- Test Thursday, 16th of February starting at 18:00 (ca. an hour), B34
- There will be a session (solving last year’s test) Tuesday 14th of February at 18:00

A Real Intertemporal Model with Investment
So far

- Work / leisure (labour supply) choice of the consumers
- Intertemporal consumption decisions of consumers
- Labour demand of firms
- Total factor productivity shocks/government expenditure shocks
- Government choice of tax timing

This week

- A complete dynamic general equilibrium model without money
- That is: ignore for the moment nominal issues
Investment

- **investment**: goods that are produced now for future use in the prod’n process (expenditure on plants/housing)

- **For the economy**: Trade off between current consumption and future consumption

- Instead of producing consumption goods engage in investment goods production thereby enhance productive capacity

- **For the firm**: Trade off between higher profits today and higher future profits in the future (via increasing productive capacity)

- Determinant: Real interest rates; opportunity cost of investment

Real interest rates

- Higher interest rates implies higher opportunity cost of investment at the margin

- A key determinant of monetary policy transmission (will be done in two weeks time!)
What we will do

- Look into the impact of exogenous shocks of capital stock (wars/earthquakes), fiscal policy, total factor productivity on equilibrium output, investment, consumption, the real interest rate, employment

Agents/ Markets

- RepCon, (provides labour supply, consumes), RepFirm (demands labour, demands investment goods, and produces), Government (consumes)

- Goods market (current/future)
- Labour market (current/future) clear
Representative Consumer

- Bring together work leisure choice and intertemporal consumption decision

- Consumer works and consumes now and in the future
  - h units of time; allocates btw work and leisure in both periods
  - \( w_1 \): current real wage, \( w_2 \): future real wage are given
  - \( r \): real interest rate
  - \( T_1 \): current real taxes, \( T_2 \): future real taxes are given
  - Decision variables \( C_1, C_2 \) and \( l_1, l_2 \) and savings subject to budget and time constraints in the current and future periods

Consumers budget constraint

**Current period**
\[
C_1 + S^p = w_1(h - l_1) + \pi_1 - T_1
\]

**Future period**
\[
C_2 = w_2(h - l_2) + \pi_2 - T_2 + (1 + r)S^p
\]

together
\[
C_1 + \frac{C_2}{1 + r} = w_1(h - l_1) + \pi_1 - T_1 + \frac{w_2(h - l_2) + \pi_2 - T_2}{1 + r}
\]
Four dimensional problem: choosing future and current consumption and leisure

- Key optimality conditions
  1. \( \text{MRS}_{l1,c1} = w_1 \)
  2. \( \text{MRS}_{l2,c2} = w_2 \)
  3. \( \text{MRS}_{c1,c2} = 1 + r \)

Current Labour Supply

- Increases in current real wage (assuming substitution effect dominates income effect)
- Increases in real interest rate
  - intertemporal substitution of leisure / labour supply
- Decreases in lifetime wealth (\( \pi - T \))
  - Current leisure and consumption are normal goods
Income and Substitution Effects of an Increase in Total Factor Productivity

The Representative Consumer’s Current Labor Supply Curve
Figure 7-2  An Increase in the Real Interest Rate Shifts the Current Labor Supply Curve to the Right

Figure 7-3  Effects of an Increase in Lifetime Wealth
Current demand for consumption

- decreases in real interest rates (if substitution effect dominates income effect) due to intertemporal substitution of consumption
- Increases in lifetime wealth

Figure 7-4  The Representative Consumer's Current Demand for Consumption Goods Increases with Income
Figure 12  An Increase in the Real Interest Rate for a Lender

Figure 7-5  An Increase in the Real Interest Rate from $r_1$ to $r_2$ Shifts the Demand for Consumption Goods Down
The representative firm

- \( Y_1 = z_1 F(K_1, N_1) \)
- \( Y_2 = z_2 F(K_2, N_2) \)

Assume that one unit of consumption is needed to produce one unit of investment.

