A Continuous Time Model for Correlated Energy Price Processes

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Correlated processes

- The need to model energy price processes under a unified multidimensional model is of major importance in optimal decisions to take, both at a corporate and at a market level.

- We quote two typical problems in electricity production: short period risk management and long period technical investments (they cannot be satisfactorily obtained by means of simulations of a single process).
Integrated corporate risk in energy markets

- We deal with corporate integrated risk management
- Our model can be applied to other economic sectors (commodities..)
- In managing the integrated risk, we consider "upstream" (natural gas and fuel) and "downstream" (electricity) commodities. Since fuel can be converted into electricity, they are strongly related and we expect a positive correlation between them.
The bivariate case

- We consider two correlated exponential mean reverting processes $p$ and $c$, which can be thought of as the selling price of a unit of energy and the corresponding fuel cost required to produce it.

- A linear correlation coefficient $\rho$ allows to capture the dependency of the two processes from common risk factors.
Exponential mean reverting processes and their volatility

An exponential mean reverting process with high volatility

An exponential mean reverting process with low volatility
The bivariate case

\[\begin{align*}
    c &= e^x \\
    p &= e^y \\
    dx &= -\lambda_x (x - f_x(t))dt + \sigma_x dW_1 \\
    dy &= -\lambda_y (y - f_y(t))dt + \sigma_y \left( \rho dW_1 + \sqrt{1 - \rho^2} dW_2 \right)
\end{align*}\]

where \( \rho = \text{corr}(dx, dy) \)

The functions \( f_x(t) \) and \( f_y(t) \) are supposed known and capture the seasonality of the process.

We determined the maximum likelihood parameters to estimate the processes.
The first problem_1

We consider the risk-return trade-off of an electricity producer who can sell part of his capacity at the forward price \( p_F \) and the remaining capacity in the spot market at the price \( p \). The decision process takes place at time \( t=0 \) (now); the forward can have any maturity \( T \) from now, but as long as the maturity is set, i.e. the forward has been signed, the producer will possibly sell the whole capacity at time \( T \): a portion of it will be devoted to honor the forward, the remaining to the spot market. We will see that the optimal strategy depends among other things on the maturity \( T \).
The first problem_2

• The producer can sign a forward/bilateral contract at price $p_F$ for the whole production: the return is certain but the risk on the cost of the input (fuel) is not “covered”, so he will devote part of his production to the spot market

• A spark spread option is embedded here: the producer has the opportunity to generate electricity or not for the spot market, depending on the cost of fuel $c$ and the spot price of electricity $p$
The role of the spot market

We show that, for a given expected profit, committing the whole production to bilateral contracts does not annihilate the risk in managing energy but, on the contrary, it is essential recurring to the spot market to reduce it.

We work in a mean variance setting.
The profit function_1

If the producer can neither buy and store the fuel nor buy it forward, for a whole production $Q$, his profit function is:

$$G = \alpha (p_F - c) + \beta \max(p - c, 0) - C_f$$
In any case the sale is risky, even in case of a total sale on forward ($\alpha = Q, \beta = 0$), since the uncertainty is given by the cost of input

$$\sigma(G)_{\alpha=Q, \beta=0} = Q \sigma(c)$$

In case of a total sale in the spot market

$$\sigma(G)_{\alpha=0, \beta=Q} = Q \sigma(max(p - c, 0))$$
The forward maturity

• All risks are time dependent and depend not only on $\sigma_x, \sigma_y$ but also on $\lambda_x$ and $\lambda_y$. 
• So spot sale may be more risky when a short maturity forward is considered, but less risky when the forward has a longer maturity.
Forward: a benchmark

- We identify a threshold value for the forward contract: it is given by the expected production cost plus the expected value of a spark spread option (i.e. the opportunity to produce energy when it is economically profitable).

\[ p_F = \mathbb{E}(\max(p-c,0)) + \mathbb{E}(c) \]
The optimal choice_1

- The efficient frontier is an arc of hyperbola in the \((\sigma(G), E(G))\) plane. Defining \(v = \frac{\alpha}{Q}\), the fraction of production devoted to the forward, the extremes of the hyperbola correspond to \(v=1\) and \(v=0\).

