Options on Energy Portfolios in an HJM Framework

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Energy Options in an HJM Framework - I

- **TOPIC:** Modeling of options on portfolios of energy futures and flow forwards with stochastic spot prices, convenience yields and exchange rates
Energy Options in an HJM Framework - I

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a flow magnitude, i.e. something to delivered over a future time interval and not as a future spot transaction
Options on Energy Portfolios in an HJM Framework - II

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Approximate closed-form solutions for options prices on portfolios of futures when the diffusion functions for the three stochastic processes are deterministic.
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Extensions to more general dynamics for the stochastic processes.
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• Extensions to more general dynamics for the stochastic processes.

• Other applications of the modeling approach.
Options on Energy Portfolios in an HJM Framework - IV

- HJM AND OPTION PRICING: BASICS
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For the purpose of option pricing, it is only relevant to consider an “equivalent martingale measure”, equivalent to the physical probability measure $P$. 
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Time line showing the time of option pricing ($0$), the option expiration time ($t$) and the expiration time of the underlying futures contract ($T$).
Options on Energy Portfolios in an HJM Framework - V

- HJM AND OPTION PRICING: Convenience Yields
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The forward interest rate process \( (f(t, s))_{t \leq s} \)

\[
P(t, T) = E^Q_t \left[ e^{- \int_t^T r_s ds} \right] = e^{- \int_t^T f(t, s) ds}
\]
HJM AND OPTION PRICING: Convenience Yields

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The forward convenience yield process \((\delta(t, s))_{t \leq s}\)

\[ G(t, T) = \frac{S_{t}}{P(t, T)} e^{-\int_{t}^{T} \delta(t, s) ds} = S_{t} e^{\int_{t}^{T} (f(t, s) - \delta(t, s)) ds} \]
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- **HJM AND OPTION PRICING: Convenience Yields**
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\]

- The futures convenience yield process \((\epsilon(t, s))_{t \leq s}\)

\[
F(t, T) = \frac{S_t}{P(t, T)} e^{-\int_t^T \epsilon(t, s) ds} = S_t e^{\int_t^T (f(t, s) - \epsilon(t, s)) ds}
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Options on Energy Portfolios in an HJM Framework - VI

• HJM AND OPTION PRICING: Forward and Futures Prices
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- Let $x_t$ be the exchange rate and let * denote values expressed in foreign currency
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\[
G(t, T) = S_t e^{\int_t^T (f(t,s) - \delta(t,s)) ds} = x_t S_t^* e^{\int_t^T (f(t,s) - \delta(t,s)) ds}
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F(t, T) = S_t e^{\int_t^T (f(t,s) - \epsilon(t,s)) ds} = x_t S_t^* e^{\int_t^T (f(t,s) - \epsilon(t,s)) ds}
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\[
G^*(t, T) = S_t^* e^{\int_t^T (f^*(t,s) - \delta^*(t,s)) ds} = S_t^* e^{\int_t^T (f^*(t,s) - \delta^*(t,s)) ds}
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Options on Energy Portfolios in an HJM Framework - VII

- HJM AND OPTION PRICING: Forward and Futures Prices
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- **HJM AND OPTION PRICING: Forward and Futures Prices**
- **Result:** Forward convenience yields in two currencies are identical
- **Result:** Forward prices are related by the forward exchange rate:

\[
G(t, T) = \left( x_t \frac{P^*(t, T)}{P(t, T)} \right) G^*(t, T)
\]
Options on Energy Portfolios in an HJM Framework - VII

- **HJM AND OPTION PRICING: Forward and Futures Prices**
- **Result:** Forward convenience yields in two currencies are identical.
- Forward prices are related by the forward exchange rate:

\[
G(t, T) = \left( x_t \frac{P^*(t, T)}{P(t, T)} \right) G^*(t, T)
\]

- Whenever forward and futures prices are identical, the futures convenience yields are also identical and the futures prices are related by the forward exchange rate:

\[
F(t, T) = \left( x_t \frac{P^*(t, T)}{P(t, T)} \right) F^*(t, T)
\]
HJM AND OPTION PRICING: BASIC FACTS
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However, when this is not the case the futures convenience yields will only be equal by coincidence.
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This is due to the continuous resettlement of the contract and possible correlation under $Q$ between the spot price process and the interest rate process.
HJM AND OPTION PRICING: BASIC FACTS

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This is due to the continuous resettlement of the contract and possible correlation under Q between the spot price process and the interest rate process.

