A forward contract to manage market power: the case of Italy

Rita L. D'Ecclesia
Dipartimento di Teoria Economica e MQSP,
University of Rome "La Sapienza"
Piazza Aldo Moro, 5. Rome. Italy
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Abstract

In a deregulated electricity market the use of derivative contracts became crucial to guarantee a competitive behavior of the spot market. Moving from a centrally owned system to a competitive one has been the main focus of most international electricity markets.

In most countries the electricity markets still do not operate in a competitive framework but experience market power. In this context the kind of contracts designed to trade electricity became a crucial tool to manage the market power risk.

Given the non-storability of the electricity the familiar arbitrage-based methods are not applicable for pricing derivative contracts. In this paper we examine an equilibrium forward contract on a nonstorable commodity when forward market participants have market powers. An equilibrium forward contract is defined using the Nash bargaining approach, in line with Dong and Liu, '05. We consider the introduction of a forward market as an effective tool to increase both the production of the commodity and the trading volume in the spot market and therefore reduce market power. We calibrate the suggested model using a real data set from the Italian market. The simulation shows that the forward price can be a downward or an upward biased predictor of the spot price, depending on the presence of market power. We also show that the forward contract may stimulate trading on the spot market.

EL Classification Numbers: D11, D91, G11, C61.
Keywords: Derivative Contract, Open Interest, Nonstorable, Electricity.
1 Introduction

Markets for physically or economically nonstorable commodities have grown rapidly in recent years thanks to the deregulation of the industry. Most of the deregulation scenarios imply the separation of power production from transmission and retailing, with production and retailing opened to competition. Over the last ten years, major countries have been experiencing deregulation in generation and supply activities (transmission, distribution activities generally remaining under the state authority). One of the important consequences of this restructuring is that trading volume, commodity variety, and innovative contract specification have grown tremendously. A large increase of trading activity, both in the spot and derivative markets occurred. The United States and the main European countries have shown a large increase of wholesale power transactions during the last decade. In the 2004 the U.S. wholesale power transactions amounted to over 503 billion megawatt hours (MWh) or about $198 billion and the Italian wholesale transactions amounted to over 323,9 million megawatt hours (MWh) or about 12,9 billion Euro in the 2005.

In this markets of non storable commodities prices are now determined according to the fundamental economic rule of supply and demand. Supply is provided by generators and demand is represented by industrial consumers, power marketers and distributors buying electricity in the pool to sell it to end-users. There is a "market pool" where bids placed by generators to sell electricity for the next day are confronted to purchase orders and equilibrium prices are defined as the intersection of the aggregate demand and supply curves for each hour (or half-hour) in the day. Unlike market participants in a typical market for storable commodities, buyers and sellers in a market for nonstorable commodities usually have significant market powers. These powers stem mainly from the limited market participation caused by nonstorability and also from

market frictions such as location preferences, transportation costs, long-term business relationships, and asymmetric information. Furthermore, in many industries where storage is physically or economically infeasible, there often exists an almost exclusive supply relationship between a supplier and a manufacturer to assure timely delivery, and accordingly, supply contracts for future delivery are usually negotiated bilaterally.  

A vast literature exists on pricing derivatives written on storable commodities. In contrast, even though the literature on pricing derivatives on nonstorable commodities has grown considerably since the deregulation of the electricity market, it is still relatively limited partly due to the inapplicability of the well known no-arbitrage argument. In addition, in this limited literature, derivative markets are usually assumed to be perfectly competitive and hence market powers of the participants are ignored. For example, both Kawai (1983) and Bessembinder and Lemmon (2002) assume that both the producers and the consumers are price takers in derivative markets.


For the European side Green and Newbery (1992), Newbery (1995) and Wolfram (1999) and that participants in the British power markets have market powers. Also, Krapels (2000) and Smeers (2004) point out how the level of competition in the European electricity system remain far from being satisfactory.

The majority of electricity trading is through bilateral contracts. In particular, only about 25% of all traded electricity in Norway-Sweden is managed by Nord Pool; the rest is handled by physical contracts in the bilateral wholesale market (Hjalmarsson (2000)).

