A Markov Regime Switching Approach for Hedging Energy Commodities

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ABSTRACT

In this paper we employ a Markov Regime Switching (MRS) approach for determining time-varying minimum variance hedge ratio in energy futures markets. The hedging effectiveness of New York Mercantile Exchange (NYMEX) petroleum futures contracts is examined using univariate MRS and bivariate MRS – VAR with GARCH error structure. The rationale behind the use of MRS models stems from the fact that the dynamic relationship between spot and futures returns may be characterized by regime shifts, which, in turn, suggests that by allowing the hedge ratio to be state dependent upon the “state of the market”, one can obtain more efficient hedge ratios and hence, superior hedging performance compared to other methods in the literature. Regime switching in GARCH processes reduces volatility persistence and improves the forecast ability. The performance of the MRS hedge ratios is compared to that of alternative models such as GARCH, Error Correction and OLS in the West Texas Intermediate Crude oil, Unleaded Gasoline and Heating oil markets. In and out-of-sample tests indicate that MRS hedge ratios outperform the other methods in reducing portfolio risk in the petroleum products markets. In the crude oil market, the MRS models outperform the other hedging strategies only within sample. Overall, the results indicate that by using MRS models market agents may be able to increase the performance of their hedges, measured in terms of variance reduction and increase in utility.
1 INTRODUCTION

Market participants in all financial and commodity markets operate in an environment subject to some degree of variability. The energy commodities group, prone to large price fluctuations and uncertainty in both the physical and the financial market, attracts the interest of this paper. It was not until after the second 1970’s oil price crisis that oil derivative contracts were introduced and thereafter developed widely. Since then, oil price risk management became an inevitable challenging task because of the global nature of oil and its implications in the international political arena. The primary crude oil distillates, gasoline, heating oil, aviation fuel and fuel oil are indispensable for transportation, industrial and residential uses. As a result, crude oil is the world’s most actively traded commodity. Physical oil trade movements in the year 2004 reached 48.11 million barrels per day between export and import regions compared to 31.44 million barrels per day in the year 1990. Oil products accounted for 10.96 million barrels per day of the trade figure whereas the latter increased by 2.38 million barrels per day in 2004 for inter-area movements.\(^1\) Global economic and political activity has proven to play a crucial role in the stability of oil prices, driving the market to relatively high levels of volatility. Even though recent technological advances enhanced the development of alternative energy sources, oil still delivers superior efficiency of use and thus, industries experience large amount of risk.

Nowadays there are two major exchanges providing oil derivative contracts, the New York Mercantile Exchange (NYMEX) and the Intercontinental Exchange (ICE), formerly International Petroleum Exchange (IPE). Also, since 1999, another marketplace for trading oil-related contracts (crude oil, gas oil, gasoline and kerosene futures), is Tokyo Commodity Exchange (TOCOM). In 2004 more than 78 million futures contracts were traded on NYMEX, of which 52.88, 12.78 and 12.88 million reflect the individual traded volumes of crude oil, gasoline and heating oil, respectively. Derivative markets allow market agents to minimize their exposure to risk by reducing the variance of their portfolio; hence risk management tools and their effectiveness in terms of hedging are of utmost importance. In this line, it is essential to evaluate

\(^1\) British Petroleum Statistical Review 2004
different approaches that are employed to construct efficient and reliable hedging strategies.

The hedge ratio i.e. the ratio of futures contracts to buy or sell for each unit of the underlying asset on which the hedger bears risk, is one of the tools that are used to manage potential adverse effects of price changes in the physical market. Earlier studies in the literature (Johnson, 1960; Stein, 1961; Ederington, 1979) derive hedge ratios that minimize the variance of the hedged portfolio, based on portfolio theory. Let $\Delta S_t$ and $\Delta F_t$ represent the price changes in spot and futures prices between period $t$ and $t-1$. Then, the minimum-variance hedge ratio is the ratio of the unconditional covariance between cash and futures price changes over the variance of futures price changes; this is equivalent to the slope coefficient, $\gamma_1$, in the following regression:

$$
\Delta S_t = \gamma_0 + \gamma_1 \Delta F_t + u_t,
$$

$$u_t \sim iid(0, \sigma^2)
$$

(1)

Within this specification, the estimated $R^2$ of Equation (1) represents the hedging effectiveness of the minimum variance hedge. Empirical studies that have employed the above methodology to estimate hedge ratios and measures of hedging effectiveness include: for T-Bill futures Ederington (1979) and Franckle (1980); for oil futures, Chen et al. (1987); for stock indices, Figlewski (1984) and Lindahl (1992); for currencies Grammatikos and Saunders (1983). However, the implicit assumption of Equation (1) that the risk in spot and futures markets and thus the optimal hedge ratio are constant over time, does not take into account the fact that since many asset prices follow time-varying distributions, the minimum variance hedge ratio should be time-varying (Myers and Thompson, 1989; Kroner and Sultan, 1993). This, in turn raises concerns regarding the risk reduction properties of hedge ratios generated from Equation (1).

To address this issue, a number of studies apply multivariate GARCH (Generalised Autoregressive Conditional Heteroscedasticity) (Engle & Kroner, 1995) models and derive time-varying hedge ratios directly from the second moments (variances of futures and spot price changes and their covariance). Examples in the literature include: for currency futures Kroner and Sultan (1993); for stock index futures Park and Switzer (1995); for interest rate futures Gagnon and Lypny (1995), for freight futures Kavussanos and Nomikos (2000); for corn futures Moschini and Myers (2002); for
electricity futures Bystrom (2003); for petroleum futures Alizadeh et al. (2004). The consensus from these studies is that GARCH-based hedge ratios change as new information arrives to the market and on average tend to outperform, in terms of risk reduction, constant hedge ratios derived from Equation (1). However, these gains are market specific and vary across different contracts while, occasionally, the benefits in terms of risk reduction seem to be minimal (Lien and Tse, 2002).

The rationale behind the use of the GARCH models lies in the fact that asset returns tend to exhibit volatility clustering; in other words, large (small) price changes tend to be followed by large (small) price changes (Mandelbrot, 1963). This pattern of volatility behaviour suggests that although actual price changes might be uncorrelated, the conditional second moments could be time dependent. The most widely used approaches for modelling time-varying volatility are the GARCH family models. Empirically, a common feature of GARCH models is that they tend to impute a high degree of persistence to the conditional volatility. This means that shocks to the conditional variance that occurred in the distant past continue to have a nontrivial impact in the current estimate of volatility. Lamoureux and Lastrapes (1990) associate these high levels of volatility persistence with structural breaks or regime shifts in the volatility process. They demonstrate this by introducing deterministic shifts in the conditional variance equation and find that this leads to a marked reduction in the degree of volatility persistence, compared to that implied by the GARCH models. As an alternative to GARCH models, Wilson et al. (1996) employ an iterative cumulative sums-of-squares (ICSS) methodology and show evidence of sudden changes in the unconditional volatility of oil futures contracts. With data covering the period 1984 to 1992, three major volatility shifts are detected and the reasons are attributed to the nature and magnitude of exogenous shocks (OPEC policy, Iran-Iraq conflict, Gulf War and extreme weather conditions). By accounting these shifts in the ARCH framework they find similar conclusions with Lamoureux and Lastrapes (1990). Fong and See (2002; 2003) also report significant regime shifts in the conditional volatility of crude oil futures contracts, which dominate the GARCH effects. In addition, they find that in a high variance regime a negative basis is more likely to increase the regime persistence than a positive basis and associate volatility regimes with specific market events. The existence of regime shifts in the relationship between spot and futures returns is also demonstrated by Sarno and Valente (2000) who show that regime switching models
explain the relationship between spot and futures prices better than simple linear models in the FTSE-100 and S&P 500 stock index futures markets.

The evidence presented above suggests that by allowing the volatility to switch stochastically between different processes under different market conditions, one may obtain more robust estimates of the conditional second moments and, as a result, more efficient hedge ratios compared to the methods which are currently being employed, such as GARCH models or OLS. Whether this is the case is an issue that is examined empirically in this paper for the crude oil, gasoline and heating oil futures contracts, traded on NYMEX.

Alizadeh and Nomikos (2004) examined the hedging effectiveness of FTSE-100 and S&P 500 stock indices, taking into account regime shifts in the state of the market, by introducing Markov Switching Models for the estimation of dynamic hedge ratios. Allowing Equation (1) to switch between two state processes, they provided evidence in favour of those models in terms of variance reduction and increase in utility, both in- and out-of-sample. Following Gray (1996), Lee and Yoder (2006) extend the multivariate MRS model, to a state dependent bivariate GARCH model. They apply their model in the corn and nickel futures markets and they report higher, yet insignificant, variance reduction compared to OLS and the single-regime GARCH hedging strategy.