The relation between the two futures prices involve the (Q)-covariance properties between the exchange rate and the domestic discount factor. The futures convenience yields are identical if and only if

\[ x_t P^*(t, T) F^*(t, T) = P(t, T) F(t, T) \iff \]

\[ E^Q_t \left[ x_t P^*(t, T) \frac{S_T}{x_T} \right] = E^Q_t \left[ P^*(t, T) e^{\int_t^T (r_s^* - r_s) ds} S_T \right] = E^Q_t [P(t, T) S_T] \]
HJM AND OPTION PRICING: BASIC FACTS

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x_t P^*(t, T) F^*(t, T) = P(t, T) F(t, T) \quad \Leftrightarrow \\
E^Q_t \left[ x_t P^*(t, T) \frac{S_T}{x_T} \right] = E^Q_t \left[ P^*(t, T) e^{\int_t^T (r_s^* - r_s) ds} S_T \right] = E^Q_t \left[ P(t, T) S_T \right]
\]

If e.g. \( r_s^* - r_s \) is deterministic we have

\[
P^*(t, T) = e^{-\int_t^T (r_s^* - r_s) ds} P(t, T) + Cov_t^Q \left( \frac{x_T}{x_t}, e^{-\int_t^T r_s ds} \right)
\]
HJM AND OPTION PRICING: DYNAMICS OF THE BASIC STOCHASTIC PROCESSES

The spot price process $S_t$

$$S_t = s_0 + \int_0^t S_u \mu_S(u) du + \int_0^t S_u \sigma_S(u) \cdot dW^Q_u$$
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Options on Energy Portfolios in an HJM Framework - IX

- **HJM AND OPTION PRICING: DYNAMICS OF THE BASIC STOCHASTIC PROCESSES**

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  - The futures convenience yield process \((\epsilon(t, s))_{t \leq s}\)

  \[ \epsilon(t, s) = \epsilon_0 + \int_0^t \mu_\epsilon(u, s) du + \int_0^t \sigma_\epsilon(u, s) \cdot dW^Q_u \]
• HJM AND OPTION PRICING: IMPLIED DYNAMICS OF THE FUTURES PRICE PROCESSES
Options on Energy Portfolios in an HJM Framework - X

- HJM AND OPTION PRICING: IMPLIED DYNAMICS OF THE FUTURES PRICE PROCESSES
- The dynamics of $F(t, T)$ can be shown to be

\[ F(t, T) = F(0, T) + \int_0^t F(u, T)s_F(u, T) \cdot dW_u^Q \]
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$$F(t, T) = F(0, T) + \int_0^t F(u, T)s_F(u, T) \cdot dW_u^Q$$

where the volatility function $s_F$ is given as

$$s_F(u, T) = \sigma_S(u) + \int_u^T (\sigma_f(u, s) - \sigma_\epsilon(u, s)) \, ds$$
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• Exchange rate risk is captured by $\sigma_S(u)$ and may make up a significant part of the spot price volatility in domestic currency
• HJM AND OPTION PRICING: PRICE FORMULA FOR A EUROPEAN CALL OPTION
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The pay-off function at time \( t < T \) for a European strike-\( K \) call option with the futures price \( F(t, T) \) as underlying variable is

\[
C_t = \max\{0, F(t, T) - K\}
\]

The pricing function at time \( 0 < t < T \) can be shown to be

\[
C_0 = P(0, t) \left( F(0, T)\Phi(d_1) - K\Phi(d_2) \right)
\]

where

\[
d_1 = \frac{\log \frac{F(0, T)}{K} + \frac{1}{2} \Sigma^2_F}{\sqrt{\Sigma^2_F}}, \quad d_2 = d_1 - \sqrt{\Sigma^2_F}, \quad \Sigma^2_F = \int_0^t ||s_F(u, T)||^2 du
\]
Options on Energy Portfolios in an HJM Framework - XII

