility, MN has the condition that the received portion of equity exactly covers the amount of the face value of the debt forgiven,

\[
A = e^{-rs} \mathbb{E}_T \left[ \theta \left[ V_S - (F - A) e^{rs} \right] 1_{V_T \geq (F - A) e^{rs}} \right]
\]

\[
= \theta \left[ V_T N (d_1 ((F - A) e^{rs})) - (F - A) N (d_2 ((F - A) e^{rs})) \right].
\]

(58)

Denote Eq.(58) by \( f(A, s) \). They argue that this condition may be viewed as an equilibrium condition: for bondholders, the amount \( A \) forgiven is the maximum for a given portion \( \theta \) of equity received, while for equityholders, it is the minimum amount acceptable. We believe that this is not the case. In their Table 1 they simulate the optimal \((A^*, s^*)\) for \( \theta = 0.5 \) and different values of \( \beta \) and \( V_T \). Then for \( \beta = 0.8 \) and \( V_T = 30 \), their admissible \((A^*, s^*)\) are computed as \((1.25, 2.48)\). In this case \( f(1.25, 2.48) = 2.25 \), i.e. the PV \( V \) of the equity received (in their set-up - see footnote 23) equals the forgiven amount, as assumed. Then \( V_{B_T}^{DES} = 24.07 + 1.25 = 25.32 \), where \( 24.07 \) is the PV of the restructured bond. However for \( A = 7.04 \), it can be computed that \( f(7.04, 2.48) = 2.25 \) and \( V_{B_T}^{DES} = 23.27 + 2.25 = 25.52 \). In other words, the bondholders are able to increase their value by forgiving \( A \) higher than the PV of the equity received. The equityholders are also better off as \( V_{E_S}^{DES} \) increases from 1.25 to 2.25, and this is therefore a Pareto-improving agreement. As such, MN’s admissibility condition does not yield a Pareto efficient outcome. Intuitively, by increasing \( A \) the bondholders are able to increase the value of their equity holding more than the loss in the value of the remaining debt.
maximise the value of bondholders’ holdings:

\[
(A^*, \theta^*, s^*) = \arg \max_{A, \theta, s} V_{B_T}^{DES} \text{ subject to } V_{B_T}^{DES} \geq \beta V_T
\]

where \( A \in [0, F], \ \theta \in [0, 1] \) and \( s \geq 0 \), and \( V_{B_T}^{DES} \) is given by Eq.(57).

This problem can be solved in two stages: first, find the optimal \((A^*, \theta^*)\) given \( s^* \), and second, find \( s^* \) with the maximum \( V_{B_T}^{DES} \). In focussing on the first stage, the first-order conditions are:

\[
\frac{\partial V_{B_T}^{DES}}{\partial A} = V_T N (d_1 ((F - A) e^{rs})) - (F - A) N (d_2 ((F - A) e^{rs})) - \frac{k}{(1-\theta)^2} = 0
\]

\[
\frac{\partial V_{B_T}^{DES}}{\partial \theta} = \frac{(1-\beta) N' (d_2 ((F - A) e^{rs})) - (1 - \theta) N (d_2 ((F - A) e^{rs}))}{\sigma \sqrt{2N'}} = 0
\]

(60)

Solving the simultaneous equations yields the optimal \( A^* \) as the solution to,

\[
V_T N (d_1 ((F - A) e^{rs})) - (F - A) N (d_2 ((F - A) e^{rs})) - \frac{k \sigma^2 s}{(1-\theta)^2} \left[ \frac{N (d_2 ((F - A) e^{rs}))}{N' (d_2 ((F - A) e^{rs}))} \right]^2 = 0
\]

(61)

and \( \theta^* \) given by,

\[
\theta^* = 1 - \frac{(1-\beta) N (d_2 ((F - A) e^{rs}))}{\sigma \sqrt{2N'} (d_2 ((F - A) e^{rs}))},
\]

(62)

Then:

1. \( A^* < F \), as \( A = F \) cannot be the solution for Eq.(61) as then the last term goes to \( \infty \).
2. \( \theta^* < 1 \).