Therefore by investigating the hedging effectiveness of Markov regime switching models we contribute to the existing literature in a number of ways. First, NYMEX oil futures are used to generate hedge ratios that are regime dependent and change as market conditions change. Second, along with simple MRS univariate models, we extend this approach to a bivariate Regime Switching VAR model with GARCH error structure. The regimes of the models are treated as latent variables since they are estimated along with the other parameters of the model using maximum likelihood techniques. Third, we evaluate the hedging effectiveness of these models in the United States energy market, using both in- and out-of-sample tests. The out-of-sample tests, in particular, are performed by forecasting the regime probabilities using the estimated transition probability matrices, and calculating hedge ratios based on these forecasts. Finally, the performance of the regime switching hedge ratios is compared to that of
alternative hedge ratios generated from a variety of models that have been proposed in the literature such as GARCH and error-correction models. This way we provide robust evidence on the performance of the proposed hedging strategy.

Our paper is different from the Lee and Yoder (2006) Regime Switching - BEKK study in the sense that for more parsimonious representation we employ the diagonal BEKK parameterization of variance covariance matrix reducing the computational burden and number of parameters to be estimated. Furthermore we allow for lagged cross terms in the mean equation in order to capture the interactions of spot-futures prices, since exclusion of any relevant explanatory variables from the mean equation could increase the variance of the error term, threatening the GARCH results. At last, we employ a battery of alternative models, to provide evidence for the robustness of MRS hedge ratios.

The structure of this paper is as follows. We next present the minimum-variance hedge ratio methodology and illustrate the univariate MRS models used in this study. We then present the MRS-BEKK model estimation procedure. Data and their properties with our empirical results are reported and discussed. This is followed by an evaluation of the hedging effectiveness of the proposed strategies; conclusions are given in the last section.

2 MARKOV REGIME SWITCHING MODELS AND HEDGING

Market participants in futures markets choose a hedging strategy that reflects their individual goals and attitudes towards risk. The degree of hedging effectiveness in futures markets depends on the relative variation of spot and futures price changes as well as the hedge ratio, that is the ratio of futures contracts to buy or sell for each unit of the underlying asset. The hedge ratio that minimises the variance of the hedge portfolio is derived as the slope coefficient of spot price changes on futures price changes, as in Equation (1). This can also be expressed as:
\[ \gamma_1 = \frac{Cov(\Delta S_t, \Delta F_t)}{Var(\Delta F_t)} \]  

(2)

Therefore, the minimum variance hedge ratio of Equation (2) is the ratio of the unconditional covariance between cash and futures price changes over the variance of the futures price changes.\(^2\) Equation (2) can also be extended to accommodate the conditional minimum-variance hedge ratio, \(\gamma_{1,t}\), which is the time varying equivalent of the conventional hedge ratio \(\gamma_1\), in Equation (1). This is believed to be more efficient in reducing the risk of a hedged position, compared to the conventional hedge ratio, because it is updated as it responds to the arrival of new information in the market. To estimate such a dynamic hedge ratio, the second moments of spot and futures returns in Equation (2) are conditioned on the information set available at time \(t - 1\), primarily using multivariate GARCH models.

Sarno and Valente (2000) provide a further dimension to the literature by showing that changes in market conditions may affect the relationship of spot and futures prices. Using a multivariate extension of the Markov Switching Model (MRS) proposed by Hamilton (1989) and Krolzig (1999), they find that the relationship between spot and futures returns in the S&P 500 and FTSE-100 market is regime dependent. This in turn suggests that shifts in the spot-futures relationship may have an impact on the magnitude of the hedge ratio and consequently, on the hedging effectiveness of the futures market. Alizadeh and Nomikos (2004) allow for changes in the market conditions to affect the hedge ratios. They extend Equation (1) to a two-state MRS model in order to allow for switches between two different processes, dictated by the state of the market. This can be shown mathematically as:

\[ \Delta S_t = \gamma_{0,t} + \gamma_{1,t} \Delta F_t + \epsilon_{t} \quad \epsilon_{t} \sim iid(0, \sigma^2_{\epsilon,t}) \]  

(3)

\(^2\) It can be shown that if expected futures returns are zero, i.e. if futures follow a martingale process \(E_t(F_{t+1}) = F_t\), then, the minimum variance hedge ratio of Eq. (2) is equivalent to the utility-maximizing hedge ratio. A proof of this result is available at Benninga et al. (1984) and Kroner and Sultan (1993). The martingale assumption of futures returns implies that the expected returns from the hedged portfolio are unaffected by the number of futures contracts held, so that risk minimization becomes equivalent to utility maximization. The assumption of zero expected returns is also in line with the descriptive statistics presented in Table I, which show that the unconditional futures returns have a mean of zero.
where, \( s_t = \{1, 2\} \) indicates the state in which the market is in. The link between the two states of the market in Equation (3) is provided through a first-order Markov process with the following transition probabilities:

\[
\begin{align*}
\Pr(s_t = 1 | s_{t-1} = 2) &= P_{12} \\
\Pr(s_t = 2 | s_{t-1} = 2) &= P_{22} = (1 - P_{12}) \\
\Pr(s_t = 2 | s_{t-1} = 1) &= P_{12} \\
\Pr(s_t = 1 | s_{t-1} = 1) &= P_{11} = (1 - P_{12})
\end{align*}
\] (4)

where the transition probability \( P_{12} \) gives the probability that state 1 will be followed by state 2, and the transition probability \( P_{21} \) gives the probability that state 2 will be followed by state 1. Transition probabilities \( P_{11} \) and \( P_{22} \) give the probabilities that there will be no change in the state of the market in the following period. These transition probabilities are assumed to remain constant between successive periods and can be estimated along with the other parameters of the model.

Once the density functions for each state of the market and probabilities of being in respective states are defined, the likelihood function for the entire sample is formed by eliminating the unobserved term \( s_t \), and summing up the possible values of it. The corresponding log-likelihood is constructed as:

\[
L(\theta) = \sum_{i=1}^{T} \log \left( \frac{\pi_{1,t}}{\sqrt{2\pi\sigma_{1,t}^2}} \exp \left[ -\frac{(\Delta s_i - \gamma_{0,1} - \gamma_{1,1} \Delta F_i)^2}{2\sigma_{1,t}^2} \right] + \frac{\pi_{2,t}}{\sqrt{2\pi\sigma_{2,t}^2}} \exp \left[ -\frac{(\Delta s_i - \gamma_{0,2} - \gamma_{1,2} \Delta F_i)^2}{2\sigma_{2,t}^2} \right] \right)
\] (5)

where \( \theta = (\gamma_{0,\text{st}}, \gamma_{1,\text{st}}, \sigma_{\text{st}}^2, s_t = 1, 2) \) is the vector of parameters to be estimated\(^3\) and \( \pi_{1,t} \), \( \pi_{2,t} \) are the probabilities of the regime being in state 1 or 2, respectively. \( L(\theta) \) can be

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\(^3\) Some studies argue that constant transition probabilities and/or variances are restrictive assumptions. Studies such as Diebold et al. (1994), Marsh (2000), Fong and See (2002; 2003) and Alizadeh and Nomikos (2004) condition transition probabilities on observable variables that are part of the information set e.g. the basis. Moreover, Perez-Quiros and Timmerman (2000) condition the variances of stock returns on the lagged levels of treasury bills. Alizadeh and Nomikos (2004) use the lagged average basis, to condition the variances. Other studies such as Hamilton and Susmel (1994), Gray (1996) and Dueker (1997) use ARCH and GARCH specifications to model the dynamics of volatilities. In this study, we use constant transition probabilities, although the models can easily be extended to allow for a time-varying transition probability matrix \( P \). As for the variances, the focus is on the bivariate MRS GARCH case. The univariate MRS is used as benchmark, thus variances are not conditioned on the available information set and considered constant within each regime. However, two alternative models were estimated as in Alizadeh and Nomikos (2004); a) an MRS model in which the transition probabilities are conditioned on the lagged basis and b) an MRS model in which both the transition probabilities and variances are conditioned on the lagged basis. These results are not presented here and are available from the authors.
maximised using numerical optimization methods, subject to the constraints that $\pi_{1,t} + \pi_{2,t} = 1$ and $0 \leq \pi_{1,t}, \pi_{2,t} \leq 1$.

