A Supply and Demand Based Volatility Model
for Energy Prices

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Agenda

1. The objectives and results

2. A Supply and Demand based Volatility Model for Energy Prices (SDV model)

3. Empirical studies on the volatility in the U.S. natural gas prices

4. Conclusions and directions for future research
1. The objectives and results
Volatility in energy prices

- Deregulation of energy markets causes time-varying volatility in the prices.
- It is important for energy companies to capture the volatility appropriately.
Existing volatility models in energy prices

Financial market models for volatility have been applied to energy markets directly.
For example

As continuous time models,

- Heston model (Heston (1993)) and C.E.V. models (Cox(1975) and Emanuel and MacBeth(1982))

⇒ directly introduced to Eydeland and Wolyniec (2003).
For another example

As discrete time models,

- ARCH model (Engle (1982)), GARCH model (Bollerslev (1986)), and ARCH-M model (Engle, Lilien, and Robins (1987))

⇒ used in Duffie, Gray, and Hoang (1999), Pindyck (2004), and Deaves and Krinsky (1992).
Energy prices and demands

Energy prices are strongly affected by the supply-demand relationship such as consumer behavior and production technology.
The characteristics of energy price volatility

- \( \uparrow \) Price \( \Rightarrow \uparrow \) Volatility
It is called “Inverse leverage effect” (Eydeland and Wolyniec (2003)).
The characteristics of energy price volatility

- The drift of price returns is influenced by the volatility. It is called “Volatility-in-mean effect” (Deaves and Krinsky(1992)).
The objectives

We propose a volatility model for energy prices explicitly characterized by the supply-demand relationship, which we call a Supply and Demand based Volatility (SDV) model.

With this model, we empirically analyze the volatility in the U.S. natural gas prices.
The results

(1) The SDV model can produce the “Inverse leverage effect” and the “Volatility-in-mean effect”.

(2) The (G)ARCH-M model is (with some approximation) derived from the SDV model. In this sense, the (G)ARCH-M model has foundation on the supply-demand relationship.
(3) The empirical studies which analyze the volatility in the U.S. natural gas prices by using the SDV model show that there exist both the “Inverse leverage effect” and the “Volatility-in-mean effect”. 
2. SDV model

As we see, energy prices are strongly affected by the supply and demand relationship.
Price and Demand Relationship in the U.S. NG market

The plots seem scattered around a single monotone increasing convex curve.
A Simple Model for Energy Prices

The supply and demand relationship determines the price.

![Graph showing supply and demand curves with equilibrium price and volume.](image-url)
A Simple Model for Energy Prices

Suppose curve $g$: Fixed in a short term $P_t = g(S_t)$

Demand curve: Vertical (inelastic to prices) and stochastically fluctuates with demand

Energy demand $D_t$ is represented by the sum of $n$ kinds of demand $D_t^i$ which follows a stochastic process, respectively:

$$D_t = \sum_{i=1}^{n} D_t^i$$

$$dD_t^i = \mu^i_D dt + \sigma^i_D dw^i_t \quad \text{where} \quad E_t[dw^i_t dw^j_t] = \rho_{ij} dt.$$  \hspace{1cm} (2)

Equilibrium: The intersection ($S = D$) between the supply and demand curves determines the equilibrium price.

$$P_t = g(D_t)$$  \hspace{1cm} (3)
Proposition 1 Suppose that equilibrium prices for energy are given by the function of the demand as \( P_t = g(D_t) \) where the demand process is generated by Eqs. (1) and (2). Then, a Supply and Demand based Volatility (SDV) model for energy prices is expressed as follows:

\[
\frac{dP_t}{P_t} = \mu_t dt + \sigma_t dw_t
\]  

(4)

\[
\sigma_t = \frac{g'(D_t)}{g(D_t)} \sigma_D
\]  

(5)

\[
\mu_t = \left( \frac{\mu_D}{\sigma_D} + \frac{1}{2} \frac{g''(D_t)}{g'(D_t)^2} \right) \sigma_D
\]  

(6)

\[
\mu_D = \sum_{i=1}^{n} \mu_i^D, \quad \sigma_D = \sqrt{\sum_{i,j=1}^{n} \sigma_{iD}^i \sigma_{jD}^j \rho_{ij}}, \quad dw_t = \frac{1}{\sigma_D} \sum_{i=1}^{n} \sigma_{iD}^i dw_t^i.
\]

Proof We apply Ito's Lemma to Eq. (3) by using Eqs. (1) and (2).
The volatility $\sigma_t$ is expressed by the supply curve function $g$ and demand $D_t$.