In the energy market covered by PJM, 54% of the trading is through bilateral transactions in 2001, and even for the rest of the traded contracts, telephone negotiation is usually involved (see Laughlin (2003) for example). In the Italian market bilaterally contracts amounted to 120.3 billion of MWh in the 2005 representing 1/3 of the total traded volume.

As opposed to the assumptions made in this paper, Bessembinder and Lemmon (2002) assume that the forward market is perfectly competitive, the retailer’s resale price is fixed, and the producer and the retailer have the same risk aversion.
as hedging, use of derivatives and, quite importantly, to identify and price the options embedded in energy contracts that have been written for decades.

The issue of how electricity is priced in spot and forward wholesale power markets has become one of the most controversial topics facing utilities, power producers, regulators, political officials, accounting firms, and a broad array of financial market participants. An important complication that makes this issue particularly difficult to address is the unique nature of electricity as a commodity, since it is virtually nonstor-able. This feature eliminates the buffering effect associated with holding inventories, and makes the possibility of sudden large price changes more likely. In addition, the standard no-arbitrage approach to pricing financial securities cannot be applied.

This paper examines the role of the forward contract in the electricity market, given forward contracts have been rapidly growing in importance as both financial risk management tools for hedgers as well as liquid investment vehicles for energy trading firms. Following Dong and Liu (2005) we calibrate the equilibrium forward contract on Italian market data using an extensive set of hourly spot and day-ahead electricity prices from the wholesale Italian Power Exchange for the period from June 2004 to November 2006.

Dong and Liu derive the unique equilibrium forward contract in closed form, and show that a forward contract can help a participant both improve profitability (speculation benefit) and reduce profit risk (hedging benefit). They also show that the market powers of the participants may only affect the equilibrium forward price, but not its contract size (which can also be interpreted as open interest). Furthermore, the introduction of a forward market may increase both the production of the commodity and the trading volume in the spot market. The unique forward contract is found assuming both the supplier and the manufacturer have significant market powers and negotiate the forward contract through a Nash bargaining process.

As Dong and Liu who calibrated the model on the PJM electricity market we also find that forward price on a nonstor-able can be non-

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4 Some examples include Eydeland and Geman (1999), Pirrong and Jermaykyan (1999), and Bessembinder and Lemmon (2002).
6 As Nash (1950) shows, the Nash bargaining solution is the only one that is Paretian, symmetric, and that is independent of utility units and irrelevant alternatives (see also Proposition 22.E.1 of Mas-Colell, Whinston, and Green 1995).
monotonic in the spot price.

For the calibrated model, we find that the forward price acts differently as predictor of the spot price for the various seasons. The difference across seasons may be explained by a change in the convenience yield for the commodity and the market power of the manufacturer. In winter, spring, and fall, the convenience yield is high, which implies that the supplier’s capacity reservation cost is only a fraction of the spot price. The obtained results on the relationship between the forward price and the expected spot price is consistent with that of Bessembinder and Lemmon (2002), who attribute the finding to the difference in the spot price skewness.

The outline of the paper is as follows. Section II briefly recall the main model’s assumptions. Section III introduces the Italian Power Exchange. Section IV contains the description of data for the empirical calibration of the model. Section V analyzes the results and Section VI concludes with a few comments and suggestions for future research.

2 The model

We refer to the model developed by Dong and Liu (2005), according to which a closed form solution for a forward contract is derived. We here shortly recall the main assumptions and development of the model and refer to Dong and Liu (2005) for further details.

A manufacturer uses an input commodity to produce a final product at time $T$ and sells it in a final product market. At time $t \in [0; T)$, the time $T$ final product demand $D_T$ is uncertain, with mean $\mu_D$ and standard deviation $\sigma_D$. This demand will only be realized and observed at time $T$ just before the production. At time 0 the manufacturer can negotiate a supply contract that matures at time $T$ with only one supplier. This is assumed in order to model the bilateral nature of the supply contracts. In particular, at time $t$ it is assumed that the manufacturer buys a forward contract $(f, Q)$, where a positive $Q$ means that the manufacturer agrees to buy $Q$ units of the input commodity at time $T$ from the supplier at a price of $p_f$ per unit. The supplier and the manufacturer incur fixed negotiation costs, $F_s$ and $F_m$ respectively. In addition they both can trade in the input commodity spot market.