Compare now this DES scheme with the equity-conversion CoCo bail-in-bail-in structure.

For the latter, the CoCo bail-in kicks in at time \( T \) as described in earlier sections. Unlike the earlier sections we introduce future default risk at time \( S > T \). For simplicity we assume that when the CoCo bonds are partially (but not wholly) converted at their maturity \( T \), then the
remaining matured CoCo bonds are replaced by vanilla bonds of the same face value. This means that there will be no CoCo bail-in at time $S$. Then the payoffs at $S$ are,

$$\begin{align*}
\text{No CoCo trigger} & \quad \left( \frac{F}{1-\tau}, \infty \right) \\
& \quad \begin{align*}
V_S & \geq Fe^{rs} \\
D_S & = Fe^{rs} \\
E_S & = V_S - Fe^{rs}
\end{align*} \\
& \quad \begin{align*}
V_S & < Fe^{rs} \\
D_S & = \beta V_S \\
E_S & = 0
\end{align*}
\end{align*}$$

$$\begin{align*}
\text{Partial CoCo trigger} & \quad \left( \frac{F_B}{1-\tau}, \frac{F}{1-\tau} \right) \\
& \quad \begin{align*}
V_S & \geq (1 - E) V_T e^{rs} \\
D_S & = (1 - E) V_T e^{rs} + \frac{E - \tau}{E} [V_S - (1 - E) V_T e^{rs}] \\
E_S & = \frac{\tau}{E} [V_S - (1 - E) V_T e^{rs}]
\end{align*} \\
& \quad \begin{align*}
V_S & < (1 - E) V_T e^{rs} \\
D_S & = \beta V_S \\
E_S & = 0
\end{align*}
\end{align*}$$

$$\begin{align*}
\text{Full CoCo trigger} & \quad \left[ \frac{F_B}{1-\tau}, \frac{F}{1-\tau} \right] \\
& \quad \begin{align*}
V_S & \geq Fe^{rs} + \frac{(1-\tau)VT e^{rs}}{E - \tau} (V_S - Fe^{rs}) \\
D_S & = Fe^{rs} + \frac{(1-\tau)VT e^{rs}}{E - \tau} (V_S - Fe^{rs}) \\
E_S & = \frac{\tau}{E} [V_S - (1 - E) V_T e^{rs}]
\end{align*} \\
& \quad \begin{align*}
V_S & < Fe^{rs} \\
D_S & = \beta V_S \\
E_S & = 0
\end{align*}
\end{align*}$$

$$\begin{align*}
\text{Forced bail-in by vanilla bondholders} & \quad \left[ 0, \frac{F_B}{1-\tau} \right] \\
& \quad \begin{align*}
V_S & \geq (1 - E) V_T e^{rs} \\
D_S & = (1 - E) V_T e^{rs} + \frac{E - \tau}{E} [V_S - (1 - E) V_T e^{rs}] \\
E_S & = \frac{\tau}{E} [V_S - (1 - E) V_T e^{rs}]
\end{align*} \\
& \quad \begin{align*}
V_S & < (1 - E) V_T e^{rs} \\
D_S & = \beta V_S \\
E_S & = 0
\end{align*}
\end{align*}$$

(63)

For example, as established in Section 2.5.3, when $V_T \in \left( \frac{F_B}{1-\tau}, \frac{F}{1-\tau} \right)$ the CoCo bondholders end up with $D_C = (1 - E) V_T - F_B$ of the unconverted (and now matured) bond and $E_C = (E - \tau) V_T$ of equity. Therefore in aggregate the bondholders’ (the original vanilla bondholders and the CoCo bondholders) position is $F_B + [(1 - E) V_T - F_B] = (1 - E) V_T$ of vanilla bond and $(E - \tau) V_T$ of equity. At $S$ the firm will remain solvent if $V_S \geq (1 - E) V_T e^{rs}$, in which case the bondholders’ position is the total of $(1 - E) V_T e^{rs}$ of bond and a share $\frac{E - \tau}{E}$ of the equity $V_S - (1 - E) V_T e^{rs}$.

