Hydropower with Financial Information

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Production optimization

- Many Operations Management papers show that financial market data should be used in operational decisions
  - However, it is unclear if the information is used in practice.

- Hydropower producer should consider
  1. current spot price and expected future prices
  2. water reservoir level and expected inflow
  3. production constraints

- For instance,
  - the higher the forward prices the more should be produced later
  - the higher the water level the more should be produced now
Problems

- What is the optimal production strategy of a hydropower plant with forward curve information?
  - can the optimal strategy be simplified?
- How well does the model explain the realized production?
Results

- Long- and medium-term production strategy
  - simple parameterization of the optimal policy
  - the accuracy of the parameterization from the corresponding upper bound

- Test with a Norwegian hydropower producer’s realized strategy
  - model earnings are within 2.6% from the theoretical upper bound
    => model close to the optimal
  - significant part of the actual production strategy can be explained with our method
    => the forward information is used in the production optimization
Related papers

- Pereira (1989), Pereira and Pinto (1991)
  - hydro scheduling in regulated markets

  - hydro scheduling in deregulated markets

  - electricity forward curve dynamics

- Kall and Wallace (1997), Broadie and Glasserman (1997)
  - approximation error from the value of perfect information (perfect foresight solution, wait-and-see solution)
HJM-type forward dynamics

- $T$-maturity forward price dynamics:

$$dS(t,T) = S(t,T) e^{-\alpha(T-t)} \sigma(T) dB(t,T) \quad \forall \ t \in [0,T]$$

where $\alpha$ models the decrease in the volatility as a function of maturity

- decrease is caused by mean-reversion

- Brownian motions have the following correlation structure

$$dB(t,T_1)dB(t,T_2) = e^{-\rho|T_1-T_2|} dt \quad \forall \ T_1,T_2 \in [0,\tau]$$

where $\rho$ models the correlation between forwards

- the correlation decreases as a function of the maturity difference
Forward curve in Scandinavian

Risk adjusted expected spot price during the 16th week
Volatility structure

- Reflects uncertainty in the spot price

Spot volatility during the 16th week

Volatility

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

Contract, weekly
Forward curve, winter 01-02
Forward parameters

- Volatility parameter:

<table>
<thead>
<tr>
<th></th>
<th>Year 1999</th>
<th>Year 2000</th>
<th>Year 2001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility parameter $\alpha$</td>
<td>2.31</td>
<td>6.06</td>
<td>3.67</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.25</td>
<td>0.32</td>
<td>0.19</td>
</tr>
</tbody>
</table>

- Correlation parameter:

<table>
<thead>
<tr>
<th></th>
<th>Year 1999</th>
<th>Year 2000</th>
<th>Year 2001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation parameter $\rho$</td>
<td>3.62</td>
<td>5.30</td>
<td>4.61</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.04</td>
<td>0.26</td>
<td>0.17</td>
</tr>
</tbody>
</table>

- These changes can be explained by reservoir and snow levels
Objective function

- The information the company has at time $t$: 
  (i) electricity forward curve, $S(t, \cdot)$
  (ii) expected inflow curve, $\nu(t, \cdot)$
  (iii) water level, $x(t)$

- We maximize the expected discounted cash flows:

$$\sup_{u(s) \in [0, \bar{u} \land (x(s) + d\nu(s))] \forall s \in [t, \tau]} \mathbb{E} \left[ \eta \int_t^\tau e^{-r(s-t)} u(s) S(s) ds \mid F_t \right] \forall t \in [0, \tau]$$

\[ dx(t) = \nu(t) - u(t) - I \{ x(t) + \nu(t) - u(t) > \bar{x} \} (x(t) + \nu(t) - u(t) - \bar{x}) \]
Optimal strategy

Since the objective function is linear with respect to $u$ we get a bang-bang control:

$$u^K(t) = I \{S(t) \geq K(t, x(t), S(t, \cdot), \nu(t, \cdot))\}(\bar{u} \land (x(t) + \nu(t)))$$

where $K$ is the production threshold and it depends on the forward curve, inflow estimate, and the water level.

$=>$ Once we know the production threshold, we know the optimal policy.
Production threshold characteristics

1) The production threshold converges to zero as the time approaches the end of the planning period.
   - we simply assume here that the reservoir value after the planning horizon is zero (or constant)

2) The probability of spillage is an increasing function of the water level
   => the threshold must fall as a function of the water level.

3) Similarly, if the expected future inflow increases the threshold falls.