Both the supplier and the manufacturer are small relative to other firms in these markets and so they are only price takers. In addition, following Rolfo (1980), Hershleifer and Subrahmanyam (1993) and Bessembinder and Lemmon (2002), both the manufacturer and the supplier have mean-variance preferences over their own risky pro...ts. The manu-
The manufacturer's utility at time 0 is
\[ U_m = E[\pi_m] + \lambda_m \text{var}[\pi_m] \tag{1} \]
where \( \pi \) is a manufacturer's profit at time T and \( \lambda_m \) is the corresponding risk aversion coefficient and \( E \) and \( \text{var} \) denote the expectation and variance operators over the time T distributions of random variables as the input spot price, \( p \), the final product demand, \( D \), the final product sale price, \( s \). The supplier's utility at time 0 is
\[ U_s = E[\pi_s] + \lambda_s \text{var}[\pi_s] \tag{2} \]

The model is developed for regions where the utility functions are increasing in profits: \( \lambda_j < 1/\bar{\pi}_j \), where \( \bar{\pi}_j \) is the maximum possible profits for \( j \in [m,s] \).

Precisely, given the production level \( q \) of the final product, the manufacturer's time T profit is equal to the revenue from final product market \( s \min(q,D) \), minus the cost of forward contract \( fQ \), minus the cost of spot trading \( p(q-D) \), minus the goodwill cost \( g(D-q) \) from not satisfying demand \( D \) and minus the negotiation cost \( F_m1_{fQ \not= 0} \)
\[ \pi_m(f,Q,D,s,p) = \max_{q \not= 0} s \min(q,D) - fQ - p(q-D) - g(D-q) + \text{\^{}} \tag{3} \]

At time T without a forward contract (manufacturer's operational profit) is given by
\[ \pi_{m0}(f,Q,D,s,p) = (s + g - p) + D - gD \tag{4} \]
so the profit realized from the forward contract is given by:
\[ E[pQ + F_m1_{fQ \not= 0}] \tag{5} \]

The optimal production level
\[ q^* = \begin{cases} \frac{1}{2}D, & \text{for } s + g > p \\ 0, & \text{for } s + g \leq p \end{cases} \]
is always independent of the forward contract \( (f,Q) \) and this in the presence of a spot market a forward contract does not affect time T production decisions.

Given a forward contract \( (f,Q) \) the expected profit at time T is given by
\[ E[\pi_m] = E[\pi_{m0}] + \mu_p fQ + F_m1_{fQ \not= 0} \]
\[ \text{var}[\pi_m] = \text{var}[\pi_{m0}] + \sigma_p^2Q^2 + 2Q \text{Cov}_t \tag{6} \]
and
\[ \text{cov}_t = \text{cov}(s_T, p_T) + D_T, p_T \] (7)
is the time t conditional covariance between the profit from trading in
the spot market and per unit profit from the forward.

In line with other findings in the existing literature on derivative mar-
kets for nonstorable commodities (Green and Newbery (1992), Newbery
(1995), Wolfram (1999), and Krapels (2000)), Dong and Liu assume that
the forward market is imperfectly competitive and this usually dictates
the market powers of both manufacturer and supplier. Using the stan-
dard Nash bargaining game to model the contract negotiation process of
two parties to capture the various degrees of market powers, it is possible
to define the forward contract maximizing a power-weighted product of
manufacturer and supplier utility gains:
\[ \max (U_m - U_{m0})^\theta (U_s - U_{s0})^{\frac{1}{\theta}} \] (8)
subject to the constraint:
\[ U_m - U_{m0} \geq 0 \] (9)
\[ U_s - U_{s0} \geq 0 \] (10)
and \( \theta \in [0, 1] \) measures the manufacturer’s market power. The sequence
of events can be summarized in:

1. At \( t = 0 \) the supplier and the manufacturer negotiate a forward
   contract, \((Q_T, f_T)\);

2. Immediately after the forward transaction, the supplier reserves
   the capacity \( K_t \), at the unit capacity reservation cost of \( c_t \).

