A modification of Honoré’s triple-link model in the synoptic problem

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Summary. In New Testament studies, the synoptic problem is concerned with the relationships between the gospels of Matthew, Mark and Luke. In an earlier paper a careful specification in probabilistic terms was set up of Honoré’s triple-link model. In the present paper, a modification of Honoré’s model is proposed. As previously, counts of the numbers of verbal agreements between the gospels are examined to investigate which of the possible triple-link models appears to give the best fit to the data, but now using the modified version of the model and additional sets of data.

Keywords: New Testament; synoptic problem; triple-link model; probability model

1 Introduction

In an earlier paper (Abakuks(2006a)), a detailed specification was made of the assumptions underlying the so-called triple-link model for the synoptic problem in New Testament studies that was proposed by Honoré (1968). A statistical analysis was carried out, based on the numbers of verbal agreements between the synoptic gospels, that is, the numbers of common occurrences in the same section of material of the same Greek word in the same grammatical form, where Honoré’s sections (or “pericopes”, in the terminology used by biblical scholars) were based on those of Huck (1949). In the present paper we shall argue for a modification of Honoré’s assumptions which corresponds better to current views about how a later evangelist would have made use of the gospels of his predecessors in writing his own gospel. We shall then re-analyse some of the data using the modified model.

There have been other major compilations of the statistics of verbal agreements between the synoptic gospels according to a number of variant definitions of verbal agreement. Most notably there is the work of Solages (1959), who also attempted an analysis of synoptic relationships using ideas from combinatorial mathematics, and of Morgenthaler (1971). However, the compilation that provides data in an appropriate and readily usable form for testing the Honoré model, either in its original or its modified form, is that of Tyson and Longstaff (1978). We shall use their data, in addition to those of Honoré, in

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order to check the robustness of our conclusions to alterations in the method of counting verbal agreements.

In the triple-link model, the terms Gospel A, B and C refer to any permutation of the synoptic gospels, Mark (Mk), Matthew (Mt) and Luke (Lk). It is supposed that Gospels B and C both use Gospel A and that Gospel C also uses Gospel B. Let $x$ be the probability that a given word in A is transmitted unaltered to B. Let $y$ be the probability that a given word in B is transmitted unaltered to C. Let $z$ be the probability that a given word in A is transmitted unaltered directly to C. The relationship is illustrated in Fig. 1 below, which essentially belongs to a family of diagrams of possible synoptic relationships that had been produced by Farmer (1964), pp 208-09. (Farmer was instrumental in the revival of the Griesbach hypothesis, which corresponds to the identification A = Mt, B = Lk, C = Mk.)

![Fig. 1. The triple-link model](image)

2 The data

Honoré’s data were discussed in Abakuks (2006a) and it was argued there that the particular data for which the triple-link model would be most appropriate came from the double and triple tradition combined, that is, sections of material that are present in either two or all three of the gospels, but excluding the sections that are unique to any one of the gospels. Furthermore, it turned out that this set of data did indeed provide the best fit to the model. In the present paper we shall restrict attention to such sets of data. Some discussion and examples of single, double and triple tradition material, together with some historical background about the synoptic problem, may be found in Abakuks (2006b).

In addition, we shall use two sets of data obtained from Tyson and Longstaff (1978). The first set of counts of verbal agreements of words that are identical in grammatical form by its definition corresponds very closely to Honoré’s. The resulting numbers are somewhat different, however, partly because there are some slight differences in what is counted as an agreement within a section and partly because Tyson and Longstaff use somewhat different section divisions and a different version of the text, that of Aland (1971).

The second set of data is of counts of words that are in Tyson and Longstaff’s terminology “identical words in continuous agreement.” These are counts of words in comparable passages that are identical in form and that are immediately preceded or followed by words that are in identical agreement, i.e., they are part of a block of identical words. (See Tyson and Longstaff (1978), pp 9-11.) This then amounts to a more stringent definition of agreement that takes into account the order in which words occur in a way
that had been put forward by Farmer in his *Synopticon* (1969). This stricter definition of agreement naturally leads to smaller counts of agreements — isolated words that are in identical agreement are not counted as agreements according to this definition.

