A Monetary Intertemporal Model:
The Neutrality of Money, Long-Run Inflation, and Money Demand

- Extend the intertemporal model to capture money (monetary policy!)
- Monetary neutrality: a one time change in the level of money supply has no (long term) real consequences
- Monetary non-neutrality: a change in the GROWTH rate of money supply has (long term) real consequences
- Money is very closely correlated with price inflation that is costly
- Money demand function
Monetary Policy: Central Bank Targets

- Can Vary
- Can be Multiple
  - Inflation
  - Output Gap
  - Employment
  - Interest Rates
  - Exchange Rates
  - etc.
  - Some of them (or all together !!)
- Business Cycle

Potential Policy Instruments

- CB’s MAY control..
- Short term interest rates
- Reserve requirement ratios
- Monetary aggregates (Narrow or Broad)
  - Cash component of monetary aggregates, M1C
  - Monetary Base
What is Money?

- Medium of exchange
- Store of value
- Unit of account

US Monetary Aggregates (mln $)

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UK Monetary Aggregates

Monetary Intertemporal Model

Double coincidence of wants: Solution money

- Cash in Advance model (Clower 1967)
  - You need cash-in-advance to go shopping
Monetary Intertemporal Model

- Two periods: current and future
- Two assets: money (numeraire) and nominal bonds (can be issued by consumers or government and pays R)
  - Both fiscal (G,T) and monetary policy (M) possible
- No default on debt!
- No intermediaries: (banks)

Markets

- Labour market
- Goods market
- Money market
  - Credit market: implied by the eq’m
Real and Nominal Interest Rates and Fisher Relation

- Nominal bond: asset that sells for one unit of money in the current period and pays off $1+R$ unit of money in the future period
- $R$: nominal interest rate; $r$: real interest rate
- Inflation rate: $\text{INF} = (P_2 - P_1)/P_1$
- Fisher relation: $1+r = (1+R)/(1+\text{INF})$
- After substitution and collecting terms
- $r \approx R-\text{INF}$

Figure 10.1  **Real and Nominal Interest Rates, 1948–2003**
Representative Consumer

- Transactions are of particular sequence!
  - Inherits two assets from previous period (M and B) and pays taxes
  - Credit Market: Go to credit market and rearrange asset portfolio (by using money) and buy new bonds
  - Labour market: Go to firm offer employment at a market real wage \( w \) but will be paid only after the goods are sold!
  - Goods Market: Purchase goods in the goods market ONLY with money
  - At the end of the period: after goods are purchased, obtain real wages, and dividend income (all paid in money) and

Bank of England November 2005 Forecast
Current GDP projection based on market interest rate expectations
Current CPI inflation projection based on market interest rate expectations

The fan charts depict the probability of various outcomes for CPI inflation in the future. If economic circumstances identical to today’s were repeated over 100 occasions, the MPC’s best collective judgement is that inflation over the subsequent three years would lie within the darkest central band on only 10 of those occasions. The fan charts are constructed so that outcomes of inflation are also expected to lie within each pair of the lighter red areas of 10 occasions. Consequently, inflation is expected to lie somewhere within the entire fan charts on 90 out of 100 occasions. The bands widen as the time horizon is extended, indicating the increasing uncertainty about outcomes. See the box on pages 48–49 of the May 2002 Inflation Report for a fuller description of the fan chart and what it represents.

Consumer Decisions

- C,
- L,
- B^d,
- M

To do her own well being as good as possible!
Key Constraints


\[ PC_1 \leq M_0 + B_0 (1 + R_0) - P_1 T_1 - B_1^d \]

Consumer Budget Constr.

\[ P_1 C_1 + B_1^d + M_1^d = M_0 + (1 + R_0) B_0 + P_1 w_1 (h_i - l_i) + P_1 \pi_1 - P_1 T_1 \]

Figure 10.2 The Sequence of Transactions During a Period in the Monetary Intertemporal Model
Cash in advance constraint binds!

