VALUATION OF ELECTRICITY FUTURES
REDUCED-FORM vs. DYNAMIC EQUILIBRIUM MODELS

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I. Motivation

DAILY SPOT PRICE AT NORD POOL

[NOK/MWh]

Daily Spot Price

First Differences

03/31/1999 03/31/2000 03/31/2001 03/31/2002 03/31/2003 03/31/2004
I. Motivation

DAILY ELECTRICITY DEMAND

First Differences
I. Motivation

DAILY SPOT PRICES AND DAILY ELECTRICITY DEMAND

Daily Spot Price [NOK/MWh]

Daily Electricity Demand [GW]
I. Motivation

Valuation of Electricity Futures

Econometric Models
- Johnsen (2001)
- Elliot et al. (2003)
- Burger (2003)

Reduced-Form Models
- Geman/Roncoroni (2006)
- Benth et al. (2006)

Equilibrium Models
- Routledge et al. (2001)
- Bessembinder/Lemmon (2002)
- Bühler/Müller-Merbach (2007)
I. Motivation

CONTRIBUTION

• Dynamic Extension of Bessembinder/Lemmon (2002)
  - Autocorrelation of Demand Process
  - Term Structure of Futures Prices and Risk Premia
  - Humped, Upward–Downward–Sloping Term Structures
  - Valuation of Cascade Futures

• Empirical Study for Nord Pool Data
  - First Empirical Test of an Equilibrium Model
  - Comparison with One-Factor Reduced Form Model by Lucia/Schwartz

DOES IT PAY OFF?
AGENDA

I. Motivation

II. Dynamic Equilibrium Model

III. Design of the Empirical Study

IV. Data and Estimation Results

V. Valuation Results
II. Dynamic Equilibrium Model

Equilibrium Model

Producers  Market  Retailers  End User
II. Dynamic Equilibrium Model

**PROFIT PRODUCERS**

\[
MC(\tilde{D}_t)\tilde{D}_t + \sum_{T} (\tilde{F}_{t,T} - \tilde{F}_{t-1,T})\tilde{Q}_{t-1,T} - C(\tilde{D}_t)
\]

- **Spot Price**
- **Demand**
- **Short Position Producers**
- **Production Costs**

**PROFIT RETAILERS**

\[
p\tilde{D}_t - MC(\tilde{D}_t)\tilde{D}_t - \sum_{T} (\tilde{F}_{t,T} - \tilde{F}_{t-1,T})\tilde{Q}_{t-1,T}
\]

- **Revenues**
- **Costs**
- **Mark-to-Market**
II. Dynamic Equilibrium Model

EQUILIBRIUM

\[ F_{t,T} = E_t \{ \tilde{F}_{t+1,T} \} - \xi \cdot \text{Cov}_t \left\{ \sum_{\tau=t+1}^{\Theta} e^{r(\Theta-\tau)}(p \cdot \tilde{D}_\tau - C(\tilde{D}_\tau)); \tilde{F}_{t+1,T} \right\} \]

- Risk Aversion Factor
- Retail Price
- Demand in Period \( \tau \)

\[ \tilde{F}_{t,T} = \tilde{S}_T = MC(\tilde{D}_T) \]

Quantity \( _T \sim \sum_{\tau} (\cdot) \cdot \text{Cov}_t (\tilde{F}_{r,\tau} \cdot \tilde{D}_\tau - C(\tilde{D}_\tau); \tilde{F}_{t+1,T}) \]

- Hedging Comp.

\[ + \ F_{t,T} - E(\tilde{F}_{t+1,T}) \]

- Speculat. Comp.

\[ - \sum_{\tau} \sum_{\delta} \text{Cov}_t (\Delta \tilde{F}_{r,\delta} \cdot \tilde{Q}_{r-1,\delta}; \tilde{F}_{t+1,T}) \]

- Mark-to-Market

Wolfgang Bühler
II. Dynamic Equilibrium Model

RISK PREMIA $F_{0,T} - E(\tilde{S}_T)$

ONE PERIOD EQUILIBRIUM AND REDUCED FORM MODEL

Reduced Form Model

$\sigma_x = 22.25 \text{ NOK/Week}$

$\kappa = 0.1 \text{ per Week}$

$\lambda = -0.02$

High Demand 60 GW

Medium Demand 40 GW

Low Demand 20 GW $\Delta = 3036 \text{ TWh}$
II. Dynamic Equilibrium Model

RISK PREMIA \( \tilde{F}_{0,T} - E(\tilde{S}_T) \)

DYNAMIC EQUILIBRIUM MODEL

\( \sigma_D = 5 \text{ GW/Week} \)
\( E(S_T) = 151 \text{ NOK/MWh} \)
\( p = 150 \text{ NOK/MWh} \)
\( \xi = 0.0002 \text{ 1/NOK} \)
\( r = 5\% \)
\( \rho = 0.6 \)