For the double and triple tradition combined, Table 1 gives the counts of words classified according to their presence or absence in each of the synoptic gospels. In any row, the count refers to the number of words that are present in the gospels marked with the number 1 but absent in the gospels marked with the number 0. The first column of counts, using Honoré’s data, is identical to one of the sets of data used in Abakuks (2006a). The second and third columns of counts are from Tyson and Longstaff (1978), abstracted from the tables in their Chapter 7, the second column using a similar definition of agreement to that of Honoré but the third column using the more stringent definition of identical words in continuous agreement.

<table>
<thead>
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<th>Honoré</th>
<th>Tyson &amp; Longstaff</th>
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<td>Lk</td>
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<td>8520</td>
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3 A probability model

Recapitulating the development in Abakuks (2006a), in the notation of probability theory we shall denote by \( A, B \) and \( C \) the events that a given word is in Gospel A, Gospel B and Gospel C, respectively. We shall further denote by \( C_1 \) the event that the given word is in Gospel C and has been transmitted via Gospel B and denote by \( C_2 \) the event that the given word is in Gospel C and has been transmitted directly from Gospel A. According to the triple-link model, any word that Gospel C has in common with either Gospel A or Gospel B has been transmitted to Gospel C from either Gospel A or Gospel B. It follows that

\[
A \cap C = A \cap (C_1 \cup C_2)
\]

and

\[
B \cap C = B \cap (C_1 \cup C_2).
\]

We also note that the occurrence of the event \( C_1 \) implies \( B \), so that we have \( C_1 \subseteq B \). Similarly, the occurrence of the event \( C_2 \) implies \( A \), so that \( C_2 \subseteq A \).

We shall evaluate the conditional probability,

\[
\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)},
\]

directly from the data by the corresponding relative frequency, that is, the ratio of the number of words that are in both Gospels A and B to the number of words that are in
Gospel A. The conditional probability so evaluated is precisely the probability that, for that part of the synoptic material being considered, that is, the double and triple tradition combined, a word chosen at random from Gospel A is also found in the same context in Gospel B. Similar direct evaluations can be made for all conditional probabilities involving A, B and C, but conditional probabilities that involve C₁ and C₂ will have to be evaluated indirectly.

In terms of the notation that we have introduced, the probabilities x, y and z may be expressed as

\[ x = \Pr(B|A), \]
\[ y = \Pr(C_1|B), \]
\[ z = \Pr(C_2|A). \]

We shall want to evaluate the probabilities x, y and z, which we shall do by expressing them in terms of conditional probabilities which can be evaluated directly. It is straightforward to evaluate x directly from equation (1), but expressions for y and z are not so immediate and depend on further assumptions that are made about the probabilities of word transmission.

In Abakuks (2006a), following Honoré, three conditional independence assumptions were made, the last of which had not been explicitly recognised by Honoré but was implicit in formulae that he arrived at. In the present paper we shall retain the first two assumptions but change the third assumption of conditional independence of C₁ and C₂ to the assumption that they are mutually exclusive. This, as we shall argue, corresponds better to the way in which the gospel writer may be supposed to have worked. It also leads to somewhat simpler formulae and an improvement in the fit of the model.

**Assumption 1** – given that a word is in Gospel A, the event that it is transmitted to Gospel B and the event that it is transmitted directly from Gospel A to Gospel C are independent. Thus

\[ \Pr(B \cap C_2|A) = \Pr(B|A) \Pr(C_2|A), \]

which is equivalent to

\[ \Pr(C_2|A \cap B) = \Pr(C_2|A). \]

**Assumption 2** – given that a word is in Gospel B, the event that it is in Gospel A and the event that it is transmitted from Gospel B to Gospel C are independent. Thus

\[ \Pr(A \cap C_1|B) = \Pr(A|B) \Pr(C_1|B), \]

which is equivalent to

\[ \Pr(C_1|A \cap B) = \Pr(C_1|B). \]