4) If the forward curve rises, the value of waiting increases
   => the production threshold is an increasing function of the future electricity prices.
Production threshold

- We use the following parameterization

\[ \tilde{K}(t, x(t), \tilde{\nu}(t), \tilde{s}(t)) = \alpha_s \tilde{s}(t) e^{-\alpha_x x(t) - \alpha_v \tilde{\nu}(t) - \alpha_t \frac{1}{t-t}} \]

where \( \tilde{\nu}(t) \) and \( \tilde{s}(t) \) are the expected average future inflow and the average forward price
- the parameterization satisfies the threshold characteristics 1-4

- The parameters \( \alpha_S, \alpha_x, \alpha_v, \) and \( \alpha_t \) are optimized by using Monte Carlo simulation

- Clearly, the parameterization gives a lower bound for the objective function
Upper bound

The objective function’s upper bound is got by calculating the “average best strategy”:

\[ \eta E \left[ E \left[ \sup_{u(s) \in [0, \bar{u} \land (x(s) + \nu(s))] \forall s \in [t, \tau]} \int_{t}^{\tau} e^{-r(s-t)} u(s) S(s) ds \mid F_\tau \right] \mid F_t \right] \]

- for each path we calculate the best policy and the corresponding objective function
- then the upper bound is the average of the objective functions

If the upper and lower bounds are close to each others then the production threshold parameterization approximates weakly the optimal policy
Empirical analysis

- Usually the testing of production models is hard because there is no data
  - Luckily, we have the production data of Driva Kraftverk’s biggest hydro reservoir during 1997-2003

- Even though we have the production data, we have a problem: we don’t have data on historical water levels and inflow forecasts
  - historical forward data is naturally available
Testing period

- We test the model only during winter months – then the inflow can be assumed to be zero and the water level equals the minimum level at the end of the period
- Driva’s inflow data:
Threshold parameters

- On average the parameterized threshold gives a production strategy with earnings less than 2.6% from the earnings of the optimal production.
- Note that only the parameter for water level changes significantly.

<table>
<thead>
<tr>
<th>Winter</th>
<th>$\alpha_s$</th>
<th>$\alpha_t$</th>
<th>$\alpha_x$</th>
<th>max error</th>
</tr>
</thead>
<tbody>
<tr>
<td>97-98</td>
<td>1.1</td>
<td>0.011</td>
<td>0.0010</td>
<td>3.4%</td>
</tr>
<tr>
<td>98-99</td>
<td>1.2</td>
<td>0.013</td>
<td>0.0015</td>
<td>2.4%</td>
</tr>
<tr>
<td>99-00</td>
<td>1.2</td>
<td>0.010</td>
<td>0.0020</td>
<td>2.9%</td>
</tr>
<tr>
<td>00-01</td>
<td>1.2</td>
<td>0.010</td>
<td>0.0015</td>
<td>2.2%</td>
</tr>
<tr>
<td>01-02</td>
<td>1.1</td>
<td>0.012</td>
<td>0.0010</td>
<td>2.9%</td>
</tr>
<tr>
<td>02-03</td>
<td>1.2</td>
<td>0.013</td>
<td>0.002</td>
<td>1.9%</td>
</tr>
</tbody>
</table>
Threshold, Cont’d

![3D graph showing the relationship between $K_s$ in NOK/MWh and the average forward curve over weeks. The graph highlights specific volumes of $x$.](image)
Model and realized earnings

- Cumulative earnings are close to each other:
Correlation analysis

- The model explains better the realized production than the spot price
  \( \Rightarrow \) forward information is used in the production

- \( p \)-value for that the spot correlation equals the model’s correlation is \( 3.18 \cdot 10^{-5} \)

- Model correlation can be improved by adding water level (and other factors, e.g. inflow forecast)
  - during normal years, 00-01 and 01-02, the average correlation is more than 50%

<table>
<thead>
<tr>
<th>Winter</th>
<th>Our model</th>
<th>( p )-value of no correlation</th>
<th>Spot price</th>
<th>( p )-value of no correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>97-98</td>
<td>-56.17%</td>
<td>0.0081</td>
<td>-22.34%</td>
<td>0.3304</td>
</tr>
<tr>
<td>98-99</td>
<td>35.75%</td>
<td>0.1119</td>
<td>22.27%</td>
<td>0.3319</td>
</tr>
<tr>
<td>99-00</td>
<td>3.46%</td>
<td>0.8815</td>
<td>-30.51%</td>
<td>0.1786</td>
</tr>
<tr>
<td>00-01</td>
<td>49.29%</td>
<td>0.0232</td>
<td>-41.09%</td>
<td>0.0643</td>
</tr>
<tr>
<td>01-02</td>
<td>60.56%</td>
<td>0.0036</td>
<td>50.77%</td>
<td>0.0188</td>
</tr>
<tr>
<td>02-03</td>
<td>14.22%</td>
<td>0.5386</td>
<td>-35.76%</td>
<td>0.1114</td>
</tr>
</tbody>
</table>
Summary

- Stylized model for medium- and long-term planning
- Model’s earnings are close to the maximum earnings
  => a good approximation
- The model explains better the realized production than spot price