Estimating Equation (3), using the MRS specification outlined above, yields two hedge ratios, $\gamma_{1,1}$ and $\gamma_{1,2}$, which represent the minimum variance hedge ratios, given the state of the market. In fact, these two hedge ratios can be considered as the upper and lower bounds of the optimal hedge ratio. Since the probability of the market being in state 1 or 2 at any point in time is given by $\pi_{1,t}$ and $\pi_{2,t} = 1 - \pi_{1,t}$, where $0 \leq \pi_{1,t} \leq 1$ and $0 \leq \pi_{2,t} \leq 1$, the optimal hedge ratio at any point in time can be determined as the weighted average of the two estimated hedge ratios, weighted according to their respective probabilities. Hence, the optimal hedge ratio at time $t$ will be dependent on the probability of the market being in state 1 or 2 and can be expressed as:

$$\gamma^*_t = \pi_{1,t}\gamma_{1,1} + (1 - \pi_{1,t})\gamma_{1,2}$$  \hspace{1cm} (6)

Estimating the optimal hedge ratio using the Markov Regime Switching model outlined above allows for shifts in the mean and volatility processes and recognizes any changes in the relationships between them. This ensures a better estimate of optimal hedge ratio compared to OLS or GARCH models as the first estimates a constant hedge ratio and the second estimates a hedge ratio which is time-varying but mainly autoregressive in nature.

### 3 MARKOV REGIME SWITCHING ARCH MODELS AND HEDGING

An alternative way to estimate the optimum hedge ratio would be to use Equation (2). Conditional second moments of spot and futures returns are measured by the family of ARCH models, introduced by Engle (1982). For this purpose we employ a VAR model for the conditional means of spot and futures returns with a multivariate GARCH error
structure. Allowing for regime shifts in the intercept term\(^4\), the conditional means of spot and futures returns are specified using the following VAR:

\[
\Delta X_t = \mu_{st} + \sum_{i=1}^{p-1} \Gamma_i \Delta X_{t-i} + \epsilon_{t,st} \quad ; \quad \epsilon_{t,st} = \begin{pmatrix} \epsilon_{S,t,st} \\ \epsilon_{F,t,st} \end{pmatrix} \sim \text{IN}(0, \Sigma_{t,st})
\]

(7)

where \(X_t = (\Delta S_t, \Delta F_t)\) is the vector of spot and futures returns, \(\Gamma_i\) is a 2x2 coefficient matrix measuring changes in \(X_t\) and \(\epsilon_{t,st} = (\epsilon_{S,t,st}, \epsilon_{F,t,st})\) is a vector of Gaussian white noise processes with time varying state dependent covariance matrix \(H_{t,st}\). The unobserved state variable \(st\) follows a two-state, first order Markov process with constant transition probabilities, as they are specified in Equation (4).

The conditional second moments of spot and futures returns are specified as a GARCH(1,1) model (Bollerslev, 1986). However, in the regime-switching framework, the GARCH model in its basic form would be intractable because both, conditional variance and conditional covariance would be a function of all past information, rather than a function of the current regime alone. This path-dependency problem would require the integration of an exponentially increasing number of regime paths, at each step, delivering an infeasible model to estimate. Hamilton and Susmel (1994) and Cai (1994) solve the path dependency problem by eliminating the GARCH term. The main drawback of their model is that many lags of ARCH terms are needed in order to capture the dynamics of volatilities. Gray (1996) suggests a possible formulation for the conditional variance process by using the conditional expectation of the variance. Lee and Yoder (2006) extend Gray’s model to the bivariate case and fully solve the path dependency problem by developing a similar collapsing procedure for the covariance.

Following augmented Baba et al. (1987) (henceforth BEKK) representation (see Engle and Kroner, 1995), the GARCH-like formulation of the variance/covariance matrix can be written as:

\[^4\text{In this paper we allow for regime shifts only in the intercept term of the VAR system in Equation (7). Further extension of this model to allow switching in the autoregressive terms is straightforward. In addition to this model, we also estimate models allow for switching in all the parameters of the mean equation, or even include the error correction term as estimated following Johansen procedure (1988). However, the above model selected as the more parsimonious, overcoming convergence problems, whereas the gains of its extension were minimal and its still an under research area.}\]
\[
H_{st} = C_{st}' C_{st} + A_{st}' e_{t-1} e_{t-1}' A_{st} + B_{st}' H_{t-1} B_{st}
\]  

(8)

for \(st = \{1,2\}\), where, \(C_{st}\) is a 2x2 lower triangular matrix of state dependent coefficients, \(A_{st}\) and \(B_{st}\) are 2x2 state dependent coefficient matrices restricted to be diagonal\(^5\), with \(\alpha_{i,i,st}^2 + \beta_{i,i,st}^2 < 1\), \(i=1,2\), for stationarity within each regime. This formulation, guarantees \(H_{st}\) to be positive definite for all \(t\) and, in contrast to the constant correlation model of Bollerslev (1986) it allows the conditional covariance of spot and futures returns to change signs over time\(^6\). Moreover, in this diagonal representation the state dependent conditional variances are a function of the lagged values of both the lagged aggregated variances and aggregated error terms (after integrating the unobserved state variable). Similarly, the state dependent conditional covariance is a function of lagged aggregated covariance and lagged cross products of the aggregated error terms.

Gray’s (1996) integrating method of the state dependent variances as applied for both spot-futures returns can be expressed as:

\[
h_t = E[X_t^2|\Omega_{t-1}] - E[X_t|\Omega_{t-1}]^2 = \pi_{1,t}(\mu_{1,t}^2 + h_{1,t}) + (1 - \pi_{1,t})(\mu_{2,t}^2 + h_{2,t}) - \left[\pi_{1,t}\mu_{1,t} + (1 - \pi_{1,t})\mu_{2,t}\right]^2
\]

(9)

where \(X_t\) represents the dependent variable, futures or spot returns, \(h_t\) is the aggregate variance, \(\mu_{st,t}\) the state dependent mean equation, \(\pi_{st,t}\) the regime probabilities as defined in Equation (4) and \(h_{st,t}\) the state dependent variances, for \(st = \{1,2\}\). Similarly, the collapsing procedure for the state dependent residuals can be written as:

\[
\varepsilon_t = X_t - E[X_t|\Omega_{t-1}] = X_t - \left[\pi_{1,t}\mu_{1,t} + (1 - \pi_{1,t})\mu_{2,t}\right]
\]

(10)

Additionally, in a bivariate model under the formulation of Equation (8) the path dependency problem must be solved for the covariance as well. We follow the method

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5 Coefficients matrices A and B are restricted to be diagonal for a more parsimonious representation of the conditional variance (see Bollerslev et al. 1994).

6 For a discussion of the properties of this model and alternative multivariate representations of the conditional variance matrix see Bollerslev et al. (1994) and Engle and Kroner (1995).
proposed by Lee and Yoder (2006) to integrate the regime paths at each step. The collapsing procedure for the covariance is specified as:

\[
h_{sf,t} = E \left[ \Delta S_t, \Delta F_t \mid \Omega_{t-1} \right] - E \left[ \Delta S_t \mid \Omega_{t-1} \right] \cdot E \left[ \Delta F_t \mid \Omega_{t-1} \right] = \pi_{1,t} \left[ \mu_{1,t} \mu_{1,t} + h_{st,t} \right] + \left( 1 - \pi_{1,t} \right) \left[ \mu_{2,t} \mu_{2,t} + h_{st,t} \right] - \left[ \pi_{1,t} \mu_{1,t} \left( 1 - \pi_{1,t} \right) \mu_{2,t} \right] \left[ \pi_{1,t} \mu_{1,t} \left( 1 - \pi_{1,t} \right) \mu_{2,t} \right]
\]

where \( h_{sf,t} \) is the aggregate covariance, \( \mu_{st,t} \) and \( \mu_{st,t} \) are the mean equations for spot and futures returns, respectively and \( h_{st,t} \) the state dependent covariances for \( st = \{1,2\} \).

Under the specifications of Equations (8), (9), (10) and (11) the Markov Regime Switching BEKK model becomes path-independent because variance/covariance matrix depends on the current regime alone and not on the entire history. Consequently, the Markov property for a first order Markov process is not violated and we can allow for the conditional means of spot and futures a multivariate GARCH error structure. Finally, assuming that the state dependent residuals follow a multivariate normal distribution with mean zero and time varying state dependent covariance matrix \( H_{st,t} \), the density function for each regime (state of the market) can be written as follows:

\[
f(X_t \mid s_t; \theta) = \frac{1}{2\pi} \left| H_{st,t} \right|^{-\frac{1}{2}} \exp \left( -\frac{1}{2} e_{t,st}' H_{st,t}^{-1} e_{t,st} \right) \quad ; st = \{1,2\}
\]

where \( \theta = (\alpha_{S,t}, \beta_{S,t}, \alpha_{F,t}, \gamma_{F,t}, \epsilon_{1,t,1}, \epsilon_{2,t,1}, \epsilon_{2,t,2}, \alpha_{2,t,1}, \alpha_{2,t,2}, \beta_{1,t,1}, \beta_{2,t,2}, P_{12}, P_{12}) \) is the vector of parameters to be estimated, \( e_{t,st} \) and \( H_{st,t} \) are defined in Equations (7) and (8), respectively.