3. The demand \( D \), the product price \( s_T \), and the commodity spot
   price \( p_T \) are observed at time \( T \). The delivery of \( Q_t \) units of the
   commodity and the payment of \( f_tQ_t \) are then made.

4. At time \( T \), the manufacturer and the supplier can also trade in
   the commodity spot market at the spot price \( p_T \) to maximize their
   profits.

5. The manufacturer then produces the final product and sells it to
   the final product market at the unit price of \( s_T \).

Using the Nash bargaining game the manufacturer and supplier utility
function is maximized and a unique equilibrium \((Q^*, f^*)\) is found:
\[ Q^* = \begin{cases} \mu_{pt} \phi_{i} \frac{2\lambda_m \text{cov}}{2\lambda_m \sigma_{pt}^2} \quad \text{when} \quad b < \mu_{pt} \phi_{i} \frac{\lambda_s}{\lambda_m + \lambda_s} 2\lambda_m \text{COV} \\ 0 \quad \text{otherwise} \end{cases} \] (11)
\[ f^n = (1 + \theta) \mu_{pt} \lambda_m \sigma_{p t}^2 Q_i^n + 2COV_i^Q + \theta b_i \frac{U_{s0}}{Q_i^n} \]  

(12)

and

\[ U_m^n = \theta \frac{\lambda_{s0}^2 COV_i^2}{(\lambda_m + \lambda_s) \sigma_p^2} U_{s0} \quad ; \quad U_s^n = \frac{1}{\theta} U_m^n \]  

(13)

The equilibrium forward price as well as the equilibrium utility gains of the manufacturer are also unaffected by the supplier’s risk aversion coefficient. When \( b > \mu_{pt} \), the supplier’s utility gain depends on his/her risk aversion. Accordingly, the more risk averse is the supplier, the less gain he receives from trading in the forward market and hence he demands a higher forward price in the negotiation. Finally, as the capacity reservation cost increases, the forward becomes less attractive and thus the equilibrium open interest decreases.

In this paper we want to estimate the possible effects of a forward contract in the Italian electricity market. According to the main finding of this model the equilibrium forward contract price and quantity may be derived using information on the electricity spot price and demand.

3 The Italian Electricity Market

The Italian Power Exchange originates from Legislative Decree no. 79 of 16 March 1999 (Legislative Decree 79/99). As in other international experiences, various entities are in charge of managing the electricity sector, each playing a specific role that is explicitly defined in the applicable legislation. The players in the Italian market are

1. the Ministry of Productive Activities which, among others, formulates the strategies and policies for security and cost-effectiveness of the national power system;

2. the Autorità per l’Energia Elettrica ed il Gas (AEEG, Electricity & Gas Regulator), which is vested with regulating and monitoring tasks aimed to promoting competition and efficiency of the sector;

3. Gestore della Rete di Trasmissione Nazionale (GRTN, Italian Independent Transmission System Operator), which carries out the activities of transmission and dispatching of electricity;

4. Acquirente Unico (AU, Single Buyer), which guarantees the supply of electricity to the so-called “captive customers”; and

5. Gestore del Mercato Elettrico (GME, Electricity Market Operator), which organises and manages the Electricity Market under criteria of non-discrimination, transparency, objectivity and competitiveness between producers.
The scheduling of generating units for the following day is vested in GME. GME collects the supply offers (sale offers) for each hour and each unit that are submitted by producers in the energy markets. Therefore, the expected demand in each hour of the following day is not estimated by GRTN, but directly producers and eligible customers may sell and buy electricity not only in GME’s organised market, but also through purchase and sale contracts made outside the bidding system (the so-called bilateral contracts). In the latter case, both the supplies – i.e. injection and withdrawal schedules – and the price at which electricity is valued are freely determined by the parties. However, also bilateral contracts are checked for compliance with transmission constraints.

To this end, GRTN will notify GME of the injection and withdrawal schedules resulting from bilateral contracts, in the form of supply offers and demand bids. The Electricity Market consists of: having maximum price priority, i.e. zero-price supply offers and non-price-dependent demand bids, respectively (Fig. 1).

