A Monetary Intertemporal Model

A Monetary Intertemporal Model: The Neutrality of Money, Long-Run Inflation, and Money Demand

- Extend the intertemporal model to capture money (monetary policy)
- Monetary neutrality: a one time change in the level of money supply has no (long term) real consequences
- Monetary non-neutrality: a change in the GROWTH rate of money supply has (long term) real consequences
- Money is very closely correlated with price inflation that is costly
- Money demand function

Central Bank Targets

- Can vary
- Can be multiple
  - Inflation
  - Output Gap
  - Employment
  - Interest Rates
  - Exchange Rates
  - etc.
  - Some of them (or all together !)
- Business Cycle

Potential Policy Instruments

- CB’s MAY control..
  - Short term interest rates
  - Reserve requirement ratios
- Monetary aggregates (Narrow or Broad)
  - Cash component of monetary aggregates, M1C
  - Monetary Base

What is Money?

- Medium of exchange
- Store of value
- Unit of account

Monetary Aggregates

- Narrow Monetary aggregates (can control)
  - Cash component of monetary aggregates, M1C
  - Monetary Base
- Broad Monetary aggregates (difficult but possible)
  - M1 = M1C + Traveller's Checks + Demand Deposits + Other Checkable Deposits
  - M2 = M1 + Small Denomination time deposits + Saving Deposits + Money Market Mutual Fund Shares (noninstitutional)
  - M3 = M2 + Large Denomination time deposits + Money Market Mutual Fund Shares (institutional) + Term repurchase agreements + Term Eurodollars
  - Less used
Table 10.1  Monetary Aggregates, September 2003 (in $billions)

<table>
<thead>
<tr>
<th>M0</th>
<th>720.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>1275.6</td>
</tr>
<tr>
<td>M2</td>
<td>6083.9</td>
</tr>
<tr>
<td>M3</td>
<td>8854.5</td>
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Monetary Intertemporal Model

- Double coincidence of wants: Solution money
  - Cash in Advance model (Clower 1967)
    - You need cash-in-advance to go shopping

Monetary Intertemporal Model

- Two periods: current and future
- Two assets: money (numeraire) and nominal bonds (can be issued by consumers or government and pays R)
  - Both fiscal (G,T) and monetary policy (M) possible
- No default on debt!
- No intermediaries: (banks)

Markets

- Labour market
- Goods market
- Money market
  - Credit market: implied by the eq’m
Real and Nominal Interest Rates and Fisher Relation

- Nominal bond: asset that sells for one unit of money in the current period and pays off \(1+R\) unit of money in the future period
- \(R\): nominal interest rate; \(r\): real interest rate
- Inflation rate: \(\text{INF} = \frac{(P_2 - P_1)}{P_1}\)
- Fisher relation: \(1 + r = \frac{1 + R}{1 + \text{INF}}\)
- After substitution and collecting terms
  \[ r \approx R - \text{INF} \]

Representative Consumer

- Transactions are of particular sequence!
  - Inherits two assets from previous period (M and B) and pays taxes
  - Credit Market: Go to credit market and rearrange asset portfolio (by using money) and buy new bonds
  - Labour market: Go to firm offer employment at a market real wage \(w\) but will be paid only after the goods are sold!
  - Goods Market: Purchase goods in the goods market ONLY with money
  - At the end of the period: after goods are purchased, obtain real wages, and dividend income (all paid in money) and

Consumer Decisions

- \(C\), \(L\), \(B^d\), \(M\)
- To do her own well being as good as possible!

Key Constraints

Cash-in-advance constr.
\[ P_C \leq M_0 + B_0 (1 + R_0) - P_T - B_T^d \]

Consumer Budget Constr.
\[ PC + R_T + M_T = M_0 + (1 + R_0)B_0 + P_T(l_h - l_r) + P_T\pi_t - P_T^d \]
Cash in advance constraint binds!

- That is, $R > 0$!
- If $R < 0$ consumer is willing to hold more money than needed

Money Demand

$$\frac{M^d}{P} = L(Y, R)$$

Using Fisher relation

$$\frac{M^d}{P} = L(Y, r + \infty)$$

Or in terms of nominal money demand

$$M^d = L(Y, r + \infty) * P$$

Money Demand

Demand for money is determined by:

1. current real demand for money increases when real income increases (lifetime wealth increases, increasing the demand for future cash goods)
2. current real demand for money decreases when nominal interest rate $R$ increases (opportunity cost of holding money increases)

Figure 9-2 The Nominal Money Demand Curve in the Monetary Intertemporal Model

Figure 9-3 The Effects of an Increase in Current Real Income on the Nominal Money Demand Curve

Representative Firm

- Production function
  $$Y = zF(K, N)$$
- It hires labour in the current period until the marginal product of labour is equal to the real wage
  $$MP_N = w_1$$
  $$MP_K = r$$
Government

- Responsible for both monetary and fiscal policy
- Government Budget Constraint
  - LHS: government spending
  - RHS: government receipts

\[ PG_t + (1 + R_t)B_0 = PT_t + B_t + M_t - M_0 \]

Competitive Equilibrium

- Equivalent to the real intertemporal model with the exception that now money matters
- For this purpose we need to add another market next to labour and goods markets
  - Money market

Labour Market

- \( L^d \) shifts right if \( z_1 \uparrow \) or \( K_1 \uparrow \)
- \( L^s \) shifts right as \( w, r, \uparrow \) (substitution btw current and future leisure and current and future consumption goods) or life time \( Y \downarrow \) (income effect)