Then,

**Proposition 10** CoCo bond bail-in is a non-admissible DES.

**Proof.** Compare the CoCo payoffs for the bondholders in Table (63) with those of DES in Table (51). The two are equivalent when $V_T \geq \frac{F}{1-\tau}$ (no DES, no bail-in). When $V_T \in \left( \frac{F_B}{1-\tau}, \frac{F}{1-\tau} \right)$
and $V_T \in \left[0, \frac{F_B}{r}\right)$, the DES payoff is equivalent to the CoCo payoff when $A = F - (1 - E) V_T$ and $\theta = \frac{E - \tau}{\tau}$. Similarly when $V_T \in \left[\frac{F_B}{1 - \tau}, \frac{F_B}{\tau}\right)$, the two are equivalent when $A = F_C$ and $\theta = \frac{(1 - \tau)V_T - F_B}{V_T - F_B}$. This means that the CoCo outcome is within the (unconstrained) feasible set of possible DESs, but they do not satisfy the first-order conditions (61) and (62). Hence the CoCo outcome cannot be the admissible DES.

Basically, in contrast to the DES, in equity-conversion CoCo bail-in the bondholders are unable to negotiate $A$ or $\theta$ to achieve their optimal debt-to-equity conversion as these parameters are pre-defined at the CoCo bond inception. Indeed the write-off bond is the worst case scenario where $A$ and $\theta$ are pre-set at $(F,0)$.

Fig.16 graphs the present values ($PV$) at $T$ of the bondholders’ payoffs for both admissible DES and CoCo structures. The regions divided by vertical lines correspond to those specified in Table (63). The admissible DES $PV$ is simulated by solving Eq.(61) numerically for the optimal $A^*$ for each $V_T$, which is then substituted in Eq.(62) to compute $\theta^*$. As shown, the
bondholders are able to attain higher values by their choice of $A$ and $\theta$ than under the CoCo structure.

Fig. 17 graphs the $PV$ at $T$ of the equityholders’ payoffs for both admissible DES and CoCo structures. For them their $PV$ is higher under CoCo than under DES. The graph suggests that if the equityholders knew for certain that a DES would be implemented, then under DES too they would have an incentive to take higher risks and sacrifice firm value than would be under no bail-in case. However as shown, these agency costs are still higher under CoCo bail-in than for the traditional DES.

6. Concluding Remarks

The new financial regulation has been articulated to dampen moral hazard and to minimise the chances of another financial crisis that could jeopardise again the integrity of the banking system. However in reality, the regulator is “swapping” bail-out for bail-in, which is in essence
a replacement of moral hazard (banks relying on the inherent guarantee by the government) with agency costs. If the burden of an ailing bank fell to the taxpayers in the past, it will now fall to the bondholders who will be required to be very mindful about the investments they own in a bank. Historically, apart from the very few cases where the bank was fully nationalised (e.g. Bankia in 2012; SNS in 2013), the equityholders would simply suffer dilution (e.g. Lloyds and ING, both in 2008), or, in many cases, were unaffected with the injection of new equity in the form of preference shares with CT1 qualification (Goldman Sachs, Morgan Stanley, etc.).

Under the new bail-in regime, the equityholders take the first losses up to the CoCo trigger point where bondholders get written-down/off or converted into a non-admissible DES, whilst there is still at least 7.0% (the CoCo trigger ratio) of assets in equity. This going-concern DAPR accentuates the agency costs that the bail-in structure is introducing into the banking industry.