The following are the general elements common to the three markets:

1) Day-Ahead Market (MGP) for the wholesale trading of energy between producers and wholesale customers (or eligible customers). This market usually takes place in the morning of the day ahead of delivery day. All Market Participants in respect of all supply points may participate in this market.

2) Adjustment Market (MA), where Market Participants may revise the schedules resulting from the Day-Ahead Market, by submitting additional demand bids or supply offers. This market takes place immediately after the Day-Ahead Market, usually in the afternoon, and

3) Ancillary Services Market (MSD), where Market Participants submit offers/bids for increase or decrease of injection or withdrawal in each hour.

The following are the general elements common to the three markets:
Oﬀers/bids. Market Participants submit demand bids or supply oﬀers (sale or purchase oﬀers) into the market. Oﬀers/bids consist of “quantity-unit price” (MWh, €/MWh) pairs. Market Participants. Each Market Participant may submit oﬀers/bids in respect of diﬀerent supply points, just as diﬀerent Market Participants may submit oﬀers/bids for the same point. Consequently, Market Participants should not necessarily coincide with dispatching users. Validation of oﬀers/bids. The oﬀers/bids that GME receives are checked for the adequacy of formulation and then are validated by the information system upon receipt and the outcome of the validation is timely notiﬁed to the relevant Market Participant.

Acceptance of oﬀers/bids. Oﬀers/bids in the Day-Ahead Market and Adjustment Market are accepted by GME, whereas oﬀers/bids in the Ancillary Services Market are accepted by GRTN. In the latter market, GME gathers the oﬀers/bids submitted by Market Participants and notiﬁes them of the results. GME’s Electricity Market does not work as a continuous trading power exchange, but is more similar to an auction. The acceptance of oﬀers/bids in respect of each hour is a separate process, which ends independently of the processes in other hours. After determining the market results, GME: notiﬁes to each Market Participant the result of the respective oﬀers/bids.

Day-Ahead Market (MGP). The Day-Ahead Market is a market for wholesale trading of energy between Market Participants, where prices, traded quantities, injection and withdrawal schedules for the following day are deﬁned. GME notiﬁes the schedules to GRTN in order to enable it to check their consistency with the transmission limits of the grid and to determine the expected demand. All entities qualifying as “Market Participants” may opt to participate in the MGP. The central counterparty for purchase and sale transactions in the MGP is GME. Part of electricity may be also traded in the MGP through Bilateral Contracts. The electricity traded under bilateral contracts participates in the entire process, because: i) it uses part of the available transmission capacity; and ii) it contributes to the quantities used for weighting the single national price. For this purpose, GRTN notiﬁes to GME the schedules arising from this trading.

Adjustment Market (MA). The Adjustment Market is a market for wholesale trading of electricity between Market Participants, for determining prices and traded quantities and for revising the injection and withdrawal schedules resulting from the Day-Ahead Market for the following day. GME notiﬁes these schedules, too, to GRTN, which will check them for consistency with the transmission limits on the grid and will determine the amount of demand. The need for an Adjustment Market after the Day-Ahead Market arises from the use of simple oﬀers/bids:
as the 24 hourly injection or withdrawal schedules in each supply point are determined independently of each other, they are not guaranteed to be jointly consistent with the dynamic constraints of the power plants that are related to such points. The availability of an Adjustment Market allows Market Participants to submit appropriate demand bids or supply offers in order to accommodate their own schedules. All entities qualifying as “Market Participants” may opt to participate in this Market and the central counterparty for sale and purchase transactions will be GME.

4 The dataset

The dataset has been provided by the Italian Power Exchange. It refers to the daily spot price for the period 2004-2006, the average trading volumes, and the average monthly demand. Figure 2 shows the average price dynamic over the last two years. Figure 3 reports the outstanding volume of sales in millions of Euros for the last two years. The average electricity purchasing price (PUN) in IPEX was equal to 76.28 €/MWh, up by 10.69 €/MWh on the same month of 2005 (+16.3%).