Once the density functions for each state of the market and probabilities of being in respective states are defined, the likelihood function for the entire sample is formed in the same way as in the univariate case, by a mixture of the probability distribution of the state variable and the density function for each regime as follows:

\[
f(X_t; \theta) = \frac{\pi_{1,t}}{2\pi} \left| H_{t,1} \right|^{-\frac{1}{2}} \exp \left( -\frac{1}{2} e_{t,1}' H_{t,1}^{-1} e_{t,1} \right) + \frac{\pi_{2,t}}{2\pi} \left| H_{t,2} \right|^{-\frac{1}{2}} \exp \left( -\frac{1}{2} e_{t,2}' H_{t,2}^{-1} e_{t,2} \right)
\]
The log-likelihood of the above density function can then be defined as:

\[ L(\theta) = \sum_{t=1}^{T} \log f(X_t; \theta) \]  

(14)

\( L(\theta) \) can be maximised using numerical optimization methods, subject to the constraints that \( \pi_{1,t} + \pi_{2,t} = 1 \) and \( 0 \leq \pi_{1,t}, \pi_{2,t} \leq 1 \).

Estimating Equations (7), (8), (9), (10) and (11) using the MRS specifications outlined above, the second moments of spot and futures returns are conditioned on the information set available at time \( t-1 \). Based on Equation (2) the estimated hedge ratio at time \( t \), given all the available information up to \( t-1 \) can be written as:

\[ \gamma^*|_{\Omega_{t-1}} = \frac{Cov(\Delta S_t, \Delta F_t | \Omega_{t-1})}{Var(\Delta F_t | \Omega_{t-1})} \]  

(15)

where \( Cov(\Delta S_t, \Delta F_t | \Omega_{t-1}) \) and \( Var(\Delta F_t | \Omega_{t-1}) \) are calculated from the collapsing procedure, as presented in Equations (11) and (9), respectively.

Estimating the optimal hedge ratio using the Markov Regime Switching BEKK model outlined above further allows for structural changes in the GARCH processes and overcomes some of the limitations that traditional GARCH models exhibit. First, by allowing the volatility equation to switch across different states, we relax the assumption of constant parameters throughout the estimation period improving the ‘fit’ of the data. Second, accounting for regime switching, the high volatility persistence imposed by single regime models decreases and the forecasting performance is expected to be better (see for example Lamoureux and Lastrapes, 1990; Cai, 1994 and Dueker, 1997). Moreover, the estimated hedge ratio is dependent on the state of the market and its Markovian formulation abolishes the autoregressive nature of GARCH hedge ratios. Consequently, one expects MRS hedge ratios estimated by the variance/covariance matrix to outperform the conventional hedging strategies.
4 DESCRIPTION OF THE DATA
AND PRELIMINARY ANALYSIS

The data set for this study comprises of weekly spot and futures prices for three energy commodities: WTI crude oil, Unleaded Gasoline and Heating oil, covering the period January 23, 1991 to July 28, 2004, resulting 706 weekly observations. Spot and futures prices are Wednesday prices; when a holiday occurs on Wednesday, Tuesday’s observation is used in its place. The above dataset was obtained from CRB-Infotech CD and Datastream along with volume and open interest data. Data for the period January 23, 1991 to July 30, 2003 (654 observations) are used for the in-sample analysis; out-of-sample analysis is carried out using the remaining data from the period August 6, 2003 to July 28, 2004 (52 observations). All the commodities under study are traded on the New York Mercantile Exchange (NYMEX).

WTI contracts are traded for all deliveries within the next 30 consecutive months as well as for specific long-dated deliveries such as 36, 48, 72 and 84 months from delivery. Each contract is traded until the close of business on the third business day prior to the 25th calendar day of the month preceding the delivery month. If the 25th calendar day of the month is a non-business day, trading shall cease on the third business day prior to the business day preceding the 25th calendar day. Unleaded Gasoline contracts are traded for all deliveries up to 12 consecutive months; NYMEX Heating oil contracts are traded for all deliveries within the next 18 months. Each contract of either Gasoline or Heating oil is terminated the last business day of the month preceding the delivery month. WTI contract is quoted in US dollars per barrel (US$/bbl) whereas the two crude products are quoted in US dollar per US gallon. The size of each contract is 42,000 gallons (1,000 barrels). While for the two crude oil products a reliable cash market exists, reformulated gasoline and heavy fuel oil in New York harbour, respectively, crude oil spot data lack consistency, so instead we used the global spot index published by the US Department of Energy (DOE) (obtained from the CRB Infotech database).

One problem encountered in the analysis of futures contracts is that individual contracts expire. In order to deal with thin trading and expiration effects, it is assumed that the
hedger will switch contracts, the next business day after trading activity has shifted from the nearest to the second nearest to maturity contract. Therefore, we utilize the volume and open interest dataset to discriminate liquidity between the first and second nearest to maturity contracts since the ‘most effective hedge is the nearby contract’ (Chen et al, 1987). Consequently, in all cases, the nearest contract available is chosen as the appropriate hedge mechanism, and rolling over to the front month contract occurs the business day following the day that both trading volume and open interest exceed that of the nearest to expiry contract7.

Having constructed a continuous time series for the futures contracts prices, spot and futures prices are then transformed into natural logarithms. Summary statistics of the levels and return series are presented in Table I, Panel A. As expected, spot prices are more volatile than the futures prices. Jarque-Bera (1980) tests indicate significant departures from normality for all the commodities and for both spot and futures prices, with the exception of Heating oil spot prices at 10% significance level. The distributions of the returns appear to be normal. The Ljung-Box (1978) Q statistic on the first six lags of the sample autocorrelation function is significant for all spot and futures prices revealing that serial correlation is present in both spot and futures prices. The same is evident for the spot returns time series. Futures prices returns show no significant signs of serial correlation with the exception of Unleaded Gasoline contract at 10% significance level. Engle’s (1982) ARCH test, carried out as the Ljung-Box Q statistic on the squared series, indicates the existence of heteroscedasticity for all the level and return series, with the exception of WTI futures returns. Finally, Phillips and Perron (1988) non-parametric unit root tests on the levels and first differences indicate that the spot and futures prices are first difference stationary.

Having identified that spot and futures prices are $I(1)$ variables, cointegration techniques are used next to investigate the existence of a long run relationship between these series. Johansen (1988) cointegration tests, presented in Table I, Panel B, indicate that all physical commodity prices stand in a long-run relationship with the corresponding futures contracts. The normalized coefficient estimates of the

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7 For instance the November 2002 WTI futures contract expires on October, 22. The rollover to the December 2002 contract takes place five business days prior expiry, on the 15th of the same month, because open interest crossover between the two nearby contracts occurred on the 10th, while volume crossover on October, 14.
cointegrating vector $\beta' = (\beta_1 \beta_0 \beta_2)$ represent this long-run relationship between the series. Furthermore the results of likelihood ratio tests on the hypothesis that there is a one-to-one relationship between spot and futures prices; that is, the cointegrating vector is the lagged basis: $H_0: \beta' = (1, 0, -1)$, show that the hypothesis can be rejected at all significant levels, with the exception of Heating oil. Therefore, for WTI crude oil and Unleaded Gasoline we use the unrestricted cointegrating vectors, in the joint estimation of the conditional mean and the conditional variance (VECM-GARCH), whereas for the Heating oil market the cointegrating vector is restricted to be the lagged basis.

5 EMPIRICAL RESULTS

Markov Regime Switching Models with different specifications are estimated assuming two states. The choice of a two-state process is motivated by the fact that this model captures the dynamics of the spot and futures returns in a more efficient way than a Markov process with more than two regimes. For instance, Fong and See (2002;2003) use a two-state process in the crude oil futures market. On the other hand, Sarno and Valente (2000) use a three-state process to model spot-futures relationship in stock indices; nonetheless, in their study the third state seems to capture only jumps in the futures prices at the time of switching between contracts of different maturities and does not reflect fundamental changes in market conditions. Finally, the two-state process is intuitively appealing since it allows for periods of low and high volatility. The MRS model of Equation (3) is then estimated by maximizing the log-likelihood function of Equation (5); we call this the univariate MRS model. Furthermore, we extend the regime switching model to a bivariate MRS-VAR allowing for a GARCH error structure, with constant transition probabilities; we call this model, the MRS-BEKK model. The results of both the univariate and bivariate models are presented in Table II.