**Assumption 3** – the event C₁ that a word is transmitted from Gospel B to Gospel C and the event C₂ that it is transmitted directly from Gospel A to Gospel C are mutually exclusive. It follows that

\[ \Pr(C_1 \cup C_2|A) = \Pr(C_1|A) + \Pr(C_2|A) \]

and

\[ \Pr(C_1 \cup C_2|B) = \Pr(C_1|B) + \Pr(C_2|B). \]
As pointed out by Honoré and others and discussed by Abakuks (2006a), such assumptions will by no means hold exactly, but we have to make some such assumptions in order to make some progress in our analysis. In attempting to justify our new Assumption 3 it is useful to consider the physical conditions under which the gospel writers worked. Much relevant material is reviewed in Chapter 5 of Neville (2002). This is an area in which Downing has published a number of papers, many of which are collected together in Downing (2000), although it should be noted that he is a staunch defender of the standard two-source synoptic hypothesis as against any of the hypotheses that underlie our triple-link model. However, some of the most important points for our purposes are already to be found in the much earlier work of Sanday (1911), pp 16-19. We must not imagine the gospel writer as sitting in a study with tables or desks on which the earlier texts were immediately available to him for reference. In the first century, the gospels were written on scrolls, which were scarce and, furthermore, difficult to work with, requiring some effort to move from one section to another. It is unlikely that the gospel writer would have been using two or more scrolls simultaneously. It is more likely that, if on any occasion he was reading a section of a scroll as his primary source, or having it read to him, he would be relying on his memory to compare this with any other material, whether written or oral, that was known to him. For a discussion of this see in particular Downing (1988). So the author of Gospel C might, perhaps, at any one time be looking either at a scroll of Gospel A or a scroll of Gospel B, but not both, and possibly, from time to time, might switch from one to the other. All this is somewhat conjectural, but, in any such kind of scenario, the assumption that $C_1$ and $C_2$ are mutually exclusive would seem to be more natural than the assumption that they are conditionally independent.

To be sure, our model with its assumptions is a simplification. For instance, we are not taking into consideration the phenomenon of “doublets”, pieces of text that appear twice in a very similar form in the same gospel, of which a list is given by Hawkins (1909) and which are often taken as evidence that the gospel author has at least two sources from which he is working, both of which contain a version of the given piece of text. It seems worth noting in passing that Hawkins (1909) is a classic text for students of the synoptic problem and may be of historical interest to statisticians, as it includes a variety of tables of statistics relating to the synoptic problem, especially tables of word frequencies. Hawkins’ first edition of 1899 thus represents an early attempt at dealing with the synoptic problem in a statistical way.

It should also be remarked that there is no model of a kind similar to our triple-link model that could be used to assess the validity of the widely accepted two-source hypothesis for synoptic relationships or to compare its goodness of fit with that of the triple-link model. According to the two-source hypothesis, Matthew and Luke have two written sources in common, Mark and a hypothetical ‘Q’; but we have no surviving text of Q from which transmission probabilities of words could be calculated in the type of way that we do for the triple-link model. Any attempted reconstruction of a Q text, such as that of Robinson et al (2000), is based on words that are present in the Matthew-Luke double tradition and by its very nature will not include material that may have been present in Q but has not been utilised by both Matthew and Luke. Furthermore, it is generally supposed by proponents of the two-source hypothesis that there is some overlap between Mark and Q — see, for example, Streeter (1924), pp 186-191, or Tuckett (1996), pp 31-34. The reconstruction of any Q material where there is such an overlap is
necessarily very tentative.

4 A statistical analysis

The expression that we may use to evaluate $z$ here remains the same as the one derived in Abakuks (2006a):

$$z = \frac{\Pr(\bar{B} \cap C|A)}{\Pr(B|A)},$$

(4)

where $\bar{B}$ represents the complementary event that $B$ does not occur. Equivalently, $z = \Pr(C|A \cap \bar{B})$, so that our evaluation of $z$ is based on the counts of the words that are present in Gospel A but not in Gospel B. The expression that enables us to evaluate $y$ is simpler than the one in Abakuks (2006a):

$$y = \Pr(C|B) - z \Pr(A|B).$$

(5)

Once we have used equation (4) to evaluate $z$, we shall be able to use equation (5) to evaluate $y$. The second term on the right hand side of equation (5) represents an adjustment for words that are present in both Gospel B and Gospel C but have been transmitted directly from Gospel A to Gospel C. The derivations of equation (5) and the following equations (6) and (7) are given in Appendix A.