Electricity volumes traded on IPEX were equal to 17.3 million MWh, down by 9.7% on December 2005. Average market liquidity amounted to 61.2%.

The lowest average selling price was recorded in northern Italy (74.59 €/MWh), the highest one in Sicilia (85.69 €/MWh). The value of transactions was equal to €1.4 billion, up by 5.3% on December of last year.

\*7In Italy at the end of the trading day a National Single Price PUN is set as weighted average of the electricity prices over the various geographical zones.
The demand of electricity for the 2005 estimated by the amount traded on the IPEX and the amount of bilateral contracts summed up to 323.2 TWh. The maximum of hourly demand was 56,412 MWh in the winter and 51,766 MWh in the summer. The minimum of hourly demand was equal to 22,681 MWh. The dynamic of the average hourly monthly demand shows the standard two peaks framework (Figure 4).

Demand seasonality is also present on the market for the winter and summer time. The amount of electricity bought on the IPEX represents the 62% of the total traded electricity while bilateral contracts (38%) account for the remaining electricity bought (Figure 5).

The demand composition on the IPEX and OFF-IPEX is reported in Figure 6. For the 2005, 62% of the electricity demanded was traded on the IPEX where Single Buyers represent the main operator (69% of
the traded electricity). On the side of bilateral contracts Single Buyers trade only 21% of the electricity while the main user of bilateral contracts are other operators (78%).

5 Empirical Results

The hourly spot price information as well as day-ahead prices for each hour offers a way to study the properties of electricity spot prices.

Electricity prices are driven by spot demand and supply considerations, with demand in the short-term market being fairly inelastic. As a result, sizeable shocks in production or consumption may give rise to the price jumps which have been observed since 1998 in various parts in the United States. Spike in prices have been motivated by disruption in transmission, generation outages, extreme weather or a conjunction of these circumstances.

To understand how the equilibrium forward price and the equilibrium open interest are affected by various fundamental parameters, we perform some numerical analyses on the Italian Data. According to the Dong and Liu model both the supplier and the manufacturer have significant market powers. The supplier produces the commodity, and the manufacturer uses the commodity to produce a final product. The supplier is subject to both uncertain production costs and commodity spot price risk. The manufacturer also faces final product price risk and commodity demand risk.

We need to make some basic assumptions to use the Dong and Liu’s results presented in section 3.

1. We assume the commodity is represented by the electricity and so the $p_t$ is represented by the spot price series;

Figure 5: Role of bilateral contracts
### Demand Composition on the IPEX and OFF-IPEX, 2005

<table>
<thead>
<tr>
<th>Category</th>
<th>MWh</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Buyer</td>
<td>139,179,880.00</td>
<td>0.686</td>
</tr>
<tr>
<td>Other operators</td>
<td>47,682,936.00</td>
<td>0.235</td>
</tr>
<tr>
<td>Pumping</td>
<td>8,087,174.00</td>
<td>0.040</td>
</tr>
<tr>
<td>Foreign zones</td>
<td>2,773,208.00</td>
<td>0.014</td>
</tr>
<tr>
<td>Additional bids</td>
<td>5,262,767.00</td>
<td>0.026</td>
</tr>
<tr>
<td><strong>IPEX</strong></td>
<td><strong>202,985,965.00</strong></td>
<td><strong>1.000</strong></td>
</tr>
<tr>
<td>Foreign Bilateral</td>
<td>1,143,298.00</td>
<td>0.010</td>
</tr>
<tr>
<td>National bilateral (Single Buyer)</td>
<td>25,153,421.00</td>
<td>0.209</td>
</tr>
<tr>
<td>National bilateral (Others)</td>
<td>93,902,066.00</td>
<td>0.781</td>
</tr>
<tr>
<td><strong>OFF IPEX</strong></td>
<td><strong>120,198,785.00</strong></td>
<td><strong>1</strong></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>323,184,750.00</strong></td>
<td><strong>1</strong></td>
</tr>
</tbody>
</table>

Figure 6:

2. The final product prices, \( s_t \), is obtained by simulation assuming electricity represents a constant\(^8\) %, \( \gamma \), of the final cost of the product, so we do not consider at this stage the final product risk.