- That is, \( R > 0 \)!
- If \( R < 0 \) consumer is willing to hold more money than needed

Money Demand

\[
\frac{M^d_1}{P_1} = L(Y_1, R)
\]

Using Fisher relation

\[
\frac{M^d}{P} = L(Y, r + \text{INF})
\]

Or in terms of nominal money demand

\[
M^d = L(Y, r + \text{INF}) \times P
\]
Money Demand

Demand for money is determined by:

1. current real demand for money increases when real income increases (lifetime wealth increases, increasing the demand for future consumption goods)

2. current real demand for money decreases when nominal interest rate $R$ increases (opportunity cost of holding money increases)

Figure 9-2  The Nominal Money Demand Curve in the Monetary Intertemporal Model

$$M^d = P(Y, r + i)$$
Figure 9-3  The Effects of an Increase in Current Real Income on the Nominal Money Demand Curve

Government

- Responsible for both monetary and fiscal policy
- Government Budget Constraint
  - LHS: government spending
  - RHS: government receipts

\[ P_1G_1 + (1 + R_0)B_0 = P_1T_1 + B_1 + M_1 - M_0 \]
Competitive Equilibrium

- Equivalent to the real intertemporal model with the exception that now money matters
- For this purpose we need to add another market next to labour and goods markets
  ➔ Money market

Figure 9-7 The Current Money Market in the Monetary Intertemporal Model

Slide 25

Slide 26
Money Market

- Money demand function
  \[ M^d_1 = P_1 \times L(Y_1, R_1) \]
- Since \( M_s = M \) (exogenously set) nominal money market condition is
  \[ M_1 = P_1 \times L(Y_1, R_1) \]
- From Fisher equation
  \[ M_1 = P_1 \times L(Y_1, r_1 + \text{INF}_1) \]

Money Market (cont’d)

- Current real money demand is affected by
  - By changes in \( Y \), i.e. lifetime wealth,
  - by changes in \( r_1 \), due to intertemporal substitution effect on the future quantity of consumption goods.
  - Assume that long term inflation is constant

  \[ i.e. \, M^d_1/P_1 = L(Y_1, r_1) \]
A Level Increase in the Money Supply and Monetary Neutrality

- Suppose an increase in the money supply at period 1 and money supply level stays at the new level forever (permanent money supply shock)
- In Government BC

\[ P_1 G_1 + (1 + R_0) B_0 = P_1 T_1 + B_1 + M_1 - M_0 \]

- \( B_0, R_0 \) are predetermined based on the expectation that \( M_1 \) is not going to change
- Adjustment should take place from other variables
What causes money supply increase: Three Possibilities

- Gov’t can reduce current taxes: ‘helicopter drop’ (M. Friedman)
- Gov’t can reduce the quantity of B₁: ‘open market operations’
- Gov’t increase G₁, and to fund ΔG print money: seigniorage from inflation tax

- Assume helicopter drop (lump sum transfer of money to the representative agent)
What happens in eq’m after money level shock?

- Nothing in the labour and goods markets since none of the variables are dependent on M
- Classical dichotomy
- Real activity is orthogonal to nominal activity
- Monetary shock is compensated by a one-off price level adjustment
- Monetary neutrality!

Figure 9-10 The Effects of a Level Increase in $M$—The Neutrality of Money
Short-Run Analysis of a Temporary Decrease in Total Factor Productivity

- TFP shock

Figure 9-13 Short-Run Analysis of a Temporary Decrease in Total Factor Productivity
Figure 10.10  Relative Price of Energy

Figure 10.11  Percentage Deviations from Trend in the Price Level
Velocity of Money

- VM = PY

- V = measure of the number of times the money stock M turns over during a given current time period

- i.e. V = PY/M = Y/L(Y,R)

Figure 9-19 Scatter Plot of the Velocity of M1 vs. the Nominal Interest Rate, 1947-1999
Quantity Theory of Money and Monetarism