We use our values for $x$, $y$ and $z$ to calculate $\Pr(B \cap C|A)$ and $\Pr(C|A)$ and then attempt to validate our model by checking how close these calculated values are to the direct evaluations obtained from the observed word counts. We can express $\Pr(B \cap C|A)$ and $\Pr(C|A)$ in terms of $x$, $y$ and $z$ as follows:

$$\Pr(B \cap C|A) = x(y + z)$$

(6)

and

$$\Pr(C|A) = xy + z.$$  

(7)

The expressions of equations (6) and (7) are simpler than the corresponding expressions in Honoré (1968) and Abakuks (2006a), which in both equations included an additional term, $-xyz$, on the right hand side.

We now evaluate the quantities described above for each of the six possible permutations of the three synoptic gospels. The results for the Honoré data are given in Table 2 and for the two sets of the Tyson and Longstaff data in Tables 3 and 4, respectively.

**Table 2.** Honoré data

| A-B-C      | $x$ | $y$ | $z$ | $x(y + z)$ | $\Pr(B \cap C|A)$ | $\Pr(C|A)$ |
|------------|-----|-----|-----|------------|-------------------|-----------|
| Mt-Mk-Lk   | 0.315 | 0.174 | 0.239 | 0.130 | 0.127 | 1.024 | 0.294 | 0.291 | 1.010 |
| Lk-Mk-Mt   | 0.239 | 0.348 | 0.248 | 0.142 | 0.147 | 0.972 | 0.331 | 0.335 | 0.988 |
| Mk-Mt-Lk   | 0.416 | 0.234 | 0.181 | 0.173 | 0.168 | 1.028 | 0.278 | 0.274 | 1.017 |
| Lk-Mt-Mk   | 0.335 | 0.275 | 0.139 | 0.130 | 0.147 | 0.946 | 0.231 | 0.239 | 0.967 |
| Mt-Lk-Mk   | 0.291 | 0.150 | 0.265 | 0.121 | 0.127 | 0.949 | 0.309 | 0.315 | 0.980 |
| Mk-Lk-Mt   | 0.274 | 0.254 | 0.342 | 0.163 | 0.168 | 0.970 | 0.411 | 0.416 | 0.988 |
Table 3. Tyson and Longstaff data — words identical

| A-B-C        | x    | y    | z    | Pr(B ∩ C|A) | Pr(C|A) |
|--------------|------|------|------|--------|--------|
| Mt-Mk-Lk    | 0.292 | 0.179 | 0.208 | 0.113  | 0.114  | 0.991 | 0.260 | 0.261 | 0.996 |
| Lk-Mk-Mt    | 0.227 | 0.347 | 0.228 | 0.130  | 0.137  | 0.954 | 0.307 | 0.313 | 0.980 |
| Mk-Mt-Lk    | 0.407 | 0.210 | 0.177 | 0.157  | 0.159  | 0.989 | 0.262 | 0.264 | 0.993 |
| Lk-Mt-Mk    | 0.313 | 0.257 | 0.131 | 0.122  | 0.137  | 0.891 | 0.212 | 0.227 | 0.934 |
| Mt-Lk-Mk    | 0.261 | 0.151 | 0.241 | 0.102  | 0.114  | 0.899 | 0.280 | 0.292 | 0.960 |
| Mk-Lk-Mt    | 0.264 | 0.237 | 0.337 | 0.151  | 0.159  | 0.952 | 0.399 | 0.407 | 0.981 |

For each ordering of the gospels, the values of Pr(B ∩ C|A) and Pr(C|A) have been calculated from x, y and z, using equations (6) and (7), respectively. The values of Pr(B ∩ C|A) and Pr(C|A) have also been evaluated directly from the data in Table 1. In each case, the ratio of the probability as calculated from x, y and z to the probability as evaluated directly has been calculated, and we use the closeness of these ratios to one as a measure of how well the triple-link model fits the data.