Several observations merit attention. First, regarding the univariate MRS model, reported in Table II, Panel A, it can be seen that for the estimates of the volatilities (standard deviations $\sigma_1$ and $\sigma_2$) under the two states, the restriction of equal volatilities is strongly rejected in each case, according to the Likelihood Ratio (LR) tests, presented in the same table. As indicated by the LR tests, there is also marked asymmetry across
the hedge ratios, $\gamma_{t,st}$, $st = 1,2$, between the two states. This suggests that the dynamics of the spot-futures relationship are different under these two different regimes. We can note an evident association between the magnitude of the hedge ratio and the state of the portfolio volatility; as expected, a high variance state is associated with a low hedge ratio and vice versa. That is because during high volatility periods the correlation of the short run dynamics (returns) is expected to fall and thus, spot-futures prices to diverge. Furthermore, the OLS hedge ratio of Equation (1), lies between these two hedge ratios as the OLS technique estimates the average hedge ratio over the sample period, as opposed to the MRS model that allows the hedge ratio to depend on the state of the market. These findings suggest markedly different dynamics of spot-futures relationship under the two regimes and are consistent across all three models, for all three markets.

Looking at the estimated MRS-BEKK models, presented in Panel B of Table II, although the inclusion of a constant is significant only in the case of Heating oil, the signs of these coefficients are reverse, in each regime. Moreover, the inclusion of lagged cross autoregressive terms, in the mean equation displays significance only in the spot equation. This implies that spot prices respond to changes in the short run dynamics, whereas futures remain unresponsive to lagged cross returns. Spot prices are usually more sensitive to information due to the fact that they incorporate higher liquidity and thus, information is automatically absorbed in the spot markets whereas in the futures markets the speed of adjustment to the available information is a function of liquidity, maturity etc. This is expected, since oil futures prices are determined by supply and demand in the oil physical market and suggests that petroleum futures are exogenous to spot prices.

The degree of persistence in the variance in each regime is measured by the sum of $\alpha_{ii,st}^2 + \beta_{ii,st}^2$ coefficients for $st = 1, 2$. In all cases the sum less than unity indicating that the GARCH system is covariance stationary. As indicated by these sums, low variance states are characterized by lower persistency, whereas in the high variance state persistency increases. This is in line with other studies in the literature such as Fong and See (2002) in the crude oil futures market. The only exception is the heating oil equation, where the low futures variance state is associated with longer memory. This can be attributed to the fact that the high variance regime occurs infrequently at the point of the upward jumps of the basis. Visual inspection of the futures and spot prices
shows that these jumps are caused solely by spot price spikes. Descriptive statistics (Table I) show that heating oil involves the higher spot price volatility and the lowest futures price volatility of all three commodities. As a result, the low variance state is dominant throughout the sample period and occasional spot price jumps are captured by the model as the high variance state. However, for the high variance state error and GARCH coefficients are not significant, probably because the high variance regime is of short duration whereas the low variance regime is the prevailing state.

From the estimated transition probabilities $P_{12}$ and $P_{21}$ we can calculate the duration of being in each regime. For instance in the case of WTI crude oil market the transition probabilities of MRS-BEKK (Panel B, Table II) are estimated as $P_{12} = 25.4\%$ and $P_{21} = 13.5\%$; these indicate that the average expected duration of being in regime 1 is about 4 ($=1/0.254$) weeks and the average expected duration of being in regime 2 is about 7 ($=1/0.135$) weeks\(^8\). Thus, high variance states are less stable and are characterized by shorter duration compared to low variance states. The univariate MRS model (Panel A, Table II) indicates that the states are equally stable with the exception of Heating oil.

Consequently, allowing the conditional variances to be both time varying and regime switching, the persistency of regimes is reduced, in other words, the transition probabilities are higher, allowing for more frequent switching. For example, in the WTI crude oil market the transition probabilities as estimated from the MRS model are $P_{12} = 20.1\%$ and $P_{21} = 20.5\%$ indicating a duration of about 5 weeks for each regime, compared to 4 and 7 weeks as estimated by the MRS-BEKK.

The “smooth” regime probabilities for the WTI crude oil, unleaded gasoline and heating oil markets derived from the estimated MRS-BEKK model are presented in Figures 1, 4 and 7, respectively\(^9\). These indicate the likelihood of being in state 2 (low variance state). The shaded areas in the graphs identify the periods when the market is in the high

---

\(^8\) The average expected duration of being in state 1 is calculated using the formula suggested by Hamilton (1989):

$$
\sum_{t=1}^{\infty} iP_{1i} (1 - P_{1i}) = (1 - P_{1i})^{-1} = (P_{12})^{-1}
$$

\(^9\) Based upon the estimated parameter vector $\hat{\delta}$, estimated from data spanning the period $t=1$ to $T$, three estimates about the unobserved state variable $s_t$, can be made. The first is the estimated probability that the unobserved state variable at time $t$ equals 1 given the observations 1 to $t < T$ and is termed the filtered probability about $s_t$. The second is the estimated probability that the unobserved state at time $t$ equals 1 given the entire sample of observations from 1 to $T$, termed the “smooth” probability. The third is the estimated probability that the unobserved state variable at time $T+1$ equals 1 given observations 1 to $T$ and is termed the expected or predicted probability about $s_t$. See Hamilton (1994) for further details.
variance state. In the case of WTI crude oil, state two is prevailing over the periods 1991 to late 1995, mid 1996 to late 1997 and early 2001 to mid 2002. Figure 1 illustrates that WTI is characterized mainly by the low variance state until 1995, attributed to the restoration of Kuwait’s production after the Gulf war and overproduction from the OPEC countries in combination with relatively weak demand. The low variance state is then disturbed by bad weather conditions in the US and Europe as well as the tension in the Middle East, Asian crisis in 1998 etc, which creates instability in the high/low variance regimes occurrence i.e. shorter duration regimes. In the case of Unleaded Gasoline, each of the two regimes is characterized by less persistency than WTI, but the pattern, in Figure 4, is very similar to Figure 1. As expected, mainly due to seasonal factors, regimes are less stable, since backwardations and supply shortages for a light distillate are more highly likely and frequent, due to constrained refining capacity and the fact that the production is subject to the quality of the crude. Even in periods of crude oil oversupply (e.g. 1993), constrained refining capacity may disturb the supply/demand dynamics of the refined products. Concerning heating oil, the regimes are more ‘distinct’ with the low variance state dominant. Although also seasonal, the behaviour of the basis is different as it seems to be more stable, with occasional jumps that persist only for a short period of time. So, high variance periods are the early 1991, mainly due to the Persian Gulf War and the effects of the resolution of the Soviet Union. The next three jumps of the basis occur in the late 1993, late 1995 and late 1996, which can be associated with cold weather and regional supply demand imbalances. After the year 2000, the high variance state becomes more frequent. This is due to the strong demand and tight production, followed by the September 11, 2001 terrorist attacks and the following recession in the US. Inventory levels in combination with constrained refining capacity increased the volatility of the market. Figures 2, 5 and 8 plot the basis (defined as the logarithm of spot minus logarithm of futures) of the WTI crude oil, Unleaded gasoline and Heating oil, respectively. Figures 3, 6 and 9 show the in-sample OLS, GARCH and MRS-BEKK hedge ratios for all three markets. The shaded areas, which specify the periods that the market is in state 2, are also projected to these graphs to facilitate the visual comparison of the state of the market and the magnitude of the basis and the hedge ratio.

The graphs of the hedge ratios indicate that the fluctuations of the MRS hedge ratios are similar to those of the smooth probabilities, but smoother; this is expected since they are
constructed based on these regime probabilities. Turning next to the graphs of the basis in Figures 2, 5 and 8, we can note that when the basis is close to zero and the basis is relatively less volatile, the market is in the low variance state (state 2). During these periods the hedge ratio is higher and less volatile. Similarly, when the market is in state 1 (high variance state) the basis is further away from zero. This indicates that there is a positive relationship between the volatility and the magnitude of the basis. This is consistent with the findings of other studies such as Lee (1994), Chouldry (1997) and Kavussanos and Nomikos (2000) who found that when the spread between spot and futures (i.e. the basis) increases then the volatility in the market increases as well.

6 TIME VARYING HEDGE RATIOS
AND HEDGING EFFECTIVENESS

Following estimation of the univariate MRS models, smooth probability estimates are used to calculate an in-sample state-dependent hedge ratio for each market using Equation (6). Similarly, estimation of the bivariate MRS-BEKK smooth probability estimates are used to calculate an in-sample state-dependent hedge ratio for each market using Equation (15). To formally assess the performance of these hedges, portfolios implied by the computed hedge ratios each week are constructed and the variance of returns of these portfolios over the sample is calculated. In mathematical form we evaluate:

\[
\text{Var} (\Delta S_t - \gamma_t^* \Delta F_t) \quad (16)
\]

where \( \gamma_t^* \) are the computed hedge ratios. To compare the hedging performance of the MRS against alternative models that have been proposed in the literature, we also estimate and calculate hedge ratios based on the OLS model of Equation (1), on a bivariate Vector Error Correction Model (VECM) of spot and futures returns (Engle & Granger, 1987; Johansen, 1988), as well as time varying hedge ratios generated from a
bivariate VECM model with GARCH error structure. For benchmarking purposes, we also consider the use of a naïve hedge by taking a futures position which exactly offsets the spot position (i.e. setting $\gamma_t^* = 1$).