$\Rightarrow$ Demand fluctuation causes time-varying volatility.
The Characteristics of SDV model (Contd.)

\[ \mu_t = \left( \frac{\mu_D}{\sigma_D} + \frac{1}{2} \frac{g''(D_t)}{g'(D_t)} \sigma_D \right) \sigma_t \]

The drift \( \mu_t \) is time varying with volatility \( \sigma_t \).

\[ \Rightarrow \text{Volatility-in-mean effect} \]
In order to examine the volatility $\sigma_t$ for energy prices in detail, we analyze the SDV model by employing an upward sloping supply curve reflecting energy markets and one-factor demand process for simplicity, which we call one-factor SDV model.
The exponential supply curve does not fit well to the historical data.
For $a < 0$, the curvature is larger than that of the exponential.

$$P_t = \begin{cases} \left(1 + a \frac{S_t}{c} \right)^{\frac{1}{a}} & (S_t \leq c\tau) \\ \frac{(1+a\tau)^{\frac{1}{a}}}{1-a} \left[ \exp \left( \frac{1-a}{1+a\tau} \left( \frac{S_t}{c} - \tau \right) \right) - a \right] & (S_t \geq c\tau) \end{cases}$$

(7)

where $a$ is assumed to be less than or equal to 0.
Demand process

Demand $D_t$ follows one-factor stochastic process:

$$dD_t = \mu_D(D_t)dt + \sigma_D dw_t.$$  \hfill (8)

We assume that $\mu_D(D_t)$ is a linear function of $D_t$ and $\sigma_D$ is a constant.
One-factor SDV model

Supposing the inelasticity of demand to prices, the energy prices are given as the inverse Box-Cox transformation of the demand. Applying Ito’s Lemma to Eq. (7) with Eq. (8) and then replacing the drift term for $S_t \geq c\tau$ by that for $S_t \leq c\tau$ in order to guarantee the existence of the solution, we have the one-factor SDV model for energy prices that is tractable as in Eqs. (9) - (11)

\[
\frac{dP_t}{P_t} = \mu_t dt + \sigma_t dw_t \tag{9}
\]

\[
\sigma_t = \begin{cases} 
\sigma P_t^{-a} & P_t \leq (1 + a\tau)^{\frac{1}{a}} \\
\sigma \frac{1-a}{1+a\tau} \left(1 + \frac{a}{P_t} \frac{(1+a\tau)^{\frac{1}{a}}}{1-a}\right) & P_t \geq (1 + a\tau)^{\frac{1}{a}}
\end{cases} \tag{10}
\]

\[
\mu_t = k_1 \sigma_t + k_2 \sigma_t^2 \tag{11}
\]

where $\sigma = \frac{\sigma_D}{c}$, $k_1 = \frac{\mu_D}{\sigma_D}$, and $k_2 = \frac{1-a}{2}$.
One-factor SDV model (1)

The volatility term:

\[ \sigma_t = \sigma P_t^{-a} \]

- For \( a < 0 \), volatility \( \sigma_t \) increases in price \( P_t \).
  \[ \Rightarrow \text{Inverse leverage effect} \]

- For \( a = 0 \), volatility \( \sigma_t \) is constant.
One-factor SDV model (2)

\[ \mu_t = k_1 \sigma_t + k_2 \sigma_t^2 \]

- Drift \( \mu_t \) is time varying with volatility \( \sigma_t \).

\[ \Rightarrow \text{Volatility-in-mean effect} \]
(G)ARCH-M model is applied to energy prices.

Often, “goodness of fit” is the only reason to use the (G)ARCH-M model.