For Honoré’s original model, as discussed in Abakuks (2006a), it was to be expected that the ratio would be less than one in each case, since the assumptions of statistical independence would have the effect of making the predicted number of words that the gospels have in common smaller than what is actually observed. In comparing the results of the original model with the present one, using Honoré’s data and Table 2, we may note that the values of x and z remain unchanged, but from comparison of equation (5) with the corresponding equation (5) in Abakuks (2006a), it follows that the value of y is necessarily smaller for the present model. However, the values of Pr(B ∩ C|A) and Pr(C|A) as calculated from the values of x, y and z have increased in each case, bringing the corresponding ratios closer to unity, with the consequence that the present model gives a better fit. Indeed, the ratios for what were the two best-fitting gospel orderings for the original model, Mt-Mk-Lk and Mk-Mt-Lk, now have values that are greater than one, though close to it. The gospel orderings Lk-Mk-Mt and Mk-Lk-Mt, with their ratios slightly less than one, give fits which are about as good as those of Mt-Mk-Lk and Mk-Mt-Lk. The fits given by Lk-Mt-Mk and Mt-Lk-Mk are only slightly worse.

In comparing how well the model works for the different gospel orderings it is also worth considering the relative values of x, y and z as measures of the extent to which any gospel writer is making use of his predecessors. A possible criterion for the validity of the model for a particular gospel ordering arises from Honoré’s observation that if the gospels were written in the order A-B-C then, since B has only A as his source, whereas C has both A and B as sources, one would expect C to make less use of each of A and B than B does of A. This translates into the inequalities x > z and x > y. So we would...
expect a plausible model to satisfy

\[ x > \max(y, z). \]  

(8)

But it should be noted that we could have either \( z > y \) or \( y > z \), as C could be using either A or B as his main source. If we adopt the criterion (8) then we should be careful to bear in mind that it is not a logically necessary one, since B could simply be following his source material less closely than is C, but if we do adopt this criterion then it eliminates the orderings Lk-Mk-Mt and Mk-Lk-Mt which have Matthew as the last of the gospels to be written.

For both versions of the Tyson and Longstaff data, although we have not presented here the results of the analysis using the original Honoré model, it turns out that the results for the modified model as presented in Tables 3 and 4 give a better fit. It is also reassuring that our conclusions for both versions of the Tyson and Longstaff data are broadly the same as for the Honoré data. The first set of counts of word agreements for the Tyson and Longstaff data give numbers that are smaller than those of Honoré and this, with a couple of minor exceptions, results in smaller values of \( x \), \( y \) and \( z \). The ratios that are used to assess the fit of the model are also smaller, all now less than one. We have a clearer ranking of the models with Mt-Mk-Lk and Mk-Mt-Lk giving the best fit, indeed, the best that we have from among all the sets of data analysed and all the models used, followed by Lk-Mk-Mt and Mk-Lk-Mt, with Lk-Mt-Mk and Mt-Lk-Mk giving the worst fit. Again, if we use the criterion (8), Lk-Mk-Mt and Mk-Lk-Mt are eliminated. For the second set of data from Tyson and Longstaff, the more stringent definition of agreement results in smaller counts of agreements and in further reductions of the values of \( x \), \( y \) and \( z \). The ratios for assessing the fit of the model have also decreased, so that the fit of the model is now worse than for the previous sets of data. However, the comments regarding the ranking of the models are exactly the same as for the first set of data from Tyson and Longstaff.

It is encouraging that our methodology appears to be robust to variations in the method of counting agreements, for in each case we come to similar conclusions regarding the ranking of the various possible gospel orderings. The counting of agreements between words, by whatever method, is something that can be done in a fairly mechanical way to provide us with statistical data to analyse. However, this is a somewhat crude way of measuring textual agreements, which takes no account of the grammatical structure of the sentences in which the words are found or of the meaning that the words are attempting to convey. There is much scope for trying to find more subtle ways of quantifying textual agreement. It is an open question as to how robust the methods presented in this paper might be to more radical changes in the measurement of agreement.