- Milton Friedman
  - \( M = \frac{1}{V} \cdot P \cdot Y \)
  - Assumption: money demand function is stable, thus \( V \) is stable
  - Predictable relationship between nominal income and money supply

Influential in the 70’s

- If as a Central Banker you can control money supply you can control inflation
  - It relies on stable money demand function
  - What if money demand is unstable?
Statistical Evidence

Granger Causality Tests

\[ \Delta y_t = \alpha + \sum_{i=1}^{4} \beta \Delta y_{t-i} + \sum_{i=1}^{4} \delta \Delta m_{t-i} + \nu_t \]

VAR's and Variance Decompositions

Stability Tests


### Table 1 — F Statistics for Financial Variables in Nominal-Income Equations

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<tr>
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<tr>
<td>( \Delta \text{ln(base)} )</td>
<td>3.06**</td>
<td>1.85</td>
<td>0.82</td>
</tr>
<tr>
<td>( \Delta \text{ln(M1)} )</td>
<td>7.23**</td>
<td>5.70**</td>
<td>2.27**</td>
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<tr>
<td>( \Delta \text{ln(M2)} )</td>
<td>5.13**</td>
<td>4.49**</td>
<td>1.85</td>
</tr>
<tr>
<td>( \Delta \text{ln(credit)} )</td>
<td>5.54**</td>
<td>5.55**</td>
<td>0.33</td>
</tr>
<tr>
<td>( \Delta \sigma )</td>
<td>1.60</td>
<td>1.55</td>
<td>4.30**</td>
</tr>
<tr>
<td>( \sigma_{\psi} )</td>
<td>4.82**</td>
<td>4.68**</td>
<td>3.16*</td>
</tr>
<tr>
<td>( r_p = r_p )</td>
<td>4.43**</td>
<td>4.68**</td>
<td>3.16*</td>
</tr>
</tbody>
</table>

A. Three-Variable System (Nominal Income, Fiscal Variable, Financial Variable):

B. Two-Variable System (Nominal Income, Financial Variable):

\( \Delta \text{ln(base)} \) | 4.07**        | 2.107         | 0.92          |
\( \Delta \text{ln(M1)} \)     | 7.39**        | 5.75**        | 2.147         |
\( \Delta \text{ln(M2)} \)     | 5.19**        | 3.15**        | 2.273         |
\( \Delta \text{ln(credit)} \) | 4.98**        | 1.68          | 6.39          |
\( \Delta \sigma \)           | 1.91          | 5.84**        | 4.36**        |
\( \sigma_{\psi} \)          | 2.117         | 5.05**        | 3.83**        |
\( r_p = r_p \)              | 6.34**        | 5.30**        | 3.35*         |

Notes: Estimated regressions use four lags of each variable. Income is GNP in current dollars, and the fiscal variable is mid-expansion federal expenditures.

*Statistically significant at the \( P \leq 0.1 \) level; **Statistically significant at the \( P \leq 0.05 \) level; ***Statistically significant at the \( P \leq 0.01 \) level.
**Money Demand Shocks**

What can trigger money demand shift?

1. A change in the costs of using alternatives to currency as means to payment (e.g. cost of debit card falls)
2. A change in the costs of converting other assets into currency (e.g. time costs, cash dispensers)
3. A change in Gov't regulations (demand deposits paying interest)
4. A change in inflation risk
5. A change in the riskiness of alternative assets

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**Millennium Bug !! (% Change in the US Currency in Circulation )**

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Figure 9-16  A Shift in the Demand for Money

Figure 10.13  M1
Figure 9-18  Velocity of M1

Figure 9-20  Central Bank Response Stabilizes Price Level (without observing P but Y and r)
In sum

- Monetary intertemporal model
- Cash in advance constraint
- Model implications:
  - Monetary neutrality
  - Non-superneutrality
- Inflation is costly
  - Leads to misallocation of resources
- Money demand instability is a major problem