- APPROXIMATE CLOSED-FORM SOLUTIONS FOR OPTION PRICES
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• The dynamics for the “value” of a futures portfolio $H(u)$ is

$$
\begin{align*}
    dH(u) &= \sum_{i=1}^{n} \alpha_i dF(u, T_i) = \sum_{i=1}^{n} \alpha_i F(u, T_i) \sum_{j=1}^{d} s_{F_j}(u, T_i) dW^Q_j(u) \\
    &= H(u) \left( \sum_{i=1}^{n} w(u, T_i) \sum_{j=1}^{d} s_{F_j}(u, T_i) dW^Q_j(u) \right) \\
    &= H(u) \left( \sum_{i=1}^{n} \sum_{j=1}^{d} w(u, T_i) s_{F_j}(u, T_i) dW^Q_j(u) \right),
\end{align*}
$$

THE APPROXIMATION IN THE CALL CASE IS
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\[ C^H (0; t, K) \approx \varphi C^F (0, t; T; \frac{K}{\varphi}) \] (1)

where \( T \in [t, \bar{T}] \) is some chosen maturity of an approximating single-delivery futures and the scaling factor \( \varphi \) is given by

\[ \varphi = \frac{H(0)}{F(0, T)} \]
STOCHASTIC DURATION, DEF. 1 ($\delta_H(0)$ [MYOPIC])

$$\sum_{j=1}^{d} s_{F_j}(0, \delta_H(0))^2 = \sum_{j=1}^{d} \left( \sum_{i=1}^{n} w(0, T_i) s_{F_j}(0, T_i) \right)^2$$
Options on Energy Portfolios in an HJM Framework - XIV

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• STOCHASTIC DURATION, DEF. 2 ($\delta_A^H(0)$) [GLOBAL])

$$\int_{0}^{t} \sum_{j=1}^{d} s_{F_j}(u, \delta_A^H(0))^2 du = \int_{0}^{t} \sum_{j=1}^{d} \left( \sum_{i=1}^{n} w(0, T_i) s_{F_j}(u, T_i) \right)^2 du$$
Options on Energy Portfolios in an HJM Framework - XIV

- **STOCHASTIC DURATION, DEF. 1** ($\delta_H(0)$ [MYOPIC])

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\]

- **STOCHASTIC DURATION, DEF. 2** ($\delta^H_A(0)$) [GLOBAL])

\[
\int_0^t \sum_{j=1}^{d} s_{F_j}(u, \delta^H_A(0))^2 du = \int_0^t \sum_{j=1}^{d} \left( \sum_{i=1}^{n} w(0, T_i) s_{F_j}(u, T_i) \right)^2 du
\]

- The true volatility, \( \int_0^t \sum_{j=1}^{d} \left( \sum_{i=1}^{n} w(u, T_i) s_{F_j}(u, T_i) \right)^2 du \), is stochastic, but we operate with the above approximation.
## MONTE CARLO RESULTS FOR THE CALL CASE

<table>
<thead>
<tr>
<th>Strike</th>
<th>Price deviation</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$C[^H]$</td>
</tr>
<tr>
<td>70%</td>
<td>0.14</td>
</tr>
<tr>
<td>80%</td>
<td>0.02</td>
</tr>
<tr>
<td>85%</td>
<td>-0.07</td>
</tr>
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<td>90%</td>
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<td>95%</td>
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<td>100%</td>
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<td>105%</td>
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<td>115%</td>
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<td>120%</td>
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Options on Energy Portfolios in an HJM Framework - XVI

MONTE CARLO RESULTS FOR THE PUT CASE

<table>
<thead>
<tr>
<th>Strike</th>
<th>$P[\delta^H]$</th>
<th>$P[\delta^H_A]$</th>
<th>$P\left[\frac{\delta^H + \delta^H_A}{2}\right]$</th>
<th>$P[\delta^H] + P[\delta^H_A]$</th>
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<td>-0.14</td>
<td>0.16</td>
<td>-0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>120%</td>
<td>-0.05</td>
<td>0.21</td>
<td>0.06</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>130%</td>
<td>-0.07</td>
<td>0.12</td>
<td>0.02</td>
<td>0.02</td>
<td>-0.01</td>
</tr>
<tr>
<td>SSD</td>
<td>0.280</td>
<td>0.185</td>
<td>0.045</td>
<td>0.034</td>
<td>0.025</td>
</tr>
</tbody>
</table>
Options on Energy Portfolios in an HJM Framework - XVII

• EXTENSIONS TO MORE GENERAL DYNAMICS
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- All affine models, i.e. where the single-delivery log prices are affine functions of the state variables
Options on Energy Portfolios in an HJM Framework - XVIII

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Options on Energy Portfolios in an HJM Framework - XVIII

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