It is, moreover, possible that the new bail-in structure may even aggravate the moral hazard problem. Although not discussed in this paper, there is, in fact, a second moral hazard problem apart from that associated with the equityholders, which is that of the bondholders where they shirk on their monitoring effort when they know that their investments are guaranteed by the government bail-out. What the new bail-in structure does is to alleviate this second moral hazard problem, as it forces better monitoring by the bondholders that limits the risk taking of the banks. However the equityholders’ moral hazard remains, and one could argue that, in reality, the equityholders may have more incentives to “gamble for resurrection” when the wealth extraction comes from other investors (creditors) rather than taxpayers, as the media scrutiny, and hence the reputational impact, would likely be lower. This factor is enhanced by the fact that no further shareholders’ expropriation is allowed by the public fund until all possible bail-in is exhausted, as has been in the recent cases of Banco Espirito Santo, SNS Bank and Bankia bail-ins. Moreover, bail-in may not result in restrictions on dividends or bankers’
compensations as there would be with taxpayer bail-out. In summary, the bail-in structure solves the moral hazard problem of the stakeholder who cannot influence the bank performance rather than of those who can. These are issues not analysed in this paper but they enhance our case that the new financial regulation may not alleviate the incentive problems as aimed.

Traditional Corporate Finance literature has underscored the detrimental effects of agency costs on the relationships between bondholders and equityholders, especially due to the limited investment of the latter. Higher equity advocated by some (e.g. Admati et al. (2013)) does not attenuate the problem when the equityholders enjoy the implicit put of the bail-in-able balance sheet. Higher capital costs on risky investments (Risk-Weighted Asset inflation) could potentially make banks safer, but banks are volatile institutions with non-performing loans and speculative trading that makes the business unpredictable. Equityholders are aware of this and they will exploit the opportunity to pursue low Sharpe Ratio bets and speculate with the DAPR offered by the bondholders’ put. The aggravation of this agency cost will penetrate into the asset management industry (as the major owners of the bank’s debt), and ultimately into the real economy. These are broader issues that would be explored in future research. In this paper we focussed on aspects that arise within this new bail-in world. Wealth-transfer and value destruction are two consequences of the new structure, which is even more pronounced when bondholders do not have the chance to steer the restructuring to attain a fair agreement that partially compensates their losses, as would do in an admissible DES. To conclude, the new regulations do not solve the intrinsic moral hazard of the banking industry; instead they yield new unintended consequences.
References


Appendix

A. Proof of \( (1 - \tau) N' \left( d_1 \left( \frac{F}{1 - \tau} \right) \right) > N' \left( d_1 \left( F \right) \right) \) for \( V_0 > \frac{F}{1 - \tau} \)

For this to be true we require,

\[
(1 - \tau) e^{-\frac{1}{2}d_1^2 \left( \frac{F}{1 - \tau} \right)} > e^{-\frac{1}{2}d_1^2 (F)}.
\] (64)

Now,

\[
d_1 (F) = d_1 \left( \frac{F}{1 - \tau} \right) - \frac{1}{\sigma \sqrt{T}} \ln (1 - \tau).
\] (65)

Hence,

\[
e^{-\frac{1}{2}d_1^2 (F)} = e^{-\frac{1}{2}d_1^2 \left( \frac{F}{1 - \tau} \right)} e^{-\frac{1}{2} \left\{ -\frac{2}{\sigma \sqrt{T}} \ln (1 - \tau) d_1 \left( \frac{F}{1 - \tau} \right) + \frac{1}{2\sigma^2 T} \left[ \ln (1 - \tau) \right]^2 \right\}}.
\] (66)

Thus for (64) to be true,

\[
1 - \tau > e^{-\frac{1}{2} \left\{ -\frac{2}{\sigma \sqrt{T}} \ln (1 - \tau) d_1 \left( \frac{F}{1 - \tau} \right) + \frac{1}{2\sigma^2 T} \left[ \ln (1 - \tau) \right]^2 \right\}}
\]

\[
\Leftrightarrow \ln (1 - \tau) > \frac{1}{\sigma \sqrt{T}} \ln (1 - \tau) d_1 \left( \frac{F}{1 - \tau} \right) - \frac{1}{2\sigma^2 T} \left[ \ln (1 - \tau) \right]^2
\]

\[
\Leftrightarrow 1 < \frac{1}{\sigma \sqrt{T}} d_1 \left( \frac{F}{1 - \tau} \right) - \frac{1}{2\sigma^2 T} \ln (1 - \tau)
\]

\[
\Leftrightarrow \frac{1}{2\sigma \sqrt{T}} \ln (1 - \tau) < d_1 \left( \frac{F}{1 - \tau} \right) - \sigma \sqrt{T} = d_2 \left( \frac{F}{1 - \tau} \right).
\] (67)