We offer an economic reason of the model selection by using the relationship between one-factor SDV model and (G)ARCH models.
Derivation of (G)ARCH-M model from one-factor SDV model

Based on one-factor SDV model, the log return of price $r_t = \log(P_{t+1}) - \log(P_t)$ is approximately rewritten by

$$r_t = \mu_t + \sigma_t \varepsilon_t, \quad \mu_t = \mu_1 \sigma_t + \mu_2 \sigma_t^2$$

(12)

$$\mu_1 = \frac{\mu_D(D_t)}{\sigma_D}, \quad \mu_2 = -\frac{a}{2}$$

$$\sigma_t^2 = \sigma^2 P_t^{-2a} = \frac{\sigma_D^2}{(c + aD_t)^2}$$

(13)

$$P_t = \left(1 + a \frac{D_t}{c}\right)^{\frac{1}{a}}, \quad \sigma = \frac{\sigma_D}{c}.$$
Demand model in discrete time

AR(1) model

\[ D_t = (1 - \lambda_D)D_{t-1} + \mu_D + \sigma_D \varepsilon_{t-1} \]  (14)
Proposition 2 Suppose that the energy price returns are governed by one-factor SDV model and the demand is given by AR(1) model. Then, the discrete time SDV model is expressed by followings:

\[ r_t = \mu_t + \sigma_t \epsilon_t, \quad \mu_t = \mu_1 \sigma_t + \mu_2 \sigma_t^2 \tag{15} \]

\[ \eta_t = \sigma_t \epsilon_t \tag{16} \]

\[ \sigma_t^2 = \frac{\sigma_D^2}{c^2} \left( 1 + \sum_{k=1}^{\infty} (-1)^k (k + 1) \left( \frac{a}{c} \right)^k (M_t + \sum_{i=1}^{t} k_i \epsilon_{t-i})^k \right) \tag{17} \]

\[ M_t = (1 - \lambda_D)^t D_0 + \frac{\mu_D}{\lambda_D} \{ 1 - (1 - \lambda_D)^t \} \tag{18} \]

\[ k_i = \sigma_D (1 - \lambda_D)^{i-1}. \tag{19} \]

Note that \[ P_t = \left( 1 + a \frac{D_t}{c} \right)^{\frac{1}{a}}. \tag{20} \]
Discrete time SDV model

\[
\sigma_t^2 = \frac{\sigma^2_D}{c^2} \left( 1 + \sum_{k=1}^{\infty} (-1)^k (k+1) \left( \frac{a}{c} \right)^k \left( M_t + \sum_{i=1}^{t} k_i \varepsilon_{t-i} \right)^k \right)
\] (21)

If the inverse Box-Cox function parameter \((a)\) is 0 implying the exponential supply curve, then the volatility becomes constant.
Proposition 2 (Contd) Suppose, in addition, that the squares of the past demand shocks only dominate the volatility. Then the SDV model collapses to ARCH(\(t\))-M model:

\[
\sigma_t^2 \approx \frac{\sigma_D^2}{c^2} \sum_{i=1}^{t} \left( A_{i,0} + A_{i,2} \eta_{t-i}^2 \right) \tag{22}
\]

\[
A_{i,k} = \sum_{l=k}^{\infty} (-1)^l (l + 1) \left( \frac{a}{c} \right)^l a_{l-k,i,l} b_{i,k} \tag{23}
\]

\[
a_{j,i,n} = n C_j M_t^j k_i^{n-j} (j \neq n), \quad a_{j,i,n} = \frac{1}{t} M_t^n (j = n), \tag{24}
\]

\[
b_{i,k} = \left( \frac{c + a M_{t-i}}{\sigma_D} \right)^k. \tag{25}
\]

Note that \(A_{i,k}\) is positive for all \(i\) and \(k\) if \(a < 0\).
The Implication

\[ r_t = \mu_t + \sigma_t \varepsilon_t, \quad \mu_t = \mu_1 \sigma_t + \mu_2 \sigma_t^2 \]

\[ \sigma_t^2 \approx \frac{\sigma_D^2}{c^2} \sum_{i=1}^{t} \left( A_{i,0} + A_{i,2} \eta_{t-i}^2 \right) \]

- The (G)ARCH-M model is (with some approximation) derived from the SDV model. In this sense, it has foundation on the supply-demand relationship.
Summary of SDV model

(1) The SDV model can produce the “Inverse leverage effect” and the “Volatility-in-mean effect”.