High Demand 60 GW \( \Rightarrow \) 10.080 TWh per Week

Medium Demand 40 GW

Low Demand 20 GW
III. Dynamic Equilibrium Model

ESTIMATION OF THE EQUILIBRIUM MODEL

Fixed Retail Price  \rightarrow \text{Marginal Cost Function}  \rightarrow \text{Futures Prices (} \xi = 0 \text{)}  \rightarrow \text{Estimate } \xi \text{ implicity}  \rightarrow \text{Futures Prices out-of-sample}

Spot Price  \rightarrow \text{Water Reservoir}  \rightarrow \text{Demand}  \rightarrow \text{Seasonal Component}  \rightarrow \text{Stochastic Component}  \rightarrow \text{Demand Process}  \rightarrow \text{Futures Prices}

Marginal Cost Function

Water Reservoir

Demand

Seasonal Component

Stochastic Component

Demand Process

Futures Prices
III. Design

SPECIFICATION OF THE MARGINAL COST FUNCTION

\[ MC(D) = \exp (a + bD + \gamma WR) \]

Hydro-generated electricity if the reservoir level is smaller than the median.

Hydro-generated electricity if the reservoir level is larger than the median.

Hydro-generated electricity if the reservoir level equals the median.
III. Design

SPOT PRICES AND WATER RESERVOIR LEVEL

Weekly Spot Price [NOK/MWh]

Water Reservoir Level
III. Design

**SPOT PRICES AND DEVIATION FROM MEDIAN WATER LEVEL**

![Scatter plot showing spot prices and deviation from median water level.](image)
III. Design

STRUCTURE OF DAILY DEMAND (SPOT PRICE)

\[
\tilde{D} = \alpha_0 + \sum_{i}^{12} \alpha_i M_i + \sum_{i}^{3} \beta_i D_i + \sum_{h} \gamma_i H_i + \tilde{\varepsilon}^D
\]

\[M_i\] : Triangular Functions
\[D_i, H_i\] : Dummy Variables
TIME STRUCTURE OF THE DESIGN

Estimation Period:
Estimation of deterministic components of spot prices and demand volumes on a daily basis

Aggregation into weekly data
Estimation of autoregressive deviations

Forecast of the deterministic component of spot prices and of the demand on a weekly basis

+ 1 week
### DATA

<table>
<thead>
<tr>
<th></th>
<th>Spot Prices</th>
<th>Demand incl. Export</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
<td>First Diff.</td>
</tr>
<tr>
<td>No Observ.</td>
<td>2.841</td>
<td>2.840</td>
</tr>
<tr>
<td>Mean</td>
<td>171.4</td>
<td>0.0</td>
</tr>
<tr>
<td>Stand. Dev.</td>
<td>90.5</td>
<td>20.1</td>
</tr>
<tr>
<td>Skewness</td>
<td>2.4</td>
<td>3.4</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>10.5</td>
<td>121.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.961</td>
</tr>
<tr>
<td></td>
<td>39.44 [GW]</td>
<td>0.0 [GW]</td>
</tr>
<tr>
<td></td>
<td>7.0 [GW]</td>
<td>2.3 [GW]</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>-1.0</td>
<td>0.5</td>
</tr>
</tbody>
</table>
IV. Data and Estimation

DAILY SPOT PRICE DETERMINISTIC COMPONENT, RESIDUAL

- Deterministic Component: $R^2 = 0.92$
- Residual: $\sigma_x = 19.7$, $\rho = 0.91$, $\kappa = 0.09$
IV. Data and Estimation

**DAILY SPOT PRICE DETERMINISTIC COMPONENT, RESIDUAL**

- **Det. Component**: 0.99
- **Residual**: $\sigma_z = 1.3$
- **$\rho$**: 0.5

![Graph showing daily electricity demand and estimated deterministic component with residual.](image_url)
IV. Data and Estimation

EMPIRICAL RISK PREMIUM

Weekly Average $F_{t,T}$ – Weekly Average $S_T$

Without Period Dec. 02 - Jan. 03

Weeks Until Maturity

[NOK/MWh]
IV. Data and Estimation

WATER RESEVOIR

-40 %  -20 %  0 %  20 %  40 %  60 %  80 %  100 %

03/31/1999  03/31/2000  03/31/2001  03/31/2002  03/31/2003  03/31/2004

Water Reservoir Level
Deviation from Water Reservoir Median
IV. Data and Estimation

MARGINAL COST FUNCTION

08/04/2004

\[
MC = \exp(4.64 + 0.021(D - 0.45WRD))
\]

Values: \[ a = 0.377 \]
\[ b = 0.032 \]
\[ \gamma = -0.014 \]
\[ R^2 = 0.944 \]
V. Valuation Results

RISK NEUTRAL VALUATION $\lambda = \xi = 0$

<table>
<thead>
<tr>
<th></th>
<th>Lucia/Schwartz</th>
<th>Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F^\text{th} - F^\text{obs}$ abs.</td>
<td>$F^\text{th} - F^\text{obs}$ abs.</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-25.1</td>
<td>-37.0</td>
</tr>
<tr>
<td>Std.</td>
<td>105.6</td>
<td>59.7</td>
</tr>
<tr>
<td>Min</td>
<td>-527</td>
<td>-561</td>
</tr>
<tr>
<td>Max</td>
<td>1.989</td>
<td>75</td>
</tr>
</tbody>
</table>