The portfolio variances for the three energy commodities are presented in Table III. The same table also presents the incremental variance improvement of the MRS-BEKK model against the other models. It can be seen that the MRS hedging strategies outperform the other models in terms of in-sample variance reduction. Among the MRS models, the MRS-BEKK is the best model for both the WTI crude oil and Heating oil market. In the Unleaded Gasoline market the univariate MRS provides better variance reduction compared to alternative strategies. Finally, another feature of the of the in-sample results is that MRS models provide greater variance reduction in both the crude and heating oil market as opposed to the unleaded gasoline market, where MRS-BEKK fails to deliver better variance reduction compared to constant hedge ratio strategies.

However, dynamic hedging strategies are more costly to implement than static models since they require frequent updating and rebalancing of the hedged portfolio. Consequently, hedging effectiveness is more appropriately assessed by considering the economic benefits from hedging as obtained from the hedger’s utility function. Consider an investor with the following mean variance utility function as in Kroner and Sultan (1993), Gagnon et al. (1998), and Lafuente and Novales (2003):

$$E_t U(x_{t+1}) = E_t (x_{t+1}) - kVar_t(x_{t+1})$$

where $k$ is the degree of risk aversion ($k > 0$) of the individual investor and $x_{t+1}$ represents the returns from the lagged portfolio. From Table III, the average weekly variance of returns from the hedged position in the WTI crude oil market is 6.2785 when the constant hedge ratio is used and 6.0652 when the MRS-BEKK model is used. Assuming that expected returns from the hedged portfolio are equal to zero and the degree of risk aversion is 4 then, on average, one obtains a weekly utility of $U(x_{t+1}) = -4 (6.2785) = -25.114$ if the constant hedge ratio is used and $U(x_{t+1}) = -4 (6.0652) = -$

---

10 The VECM-GARCH model is specified using the Baba et al (1987) (BEKK) representation (Engle & Kroner, 1995). See Kavussanos and Nomikos (2000) for details on the specification of these models. Estimation results for these models are available from the authors upon request.
24.261 when the MRS-BEKK hedge ratio is used. Hence, by using the MRS-BEKK model, hedgers in the market can benefit from an increase in the average weekly utility of 0.853 – y, over the constant hedge ratio, where y represents the reduced returns caused by the transaction costs incurred due to portfolio rebalancing. Therefore, a strategy based on the MRS-BEKK hedge ratio will be preferred over a constant strategy if y < 0.853. Assuming transaction costs in the range of 0.01-0.02%, even though the percentage variance reduction of the univariate Markov-based hedging strategy MRS is not dramatic, MRS hedge would still result in an improvement in utility for an investor with a mean–variance utility function and $k = 4$, even after accounting for transaction costs. Therefore, an investor with a mean-variance utility function would prefer the MRS-based strategies to the constant strategy since, on average, the increase in utility more than offsets the higher transaction costs due to rebalancing. Finally, it is also worth noting that all the MRS strategies outperform the GARCH model on the basis of utility comparisons.

The in-sample performance of the alternative hedging strategies gives an indication of their historical performance. However, investors are more concerned with how well they can do in the future using alternative hedging strategies. So, out of sample performance is a more realistic way to evaluate the effectiveness of the conditional hedge ratios. For that reason, we withhold the last 52 observations of the sample for each market, for the period August 6, 2003 to July 28, 2004, and estimate the models using only data up to this date.

In the case of the univariate Markov Regime Switching models (MRS), hedge ratios at time $t + 1$ are obtained using a two step procedure. First, estimates of the transition matrix at time $t$, $\hat{P}_t$, and the estimated smooth regime probabilities at time $t$, $\text{Pr}(s_t = 1) = \hat{\pi}_{1,t}$ and $\text{Pr}(s_t = 2) = \hat{\pi}_{2,t}$, are used to forecast regime probabilities at time $t + 1$, that is, $\pi_{1,t+1}$ and $\pi_{2,t+1}$ as follows:

$$
\begin{pmatrix}
\pi_{1,t+1} \\
\pi_{2,t+1}
\end{pmatrix} =
\begin{pmatrix}
\hat{\pi}_{1,t} \\
\hat{\pi}_{2,t}
\end{pmatrix}
\begin{pmatrix}
\hat{P}_{11,t} & \hat{P}_{12,t} \\
\hat{P}_{21,t} & \hat{P}_{22,t}
\end{pmatrix}
$$

(18)

11 These assumptions are in line with most empirical studies in the literature (Kroner & Sultan, 1993; Park & Switzer, 1995; Lafuente & Novales, 2003)
Second, the one-step ahead forecasts of regime probabilities are used to determine the optimal hedge ratio at time $t+1$, $\gamma^*_{t+1}$. This is done by multiplying the one-step ahead regime probabilities by the respective mean hedge ratio for each regime; i.e. by the mean of the hedge ratio under each market condition, and adding them up (see Hamilton, 1994 for more details).

$$\gamma^*_{t+1} = \left( \pi_{1,t+1}^e \pi_{2,t+1}^e \right) \begin{pmatrix} \gamma_{1,1} \\ \gamma_{1,2} \end{pmatrix} \tag{19}$$

In the case of Markov Regime Switching BEKK model, hedge ratios at time $t+1$ are obtained using a four step procedure. First, we estimate the forecast of the regime probabilities at time $t+1$, $\pi_{1,t+1}^e$ and $\pi_{2,t+1}^e$ as described above.

Second, we perform one step ahead forecasts of the state-dependent covariance and variance as follows:

$$\mathbb{E}[h_{SF, st, t+1} | \Omega_t] = c_{11, st} c_{12, st} + a_{11, st} a_{22, st} \varepsilon_{S,t} \varepsilon_{F,t} + \beta_{11, st} \beta_{22, st} h_{SF,t}$$

$$\mathbb{E}[h_{FF, st, t+1} | \Omega_t] = c_{12, st}^2 + c_{22, st}^2 + a_{22, st}^2 \varepsilon_{F,t}^2 + \beta_{22, st}^2 h_{FF,t} \quad ; \text{for } st = \{1, 2\} \tag{20}$$

Third we perform one step ahead forecasts of the mean equation for spot and futures returns, respectively, as follows:

$$\mathbb{E}[\Delta S_{t+1} | \Omega_t] = a_{S,t} + b_{S,1} \Delta S_t + \gamma_{S,1} \Delta F_t$$

$$\mathbb{E}[\Delta F_{t+1} | \Omega_t] = a_{F,t} + b_{F,1} \Delta S_t + \gamma_{F,1} \Delta F_t \quad ; \text{for } st = \{1, 2\} \tag{21}$$

The above forecasts of Equation (21) are used to integrate the state variable $st$ at each step of the recursive estimation, in the collapsing procedure of the variance-covariance matrix. After eliminating the state variable $st$ using Equations (9) and (11), the one step ahead forecast of the hedge ratio is computed as:
The following week (August 13, 2003) this exercise is repeated with the new observation included in the dataset. As in the case of the in-sample tests, the hedging performance of MRS-based hedge ratios is compared to that of alternative competing models. In the case of the GARCH-based hedges, the model is re-estimated each week during the out-of-sample period and out-of-sample hedge ratios are generated by obtaining one-step ahead forecasts of the time varying variance-covariance matrix, similar to Equations (20) and (22) of the MRS-BEKK forecasting procedure. Also, in the case of VECM a different hedge ratio is obtained each week by re-estimating the model each week during the out-of-sample period. The results from the out-of-sample performance from alternative hedging strategies are presented in Table IV.

Looking at the results for both Unleaded Gasoline and Heating Oil market, we can see that the highest reduction in the out-of-sample portfolio variance is achieved by the two regime MRS-BEKK model. Compared to constant (OLS) hedge the gain in variance reduction is 7.52% for Unleaded Gasoline and 2.29% for Heating oil. This compares favourably with findings in other futures markets. For out-of-sample variance reductions, Kroner and Sultan (1993) report percentage variance reductions of the GARCH hedges relative to the OLS hedge ranging between 4.94% and 0.96% for five currencies; Gagnon and Lypny (1995) find 1.87% variance reduction for the Canadian interest rate futures, Bera et al (1997) estimate 2.74% and 5.70% variance reductions for the corn and soybeans futures. Kavussanos and Nomikos (2000) report variance reductions for individual freight routes using freight futures contracts (BIFFEX) ranging between 0.43% and 5.7%. Alizadeh et al. (2004) report a gain of 1.16% to 6.81%, assessing the effectiveness of petroleum futures to hedge bunker price fluctuations. In addition, the univariate MRS models outperform the alternative non-Markovian strategies in the Heating oil market. However, in the case of WTI crude and Unleaded Gasoline, the univariate MRS fails to deliver better variance reduction than both the naïve and GARCH hedges, but it outperforms the conventional OLS hedge.