We need to choose a model to describe electricity price dynamic. An important fraction of the literature on electricity belongs to the economics and industrial economics arena, and analyzes deregulated electricity markets from the regulatory and industrial organization viewpoints (see for instance Joskow and Kahn (1999)). Different approaches to model the power price processes include Deng (1999), Eydeland Ge- man (1999), Bhanot (2000), Barone-Adesi and Gigli (2002), Lucia and Schwartz (2000), Knittel and Roberts (2001), Barlow (2002), Escribano et al. (2002). As a first approach we select the Lucia and Schwartz (2002) model to replicate the dynamics of the natural logarithm of the spot price. We denote by \( p_t \) the spot price at time \( t \) of one megawatt-hour (MWh) of electricity and by \( x(t) \) its natural logarithm \( x(t) = \ln P(t) \). We assume that the dynamics of \( x(t) \) can be viewed as the sum of two processes

\[
x(t) = f(t) + y(t)
\]

where \( f(t) \) is a highly predictable component accounting for the seasonality effects and \( y(t) \) is the random component reflecting unpredictable movements:

\[
dy_t = \alpha(\mu + \ln(y_t))dt + \sigma y_t dz_t
\]

\(^8\)A variable \( \gamma(s) \) will be introduced in a next development.
where \( \alpha, \mu, \sigma \) are all constants and \( z_t \) is a one-dimensional Brownian motion. This implies that the spot price \( p_t \) at time \( t \in [0; T] \) is log-normally distributed with

\[
E[p_t | F] = \exp \left( \mu t \ln(p_0) + \sigma z_t \right), \tag{15}
\]

\[
\text{var}[p_t | F] = \exp \left( 2 \sigma^2 t \ln(p_0) + \sigma^2 z_t \right), \tag{16}
\]

The estimation is performed by Maximum Likelihood on the discretized version\(^9\) of the model

\[
x_t = \eta + \beta_1 D_1 t + \beta_2 D_2 t + \alpha y_t + \sigma \varepsilon_t
\]

where the deterministic component

\[
f_t = \eta + \beta_1 D_1 t + \beta_2 D_2 t
\]

takes into account the weekly seasonal component assuming

\[
D_1 t = \begin{cases} 1 & t = \text{sat} \\ 0 & \text{else} \end{cases}, \quad D_2 t = \begin{cases} 1 & t = \text{sun} \\ 0 & \text{else} \end{cases}
\]

The estimation results are shown in Figure 7, where the values of the model parameters\(^10\), the values of the loglikelihood (LL) and the values of the Schwartz criterion (SC) are reported. At this stage of the work we do not account for jumps and spikes, multi-factor jump-diffusion models (Villaplana, 2003) will be introduced in a next step. The quantity demanded \( D_T \) and sales price \( s_T \) for the manufacturer’s final product are also log-normally distributed with means \( \mu_{D_T}, \mu_s \) and variances \( \sigma^2_D, \sigma^2_s \), and \( \rho_{pD}, \rho_{ps}, \) and \( \rho_{sD} \) be the respective correlations among \( p_T, D_T, s_T \). To minimize the impact of seasonality, we divide the data into four seasons: winter, spring, summer, and fall. We then estimate the parameters separately for these seasons. The equilibrium forward contract on a nonstorable commodity in an imperfectly competitive forward market according to (11) and (12) is then estimated. The parameter estimates are reported in Figure 8.

\(^9\)Using the Euler discretization of the model (Bergstom 1988, Melino 1994, Das 2001)

\(^10\)The parameters have been estimated using Matlab software.
### Estimation results (winter)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (SE)</th>
<th>Parameter</th>
<th>Value (SE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_w$</td>
<td>0.31 (0.022)</td>
<td>$a_s$</td>
<td>0.21 (0.032)</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>0.21 (0.002)</td>
<td>$\sigma_s$</td>
<td>0.19 (0.012)</td>
</tr>
<tr>
<td>$\mu_w$</td>
<td>2.984 (0.019)</td>
<td>$\mu_s$</td>
<td>3.184 (0.019)</td>
</tr>
<tr>
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<tr>
<td>LL</td>
<td>73.44</td>
<td>LL</td>
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</tr>
<tr>
<td>SC</td>
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### Estimation results (spring)

