The application of option techniques for electricity price distributions, with particular reference to variable low load factor producers

Chris Harris

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System Setup

- Power Station Unit Owner
- Option Buyer
- System Operator
- Single Buyer / Wholesale Market

Options → Dispatch
Short term forward sales
Trade notification
Cost / Payoff diagrams

Unit cost

Option payoff

£/hr

b

B

£/hr

MW

£/hr

£/MWh

-b

B
Using the basic formulae

\[ C = F \times N(d_1) - KN(d_2) \]

\[ d_1 = \frac{\ln(F/K) + \frac{1}{2} \sigma^2 (T-t)}{\sigma \sqrt{T-t}} \]

\[ d_2 = \frac{\ln(F/K) - \frac{1}{2} \sigma^2 (T-t)}{\sigma \sqrt{T-t}} = d_1 - \sigma \sqrt{T-t} \]

\[ N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{\varepsilon^2}{2}\right) d\varepsilon \]
Cost of Risk Adjustment

\[ F_{T,t} = E[S_T] - \lambda_a(T,t)\sigma_a^n(T-t) \]

Actually path dependent

Cost of risk driven by balancing mechanism rules has higher dimensionality than \(\sigma^2\)

Positive due to producer/consumer risk asymmetry or negative due to negative oil/equity correlation?

Market "gate closure" at \(t=1\) hour

\(\tau = T-t\)
Market price, conditional on exercise

\[
E(S) \mid S > K = K + \frac{\int_{S=K}^{S=\infty} P(S) * (S - K) dS}{\int_{S=K}^{S=\infty} P(S) dS}
\]

This is closely related to the cost of unit failure
Asset value risk

\[ Variance(x) = E[x^2] - (E[x])^2 \]

\[ x = \max(S - K, 0) \]

\[ E[x] = C = \int_K^\infty SP(S)dS - K\int_K^\infty P(S)dS \]

\[ Variance(x) = V = \int_K^\infty (S - K)^2 P(S)dS - C^2 \]

\[ V = F^2 \left[ \exp(\sigma^2 T)N(d_3) - N(d_1)^2 \right] - N(-d_2)K\left[ 2FN(d_1) - KN(d_2) \right] \]

\[ V = F \left[ FN\exp(\sigma^2 T)N(d_3) - K N(d_1) \right] - C(K + C) \]

\[ d_3 = \frac{\ln(\frac{F}{K}) + \frac{3}{2} \sigma^2 T}{\sigma \sqrt{T}} \]
Derivation of the variance calculation

\[ C = E\{|S - K|S > K\} = E\{F \exp\left(-\frac{1}{2} \sigma^2 T + \varepsilon \sigma \sqrt{T}\right) - K|\varepsilon > d\} \]

\[ d = \frac{\ln(K / F) + \frac{1}{2} \sigma^2 T}{\sigma \sqrt{T}} \]

\[ C = \frac{1}{\sqrt{2\pi}} \int_{d}^{\infty} F \exp\left(-\frac{1}{2} \sigma^2 T + \varepsilon \sigma \sqrt{T}\right) - K \exp\left(-\frac{\varepsilon^2}{2}\right) d\varepsilon \]

\[ C = \frac{1}{\sqrt{2\pi}} \int_{d}^{\infty} F \exp\left(-\frac{1}{2} \sigma^2 T + \varepsilon \sigma \sqrt{T} - \frac{\varepsilon^2}{2}\right) d\varepsilon - \frac{1}{\sqrt{2\pi}} \int_{d}^{\infty} K \exp\left(-\frac{\varepsilon^2}{2}\right) d\varepsilon \]

\[ C = \frac{1}{\sqrt{2\pi}} \int_{d}^{\infty} F \exp\left(-\frac{1}{2} \sigma^2 T + \varepsilon \sigma \sqrt{T} - \frac{\varepsilon^2}{2}\right) d\varepsilon - KN(d_2) \]
Asset risk versus “merit” (strike)
Asset value adjustment versus merit
Non quadratic utility functions – power law

\[ W^{1-\lambda}/(1-\lambda) - 1/(1-\lambda) \]

\[ \rightarrow \ln(W) \quad \text{as} \quad \lambda \rightarrow 1 \]
Asset value adjustment using power law utility
Basic physical relationships

(1) Demand $Q$ vs. Temperature

(2) Probability vs. Temperature

(3) £/MWh vs. MW
- Cheapest cost for equilibrium $B+U(b)$
- Marginal costs $B$

(4) $P(S)$ vs. Price $S$
- Prices at $B_i$
- Prices at $B_i+U_i$
Bootstrapping installed capacity and price distributions

*Assumes single buyer!
B-b technology frontiers

B Marginal costs

b’ fixed costs – excluding cost of risk
Individual plant frontier

- Marginal costs
- Ageing
- New

b’ fixed costs – excluding cost of risk
First iteration of the U vector (uplift above short run marginal cost)

Deterministic demand schedule

B+U stack from unit 1 to unit n (lowest merit)

Clearing price schedule

Raise $U_i$ until fixed costs recovered

Annual net revenue for each unit

$i = n$

$i = i - 1$
Impact of assumptions

- We have assumed “perfect market order”
- Option holders have call options struck at Bi, and release power at Bi+Ui
- There is no violation of this order, despite individual incentive to offer at Bi<P<Bi+Ui
Conclusions

- We can construct a simple market model using installed capacity real options
- This can include cost of risk – a critical feature
- For a benign monopoly or single buyer under specific rules, the model works well
- The model highlights the issue of instability of market order in a competitive market and offers a derivative based model for considering the evolution of supply function equilibria