Concerning the crude oil market, the greatest variance reduction is provided by the single regime VECM-GARCH hedging strategy whereas MRS-BEKK model achieves
almost the same levels of variance reduction with Constant (OLS) hedge ratio. The univariate MRS models perform better than the MRS-BEKK in that case, but still the variance reduction is greater for the GARCH model. One possible explanation for this surprising result may be the fact that occasionally MRS models do not provide accurate forecasts on an out-of-sample basis. This may be due to parameter instability between in-sample and out-of-sample periods as well as uncertainty regarding the unobserved regime, as mentioned in Engle (1994) and Marsh (2000).

7 CONCLUSIONS

In this paper we examined the performance of hedge ratios generated from Markov Regime Switching (MRS) models in energy futures markets. The rationale behind the use of these models stems from the fact that the dynamic relationship between spot and futures prices may be characterized by regime shifts. This, in turn, suggests that by allowing the hedge ratio to be dependent upon the “state of the market”, one may obtain more efficient hedge ratios and hence, superior hedging performance compared to the methods which are currently being employed.

The effectiveness of the MRS time-varying hedge ratios is investigated in the NYMEX WTI light sweet crude oil, Unleaded Gasoline and Heating oil market. LR tests on the estimated models indicate that there is marked asymmetry in unconditional and conditional volatilities under different market conditions. Additionally, hedge ratios are significantly different across different states of the market. The suggested MRS-BEKK based hedge ratios seem to capture reasonably well the data. Allowing for regime shifts, forecasting performance of GARCH models is improved by reducing volatility persistence and introducing time varying GARCH parameters via the Markovian specification. Moreover, all the MRS based hedge ratios appear to be higher when the volatility in the market is low, a finding that is in line with theory.

In and out-of-sample tests indicate that the MRS hedge ratios outperform the GARCH, VECM, and the OLS hedges in all the energy commodities markets, within sample. The same is true in the out-of-sample analysis, overall. By allowing the variances of spot-
futures returns, as well as the covariance to be state dependent, in and out of sample hedging effectiveness is further improved. However, in the WTI crude oil market the "single regime" VECM-GARCH hedging strategy reflects the best performance, whereas the MRS hedge ratios provide variance reduction similar to constant hedge ratios. Overall, the results indicate that using MRS models market agents may be able to obtain superior gains, measured in terms of variance reduction and increase in utility.

BIBLIOGRAPHY


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<th>WTI light sweet crude oil</th>
<th>Unleaded Gasoline</th>
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Panel A – Descriptive Statistics

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</tr>
<tr>
<td>Heating Oil #2</td>
<td>2</td>
<td>r = 0</td>
<td>r = 1</td>
<td>72.872</td>
<td>19.96</td>
<td>24.60</td>
<td>[1.00    -0.019 -1.031]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>r ≤ 1</td>
<td>r = 2</td>
<td>4.2065</td>
<td>9.24</td>
<td>12.97</td>
<td>[1.00    -0.019 -1.031]</td>
</tr>
</tbody>
</table>

Sample period is from January 23, 1991 to July 30, 2003 (654 weekly observations).

- JB test is the Jarque-Bera (1980) test for Normality. The test follows a $\chi^2$ distribution with 2 degrees of freedom.
- Q(6) and Q2(6) are Ljung-Box (1976) tests for 6th order autocorrelation in the level and squared series, respectively. The statistics are $\chi^2(6)$ distributed.
- PP test is the Phillips and Perron (1988) unit root test. 1%, 5% and 10% critical values for this test are -3.4402, -2.8658 and -2.5691, respectively.
- The LR statistic is used to test the hypothesis that the cointegrating vector ($\beta_0 \beta_1 \beta_2$) is (1 0 -1). The statistic follows $\chi^2$ distribution with 9 degrees of freedom.

Cointegration tests are based on Johansen (1988) procedure; the model is specified with an intercept term in the cointegrating vector and the VAR. Critical values obtained from Osterwald-Lenum (1992)
### TABLE II - Estimates of Markov Regime Switching BEKK Hedge Ratios for NYMEX Energy Commodities

Sample Period: January 23, 1991 to July 30, 2003

- **Univariate:**
  \[ \Delta S_t = \gamma_{0,t} + \gamma_{1,t} \Delta F_t + \varepsilon_{t-1} ; \quad \varepsilon_{t-1} \sim iid (0, \sigma_{t-1})^2 \]

- **Bivariate:**
  \[ \Delta S_t = a_{S,t} + \sum_{i=1}^{c_1} b_{S,i} \Delta S_{t-i} + \sum_{i=1}^{c_2} \gamma_{S,i} \Delta F_{t-i} + \varepsilon_{t,S,t} ; \quad \Delta F_t = a_{F,t} + \sum_{i=1}^{c_1} b_{F,i} \Delta S_{t-i} + \sum_{i=1}^{c_2} \gamma_{F,i} \Delta F_{t-i} + \varepsilon_{t,F,t} ; \quad \varepsilon_t = \begin{pmatrix} \varepsilon_{S,t} \\ \varepsilon_{F,t} \end{pmatrix} \Omega \sim IN \left(0, H_t \right) \]

\[ H_t = \begin{pmatrix} h_{S,S,t} & h_{S,F,t} \\ h_{F,S,t} & h_{F,F,t} \end{pmatrix} = \begin{pmatrix} c_{11,t} & c_{12,t} \\ c_{21,t} & c_{22,t} \end{pmatrix}^{-1} \begin{pmatrix} c_{11,t} & c_{12,t} \\ c_{21,t} & c_{22,t} \end{pmatrix} + \begin{pmatrix} a_{11,t} & a_{12,t} \\ a_{21,t} & a_{22,t} \end{pmatrix} + \begin{pmatrix} \beta_{11,t} & 0 \\ 0 & \beta_{22,t} \end{pmatrix} H_{t-1} \begin{pmatrix} \beta_{11,t} & 0 \\ 0 & \beta_{22,t} \end{pmatrix} \]

#### Panel A: Univariate (MRS)

<table>
<thead>
<tr>
<th></th>
<th>West Texas Intermediate</th>
<th>Unleaded Gasoline</th>
<th>Heating Oil # 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean Equation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma_{0,t} )</td>
<td>-0.043 (0.145)</td>
<td>0.075 (0.108)</td>
<td>-0.017 (0.361)</td>
</tr>
<tr>
<td>( \gamma_{1,t} )</td>
<td>0.817 (0.011)*</td>
<td>0.988 (0.048)</td>
<td>0.901 (0.002)*</td>
</tr>
<tr>
<td>( \gamma_{0,2} )</td>
<td>0.006 (0.001)</td>
<td>-0.103 (0.057)</td>
<td>-0.032 (0.001)*</td>
</tr>
<tr>
<td>( \gamma_{1,2} )</td>
<td>1.004 (0.005)*</td>
<td>1.113 (0.023)*</td>
<td>1.036 (0.012)*</td>
</tr>
<tr>
<td><strong>Variance Equation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>3.479 (0.366)*</td>
<td>3.873 (0.252)*</td>
<td>7.068 (1.613)*</td>
</tr>
<tr>
<td>( \sigma_2 )</td>
<td>0.301 (0.081)*</td>
<td>1.005 (0.012)*</td>
<td>0.859 (0.001)*</td>
</tr>
<tr>
<td><strong>Transition Probabilities</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_{12} )</td>
<td>0.2041 (0.045)*</td>
<td>0.2136 (0.018)*</td>
<td>0.1711 (0.001)*</td>
</tr>
<tr>
<td>( P_{21} )</td>
<td>0.2047 (0.056)*</td>
<td>0.2264 (0.017)*</td>
<td>0.0344 (0.001)*</td>
</tr>
</tbody>
</table>