Average Spot Price (8 weeks)

F$^\text{th} - F^\text{obs}$ Lucia/Schwartz Model
MAD Lucia Schwartz
MAD Equilibrium Model
F$^\text{th} - F^\text{obs}$ Equilibrium Model
### V. Valuation Results

**REGRESSION ANALYSIS $F_{th} - F_{abs}$**

<table>
<thead>
<tr>
<th>Exog. Variable</th>
<th>Equilibrium Model (Lucia/Schwartz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Univariate Models</td>
</tr>
<tr>
<td>Spot Price</td>
<td>-0.30* (0.25*)</td>
</tr>
<tr>
<td>Volatility</td>
<td>-0.88* (0.43*)</td>
</tr>
<tr>
<td>Dev. Water Res. Lev.</td>
<td>1.83* (0.39)</td>
</tr>
<tr>
<td>Futures Price</td>
<td>-0.43* (-0.20*)</td>
</tr>
<tr>
<td>Maturity</td>
<td>0.60* (0.42)</td>
</tr>
<tr>
<td>Contract Type</td>
<td>10.41* (2.91)</td>
</tr>
<tr>
<td>Maturity Season</td>
<td>9.34* (-12.10*)</td>
</tr>
</tbody>
</table>

* 5%  * 1%  * 0.1%
V. Valuation Results

RISK-ADJUSTED EVALUATION

<table>
<thead>
<tr>
<th></th>
<th>Lucia /Schwartz</th>
<th></th>
<th>Equilibrium</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F^\text{th} - F^\text{obs}$</td>
<td>abs.</td>
<td>$F^\text{th} - F^\text{obs}$</td>
<td>abs.</td>
</tr>
<tr>
<td>Mean</td>
<td>-3.6</td>
<td>38.0</td>
<td>-15.6</td>
<td>33.6</td>
</tr>
<tr>
<td>Std.</td>
<td>52.7</td>
<td>80.3</td>
<td>29.9</td>
<td>21.8</td>
</tr>
<tr>
<td>Min</td>
<td>-247</td>
<td>5.2</td>
<td>-119</td>
<td>12.8</td>
</tr>
<tr>
<td>Max</td>
<td>238</td>
<td>470</td>
<td>32.6</td>
<td>135</td>
</tr>
</tbody>
</table>
V. Valuation Results

RISK PREMIA LUCIA/SCHWARTZ MODEL

[Diagram showing data points with Time to Maturity on the x-axis and [NOK/MWh] on the y-axis. The data points are categorized by dates: 18-Dec-02, 23-Oct-02, 31-Jan-01, and All Estimates.]
V. Valuation Results

RISK PREMIA EQUILIBRIUM MODEL

[Diagram showing the valuation results with various data points and labels for different dates and maturities in weeks, with [NOK/MWh] on the y-axis and Time to Maturity on the x-axis.]
CONCLUSIONS

THEORETICAL COMPARISON

<table>
<thead>
<tr>
<th>Fundamental Variables</th>
<th>Reduced-Form Model</th>
<th>Dyn. Equilibr. Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot Price</td>
<td>Spot Price</td>
<td>Demand, Cost Function, Water Reservoir Level</td>
</tr>
<tr>
<td>Spikes</td>
<td>normal</td>
<td>right skewed</td>
</tr>
<tr>
<td>“normal“</td>
<td>depend on cost function</td>
<td></td>
</tr>
<tr>
<td>Risk Premia (without Season.)</td>
<td>monotonic in T</td>
<td>monotonic, humped in T</td>
</tr>
<tr>
<td>Volatility of Futures Prices</td>
<td>declines in T</td>
<td>declines in T</td>
</tr>
<tr>
<td>Correlation</td>
<td>perfect</td>
<td>not perfect</td>
</tr>
</tbody>
</table>
## CONCLUSIONS

**EMPIRICAL COMPARISON**

<table>
<thead>
<tr>
<th></th>
<th>Reduced-Form Model</th>
<th>Dyn. Equilibr. Model</th>
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</thead>
<tbody>
<tr>
<td>Stationarity of Exog. Variable</td>
<td>not always</td>
<td>always</td>
</tr>
<tr>
<td>Residuals</td>
<td>systematic component</td>
<td>no syst. component</td>
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<tr>
<td>Risk Parameters</td>
<td>improvement</td>
<td>improvement</td>
</tr>
<tr>
<td>Risk Premia</td>
<td>mostly flat</td>
<td>more in line with “observed”</td>
</tr>
<tr>
<td>Estimates</td>
<td>more volatile</td>
<td>less volatile</td>
</tr>
<tr>
<td>Adaption to Spikes</td>
<td>worse</td>
<td>better</td>
</tr>
</tbody>
</table>

**DOES IT PAY OFF?**