(2) The (G)ARCH-M model is (with some approximation) derived from the SDV model. In this sense, it has foundation on the supply-demand relationship.
3. Empirical studies on the volatility in the U.S. natural gas prices
Two analyses in empirical studies

Analysis 1: Does the inverse leverage effect exist in the U.S. NG market?

Analysis 2: Does the volatility-in-mean effect exist in the U.S. NG market?
Data for the empirical studies

Monthly price and demand for NG in the U.S.

Energy Information Administration (EIA)

339 observations of monthly data starting from January 1976 to March 2004

Wellhead prices for the proxy of spot prices

Total demand approximates to the sum of storage and consumption demands
Analysis 1: Existence of inverse leverage effect

Nonlinear least square estimation (NLSE) of the supply curve $g$ of the U.S. NG market: Historical price $P_t$ and demand $D_t$ are applied to the estimation.

$$P_t = \left(1 + a \frac{D_t}{c}\right)^{\frac{1}{a}} + \varepsilon_t$$ (26)

If $a < 0$, then there exists the inverse leverage effect by $\sigma_t = \sigma P_t^{-a}$. 
## Results of Analysis 1

<table>
<thead>
<tr>
<th>parameter</th>
<th>(a)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>estimate</td>
<td>-1.816</td>
<td>(4.136 \times 10^6)</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>-23.18</td>
<td>27.69</td>
</tr>
<tr>
<td>log-likelihood</td>
<td>-449.29</td>
<td></td>
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<tr>
<td>AIC</td>
<td>902.59</td>
<td></td>
</tr>
<tr>
<td>SIC</td>
<td>910.23</td>
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</table>
Implication of Analysis 1

• $a$ is statistically significant. (-1.816)

• $\sigma_t = \sigma P_t^{1.816} = \sigma P_t$

⇒ Volatility increases in price.

⇒ The inverse leverage effect exists in the U.S. NG market.
We estimate the parameters of the GARCH(1,1)-M model approximately derived from the discrete time SDV model.

\[ r_t = \mu_t + \sigma_t \varepsilon_t, \quad \mu_t = k \sigma_t \]  \hspace{1cm} (27)

\[ \eta_t = \sigma_t \varepsilon_t \]  \hspace{1cm} (28)

\[ \sigma_t^2 = \alpha + \beta \eta_{t-1}^2 + \gamma \sigma_{t-1}^2 \]  \hspace{1cm} (29)

If coefficients \( k, \beta, \) and \( \gamma \) are statistically significant, the volatility-in-mean effect exists.
Parameter estimation by MLE

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$k$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.092</td>
<td>$2.64 \times 10^{-5}$</td>
<td>0.270</td>
<td>0.775</td>
</tr>
<tr>
<td>Standard error</td>
<td>0.043</td>
<td>$2.32 \times 10^{-5}$</td>
<td>0.068</td>
<td>0.042</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>$4.73 \times 10^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>$-9.38 \times 10^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SIC</td>
<td>$-9.23 \times 10^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\Rightarrow$ Coefficients $k$, $\beta$, and $\gamma$ are statistically significant.

$\Rightarrow$ Volatility-in-mean effect exists in the U.S. NG market.
Summary of empirical studies

Analysis 1 implies that there exists the inverse leverage effect in the U.S. NG market.

Analysis 2 implies that there exists the volatility-in-mean effect in the U.S. NG market.
4. Conclusions and Directions for Future Research
Conclusions

(1) The SDV model can produce the “Inverse leverage effect” and the “Volatility-in-mean effect”.

(2) The (G)ARCH-M model has foundation on the supply-demand relationship because it is (with some approximation) derived from the SDV model.
(3) The empirical studies have shown that the U.S. NG market possesses both the “Inverse leverage effect” and the “Volatility-in-mean effect”.

Conclusions (Contd.)
Directions for Future Research

The empirical studies using more frequent natural gas price data like weekly and daily ones

The applications to other energy prices like crude oil and heating oil
Thank you.

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