Noting that \( \ln (1 - \tau) < 0 \) for \( \tau > 1 \), this is unambiguously satisfied when \( d_2 \left( \frac{F}{1 - \tau} \right) > 0 \Leftrightarrow \)

\[
V_0 > \frac{F}{1 - \tau} e^{-\left( r - \frac{\sigma^2}{2} \right) T},
\]

or definitely when \( V_0 \) is above the critical level \( \frac{F}{1 - \tau} \).
B. Proof of Eq.(49) being Negative

Consider the following derivative of Eq.(49) with respect to \( \Phi \) while keeping \( \Phi \) constant:

\[
\frac{\partial}{\partial F_W} \left[ -\frac{F_W e^{-rT}}{V_0 \sigma \sqrt{T}} N'(d_2 \left( \frac{F}{1-\tau} \right)) + (1-\tau) N \left( d_1 \left( \frac{F_B}{1-\tau} \right) \right) - (1-\tau) N \left( d_1 \left( \frac{F}{1-\tau} \right) \right) \right] = -\frac{e^{-rT}}{V_0 \sigma \sqrt{T}} N' \left( -d_2 \left( \frac{F}{1-\tau} \right) \right) + \frac{1-\tau}{F_B \sigma \sqrt{T}} N' \left( d_1 \left( \frac{F_B}{1-\tau} \right) \right) - \left( \frac{1-\tau}{F} \right) \frac{1}{\sigma \sqrt{T}} N' \left( -d_1 \left( \frac{F}{1-\tau} \right) \right) + \frac{1-\tau}{F_B \sigma \sqrt{T}} N' \left( d_1 \left( \frac{F_B}{1-\tau} \right) \right).
\]

(68)

The second identity uses a property of the Black-Scholes put option pricing formula, \( S_0 N'(-d_1(K)) = Ke^{-rT} N'(-d_2(K)) \). This is strictly negative if and only if,

\[
\frac{1}{F} N' \left( -d_1 \left( \frac{F}{1-\tau} \right) \right) > \frac{1}{F_B} N' \left( d_1 \left( \frac{F_B}{1-\tau} \right) \right).
\]

(69)

Analyse this:

\[
\Leftrightarrow \frac{1}{F} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_1^2 \left( \frac{F}{1-\tau} \right)} > \frac{1}{F_B} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_1^2 \left( \frac{F_B}{1-\tau} \right)}
\]

\[
\Leftrightarrow -d_1^2 \left( \frac{F}{1-\tau} \right) + d_1^2 \left( \frac{F_B}{1-\tau} \right) > 2 \ln \left( \frac{F}{F_B} \right)
\]

\[
\Leftrightarrow \left[ \ln \left( \frac{V_0 (1-\tau)}{F_B} \right) \right]^2 - \left[ \ln \left( \frac{V_0 (1-\tau)}{F} \right) \right]^2 + 2 \left( r + \frac{\sigma^2}{2} \right) T \ln \left( \frac{F}{F_B} \right) > 2\sigma^2 T \ln \left( \frac{F}{F_B} \right)
\]

\[
\Leftrightarrow \ln \left( \frac{V_0^2 (1-\tau)^2}{FF_B} \right) + 2 \left( r + \frac{\sigma^2}{2} \right) T > 2\sigma^2 T
\]

(70)

\[
\Leftrightarrow V_0 > \left( \frac{FF_B}{1-\tau} \right)^{\frac{1}{2}} e^{-\left( r - \frac{\sigma^2}{2} \right) T}.
\]

This is certainly satisfied for \( V_0 > \frac{F}{1-\tau} \) when \( r > \frac{\sigma^2}{2} \). Thus Eq.(49) is decreasing in \( F_W \) for this range of \( V_0 \). As Eq.(49) equals 0 for \( F_W = 0 \), this means that it is negative for all positive values of \( F_W \) when \( V_0 > \frac{F}{1-\tau} \).