#### Panel B: Bivariate (MRS-BEKK)

<table>
<thead>
<tr>
<th></th>
<th>West Texas Intermediate</th>
<th>Unleaded Gasoline</th>
<th>Heating Oil # 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean Equation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_{S,1} )</td>
<td>-0.275 (0.423)</td>
<td>-0.005 (0.431)</td>
<td>-2.499 (0.843)**</td>
</tr>
<tr>
<td>( a_{S,2} )</td>
<td>0.207 (0.164)</td>
<td>0.176 (0.203)</td>
<td>0.250 (0.139)**</td>
</tr>
<tr>
<td>( b_{F,1} )</td>
<td>-0.040 (0.097)</td>
<td>-0.037 (0.062)</td>
<td>0.033 (0.076)</td>
</tr>
<tr>
<td>( b_{F,2} )</td>
<td>0.437 (0.100)*</td>
<td>0.251 (0.098)</td>
<td>0.254 (0.086)*</td>
</tr>
<tr>
<td>( \gamma_{1,1} )</td>
<td>-0.374 (0.396)</td>
<td>-0.086 (0.452)</td>
<td>-3.036 (0.654)**</td>
</tr>
<tr>
<td>( \gamma_{1,2} )</td>
<td>0.207 (0.166)</td>
<td>0.155 (0.196)</td>
<td>0.247 (0.142)**</td>
</tr>
<tr>
<td>( \gamma_{2,1} )</td>
<td>0.041 (0.121)</td>
<td>0.053 (0.296)</td>
<td>-0.029 (0.090)</td>
</tr>
<tr>
<td>( \gamma_{2,2} )</td>
<td>2.016 (0.358)*</td>
<td>2.311 (0.374)*</td>
<td>1.869 (0.920)**</td>
</tr>
<tr>
<td><strong>Variance Equation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_{11,1} )</td>
<td>1.324 (0.366)*</td>
<td>1.653 (0.243)*</td>
<td>2.068 (1.145)**</td>
</tr>
<tr>
<td>( c_{12,1} )</td>
<td>-1.182 (0.229)*</td>
<td>1.250 (0.128)*</td>
<td>0.912 (1.412)</td>
</tr>
<tr>
<td>( c_{21,1} )</td>
<td>0.056 (0.024)**</td>
<td>0.055 (0.034)</td>
<td>0.166 (0.032)</td>
</tr>
<tr>
<td>( c_{22,1} )</td>
<td>0.021 (0.016)</td>
<td>0.091 (0.054)**</td>
<td>-0.049 (0.035)</td>
</tr>
<tr>
<td>( \beta_{11,2} )</td>
<td>0.498 (0.265)**</td>
<td>0.643 (0.227)*</td>
<td>0.967 (0.288)</td>
</tr>
<tr>
<td>( \beta_{12,2} )</td>
<td>0.663 (0.173)*</td>
<td>0.653 (0.132)*</td>
<td>0.501 (0.537)</td>
</tr>
<tr>
<td><strong>Transition Probabilities</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_{12} )</td>
<td>0.2541</td>
<td>0.6304</td>
<td>0.8019</td>
</tr>
<tr>
<td>( P_{21} )</td>
<td>0.1353</td>
<td>0.3075</td>
<td>0.0513</td>
</tr>
</tbody>
</table>

\( * \), ** and *** indicate significance at 1%, 5% and 10%, respectively.

**Figures in ( )** are the estimated standard errors. Standard errors are corrected for serial correlation and heteroscedasticity using the Newey-West (1987) method.

**OLS HR** is the ratio derived from the simple OLS method of Equation (1).

\( H_{t-1} \sim \chi^2 (df) \)

\( LR_{test} \) and \( Tstat \) (df =1)
### TABLE III
In-Sample Hedging Effectiveness of Markov Regime Switching Against the Constant and Alternative Time-Varying Hedge Ratio Models

<table>
<thead>
<tr>
<th></th>
<th>WTI light sweet Crude Oil</th>
<th>Unleaded Gasoline</th>
<th>Heating Oil #2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variance</td>
<td>Variance Improvement of MRSBEKK</td>
<td>Weekly Utility</td>
</tr>
<tr>
<td></td>
<td>of MRSBEKK</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unhedged</td>
<td>21.175</td>
<td>71.36%</td>
<td>-84.700</td>
</tr>
<tr>
<td>Naïve</td>
<td>6.5618</td>
<td>7.569%</td>
<td>-26.247</td>
</tr>
<tr>
<td>Constant</td>
<td>6.2785</td>
<td>3.398%</td>
<td>-25.114</td>
</tr>
<tr>
<td>VECM</td>
<td>6.2789</td>
<td>3.404%</td>
<td>-25.116</td>
</tr>
<tr>
<td>GARCH</td>
<td>6.5995</td>
<td>8.096%</td>
<td>-26.398</td>
</tr>
<tr>
<td>MRS</td>
<td>6.0988</td>
<td>0.552%</td>
<td>-24.395</td>
</tr>
<tr>
<td>MRS-BEKK</td>
<td>6.0652*</td>
<td>-</td>
<td>-24.261*</td>
</tr>
</tbody>
</table>

*The in-sample period is from January 23, 1993 (654 observations). An asterisk (*) indicates the model that provides the greatest variance reduction and/or increase in utility. See Table II for description of the different models.

*Variance denotes the variance of the hedged portfolio. Note that the variance corresponds to logarithmic returns multiplied by 100 [Equation (16)].

*Variance Improvement of MRSBEKK measures the incremental variance reduction of the MRSBEKK model versus the other models. This is estimated using the formula: \([\text{Var(Model)} - \text{Var(MRS3)}]/\text{Var(Model)}\).

*Weekly utility is the average weekly utility for an investor with a mean-variance utility function [Equation(17)] and a coefficient of risk aversion of 4, using different hedging strategies.

*Utility gain is the increase/decrease in weekly utility by using the time-varying hedging strategies (MRS and GARCH) relative to the constant strategy.
**TABLE IV**
Out-of-Sample Hedging Effectiveness of Markov Regime Switching Against the Constant and Alternative Time-Varying Hedge Ratio Models<sup>a</sup>

<table>
<thead>
<tr>
<th></th>
<th>WTI light sweet Crude Oil</th>
<th>Unleaded Gasoline</th>
<th>Heating Oil #2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variance&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Variance Improvement of MRSBEKK&lt;sup&gt;c&lt;/sup&gt;</td>
<td>Weekly Utility&lt;sup&gt;d&lt;/sup&gt; wrt Constant&lt;sup&gt;e&lt;/sup&gt;</td>
</tr>
<tr>
<td>Unheded</td>
<td>15.468</td>
<td>86.76%</td>
<td>-61.87</td>
</tr>
<tr>
<td>Constant</td>
<td>2.0333</td>
<td>-0.689%</td>
<td>-8.133</td>
</tr>
<tr>
<td>VECM</td>
<td>2.0198</td>
<td>-1.360%</td>
<td>-8.079</td>
</tr>
<tr>
<td>GARCH</td>
<td>1.6915*</td>
<td>-21.03%</td>
<td>-6.766*</td>
</tr>
<tr>
<td>MRS-BEKK</td>
<td>2.0473</td>
<td>-8.189</td>
<td>-0.056</td>
</tr>
</tbody>
</table>

<sup>a</sup>The out-of-sample period is from August 6, 2003 (52 observations). An asterisk (*) indicates the model that provides the greatest variance reduction and/or increase in utility. See Table II for description of the different models.

<sup>b</sup>Variance denotes the variance of the hedged portfolio. Note that the variance corresponds to logarithmic returns multiplied by 100 [Equation (16)].

<sup>c</sup>Variance Improvement of MRSBEKK measures the incremental variance reduction of the MRSBEKK model versus the other models. This is estimated using the formula: (Var(Model) – Var(MRSBEKK))/Var(Model).

<sup>d</sup>Weekly utility is the average weekly utility for an investor with a mean-variance utility function [Equation (17)] and a coefficient of risk aversion of 4, using different hedging strategies.

<sup>e</sup>Utility improvement wrt Constant is the increase/decrease in weekly utility by using the time-varying hedging strategies (MRS and GARCH) relative to the constant strategy.
FIGURE 1
Smooth regime probabilities for WTI Crude Oil – Probability of being in the low variance state.

FIGURE 2
Basis(log(Spot) – log(Futures)) for WTI Crude Oil.

FIGURE 3
Constant OLS, VECM-GARCH and MRS-BEKK hedge ratios for WTI Crude Oil.
FIGURE 4
Smooth regime probabilities for Unl.Gasoline – Probability of being in the low variance state.

FIGURE 5
Basis(log(Spot) – log(Futures)) for Unl.Gasoline.

FIGURE 6
Constant OLS, VECM-GARCH and MRS-BEKK hedge ratios for Unl.Gasoline.
FIGURE 7
Smooth regime probabilities for Heating Oil – Probability of being in the low variance state.

FIGURE 8
Basis(log(Spot) – log(Futures)) for Heating Oil.

FIGURE 9
Constant OLS, VECM-GARCH and MRS-BEKK hedge ratios for Heating Oil.