Figure 7-1 The Representative Consumer’s Current Labor Supply Curve

Figure 7-2 An Increase in the Real Interest Rate Shifts the Current Labor Supply Curve to the Right

Figure 7-7 The Demand Curve for Current Labor Is the Representative Firm’s Marginal Product of Labor Schedule
Figure 7-8  The Current Demand Curve for Labor Shifts Due to Changes in Current Total Factor Productivity $z$ and in the Current Capital Stock $K$

Figure 9-5  The Current Labor Market in the Monetary Intertemporal Model

Goods Market

- $Y^s$ shifts right if $z_1 \uparrow$, $K_1 \uparrow$, lifetime wealth $\downarrow$
- $Y^d$ shifts right if $T \downarrow$, $G_1 \uparrow$, $Y_2 \uparrow$, $z_2 \uparrow$, $K_1 \downarrow$

Figure 7-12  Construction of the Output Supply Curve

Figure 7-16  Construction of the Output Demand Curve

Figure 9-6  The Current Goods Market in the Monetary Intertemporal Model
A Level Increase in the Money Supply and Monetary Neutrality

- Suppose an increase in the money supply at period 1 (permanent money supply shock)
- In Government BC

\[ P \Delta G_i + (1 + R_0)B_0 = P_i T_i + B_i + M_1 - M_0 \]

- \( B_0, R_0 \) are predetermined based on the expectation that \( M_1 \) is not going to change
- Adjustment should take place from other variables

What causes money supply increase: Three Possibilities

- Gov’t can reduce current taxes: ‘helicopter drop’ (M. Friedman)
- Gov’t can reduce the quantity of \( B_1 \): ‘open market operations’
- Gov’t increase \( G_1 \), and to fund \( \Delta G \) print money: seigniorage from inflation tax

- Assume helicopter drop (lump sum transfer of money to the representative agent)

What happens in eq’m after money level shock?

- Nothing in the labour and goods markets since none of the variables are dependent on \( M \)
- Classical dichotomy
- Real activity is orthogonal to nominal activity
- Monetary shock is compensated by a one-off price level adjustment
- Monetary neutrality!
A Shock to Money Growth rate

- Story changes!
- Changes in the growth rate of money supply will not be neutral!!
- A permanent increase in the money supply implies equivalent increase in the price level
  But
- A permanent increase in the money growth rate implies increase in the inflation RATES
- Now we will show that
  1. such shocks are costly in terms of output, and resource allocation
  2. Friedman rule of optimal monetary policy

Sustained Inflations

- Usually result of monetary policy (money supply growth)
- Figure implies positive but noisy relationship between money supply growth and inflation rate

Figure 9-11 Scatter Plot of the Inflation Rate vs. the Growth Rate in $M_2$ for the United States, 1948-1999

Modelling the Change in the Money Supply Growth

- Assume that Gov’t allows the money supply to grow by making lump-sum transfers to the representative household each period
  \[ M_2 = (1 + x) M_1 \]
- x: growth rate of money supply from period 1 to 2 (exogenous)
- At period 1 and 2, from $M=M^d$ write money market eq’ms
  - $M_1=P_1*L(Y_1,r_1+\pi_1)$ at period 1
  - $M_2=P_2*L(Y_2,r_2+\pi_2)$ at period 2

Modelling the Change in the Money Supply Growth (con’d)

- $M_2/M_1=(P_2*L(Y_2,r_2+\pi_2))/(P_1*L(Y_1,r_1+\pi_1))$
- In eq’m: $Y_1=Y_2$, $r_1=r_2$, $\pi_1=\pi_2$, implying $L(Y_1,r_1+\pi_1)=L(Y_2,r_2+\pi_2)$
  Thus
  \[ M_2/M_1=P_2/P_1=\pi=(M_2-M_1)/M_1=x \]

Short-Run Analysis of a Temporary Decrease in Total Factor Productivity

- Assume inflation=0
What can trigger money demand shift?
1. a change in the costs of using alternatives to currency as means to payment
2. A change in the costs of converting other assets into currency: (cost of time)
3. A change in Gov’t regulations
4. A change in inflation risk
5. A change in the riskiness of alternative assets (stocks, deposits)
Look at the Millennium Bug! (Percentage Change in the US M1 Currency Component)

Figure 10.14 Velocity of M1

Velocity of Money

- VM = PY

- V = measure of the number of times the money stock M turns over during a given current time period

  i.e. V = PY/M = Y/L(Y,R)

Quantity Theory of Money and Monetarism

- Milton Friedman
  - M = (1/V)*P*Y

  - Assumption: money demand function is stable, thus V is stable
  - Predictable relationship between nominal income and money supply

Influential in the 70's

- If as a Central Banker you can control money supply you can control inflation

- It relies on stable money demand function

- What if money demand is unstable?
Nominal Variables: Percentage Deviations from Trend in the Money Supply (black line) and Real GDP (colored line) for the Period 1947-1999

Figure 9-20 Central Bank Response Stabilizes Price Level

Figure 9-21 Central Bank Does Not Observe the Price Level Response to a Shift in Demand for Money

In sum

- Monetary intertemporal model
- Cash in advance constraint
- Model implications:
  - Monetary neutrality
  - Non-superneutrality
- Inflation is costly
  - Leads to misallocation of resources
- Money demand instability is a major problem