Another serious issue with regard to our method of analysis is that the results are presented essentially at the level of descriptive statistics, with no standard errors or any indication of how significant are the differences in the goodness of fit between the various gospel orderings. We might well consider the transmission probabilities \( x \), \( y \) and \( z \) as parameters to be estimated and expect to see associated confidence intervals. Given the observed transmission frequencies, we could conceive of a likelihood function being written down for \( x \), \( y \) and \( z \), from which, using standard methods of analysis, techniques of statistical inference would be derived. A huge obstacle to carrying out such a programme is that the individual words cannot even remotely be regarded as behaving independently of each other. Words tend to be transmitted unaltered from one gospel to another in
clusters of varying sizes, and there are large segments of material that are not transmitted at all. Because of this, there is no obvious way of writing down a likelihood function. Nevertheless, potentially there is scope for the development of a statistically more rigorous method of analysis of our data.

5 Conclusions

If using the criterion (8) we discard the orderings Lk-Mk-Mt and Mk-Lk-Mt, which in any case represent synoptic hypotheses that have had little significant support, then there remain the orderings Mt-Mk-Lk and Mk-Mt-Lk as the ones that give the best fit. The ordering Mt-Mk-Lk corresponds to the so-called traditional Augustinian hypothesis, which, although not very popular nowadays, still carries some support — see Butler (1951) and Wenham (1991). The ordering Mk-Mt-Lk corresponds to the Farrer theory, which has received recent support in Goodacre (2002) and Goodacre and Perrin (2004). Turning to the orderings that fit the model less well, we have Lk-Mt-Mk and Mt-Lk-Mk. The first of these has never received much support, but the second corresponds to the Griesbach hypothesis (also known in its more recent manifestation as the two-gospel hypothesis), which has an active group of followers, based in the USA, whose latest major work is Peabody et al (2002).

If we wish to rank the various synoptic hypotheses that underlie the triple-link model in terms of how well they fit the model, whether in its original or now modified form, such a ranking has to be qualified, as discussed in more detail in Abakuks (2006a), by the observation that these underlying hypotheses do not in themselves include the three assumptions that we have made. So defenders of a hypothesis which fits the triple-link model less well may argue that this is not because it is inherently a less satisfactory hypothesis but because the additional assumptions of the triple-link model are less appropriate for their theory. Nevertheless, a ranking based on the triple-link model should carry some weight in discussions of the relative merits of the corresponding synoptic hypotheses. Furthermore, given a synoptic hypothesis, the calculated values of $x$, $y$ and $z$ should provide some statistical basis for comparisons of the extent to which the gospel writers made use of their predecessors.
Appendix A

For the evaluation of \( y \), using Assumption 3,

\[
\Pr(C|B) = \Pr(C_1 \cup C_2|B) \\
= \Pr(C_1|B) + \Pr(C_2|B) \\
= \Pr(C_1|B) + \Pr(C_2|A \cap B) \Pr(A|B) + \Pr(C_2|\bar{A} \cap B) \Pr(\bar{A}|B) \\
= \Pr(C_1|B) + \Pr(C_2|A \cap B) \Pr(A|B),
\]

where we have used the fact that, conditional upon \( \bar{A} \cap B \), the event \( C_2 \) cannot occur, so that the corresponding conditional probability is zero. Using equations (2) and (3) and Assumption 1, we find that

\[
\Pr(C|B) = y + z \Pr(A|B).
\]

Rearrangement of this equation gives equation (5).

Making use of the Assumptions 1, 2 and 3, the expressions of equations (6) and (7) for \( \Pr(B \cap C|A) \) and \( \Pr(C|A) \), respectively, are obtained as follows.

\[
\Pr(B \cap C|A) = \Pr(B \cap (C_1 \cup C_2)|A) \\
= \Pr((B \cap C_1) \cup (B \cap C_2)|A) \\
= \Pr(B \cap C_1|A) + \Pr(B \cap C_2|A) \\
= \Pr(C_1|A \cap B) \Pr(B|A) + \Pr(B \cap C_2|A) \\
= xy + xz = x(y + z).
\]

\[
\Pr(C|A) = \Pr(C_1 \cup C_2|A) \\
= \Pr(C_1|A) + \Pr(C_2|A) \\
= \Pr(B \cap C_1|A) + \Pr(C_2|A) \\
= \Pr(C_1|A \cap B) \Pr(B|A) + \Pr(C_2|A) \\
= xy + z.
\]
References