<table>
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<th>Value (SE)</th>
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### Estimation results (summer)

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### Estimation results (fall)

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### Parameter Estimates

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Figure 7:

Figure 8:
Figure 9: Equilibrium Forward prices as functions of spot prices

\[ f_0 = pe^{(\lambda + \theta)T}, \]

\(\lambda, \) and \(\theta\) to be the parameters for the marginal supplier in the market. The supplier has the lowest market power in the summer, this may be caused by the higher demand and the capacity bottleneck experienced in the summer.

Estimation for the forward prices according to the estimated parameters of Figure 8 allow to analyze how possible changes in the spot price may affect the forward. In the case with storable commodities, the absence of arbitrage causes that the forward price \( f_0 = pe^{(\lambda\theta)T}, \) which implies in particular that the forward price increases with the spot price. In the case of electricity, nonstorable commodities, the forward price can be nonmonotonic in the spot price. As discussed by Dong and Liu the equilibrium forward price is largely determined by the manufacturer’s and supplier’s utility gain from the forward and their relative market powers. On the one hand, as the current spot price \( p_0 \) increases, the expected spot price \( \mu_p \) of \( p_T \) increases. On the other hand, a higher spot price \( p_0 \) tends to decrease the magnitude of the covariance \( \text{COV}_0, \) which in turn reduces the benefit from hedging for the manufacturer, this may cause a reduction in the equilibrium forward price. In Figure 9 we report the relationship between the equilibrium forward price estimated on the Italian market data as a function of the spot price. The various relationship obtained for the different seasons are plotted. An increase in the spot price \( p_0 \) has different effects on the forward price. In both spring and fall, when \( p \) is small the effect on speculation dominates, so
The forward price doesn't show relevant changes, when \( p_0 \) grows, the effect on hedging becomes more important. When \( p_0 \) grows further, the effect on speculation becomes dominant again and thus the forward price starts to increase again because of the nonmonotonic nature of these two effects. The speed of increase is much higher for summer and winter because the convenience yield in summer and winter is much lower than it is in spring and fall. Since the hedging benefit from a forward largely depends on the covariance between the forward and the spot, this nonmonotonic pattern of the forward price against the spot price suggests that the covariance is important in forward pricing on nonstorable commodities in the presence of market powers.

If we consider the risks that affect the equilibrium forward contract, we have to take into account Spot price risk and commodity demand risk. To understand the difference among their effects, we examine how the changes in these risks affect the equilibrium forward contract. Figures 10 plot respectively the equilibrium forward prices as functions of the spot price volatility. As the spot price risk increases, the forward prices for all seasons increase. Intuitively, an increase in the price risk makes it more valuable to use a forward contract to hedge against this price risk. Therefore, the manufacturer is willing to pay a higher forward price. Compared to fall, the summer forward price increases at a slower rate because for summer, the market power of the manufacturer is greater than it is for fall.
6 Conclusions and further discussion

We consider an equilibrium forward contract on a nonstorable commodity in an imperfectly competitive forward market. Due to market participants' market powers, the forward contract is defined solving a Nash bargaining process. Using Dong and Liu's approach we estimate the equilibrium forward contract using Italian electricity spot prices data. We find that in contrast to the forward price on a storable commodity, the forward price on a nonstorable one can be nonmonotonic in the spot price. We show that the forward price can be a downward or an upward biased predictor of the spot price, depending on the convenience yield level and the market power. Further numerical experiment are aimed to estimate the effects of forward contracts on open interest and how demand risks may affect the forward equilibrium price and quantity.

The framework used in the paper will be applied to study more complex bilateral contracts when market power is present. Bilateral contracts have largely increased in the Italian market, so it may be useful to estimate the exact that this market may have on the spot market volatility and liquidity. This model only considers the case with one representative seller and one representative buyer in the forward market. Therefore it is obviously only a reduced form of the forward market. An interesting but challenging extension would be to allow multiple sellers and multiple buyers to trade strategically in both the forward market and the spot market.

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