Future capital stock:

- \( K_2 = (1 - d)K_1 + I \)

Note that by the end of second period capital stock is liquidated.
Profits and Current Labour Demand

- Current profits  \( \pi_1 = Y_1 - w_1N_1 - I_1 \)
- Future profits  \( \pi_2 = Y_2 - w_2N_2 + (1-d)K_2 \)

Maximization problem:

\[ V = \pi_1 + \frac{\pi_2}{(1+r)} \]

Decision variables: \( N_1, N_2, I \)

Firm hires up to the point where \( MP_N = w \)

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Figure 7-7  The Demand Curve for Current Labor Is the Representative Firm’s Marginal Product of Labor Schedule

\[ w = \text{Current Real Wage} \]
\[ N = \text{Current Employment} \]

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Figure 7-8  The Current Demand Curve for Labor Shifts Due to Changes in Current Total Factor Productivity \( z \) and in the Current Capital Stock \( K \)

Optimal Investment Decision

- MC of investment should be equal to marginal benefit of investment
- MC(I) is engaging in investment by giving up one unit of net present value of profit (thus MC(I)=1)
- MB of investment is what one unit of extra investment yields in terms of net present value of profits
  - Two elements
    - Additional unit of investment adds one unit in to future capital stock, future output increases; additional product is = future MPK
    - Each unit of current investment implies an 1-d units of capital that will be liquidated at the end of the future period (remember there are only two periods!)
  - So with discounting
    \[
    MB(I) = \frac{MP_k^2 + 1 - \frac{d}{1 + r}}{1 + r}
    \]
Optimality requires $MC(I) = MB(I)$

$$MP_K^2 - d = r$$

- Firm invests until the net MPK is equal the real interest rate.
- Opportunity cost of investment is the real interest rate (i.e. return on alternative asset, bonds)

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Figure 7-9 Optimal Investment Schedule for the Representative Firm
Optimal investment schedule shifts to the right

- **When future TFP increases**
  - Expected future MPK higher, so increase investment

- **When the current capital stock is lower**
  - From $K_2=(1-d)K_1+I$, a lower $K_1$ means lower $K_2$ for a given investment
  - Less initial capital will be left in the future after depreciation
  - Future MPK will be higher for each level of investment

- **Implication:** variability of investment (unlike consumption)

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Figure 7-10  The Optimal Investment Schedule Shifts to the Right if Current Capital Stock Decreases or Future Total Factor Productivity Is Expected to Increase

$r^i$ = Real interest rate

$MP_k - d$ = Demand for Investment Goods

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Government

\[ G_1 + \frac{G_2}{1+r^*} = T^{**} + \frac{T^{**}_2}{1+r^*} \]

Competitive Eq’m

- Labour and goods market should clear both in the current and future periods!
- (focus on the current period)
- We need to construct the output supply and demand curves
Figure 7-11 Determination of Equilibrium in the Labor Market Given the Real Interest Rate $r$

Figure 7-12 Construction of the Output Supply Curve
What shifts Output Supply Curve to the RIGHT

- Lifetime wealth decreases (say due to an increase in intertemporal government spending) \(\Rightarrow\) income effect on labour supply
- Current TFP increases \(\Rightarrow\) more output for any level of input \(\Rightarrow\) more labour demand
- Current capital stock increases \(\Rightarrow\) more output

Figure 7-13 An Increase in Current or Future Government Spending Shifts the \(Y^s\) Curve
Figure 7-14  An Increase in Total Factor Productivity Shifts the $Y^s$ Curve

Figure 7-15  The Demand for Current Goods
What shifts Output Demand Curve to the right?

- Increase in Lifetime wealth (via changes in taxes/government expenditures; fiscal stimulus)
- Increase in Future income; (consumption demand)
- Increase in Future TFP; (investment demand)
- Decrease in Current capital stock (via investment demand)
Figure 7-17 The Output Demand Curve Shifts to the Right if Current Government Spending Increases

Figure 7-18 The Complete Real Intertemporal Model
Experiments

- A temporary increase in government expenditures
- A permanent increase in government expenditures
- Decrease in current capital stock
- Increase in current TFP
- Increase in future TFP

Figure 7-19 A Temporary Increase in Government Purchases
Empirically

- It seems that output effect is stronger when the fiscal stimulus is permanent (Aiyagari et al. (1992))
Figure 7-21  Natural Log of Real Investment, 1929-1998

Figure 7-22  The Equilibrium Effects of a Decrease in the Current Capital Stock
Figure 7-23  The Equilibrium Effects of an Increase in Current Total Factor Productivity

Figure 7-26  Percentage Deviations from Trend in Real Investment and Real GDP for the United States
Figure 7-29 The Equilibrium Effects of an Increase in Future Total Factor Productivity (future MPL and MPK increase; current investment goes up)

Summary of Comovements II

Table 3.2 Summary of Business Cycle Facts

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In sum

- Investment
- Real interest rates are key determinants
- Temporary versus permanent shocks
- Current versus future shocks