- As a further useful choice criterion to select the optimal spot/forward combination, we consider the Sharpe Ratio \(SR = \frac{E(G)}{\sigma(G)}\), i.e. the return per unit of risk.

- An optimal combination at full capacity can be found, \(v_{opt} \in [0,1]\) which maximizes \(SR\).
The optimal choice varies according to the price of the forward, its maturity and the various parameters involved in the two stochastic processes governing spot price and fuel. In particular, a higher positive correlation usually allows a greater risk reduction opportunities by diversification of the sales.
The second problem

We consider the case of a producer seeking to maximize the expected value of a producing plant depending on the time of investment.

We assume that at time $t=0$ a project is postponed up to a time $t^*$ if $t^* > 0$ is such that the discounted expected value of the cash flow generated by the project reaches its maximum in $[0, +\infty]$. 
The optimization problem

\[
\max_{t_0 \in [0, +\infty)} V(t_0) = -C(Q)e^{-rt_0} + \int_{t_0}^{+\infty} E\left[\max\left(p_t - c_t, 0\right) | F_0\right] e^{-rt} dt
\]

Let \( t^* \) be the value of \( t_0 \) maximizing \( V(t_0) \); the project will be postponed if \( t^* > 0 \).
A necessary condition for investing in the plant

$$E \left[ \text{Max} \ (p_{t_0}, c_{t_0}) | F_0 \right] > E \left[ c_{t_0} | F_0 \right] + r \frac{C(Q)}{Q}$$

It is not convenient to postpone after $t_0$ if the revenue from the plant investment in $t_0$ is higher than the cost of input plus the revenue from investing the investment cost at the risk free rate.
An application to the German Market EEX (year 2005)

We considered daily prices (the timezone 9.00-10.00) of power in EEX and one-day forward gas prices for the year 2005. Since both series show no significant trends the long run mean can be taken as a constant for both processes.
The log price of EEX electricity and the 1-day forward gas log price (Jan. - Nov. 2006) against the ±σ, ±2σ and ±3σ boundaries around the respective expectations evaluated on December 30, 2005.
Risk/return tradeoff in selling energy

We build the efficient pairs \((\alpha, \beta)\) in the mean/variance plane of the profit function and identify an optimal solution adopting the Sharpe Ratio as an optimization criterion. A sensitivity analysis is performed on the effect of the correlation coefficient \(\rho\) on the optimal strategy, as observed in different time spans.
Spot selling becomes more interesting as long as the forward maturity increases.

As long as the forward price decreases, for each maturity spot selling becomes the optimal choice (not reported in Table).

This gives a clear idea that a forward must be signed with care, considering the effect that the maturity can have on the diversification.
\( \nu_{ott} \) as a function of \( p_F \) and \( T \)

- \( c(0)=55.23 \) EURO/MWH
- \( p(0)=63.6 \) EURO/MWH
Optimal timing in plant investment for a german electricity producer

- We estimated the expected unit profit in the long run, 
  \[ \lim_{t \to +\infty} E[\max(p_t - c_t, 0)] = 15.77 \text{EURO} / \text{MWh} \]

  (this is the expected gain in the spot market)
The optimal delay will be 2.5 months

It depends mainly on the fact that the long run averages of gas and electricity prices are constant and that the speeds of reversion for two processes $\lambda_x$ and $\lambda_y$ are high with respect to $r$

\[ \tau = e^{-rt} \]

$g(0)$ is the expected unit profit in the long run

$c(0)=55$
EURO/MWH

$p(0)=65$
EURO/MWH

$rC(Q)/Q = 0.8$

$r=5\%$
A threshold value to accept/refuse a project

For the German market data we can calculate a decision value to accept/refuse a project of a gas fueled plant. The plant should be rejected if the unit (per megawatt of production capacity) cost of the investment \( C(Q)/Q \) is such that

\[
\frac{C(Q)}{Q} > \frac{g(0)}{r} = 2.8161 \times 10^6 \text{ EURO/MWh}
\]
The Sharpe Ratio

\[ \gamma(v, Q) = \gamma(v) = \frac{E[G](v, Q)}{\sigma[G](v, Q)} \]

It is the expected return per unit of risk